Some more details on Sound

There are two main ingredients to wave motion – inertia and restoring force. These are the two ingredients that go into the mathematical description of sound waves. There are three ideas that come into play, and each idea has an equation that makes the idea very precise. To do this we will consider a sound of a definite frequency $f=1/T$ and wavelength λ. All sound can be thought of as an addition of pure tones with definite wavelength and frequency, so this allows us to understand all sounds.

First ingredient: Inertia

First Idea (Newtons 2nd Law) Because of inertia (measured in units of mass) it requires a force to change the state of motion (the velocity) of a piece of material.

First Equation: F=ma (The necessary Force is equal to the mass multiplied by the rate of change of the velocity). For sound we need to think of the force per unit volume (the pressure change divided by the distance over which it changes) the mass per unit volume (the mass density) and the average velocity of air molecules.

$$\frac{p}{\lambda} = \frac{mn_0 v}{T}$$

p is the pressure change caused by the sound wave.
λ is the wavelength – the distance over which the pressure changes.
m is the average mass of an air molecule.
n_0 is the normal density of molecules per cubic meter.
v is the air velocity change due to the sound wave (not the sound speed).
T is the period – the time over which the velocity changes.

Second ingredient: Restoring force

Second idea: Continuity. If air atoms are not being created or destroyed by sound, then the sound can't change the velocity of the air in one place (thus moving air away) without changing the density too.

Second equation:

$$\frac{n}{T} = \frac{n_0 v}{\lambda}$$

λ is the wavelength – the distance over which the velocity changes.
n is the density change due to the wave.
n_0 is the normal density of molecules per cubic meter.
v is the air velocity change due to the sound wave (not the sound speed).
T is the period – the time over which the density changes.

Third idea: adiabatic compression
When air is compressed by a sound wave, work is done on the air so that its temperature will rise. Thus, the pressure will rise for two reasons. One is because there are more air molecules in a given place and the second is because the temperature will rise.

Third equation:

$$\frac{p}{p_0} = \gamma \frac{n}{n_0}$$

p is the pressure change caused by the sound wave.
p_0 is the atmospheric pressure.
γ is called the ratio of specific heats – it is equal to 7/5.
n is the density change due to the wave.
n_0 is the normal density of molecules per cubic meter.

If we put these equations together, we get a theory of sound waves.

Combining these equations we see that

$$\frac{\lambda}{T} = V = \sqrt{\frac{\gamma p_0}{n_0 m}}$$

where *V* is the speed of sound. This is called a dispersion relation. It shows you that the wavelength of a sound wave is proportional to the period of the wave oscillation.
$\lambda = V \cdot T$

One other important quantity is the wave impedance. It is the ratio of the pressure change to the velocity change of the wave. Like the sound speed, this ratio only depends on the properties of the medium (in this case air).

$$Z_s = \frac{p}{v} = n_0 m V$$

We call this impedance Z_s. See the notes on impedance for more details on the importance of impedance.

Another definition of impedance depends on the area of the pipe. This matters because when air goes down a conical pipe or reaches the end of a pipe it also goes sideways. The speed of sound is the same inside as outside of pipes, but the impedance you need to use to calculate reflections is different. Equation two has to be made more complicated. For pipes, you want to use the impedance;

$$Z_{tube} = \frac{p}{Av} = \frac{n_0 m V}{A}$$

Whenever you describe a sound wave you have a choice of variables to use. They are all proportional, but in the case of the velocity, it depends on what direction you are talking about. When the density and pressure due to a sound wave are at their maximum then the magnitude of the velocity is maximum and in the direction that the wave is going. Figure 1 shows the relative change in air density and pressure for a sound near the threshold of hearing (sound intensity level of 0 db, which equals 10^{-12} W/m^2). Because of adiabatic heating, the relative pressure change is bigger than the relative density change by a factor of 7/5. Notice that they are very small! The density is changing by a little more than one hundred millionth of a percent. Notice also that the speed of the air motion due to the wave is much smaller than the speed of sound. In this case it is about ten billion times smaller.

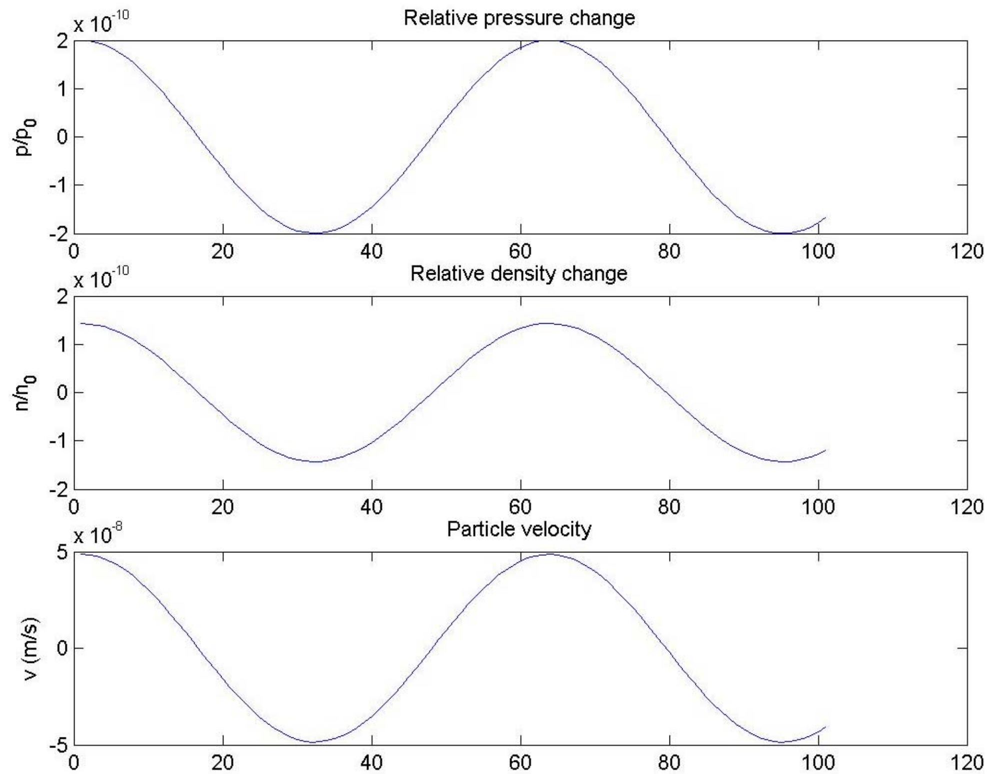

Figure 1: Variations in pressure, density and air velocity due to a sound wave moving along a positive axis.

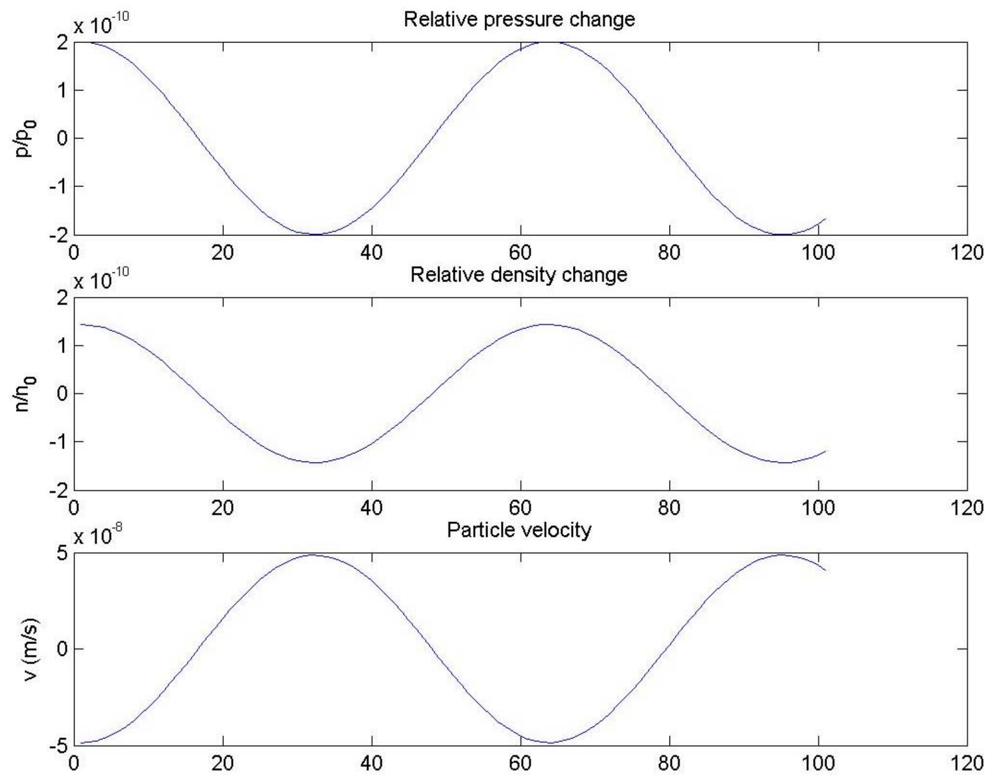

Figure 2: Variations in pressure, density and air velocity due to a sound wave moving along a negative axis. **Thus, a standing wave made up of waves traveling in opposite directions will have velocity nodes at locations of pressure antinodes and vice-versa.**

Intensity and Power

The sound intensity of a wave is the power divided by the area. It is also equal to the sound pressure times the air molecule velocity.

$$SIL(W/m^2) = p \cdot v$$

We can write this in terms of the impedance in two ways.

$$SIL(W/m^2) = \frac{p^2}{Z_s} = Z_s \cdot v^2$$

When talking about sound in pipes you often just care about the total power, which is equal to intensity times the area of the pipe.

$$Power(W) = p \cdot v \cdot A = \frac{p^2}{Z_{tube}} = Z_{tube} \cdot (A \cdot v)^2$$

Log scale for sound intensity level

In order to handle a large range of intensity levels, one often uses a log scale. This is defined by:

$$SIL(db) = 10 \cdot Log_{10}\left(\frac{Intensity}{10^{-12} W / m^2}\right)$$

Note that you can also write this in terms of the RMS pressure of the sound wave:

$$SIL(db) = 20 \cdot Log_{10}\left(\frac{p_{RMS}}{2 \cdot 10^{-5} (Pascal)}\right).$$