1. A linear transformation from $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ has eigenvalues and vectors $\lambda_1 = 0.5, \xi_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3, \xi_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.
   (a) Sketch, without any calculation, what happens to a vector in the first quadrant of the plane under this linear transformation.
   (b) Calculate what happen to a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ under this transformation.

2. Find the eigenvalues and eigenvectors of the linear operation specified by $T = \begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$.
   Note that this involves find the roots of a cubic equation. The best way to do that is to make an educated guess for one of the roots (try numbers like $-3, -2, ...3$ since usually happens on exams). Once you find a root r, factor the polynomial into $(\lambda - r)(a\lambda b\lambda + c)$ and use the quadratic equation on the remaining quadratic.

3. We define a basis ε using the basis vectors $\varepsilon_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\varepsilon_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ written in the standard basis. The linear transformation L is $L_\varepsilon = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ when written in the ε basis. What is L when written in the standard basis?

4. The linear transformation A has eigenvalue λ and eigenvector $\mathbf{v}$. Find one of the eigenvalue and eigenvector pairs for A^2 in terms of λ and $\mathbf{v}$.

5. Quantum Mechanics and other subjects make use of a quantity called the *matrix exponential.*
   $$e^M = I + M + \frac{M^2}{2} + \frac{M^3}{6} + ...$$
   If you were given a random n-by-n matrix, then this may be an unwieldy expression. Consider when M is diagonalizable ($M = P^{-1}DP$).
   (a) Write out the term for M^2 using the fact that it is diagonalizable and simplify.
   (b) What does the M^3 term look like when you do the same simplification? How about the M^n term?
   (c) Write the matrix exponential in the form $e^M = Ae^DB$. What are A and B? Why is this an easier calculation?