THE DIFFERENTIAL CROSS SECTION FOR COMPTON SCATTERING

INTRODUCTION:

Melissinos and Napolitano provide an adequate introduction to this concept in section 9.2 of their text.

The theoretical scattering cross-section for the Compton Interaction can be written (see Refs. 2 and 3):

\[
\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} \left( \frac{1 + \cos^2(\theta)}{1 + \gamma(1 - \cos\theta)^2} \right) \times \left( 1 + \frac{\gamma^2(1 - \cos\theta)^2}{(1 + \cos^2\theta)[1 + \gamma(1 - \cos\theta)]} \right)
\]

where,

\[
\gamma = \frac{E_\gamma}{m_0c^2}
\]

For the \(E_\gamma\) of 0.662 from \(^{137}\text{Cs}\) we arrive at a value of 1.29 for \(\gamma\). Also,

\[
r_0 = 2.82 \times 10^{-13} \text{ cm (classical } e^- \text{ radius)}
\]

Figure 1 shows \(E_\gamma'\) at \(\theta = 60^\circ\). The values of \((\Sigma - \beta)\) have to be corrected for the intrinsic peak efficiency of the detector. The corrected sum is given by:

\[
\left( \frac{\Sigma - \beta}{t} \right)_{\text{corrected}} = \frac{1}{\epsilon_p} \left( \frac{\Sigma - \beta}{t} \right)
\]

where \(\epsilon_p\) is the intrinsic peak efficiency for \(E_\gamma'\).
Figure 1. NaI(Tl) pulse height spectrum of Compton scattered gammas at $q = 60$ degrees from $^{137}$Cs. (Note: At $q = 0$ degrees, the scattered peak would have the full energy of the $^{137}$Cs source 662 keV.)
Figure 2. Intrinsic peak efficiency ($\varepsilon_a$) for a wide variety of NaI(Tl) crystals. The source to detector distance is 9.3 cm (Courtesy of Idaho Operations Office DOE).
For example, the \((\Sigma-\beta)\) shown in Figure 1 would be divided by the appropriate \(\varepsilon_p\) for an energy of 401 keV. The measured Compton cross-section is then given by:

\[
\frac{d\sigma}{d\Omega_m} = \left(\frac{\Sigma-\beta}{t}\right)_{\text{corrected}} \frac{1}{n\Delta\Omega\Phi_\gamma}
\]

where

\[
n = \text{the number of electrons in the scattering volume}
\]

\[
= \frac{(\text{volume}) (\text{density of Al}) (\text{Avogadros No})}{(\text{Atomic Weight})}
\]

\(\Delta\Omega\) = Solid angle of the Nal (Tl) in steradians

\[
= \frac{\text{Area of the detector (cm}^2\text{)}}{R_2^2 (\text{cm}^2)}
\]

\(o\) = The number of incident \(\gamma\)'s on the Alminum scatterer per cm \(^2\) per S.

\[
= \text{The number of } \gamma\text{'s from the source divided by} \frac{1}{4\pi R_1^2} \text{ (see Figure 9.4)}
\]

NOTE: in Eq. (15), the volume of the aluminum scatterer is given by:

\[
V = \pi R_0^2 h
\]

where

\[
R_0 = .635 \text{ cm}
\]

\[
h = R_1 \sin f_1 \text{ (see Figure 9.4)}
\]
\[ f_1 = 3.58 \text{ degrees} \]

From your experimental data \((S-b/t)\) in Table 9.1 calculate \((ds/dW)_{\text{measure}}\) Eq. (14).

Enter these experimental points on the theoretical curve 9.6 as shown in the figure. The results should show that the Klein-Nishina theory [Eq. (12)] does a good job of predicting the scattered Compton differential cross section.

References


Figure 9.6. Theoretical Compton scattering cross section vs. angle.