ASTR:7830 Homework #2

Suggested Reading: Read RLS16 Chapter 3, Sec. 3.4–3.8 (p.66–90)

Due at the beginning of class, Friday, February 2, 2024.

1. Compute the electron-ion collision frequency and electron mean free path for the following plasmas, and determine whether each plasma is strongly collisional or weakly collisional.

   (a) The plasma in the cylindrical LArge Plasma Device (LAPD) at UCLA, with a number density of $n = 10^{18}$ m^{-3}, electron temperature $T_e = 8$ eV, ion temperature $T_i = 1$ eV, and magnetic field $B = 600$ G. The magnetic field in the LAPD is axial, the cylindrical plasma chamber has $L_\parallel = 18$ m and plasma diameter $L_\perp = 40$ cm.

   (b) The solar wind has a number density of $n = 10$ cm^{-3}, electron temperature $T_e = 10^5$ K, ion temperature $T_i = 5 \times 10^4$ K, and magnetic field $B = 10$ nT. The turbulent dynamics in the solar wind are driven at the large length scales of $L \sim 10^6$ km.

2. The magnetic field strength in the Earth's magnetic equatorial plane is given by $B = B_0(R_E/r)^3$ where $B_0 = 0.3$ G, R_E is the radius of the Earth, and r is the geocentric distance.

   (a) Derive an expression for the drift period (the time it takes a particle to drift around the Earth) of a particle on the equatorial plane with a pitch angle of 90° and energy W. (Hint: Think carefully about what this implies).

   (b) Evaluate the period for a proton of 1 keV energy at a distance of $5R_E$ from the center of the earth.

   (c) Evaluate the period for an electron of 1 keV energy at a distance of $5R_E$ from the center of the earth.

   (d) Compare the answer to the drift induced by the force of gravity on the same particles.

   (e) What is the orbital period of an uncharged particle at the same position?

3. A mirror machine has a ratio $B_{max}/B_{min} = 2$. A group of electrons with an isotropic velocity distribution (Maxwellian) is released at the center of the machine. In the absence of collisions, what fraction of these electrons is confined? (Hint: Do not forget that velocity space is three-dimensional.)

4. How does the mean free path for electron-ion collisions $\lambda_{m(e-i)}$ depend on the electron temperature T_e?

5. Show, for a plasma species s with a drifting Maxwellian velocity distribution

$$f_s(\mathbf{x}, \mathbf{v}, t) = \frac{n_s(\mathbf{x}, t)}{\pi^{3/2} v_{ts}^3} \exp\left[\frac{-m_s|\mathbf{v} - \mathbf{U}_s(\mathbf{x}, t)|^2}{2T_s(\mathbf{x}, t)}\right]$$

   that the first velocity moment, $\int d^3\mathbf{v} \quad \mathbf{v} f_s(\mathbf{x}, \mathbf{v}, t)$, gives the result $n_s(\mathbf{x}, t)\mathbf{U}_s(\mathbf{x}, t)$.