

## Impedance

A very useful concept for understanding the flow of energy is called the impedance. Energy flow, or the rate at which energy is being used, is called power. If you push on a moving object with a force  $F$  and it moves with velocity  $v$  then the rate of energy transfer (the power) is equal to the force multiplied by the velocity.  $P=Fv$  Of course, just how fast the object moves will depend on the force you apply. The two are not independent. For a harmonic force, the ratio of force to velocity tells you how hard it is to move the object – this depends both on how much inertia the object has and on how strong the restoring force is.

$$Z = \frac{\text{Force}}{\text{velocity}}$$

NOTE that the velocity in this equation when we talk about waves is the "particle" velocity – the velocity of the displacement – not the propagation velocity of the wave.

### Example 1

A simple example of impedance that does not involve waves, is viscosity. If you move an object through a viscous medium (like molasses) it takes a constant force to maintain a constant velocity. This is expressed by the equation:  $F = v \cdot \eta$  The constant  $\eta$  is called the viscosity. You can see that viscous media have an impedance which is equal to the viscosity.

$$Z = \frac{F}{v} = \eta$$

## Wave impedance

As a sound wave propagates along, each piece of the material transfers the energy to the next in a kind of "bucket brigade" fashion. From a physics point of view, as long as the impedance does not change, the propagation of the wave is not disturbed. This is what makes the impedance important. At an interface between two media (for example when sound hits a wall) you often want to know how much, if any, of the wave energy is going to be reflected. If the impedance of the wall is the same as the impedance of the air then there will be no reflection of power. If the impedance of the wall is either greater or lesser than the air, then there will be significant reflection. **If the impedance changes slowly, then the wave will not reflect either.**

### Reflected power

The ratio of the reflected wave power to the wave power hitting a surface

is  $\frac{P_{refl}}{P_{inc}} = \left( \frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2$ . So when  $Z_1 = Z_2$  there is no power reflected.

## Example 2

Waves on a string.

For waves on a string the speed of the wave is given by  $v = \sqrt{\frac{F}{W}}$  where  $F$  is the tension force in the string (in Newtons) and  $W$  is the mass density (in kilograms per meter - Kg/m). The impedance of a string is given by:

$$Z_{string} = \sqrt{F \cdot W} . \text{ The units are Kg/s}$$

In musical instruments that have vibrating strings, some energy will be transferred to the body of the instrument when the wave reaches the end of the string. Just how much of the energy leaks out and how much reflects back onto the string depends on the mass and stiffness of the instrument body (its mechanical impedance) compared to the mechanical impedance of the string.

## Example 3

Sound in air (3-D).

For sound waves in air the sound speed is given by:  $v = \sqrt{\frac{\gamma p}{nm}} = \sqrt{\frac{\gamma RT}{m}}$  Here  $p$  is the air pressure,  $n$  is the number of atoms per cubic meter,  $m$  is the average mass of the air molecules,  $\gamma$  is the ratio of specific heats for air as in equal to 1.4 (no units),  $R$  is the gas constant (the equation for ideal gasses is  $p=nRT$ ) and  $T$  is the air temperature. The impedance, called the specific impedance  $Z_s$ , is given by:

$$Z_s = \sqrt{\gamma pnm} = n\sqrt{\gamma RTm} \text{ the units are Kg/(m}^2\text{s)}$$

## Example 4

Longitudinal sound in other materials.

As a general rule, if you know the sound speed  $v$  and the mass density  $\rho$ , the impedance is:  $Z = \rho \cdot v$

For steel, the mass density is  $7700\text{Kg/m}^3$  and the sound speed is  $5032\text{m/s}$ . This means that the impedance is about  $40,000,000 \text{ Kg/(m}^2\text{s)}$ . This is about 100,000 times larger than the impedance of sound waves in air. Most common materials have an impedance much higher than sound in air.

### Example 5

Sound in tubes or pipes.

When sound travels down pipes, as it does in many musical instruments, the wave is mostly 1-D (one dimensional) inside the tube. In this case there is a 1-D impedance for sound waves given by:

$$Z_{tube} = \frac{Z_s}{A} = \frac{\sqrt{\gamma pnm}}{A} = \frac{n\sqrt{\gamma RTm}}{A} \quad \text{The units are Kg/(m}^4\text{s)}$$

Here  $A$  is the area of the tube. Notice that if a tube changes in area slowly from one end to another then as the wave propagates along its impedance will change without reflections during the process. It is only when the impedance changes quickly (for example all at once at a boundary) that there is a reflection. However, once a tube is large in diameter compared to the wavelength then there will be very little reflection of a sound wave as it exits the pipe into the open air.

Note that the units of impedance depend on the situation.

### Impedance transformation

If you want to avoid a reflection of waves in passing from one medium to another it is possible to have a **transformer**, a transition which gradually takes the wave from one impedance to another.

### Example 1

A horn. It is possible to make a megaphone without an amplifier. If you place a cone to your lips and speak through the cone then your voice sounds louder. There are two related things going on.

Firstly, there is a little bit of beaming of the sound because spreading of the sound due to diffraction will be less at the open end of the horn than it would be at your mouth. This is because the open end of the horn is larger than your mouth, so diffraction will be less.

Secondly (though these two effects are linked) there will be better transmission of the sound out of your mouth because the horn will act as an impedance transformer. The more gradual change in diameter of the cone helps couple the sound to the open air.

This is the basic principle of a Victrola. A Victrola is an old fashioned record player that has no electronic amplification but instead depends on very efficient conversion of energy from the vibrating needle into sound waves.

