

29:293 Homework #1

Due at the beginning of class, Thursday, February 7, 2013.

1. Electrostatic Drift Waves

Consider a cylindrical column of plasma of radius a with an equilibrium number density that varies radially as

$$n_0(r) = \bar{n} \sqrt{1 - \frac{r^2}{4a^2}}$$

for $r \leq a$, where \bar{n} is a constant. The plasma is confined by a uniform axial magnetic field, $\mathbf{B}_0 = B_0 \hat{z}$. The low beta plasma has $m_e/m_i \ll \beta_e \ll 1$ and may be treated as a two fluid plasma with cold, singly-charged ions and isothermal electrons. Consider the plasma in the low frequency $\omega \ll \omega_{ci}$ and long wavelength $kC_i \ll \omega_{pi}$ limits, where the ion acoustic speed is defined as $C_i = T_e/m_i$ (and Boltzmann's constant has been absorbed into the temperature).

- (a) Find the magnitude of the equilibrium electron drift velocity U_d as a function of C_i , ω_{ci} , r , a , and physical constants. (Hint: Use the electron momentum equation in the small electron mass limit to obtain a constraint on the equilibrium. Then solve for the electron drift required to satisfy that constraint.)
- (b) In terms of the cylindrical coordinates (r, θ, z) , in what direction is this equilibrium drift?
- (c) Does this equilibrium drift correspond to differential or solid body rotation of the plasma? (Note that solid body rotation requires a linear variation of the velocity with radius, $U(r) \propto r$.)
- (d) To investigate the wave behavior of the plasma, a frequently used approximation is to treat the dynamics in the cylindrical plasma locally by a Cartesian coordinate system (taking $r \rightarrow x$, $\theta \rightarrow y$, and $z \rightarrow z$). For a fluctuating wave that varies in this local Cartesian coordinate system as $\exp(ik_y y + ik_{\parallel} z - i\omega t)$, find the *two* roots of the drift wave frequency in the limit $k_{\parallel} C_i \ll k_y U_d$. Express your answers in terms of k_y , k_{\parallel} , C_i , ω_{ci} , r , a , and physical constants. [Note that you do *not* simply assume that $k_{\parallel} C_i = 0$, but expand in a small parameter $\epsilon = k_{\parallel} C_i / (k_y U_d) \ll 1$.]