PHYS:7729 Homework #7

Reading: Suggested: Read GB Chapter 11, Sec 11.1 (p.428–441)
Optional: Read BS Chapter 10, Sec 10.1–10.2 (p.376–388)

Due at 5:00pm, Friday, April 16, 2021.

1. Consider the problem of the counter-streaming Cauchy distribution,

$$F_0(v_z) = \frac{C}{2\pi}\left[\frac{1}{C^2 + (v_z - V)^2} + \frac{1}{C^2 + (v_z + V)^2}\right].$$

Sketch the Nyquist diagrams for mapping the $\gamma = 0$ contour in the complex p-plane onto the complex D-plane for the following cases:

(a) $C > V$.

(b) $C = V$.

(c) $C < V$.

2. A plasma consists of a Maxwellian distribution of ions at temperature T_i and electrons at temperature T_e. The average velocity of the ions is zero and the electrons are drifting with an average velocity U_e.

(a) Make an appropriate modification of the derivation for the ion acoustic instability involving the plasma dispersion function to take into account the drifting electrons.

(b) Using the low-phase velocity approximation for the electrons and the high-phase velocity approximation for the ions, obtain an approximate expression for the real frequency of the ion acoustic mode. You may assume that $k\lambda_{De} \ll 1$.

(c) Show that the growth rate is approximately

$$\frac{\gamma}{\omega} = -\sqrt{\frac{\pi}{8}}\left[\sqrt{\frac{m_e}{m_i}}\left(1 - \frac{U_e}{C_i}\right) + \left(\frac{T_e}{T_i}\right)^{3/2}\exp\left(-\frac{T_e}{2T_i}\right)\right]$$

where the ion acoustic speed is given by

$$C_i = \sqrt{\frac{T_e}{m_i}}$$

(d) Show that the instability condition for the current-driven ion acoustic mode is

$$|U_e| > C_i\left[1 + \sqrt{\frac{m_i}{m_e}}\left(\frac{T_e}{T_i}\right)^{3/2}\exp\left(-\frac{T_e}{2T_i}\right)\right]$$