ASTR:7830 Homework #4

Suggested Reading: Read RLS16 Chapter 9 (p.275-312)

Due at the beginning of class, Friday, March 1, 2024.

1. Planetary Magnetospheres:
Compute the size of the magnetospheres of Mercury and Jupiter, taking the characteristic size of the magnetosphere to be the distance to the subsolar magnetopause. For reference, the Earth's magnetic moment has magnitude $M_E = 7.84 \times 10^{15}$ T m^3, the average density of the solar wind at 1 AU is $n_{sw_E} = 7$ cm^{-3}, and the average velocity of the solar wind at 1 AU is $v_{sw_E} = 450$ km/s.

   (a) Given that the solar wind is expanding spherically at a constant velocity beyond $10R_\odot$, the solar wind number density n_{sw} scales with heliocentric distance r as $n_{sw} \propto r^{-2}$. Compute the number density of the solar wind in the neighborhood of Mercury n_{sw_M} and Jupiter n_{sw_J} given their heliocentric distances $r_M = 0.387$ AU and $r_J = 5.2$ AU. Please give your answers in units of cm^{-3}.

   (b) Find the subsolar radius of the magnetopause for Mercury R_{MP_M} given its magnetic moment $M_M = 6.1 \times 10^{-4} M_E$. Express your answer in terms of the radius of Mercury, $R_M = 2440$ km.

   (c) Find the subsolar radius of the magnetopause for Jupiter R_{MP_J} given its magnetic moment $M_J = 2 \times 10^4 M_E$. Express your answer in terms of the radius of Jupiter, $R_J = 71500$ km.

   (d) Compare the absolute sizes of the magnetospheres of the Earth, Mercury, and Jupiter by expressing the magnetopause radius for each planet in terms of the radius of the Earth, $R_E = 6378$ km.

2. Jump Conditions at the Bowshock:
Determine the conditions on the magnetosheath side of the bowshock U_{n2}, ρ_2, p_2, and B_{t2} in terms of the incoming solar wind conditions U_{n1}, ρ_1, and B_{t1}. Make the following simplifying assumptions:

   - The thermal pressure and magnetic pressure in the solar wind may be neglected compared to the solar wind dynamical pressure, $\rho_1 U_{n1}^2 \gg p_1$ and $\rho_1 U_{n1}^2 \gg B_1^2/2\mu_0$.
   - The magnetic pressure in the magnetosheath is negligible compared to the thermal pressure, $p_2 \gg B_2^2/2\mu_0$.
   - The normal component of the velocity is much larger than the tangential component on both sides of the shock, $U_n \gg U_t$

   (a) Use the Rankine-Hugoniot jump conditions for conservation of mass, the normal component of momentum, and energy to solve for magnetosheath normal velocity U_{n2} in terms of the solar wind normal velocity U_{n1}. First, solve for the pressure p_2 using the momentum equation, and substitute this into the energy equation to determine U_{n2} in terms of U_{n1}. The adiabatic index is $\gamma = 5/3$.
   Hint: Expressing the normal mass flux $\Phi_m = \rho_1 U_{n1} = \rho_2 U_{n2}$ as a constant Φ_m in the momentum and energy equations simplifies the calculation.

   (b) Compute the magnetosheath density ρ_2 in terms of the solar wind density ρ_1.

   (c) Compute the magnetosheath thermal pressure p_2 in terms of the solar wind density ρ_1 and normal velocity U_{n1}.

   (d) Use the conservation of the tangential component of momentum and the constraint from the MHD induction equation to determine the magnetosheath tangential magnetic field B_{t2} in terms of the solar wind tangential magnetic field B_{t1}. You may assume that the tangential components of the velocity and magnetic field in the solar wind are in the same plane.

   (e) Identify the specific piece of evidence from the computations above that determines whether this compressive MHD shock is a slow shock or a fast shock.