Lecture #21 The Framework of Plasma Physics

I. Why Do We Study Plasma Physics?

A. The Fundamental Physics of Plasmas

1. We would like to understand the fundamental behavior of naturally occurring plasmas and their effect on their environment.
2. Examples: Astrophysical plasmas (intracluster medium, interstellar medium, accretion disks, stellar interiors), solar corona and solar wind, Earth's magnetosphere and ionosphere.

B. Applications

1. Exploit the properties of plasmas to achieve desirable results.
2. Examples: a. FUSION: Cheap energy!
   b. Industrial applications: plasma processing of semi-conductors, plasma sterilization, efficient lighting, electronics

II. Aim of this Lecture

A. The Basics of Plasma Physics

1. In 29:194 & 29:195, we have learned all of the fundamentals of the study of plasma physics.
2. We should now feel confident to attack any problem using some combination of these fundamental concepts and approaches,

B. The Framework

1. Today, I will attempt to provide an overview of plasma physics, highlighting:
   a. Relationships between the topics we have studied
   b. The context placing what we have learned in terms of what we have not studied.
   $\Rightarrow$ "the known unknowns."

III. Characteristic Scales in a Plasma

A. Summary

Length	Time/Frequency	Dimensionless
Particle Spacing, $n_0^{-1/3}$	Plasma Frequency, ω_p	Plasma Parameter, N_D
Debye Length, λ_D	Cyclotron Frequency, ω_c	Plasma Beta, β
Larmor Radius, r_L	Collision Frequency, ν	Magnetization r_L/L
Mean Free Path, λ_m	Observation "Frequency", $\frac{1}{T}$	Collisionality νT
System size, L		

B. Choosing an appropriate plasma description

1. Use the characteristic scales of the system of interest to determine the system of equations to use
2. Generally, we want to use the most simple system that contains the relevant physical effects
   a. Separation of timescales often allows us to ignore fast, small-scale physics if we are interested in larger times and larger scales.
      Ex: 1. $E \times B$ drift: The details of the fast Larmor motion are often not of interest — rather, we want to know the net drift of the particles.
      2. Fast charge density oscillations at the plasma frequency have negligible effect on the slow, large scale MHD fluctuations. ⇒ These fast motions average out.

IV. How do we choose an appropriate description of the plasma?

We'll begin with the most simple description, then generalize as necessary to the most complicated description.

A. Single Particle Motion:

1. Motion of Charged particles in prescribed fields $\underset{\sim}{E}(\underset{\sim}{x},t)$ & $\underset{\sim}{B}(\underset{\sim}{x},t)$.

a. This is only useful if you can guess $\underset{\sim}{E}$ & $\underset{\sim}{B}$ well, and if the effect of the charge density ρ_q and current density $\underset{\sim}{j}$ is small.

b. Gives $\underset{\sim}{x}_s$ and $\underset{\sim}{v}_s$ using the Lorentz Force Law, $m_s \frac{d\underset{\sim}{v}_s}{dt} = q_s(\underset{\sim}{E} + \underset{\sim}{v}_s \times \underset{\sim}{B})$

c. Can be a useful description of very low density plasma behavior in strong external fields. For example, laser plasma interactions.

d. Otherwise, useful for building intuition about plasma behavior.

2. Guiding Center Approximation

a. Useful for fields that vary slowly in time and smoothly in space.

Slow: $\frac{1}{|\underset{\sim}{B}|}\left|\frac{d\underset{\sim}{B}}{dt}\right| \ll \omega_{ci}$ $\qquad$ $\frac{1}{|\underset{\sim}{E}|}\left|\frac{d\underset{\sim}{E}}{dt}\right| \ll \omega_{ci}$

Smooth: $\frac{|\nabla \underset{\sim}{B}|}{|\underset{\sim}{B}|} \ll \frac{1}{r_{Li}}$ $\qquad$ $\frac{|\nabla \underset{\sim}{E}|}{|\underset{\sim}{E}|} \ll \frac{1}{r_{Li}}$

b. Adiabatic Invariants can be useful to describe the motion of particle in prescribed fields.

Ex: The motion of energetic particles in Earth's magnetosphere

i. First Adiabatic Invariant: Magnetic Moment μ (Larmor Motion)

ii. Second Adiabatic Invariant: Parallel Bounce Motion, J_2

iii. Third Adiabatic Invariant: Azimuthal Drift Motion

3. Inconsistent Model: Fields are not consistently generated.

IV. (Continued)

B. Magnetohydrodynamics (Single Fluid Theory), MHD

1. This is the most simple, consistent model of plasma evolution.
2. The MHD Approximation
   a. Strong Collisions: $\nu_{ei}\tau \gg 1$ (or $\lambda_{m}/L \ll 1$)
   b. Non-relativistic $V_0^2/c^2 \ll 1$ (where $V_0 \sim \frac{L}{\tau}$)
   c. Magnetized $r_i/L \ll 1$
3. In MHD Approximation, plasma fluctuations are quasi-neutral, $\sum_s n_s q_s = 0$.
4. When Magnetic Reynolds Number $R_{e_M} = \frac{\mu_0 L V_0}{\eta} \gg 1$, a condition often satisfied in most plasmas, the Magnetic Flux is Frozen-In to the plasma.

5. The high collisionality enables:
   a. Isotropic pressure
   b. Negligible viscosity $\Big\}\ -\nabla\cdot \underset{\approx}{P}_s = -\nabla p_s - \cancel{\nabla\cdot \underset{\approx}{\Pi}_s}^{\,0} \simeq -\nabla p_s$
   c. No Heat Flow along field lines $\Longleftarrow$ Probably the approximation most often violated in real plasmas of interest.

   $\Rightarrow$ Adiabatic $\frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right) = 0$ ← no heat flow,

6. MHD, due to its simplicity, is the most widely used plasma description
   a. WARNING: MHD is often applied even in situations where it is not formally valid ⇒ "Enter at your own risk."
7. MHD is a valid, low-frequency limit of plasma behavior, $\omega \ll \omega_{ci}$

IV. (Continued)

C. Two Fluid Equations, Cold Plasma Approximation

1. Cold Plasma Approximation: $\frac{\omega}{k} \gg v_{te}$

   a. Often a good description of high frequency plasma behavior

   b. At high frequencies, electron and ion responses are different
   $\Rightarrow$ requires a Two-Fluid Approach

   c. Since $\tau \sim \frac{1}{\omega}$ is small, $\nu_{ei}\tau \ll 1$ often, so collisional effects may be neglected.

2. In this limit, thermal spread of velocities is small, so all particles respond to fields in the same way, regardless of velocity
   $\Rightarrow$ No velocity space effects, no wave-particle interactions.

D. Two Fluid Equations, Warm Plasma

1. One may extend the validity of the Two-Fluid Approach to lower frequencies by including thermal effects (pressure).

2. Still accounts for differences in ion and electron responses.
   This occurs when: a. $r_{Li} \gtrsim L$, b. $\omega \gtrsim \omega_{ci}$ — Single-fluid MHD breaks down in these conditions.

3. Valid for a. High collisionality $\nu_{ei}\tau \gg 1$
   In this case, we have i. Isotropic pressure $\nabla \cdot \underset{\approx}{P}_s \simeq \nabla p_s$
   ii. Adiabatic Equation of State $\frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right) = 0$

   b. Moderate Collisionality $\nu_{ei}\tau \gtrsim 1$
   i. Anisotropic pressure
   $$\underset{\approx}{P}_s = \begin{bmatrix} p_{\perp s} & 0 & 0 \\ 0 & p_{\perp s} & 0 \\ 0 & 0 & p_{\parallel s} \end{bmatrix}$$
   ii. Double Adiabatic Equation of State (Chew-Goldberger-Low, CGL)
   $$\frac{d}{dt}\left(\frac{p_{\perp s}}{n_s B}\right) = 0, \quad \frac{d}{dt}\left(\frac{p_{\parallel s} B^2}{n_s^3}\right) = 0$$

IV. D. (Continued)

4. Because of moderate to high collisionality, velocity-space effects are unimportant.
   a. Collisions enforce Maxwellian, or Bi-Maxwellian, distributions.
   $\Rightarrow$ In this case, fluid approximations are a good description. (Local Thermodynamic Equilibrium)

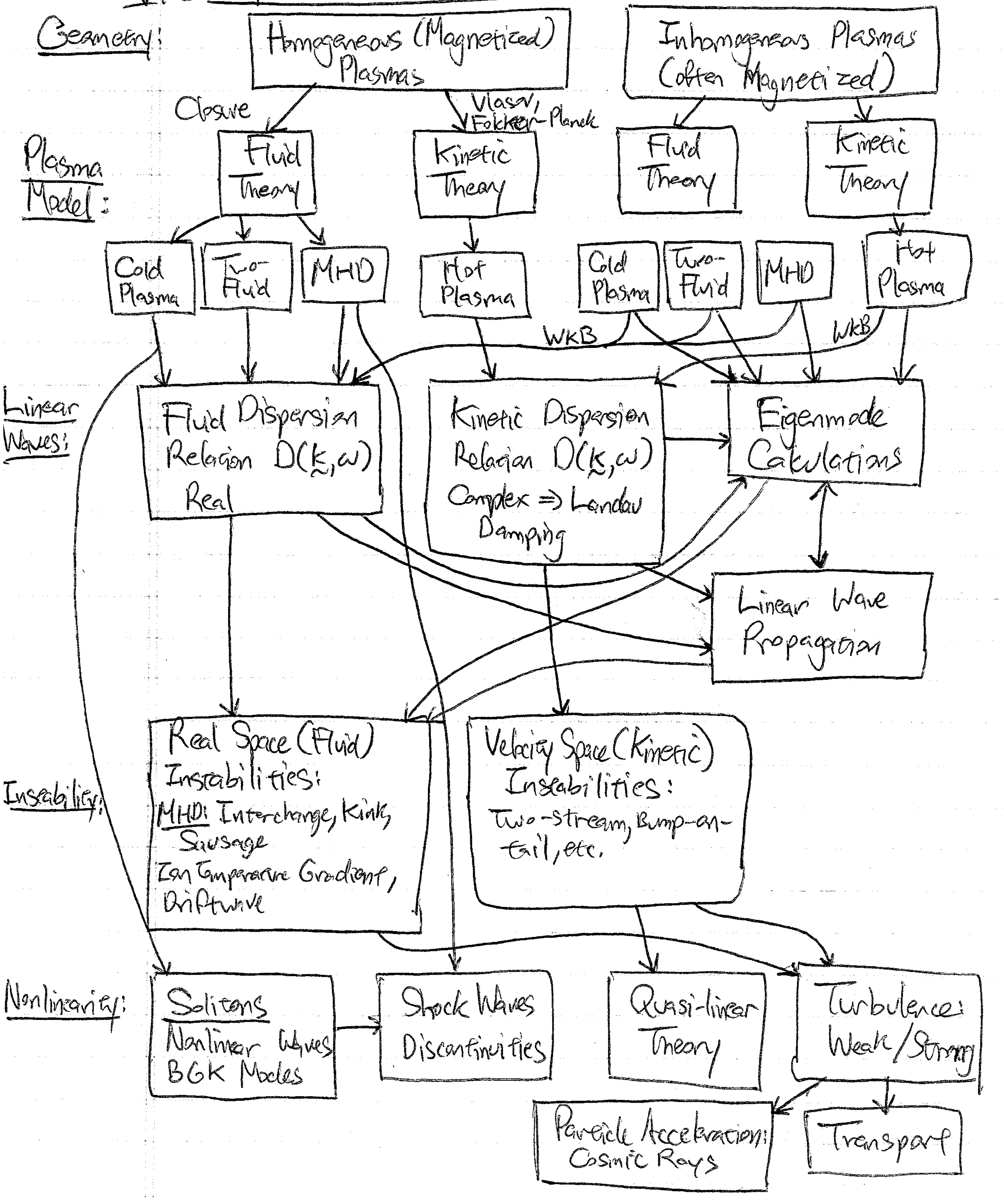
E. <u>Plasma Kinetic Equation (Boltzmann Equation)</u>

1. Required when velocity-space effects are important
   a. Collisionless plasma $\nu_{ei} T \ll 1$
   b. Finite plasma temperature (non-negligible pressure)
2. The conditions above lead to <u>strong wave-particle interaction</u> at resonant velocities, $v = \frac{\omega}{k}$.
   a. <u>Landau damping</u> (collisionless damping)
   b. Cyclotron damping (collisionless damping)
3. For a kinetic description, no Equation of State need be assumed.
   a. Equation of State is needed for <u>any</u> fluid description to close the hierarchy of moment equations.

V. <u>Waves in Plasmas</u>

A. 1. Why do we spend so much time studying waves in plasmas?
   a. Waves are the natural plasma response to an applied perturbation
   b. Small perturbations lead to linear wave behavior
   c. Unstable waves may lead to growth to nonlinear amplitudes.

V. B. Map of Waves in Plasmas



Lecture #21 (Continued)

VI. A Map of Plasma Physics

Full Problem: Klimontovich Eq.

Lorentz Force Law — Particles $\underset{\sim}{x}, \underset{\sim}{v}$ — Maxwell's Equations — Fields $\underset{\sim}{B}$ & $\underset{\sim}{E}$

Single Particle Motion

Prescribed $\underset{\sim}{E}$ & $\underset{\sim}{B}$

Solve for $\underset{\sim}{x}$ & $\underset{\sim}{v}$

Guiding Center Theory

Adiabatic Invariance

Statistical Approach: Boltzmann Equation

$$\frac{\partial f_s}{\partial t} + \underset{\sim}{v}\cdot\nabla f_s + \frac{q_s}{m_s}(\underset{\sim}{E} + \underset{\sim}{v}\times\underset{\sim}{B})\cdot\frac{\partial f_s}{\partial \underset{\sim}{v}} = \left(\frac{\partial f_s}{\partial t}\right)_{coll}$$

$$\rho_q = \int_v q_s f_s \qquad \underset{\sim}{j} = \int_v \underset{\sim}{v} q_s f_s$$

Maxwell's Equations

Closure

Two Fluid Equations:

Local Thermodynamic Equilibrium

$\Rightarrow$ Maxwellian Distribution

Useful for cold, dense plasmas

MHD: Single Fluid Theory

Frozen-in flux

Isotropic pressure

No heat flow

Plasma Equilibrium

Only nontrivial for highly magnetized plasma

$$\nabla p = \underset{\sim}{j}\times\underset{\sim}{B} + \rho g$$

Perturbations

Stable: Waves

Cold Plasma Waves

MHD Waves

Hot Plasma Waves

- Landau Damping
- Quasi-linear theory

Unstable: Instabilities

Fluid: MHD - Interchange, Kink, Sausage, Rayleigh-Taylor

Drift waves

Growth to NL amplitudes

Kinetic: - Beam, Two-Stream

- Temperature Anisotropy: Firehose, Mirror, Ion cyclotron, whistler
- Often saturate at moderate amplitudes
- Zoo of kinetic instabilities