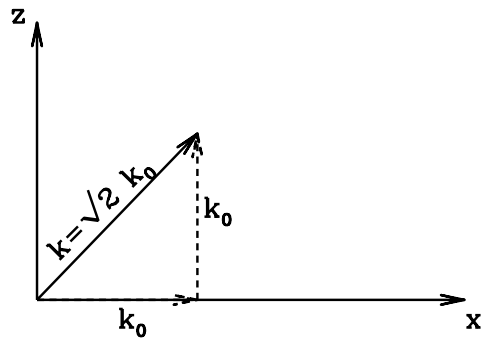
29:195 Homework #3

Due at the beginning of class, Tuesday, February 24, 2009.

1. Ray Tracing
   Consider a radio wave packet launched from an antenna located at $(x, z) = (0, 0)$ at time $t = 0$ in a plane-parallel atmosphere as depicted in the figure below.



The plasma electron density in the atmosphere increases with height as

$$n_e(z) = n_0 \frac{z^2}{H^2}.$$

The wave vector of the radio wave has components $k_x = k_z = k_0$. Please give all answers in terms of the parameters of the problem $\omega_{pe0}^2 = (n_0 q_e^2)/(\epsilon_0 m_e)$, k_0, c, and H.

(a) Calculate the frequency of the radio wave ω as a function of time.

(b) Find the rate of change of the wavevector components k_x and k_z with respect to time in terms of the problem parameters and x and z.

(c) Determine the motion of the wavepacket in time as the functions $x(t)$ and $z(t)$. Be sure to use initial conditions to solve for any unknown constants in terms of the problem parameters.

(d) Determine the trajectory in the (x, z) plane in the form $z(x)$.

(e) What is the total distance traveled in the horizontal direction before the wavepacket returns to the ground?

(f) What is the maximum height the wavepacket reaches?