29:195 Homework #6

Reading:	Required:	Read GB Chapter 7 (p.251–278)
	Optional:	Read BS Chapter 5, Section 5.6 (p.179–193)

Reading should be completed by the beginning of class, Thursday, April 2, 2009.
Problems are due at the beginning of class, Thursday, April 9, 2009.

1. In his 1959 paper, Buneman analyzed the maximum growth rate of a cold electron beam streaming at velocity V through an equal number density of ions initially at rest. The dispersion relation for this system is given by

$$D(\mathbf{k}, \omega) = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{(kV - \omega)^2} = 0.$$

   (a) If we assume that $\omega_{pi}^2 \ll \omega^2$, show that the dispersion relation can be written

$$kV = \omega + \omega_{pe} + \frac{1}{2}\frac{\omega_{pe}\omega_{pi}^2}{\omega^2}.$$

   (b) Show that the above dispersion relation has a solution for real k of the form

$$\omega = (\omega_{pe}\omega_{pi}^2 \cos\theta)^{1/3} e^{i\theta}$$

   where θ is any arbitrary angle.

   (c) Show that the imaginary part of ω reaches a maximum when $\theta = \pi/3$.

   (d) Show that, when $\theta = \pi/3$,

$$\gamma = \omega_{pe}\left(\frac{m_e}{2m_i}\right)^{1/3}\frac{\sqrt{3}}{2}$$

   and

$$\omega_r = \omega_{pe}\left(\frac{m_e}{2m_i}\right)^{1/3}\frac{1}{2}.$$

   (e) Is assumption (a) satisfied?

2. Consider the problem of the counter-streaming Cauchy distribution,

$$F_0(v_z) = \frac{C}{2\pi}\left[\frac{1}{C^2 + (v_z - V)^2} + \frac{1}{C^2 + (v_z + V)^2}\right].$$

   (a) Sketch the distribution for the choice $C = V$.

   (b) Apply Gardner's Theorem to show that stability is guaranteed for $C > \sqrt{3}V$.

   (c) Apply the Penrose Criterion to show that this equilibrium is stable for $C > V$ and unstable for $C < V$.

3. Again, consider the problem of the counter-streaming Cauchy distribution,

$$F_0(v_z) = \frac{C}{2\pi}\left[\frac{1}{C^2 + (v_z - V)^2} + \frac{1}{C^2 + (v_z + V)^2}\right].$$

Sketch the Nyquist diagrams for mapping the $\gamma = 0$ contour in the complex p-plane onto the complex D-plane for the following cases:

(a) $C > V$.

(b) $C = V$.

(c) $C < V$.

4. A plasma consists of a Maxwellian distribution of ions at temperature T_i and electrons at temperature T_e. The average velocity of the ions is zero and the electrons are drifting with an average velocity U_e.

(a) Make an appropriate modification of the derivation for the ion acoustic instability involving the plasma dispersion function to take into account the drifting electrons.

(b) Using the low-phase velocity approximation for the electrons and the high-phase velocity approximation for the ions, obtain an approximate expression for the real frequency of the ion acoustic mode. You may assume that $k\lambda_{De} \ll 1$.

(c) Show that the growth rate is approximately

$$\frac{\gamma}{\omega} = -\sqrt{\frac{\pi}{8}}\left[\sqrt{\frac{m_e}{m_i}}\left(1 - \frac{U_e}{C_i}\right) + \left(\frac{T_e}{T_i}\right)^{3/2} \exp\left(-\frac{T_e}{2T_i}\right)\right]$$

where the ion acoustic speed is given by

$$C_i = \sqrt{\frac{T_e}{m_i}}$$

(d) Show that the instability condition for the current-driven ion acoustic mode is

$$|U_e| > C_i\left[1 + \sqrt{\frac{m_i}{m_e}}\left(\frac{T_e}{T_i}\right)^{3/2} \exp\left(-\frac{T_e}{2T_i}\right)\right]$$