# 29:195 Homework #2

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| Reading: | Required: | Skim GB Chapter 6, Sections 6.1–6.6 (p.175–219) |
| | | Read GB Chapter 6, Section 6.7 (p.219–239) |
| | Optional: | Read BS Chapter 4, Sections 4.5–4.7 (p.108–130) |

Due at the beginning of class, Thursday, February 19, 2009.

1. Show that the determinant of the matrix
$$\begin{pmatrix} S - n^2\cos^2\theta & -iD & n^2\sin\theta\cos\theta \\ iD & S - n^2 & 0 \\ n^2\sin\theta\cos\theta & 0 & P - n^2\sin^2\theta \end{pmatrix}$$
can be written in the form of the Booker Quartic
$$An^4 - Bn^2 + C = 0$$
where
$$A = S\sin^2\theta + P\cos^2\theta,$$
$$B = RL\sin^2\theta + PS(1+\cos^2\theta),$$
and
$$C = RLP.$$

2. Prove that the index of refraction for cold plasma waves (the solution of the Booker Quartic above) is either purely real or purely imaginary, but never complex. Hint: Show that the discriminant $B^2 - 4AC$ is positive definite.

3. In the limit $\omega \to 0$, show that
$$R = L = S = 1 + \sum_s \frac{\omega_{ps}^2}{\omega_{cs}^2},$$
$$D = 0,$$
and
$$P = -\sum_s \frac{\omega_{ps}^2}{\omega^2}.$$

4. Assuming that the ions are infinitely massive, derive the equations for the following characteristic frequencies:
   (a) The right-hand cutoff frequency, $\omega_R$
   (b) The left-hand cutoff frequency, $\omega_L$
   (c) The upper hybrid frequency, $\omega_{UH}$

5. Whistler Waves
   (a) Assuming the wave frequency is sufficiently high that the ions do not move, that $\omega \ll \omega_p$, and that $|\omega_{ce}| \ll \omega_p$, show that the index of refraction for whistler waves with a wave vector at an angle $\theta$ with respect to the mean magnetic field is approximately
$$n^2 = \frac{\omega_p^2}{\omega(|\omega_{ce}|\cos\theta - \omega)}$$
   (b) Sketch $n(\theta)$ for $\omega \ll |\omega_{ce}|$
   (c) Sketch $n(\theta)$ for $\omega = |\omega_{ce}|/4$
   (d) Sketch $n(\theta)$ for $\omega = |\omega_{ce}|/2$