PHYS:4762 Homework #12

Reading: Read Arfken, Weber, & Harris, Chapter 22, Section 22.2–22.4 (p.1096–1121)

Due at the beginning of class, Thursday, May 5, 2016.

Homework Problems:

1. Chapter 22, Exercise 22.2.4
2. Chapter 22, Exercise 22.2.6
3. Chapter 22, Exercise 22.2.8
4. Chapter 22, Exercise 22.3.2
5. Chapter 22, Exercise 22.3.4
6. Chapter 22, Exercise 22.4.4
   HINT: Consider a cylinder of radius $r = a$ in a cylindrical coordinate system (r, θ, z) aligned with the axis of rotation. This problem is most easily done in a rotating frame of reference, in which the energy to be minimized is the sum of the gravitational potential energy of the water and the potential energy of the fluid associated with the centrifugal force in the rotating frame of reference. One wants to minimize the energy subject to the constraint of a constant volume of water. This problem is axisymmetric and the state can be described by the height of the water column $z = h(r)$ as a function of the radius, where the base of the water column is at $z = 0$.

   The gravitational potential energy per unit volume is given by $\rho g z$, where ρ is the constant mass density and g is the constant acceleration of gravity. The centrifugal potential energy per unit volume is $-\rho\omega^2 r^2/2$. Use an integral of the total potential energy density over volume to get total energy E as a functional of $h(r)$. Similarly, an appropriate integral over $h(r)$ gives the volume V. Then minimize E subject to constant V.