a) \( E_{\text{conf}} = \frac{\hbar^2}{2m^* L^2} \) Hartree units \( \frac{1}{2m^* L^2} \)

\[ \Delta E_g = E_{n,\text{conf}} + E_{e,\text{conf}} = \frac{1}{2m_n^* L^2} + \frac{1}{2m_e^* L^2} \]

\[ L = \sqrt{\frac{1}{2} \left( \frac{1}{m_n^*} + \frac{1}{m_e^*} \right) \frac{1}{\Delta E_g}} \]

\[ = \sqrt{\frac{1}{2} \left( \frac{1}{0.45 m_e} + \frac{1}{0.068 m_e} \right) \frac{1}{0.1 \text{ ev} \times 27.2 \text{ ev} \text{ Hartree}} \]

\[ = 47.98 \alpha_0 \approx 2.5 \text{ nm} \]

b) \( E_g (GaAs) = 1.519 \text{ ev} \) so this quantum well would have

\( E_g (cw) = 1.619 \text{ ev} \) which is closer to the visible range, but still infrared. Such a quantum well would be useful as a light source at this particular energy.
In 2D the sum over states is
\[ \sum_K \frac{1}{2} (\frac{\mu}{\pi \hbar^2})^2 \int_{0}^{2\pi} d^2k \]
and in 1D
\[ \sum_K \rightarrow \int_{-K_{\text{max}}}^{K_{\text{max}}} \frac{1}{2} (\frac{\mu}{\pi \hbar^2}) dE = \int_{0}^{K_{\text{max}}} \frac{2\mu}{\pi \hbar^2} dE \]

\[ L_{\perp} \int_{0}^{K_{\text{max}}} \frac{1}{2} \frac{\mu}{\hbar^2} \frac{dE}{\sqrt{E}} dE \]
\[ g_\perp(E) \]

Note: \( L_{\perp} \) above is the size in the "free direction."
The quantum well states have energies

\[ E_n = \frac{\hbar^2}{2m^* L^2} n^2 \]

energies measured wrt respective band edges.

For an electron with energy above a particular \( E_n \), the subband \( n \) will contribute a DOS

\[ g(E) = \frac{\hbar^2}{\pi^2} \sum_{n=1}^{\infty} \Theta(E-E_n) \frac{(\frac{\hbar}{m^*})^2}{n^2} \]

using atomic units with \( L = 48 \, a_0 \) (from 18.1)

electron, \( \Theta \): \[ g(E) = \frac{\hbar^2}{\pi^2} \sum_{n=1}^{\infty} \Theta(E - \frac{1}{2 \cdot 0.068 \cdot 48^2} n^2) 0.068 \frac{1}{\alpha_0^2 \text{Hart}} \]

\[ = \frac{\hbar^2}{\pi^2} \sum_{n=1}^{\infty} \Theta(E - 3.191 \times 10^{-3} n^2) \cdot 2.165 \times 10^{-2} \frac{1}{\alpha_0^2 \text{Hart}} \]

hole: \[ g(E) = \frac{\hbar^2}{\pi^2} \sum_{n=1}^{\infty} \Theta(E - \frac{1}{2 \cdot 0.45 \cdot 48^2} n^2) 0.45 \frac{1}{\alpha_0^2 \text{Hart}} \]

\[ = \frac{\hbar^2}{\pi^2} \sum_{n=1}^{\infty} \Theta(E - 4.823 \times 10^{-4} n^2) \times 0.1432 \frac{1}{\alpha_0^2 \text{Hart}} \]
6) The confinement energy for a quantum wire with square cross section (L x L)
is \[ E_{n_x n_y} = \frac{\hbar^2}{2m^* L^2} (n_x^2 + n_y^2) \]

with \( L = 30 \text{nm} \times \frac{1 \text{A}_0}{0.0529 \text{nm}} \approx 567 \text{ A}_0 \)

\[ E_{n_x n_y} = \frac{1}{2 \times 567^2 \text{ m}^*} (n_x^2 + n_y^2) \]

A subband with energy \( E_{n_x n_y} \) will contribute to the DOS \( g_s(E) \)

\[ g_s(E) = \frac{\sqrt{2m^*}}{\pi \hbar} \frac{1}{\sqrt{E - E_{n_x n_y}}} \]

so \( g_{\text{tot}}(E) \) is

\[ g_{\text{tot}}(E) = \sum_{n_x, n_y} \theta(E - E_{n_x n_y}) \frac{\sqrt{2m^*}}{\pi \hbar} \frac{1}{\sqrt{E - E_{n_x n_y}}} \]

Since \( g(E) \) diverges at each \( E_{n_x n_y} \), optical transitions will be enhanced, making the wire a potentially good lasing medium.
Since the chemical potential is higher in the n-region than in the p-region, electrons flow from n to p until diffusive equilibrium is reached. This depletes the region near the pn junction of free carriers, drastically decreasing the conductance. With an applied forward bias more minority carriers are thermally excited and these can diffuse across the depletion region, increasing the conductivity.

Assume that at equilibrium all the free electrons on the n side out to a distance \(dn\) move over to the p side, filling up all the holes to a distance \(dp\). Assuming the electrons and holes came from fully ionized donors and acceptors, the charge density looks like this:

\[
\text{p side} \begin{array}{ccc}
\text{v side} \\
-\text{en}_d & \text{en}_d & \text{en}_n \\
\end{array} = -\text{en}_a \\
\begin{array}{c}
\text{N}_d = \text{acceptor density} \\
\text{N}_d = \text{donor density} \\
\end{array}
\]

That is \( s(x) = \begin{cases} -\text{en}_a & \text{if } -\text{W}_p \leq x < 0 \\ \text{en}_d & 0 < x < \text{W}_n \\ 0 & \text{otherwise} \end{cases} \)

Charge neutrality requires \( N_d \text{W}_n = N_a \text{W}_p \)
The electrostatic potential is determined by Poisson's equation

\[ \frac{d^2 \phi}{dx^2} = -\frac{\rho(x)}{\epsilon} \]

In a region of \( \rho(x) = \rho_o = \text{const} \) the solution is

\[ \phi = -\frac{\rho_o}{2\epsilon} (x-a)^2 + \phi_o \]

where \( a, \phi_o \) are constants determined by boundary conditions.

Set boundary conditions \( \phi(W_p) = 0 \) & \( \phi(W_n) = \phi_{bi} \)

where \( \phi_{bi} \) is the built in potential, the difference in chemical potentials of the p & n regions with these BC's

\[ \phi(x) = \begin{cases} 
\frac{eN_a}{2\epsilon} (x+W_p)^2 & \text{if} \ -W_p \leq x \leq 0 \\
-\frac{eN_d}{2\epsilon} (x-W_n)^2 + \phi_{bi} & \text{if} \ 0 \leq x \leq W_n
\end{cases} \]

However \( \phi(x) \) must be continuous at \( x=0 \) so

\[ \frac{eN_a}{2\epsilon} \cdot d \phi^2 = -\frac{eN_d}{2\epsilon} W_n^2 + \phi_{bi} \]

\[ \phi_{bi} = \frac{2}{2\epsilon} \left[ N_a W_p^2 + N_d W_n^2 \right] \]

Then substitute \( W_p = \frac{N_d}{N_a} W_n \) from charge neutrality & solve for \( W_n \)

\[ W_n = \left( \frac{N_a}{N_d} \cdot \frac{1}{N_a + N_d} \cdot \frac{2\epsilon \phi_{bi}}{e} \right)^{1/2} \]
Substituting \( W_n = \frac{N_a}{N_d} W_p \) into the equation for \( \phi_{bi} \) and solving for \( W_p \)

\[
W_p = \left( \frac{N_d}{N_a} \frac{1}{N_a+N_d} \frac{2e \phi_{bi}}{e} \right)^{\frac{1}{2}}
\]

Calculation the depletion charge on the n-side

\[
q = e N_d W_n = e N_d \left( \frac{N_a}{N_d} \frac{1}{N_a+N_d} \frac{2e \phi_{bi}}{e} \right)^{\frac{1}{2}}
\]

with an applied bias \( \phi_{bi} \to \phi_{bi} - V \)

\[
q(V) = e N_d \left( \frac{N_a}{N_d} \frac{1}{N_a+N_d} \frac{2e (\phi_{bi}-V)}{e} \right)^{\frac{1}{2}}
\]

\[
C = \left| \frac{dE}{dV} \right| = e \sqrt{N_a N_d \frac{1}{N_a N_d} \frac{2e}{e}} \left| \frac{d}{dV} \sqrt{\phi_{bi}-V} \right|
\]

\[
= \sqrt{\frac{2e N_a N_d}{N_a+N_d}} \frac{1}{\sqrt{\phi_{bi}-V}}
\]

Since \( C \propto \frac{1}{\sqrt{\phi_{bi}-V}} \), it's a voltage-controlled capacitor, which could be useful.
The diagram above shows the bands with the two semiconductors unable to transfer electrons to attain equilibrium. As the materials are brought into contact, electrons will move from the n region (higher $\mu$) to the undoped region (lower $\mu$).

$E_e$ depleted

insulating the bulk n-region from the excess e's on the undoped side

$E_e$ excess e-; 2D e-gas

$\phi$ const at equil

NOTE: it says $E_{c1} < E_{c2}$!