29:293 Homework #7

Reading: Required: Read GB Chapter 11, Sec 11.1–11.2 (p.415–420)
Optional: Read BS Chapter 8, Sec 8.4–8.5 (p.307–317)

Due at the beginning of class, Thursday, April 11, 2013.

1. Show that for an electric field of the form

$$\mathbf{E}(\mathbf{x},\tau,t) = \mathbf{E}_0(\mathbf{x},\tau)\cos(\omega t - \mathbf{k}\cdot\mathbf{x})$$

the magnetic field is given by

$$\mathbf{B}(\mathbf{x},\tau,t) = -\frac{1}{\omega}\left\{[\nabla\times\mathbf{E}_0(\mathbf{x},\tau)]\sin(\omega t - \mathbf{k}\cdot\mathbf{x}) - [\mathbf{k}\times\mathbf{E}_0(\mathbf{x},\tau)]\cos(\omega t - \mathbf{k}\cdot\mathbf{x})\right\}$$

2. Calculate the numerical values of $\ln\Lambda$ for hydrogen plasmas in the range of density 1–10^{20} cm^{-3} and temperature 10^2–10^8 K. How sensitive is $\ln\Lambda$ to such a wide range of density and temperature?

3. How does the mean free path for electron-ion collisions $\lambda_{m(e-i)}$ depend on the electron temperature T_e?

4. Recall the Child-Langmuir Law for a 1-D electrostatic plasma of hydrogen with isothermal electrons with temperature T_e and cold ions,

$$j_i = \frac{4}{9}\epsilon_0\left(\frac{2e}{m_i}\right)^{1/2}\frac{\phi_W^{3/2}}{d^2}, \quad (1)$$

which expresses the space-charge limited ion current across the sheath in the limit $-e\phi_w/T_e \gg 1$ as a function of sheath width d and the potential difference ϕ_W between the wall and the potential at the sheath edge $x = d$. Recall that the potential at the sheath edge is chosen to define a potential of zero, $\phi(d) = 0$. Note that we absorb Boltzmann's constant to give temperature in units of energy.

   (a) Taking the ion current to be given by $j_i = en_dc_s$, where $c_s = \sqrt{T_e/m_i}$ and the n_d is ion density at the sheath edge, compute an expression for the sheath width d as a function of the wall potential phi_W, the electron temperature T_e, and the Debye length computed using the plasma conditions at the sheath edge $x = d$.

   (b) For typical laboratory plasma parameters of $T_e = 5\ eV$ and $n_d = 10^{18}$ m^3, compute the width of the sheath for a wall voltage of $\phi_W = -300$ V.

5. For a Langmuir probe trace using a cylindrical probe (for which the electron saturation current does not become constant), (a) compute the electron temperature using the data in the table below, and (b) estimate the plasma potential.

Probe Bias (V)	- Probe Current (A)
-65.00	-0.0001290
-60.00	-0.0001290
-55.00	-0.0000860
-50.00	-0.0000860
-45.00	-0.0000430
-40.00	-0.0000430
-35.00	0.0000000
-30.00	0.0000430
-26.00	0.0001730
-24.00	0.0003020
-22.00	0.0004740
-20.00	0.0009060
-18.00	0.0015960
-16.00	0.0032350
-14.00	0.0041410
-12.00	0.0046580
-10.00	0.0051330
-8.00	0.0055210
-6.00	0.0058230
-4.00	0.0062540
-2.00	0.0064270
0.00	0.0068150