

# ASTR-7830 Space and Astrophysical Plasmas

## Lecture #1: Introduction, Characteristic Scales in a Plasma

What is a plasma?

### I. Overall Framework of Plasma Physics

A.

Particles: Ions & Electrons

position,  $\underline{x}_s$  }  $\Rightarrow \rho$   
 velocity,  $\underline{v}_s$  }  $\downarrow$

Maxwell's Equations:

$$\nabla \cdot \underline{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\nabla \times \underline{B} = \mu_0 \underline{j} + \mu_0 \epsilon_0 \frac{\partial \underline{E}}{\partial t}$$

$$\nabla \cdot \underline{B} = 0$$

Lorentz Force Law:

$$m_s \frac{d\underline{v}_s}{dt} = q_s (\underline{E} + \underline{v}_s \times \underline{B})$$

Electromagnetic Fields:  $\underline{B}, \underline{E}$

$\underline{B}, \underline{E}$

The coupling of the particle motion & electromagnetic fields presents the challenge of plasma physics!

### B. Theoretical Description of Plasma Physics

#### 1. Inconsistent Models:

- a. Single Particle Motion: - Determine particle motion in known  $\underline{E}$  &  $\underline{B}$ .  
 - Good for developing intuition

## I.B. (Continued)

## 2. Consistent Models:

a. **Kinetic Theory** :- Statistical theory averages over motion of many particles  
 $\Rightarrow$  Distribution function  $f_s(\mathbf{x}, \mathbf{v}, t)$

- Most complete common description of plasmas
- Extremely challenging to obtain analytical results
- We won't tackle much kinetic plasma physics in this course  $\Rightarrow$  Take 029:194 & 029:293.

b. **Two-Fluid Theory** :- Evolves moments of the distribution function  
 ions &  $\uparrow$  electrons

- Density:  $n_s = \int d^3v f_s(\mathbf{x}, \mathbf{v}, t)$
- Fluid velocity,  $\mathbf{U}_s = \frac{\int d^3v \mathbf{v} f_s(\mathbf{x}, \mathbf{v}, t)}{n_s}$

- Must assume a closure (Equation of State) to obtain closed set of equations
- Allows for different behavior of ions and electrons

c. **Magnetohydrodynamics (MHD)** :- Single fluid theory is simplest consistent model.

- Probably the most widely used system in space physics & astrophysics.
- We will focus on MHD models in this course

## C. References to Texts:

I will often give references to chaps, sections, or figures in our texts:

1. [RLS16] Russell, Luhmann, & Strongaway, Space Physics: An Introduction, Cambridge Univ Press: Cambridge, 2016.
2. [TS02] Tajima & Shibata, Plasma Astrophysics, Perseus Publishing, Cambridge, Massachusetts, 2002.

## II. Vector Notation Review & Vector Calculus

- A. Why? 1. Vector notation simplifies the mathematical notation  
 2. You will get lots of practice with vector algebra & calculus.

### B. Notation:

1. Under-tilde denotes vector quantity  $\underline{B}$

a. In cartesian coordinates  $\hat{x}$ ,  $\hat{y}$ , &  $\hat{z}$ ,

$$\underline{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

2. Unit vectors:  $\hat{b} \equiv \frac{\underline{B}}{|\underline{B}|}$

3. Magnitude:  $|\underline{B}| = \sqrt{\underline{B} \cdot \underline{B}} = \sqrt{B_x^2 + B_y^2 + B_z^2}$

4. Tensor: Denoted by double under-tilde  $\underline{\underline{E}} = \begin{pmatrix} E_{xx} & E_{xy} & E_{xz} \\ E_{yx} & E_{yy} & E_{yz} \\ E_{zx} & E_{zy} & E_{zz} \end{pmatrix}$

### Vector Calculus:

5.  $\frac{\partial f}{\partial \underline{v}} = \frac{\partial f}{\partial v_x} \hat{x} + \frac{\partial f}{\partial v_y} \hat{y} + \frac{\partial f}{\partial v_z} \hat{z}$

- b.  $\nabla f = \frac{\partial f}{\partial \underline{r}} = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}$

- c. Thus  $\nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}$

## C. Vector Algebra and Calculus Review:

1. Dot Product:  $\underline{A} \cdot \underline{B} = A_x B_x + A_y B_y + A_z B_z$

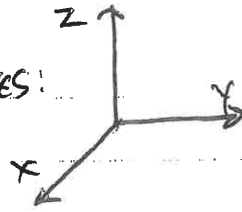
2. Cross Product:  $\underline{A} \times \underline{B} = (A_y B_z - A_z B_y) \hat{x}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + (A_x B_z - A_z B_x) \hat{y} + (A_x B_y - A_y B_x) \hat{z}$$

Lecture #2 (Continued)  
 II.2 (Continued)

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3. Right-handed coordinates:



$$\begin{aligned} \hat{x} \times \hat{x} &= \hat{z} \\ \hat{y} \times \hat{z} &= \hat{x} \\ \hat{z} \times \hat{x} &= \hat{y} \end{aligned}$$

4. Integration:  $\int d^3 \underline{v} f(\underline{v}) \equiv \int dv_x \int dv_y \int dv_z f(\underline{v})$

5.  $\underline{v} \cdot \nabla = (v_x \hat{x} + v_y \hat{y} + v_z \hat{z}) \cdot \left( \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \right) = v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}$

6.  $\underline{v} \times \nabla = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = (v_y \frac{\partial}{\partial z} - v_z \frac{\partial}{\partial y}) \hat{x} + (v_z \frac{\partial}{\partial x} - v_x \frac{\partial}{\partial z}) \hat{y} + (v_x \frac{\partial}{\partial y} - v_y \frac{\partial}{\partial x}) \hat{z}$

D. Examples:

1. MHD continuity:  $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0$

a. By NRL p.4 (7),  $\nabla \cdot (\rho \underline{v}) = \rho \nabla \cdot \underline{v} + (\underline{v} \cdot \nabla) \rho$ , so

$$\frac{\partial \rho}{\partial t} + (\underline{v} \cdot \nabla) \rho = -\rho \nabla \cdot \underline{v}$$

2. MHD momentum:  $\rho \frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \nabla) \underline{v} = -\nabla p + \underline{j} \times \underline{B}$

a. We also have  $\underline{j} = \frac{1}{\mu_0} \nabla \times \underline{B}$  (Ampere's Law, displacement current dropped)

b. Rewrite  $\underline{j} \times \underline{B}$  term in terms of only  $\underline{B}$ :

$$\underline{j} \times \underline{B} = \frac{1}{\mu_0} (\nabla \times \underline{B}) \times \underline{B} = -\frac{1}{\mu_0} \underline{B} \times (\nabla \times \underline{B})$$

From NRL p.4 (12) with  $\underline{B} = \underline{A} \Rightarrow \nabla(\underline{B} \cdot \underline{B}) = 2 \underline{B} \times (\nabla \times \underline{B}) + 2(\underline{B} \cdot \nabla) \underline{B}$

So  $\underline{B} \times (\nabla \times \underline{B}) = \frac{1}{2} \nabla(\underline{B} \cdot \underline{B}) - (\underline{B} \cdot \nabla) \underline{B}$

$$\text{Thus } \underline{j} \times \underline{B} = -\frac{\nabla B^2}{2\mu_0} + \frac{(\underline{B} \cdot \nabla) \underline{B}}{\mu_0}$$

### III. Characteristic Scales in a Plasma

#### A. Basic Parameters of a Plasma

1. Plasma consists of one, or more, ion species and electrons
2. Intensive variables:
  - a. Density,  $n_s$
  - b. Temperature,  $T_s$
  - c. Magnetic Field,  $B_0$
3. Physical properties:
  - a. mass,  $m_s$
  - b. charge,  $q_s$

#### B. Units:

1. Two Major Systems:
  - a. SI units (mks) (RLS 2016)
  - b. Gaussian units (cgs) (Tajima & Shibata, 2002)
2. I will do my best to be consistent, but may switch units from one lecture to the next based on units common to particular problems.

#### C. Length Scales: (Ordered from largest to smallest)

1. System size,  $L$ : Typical scale of system under investigation
  - a. Earth's Magnetosphere:
    - i. Magnetopause,  $L \sim 10 R_E$
    - iii. Bowshock,  $L \sim 15 R_E$
    - iv. Magnetotail,  $L \sim 100 R_E$  (see RLS16, Fig 9.2)
  - b. Solar Wind Turbulence:
    - i. Outer Scale,  $L \sim 10^{11} \text{ cm} = 10^6 \text{ km}$
  - c. Accretion Disk (around  $M = 1 M_\odot$  star)
    - i. Radius,  $R \sim 10^{10} \text{ cm}$
    - ii. Height,  $H \sim 10^8 \text{ cm}$
  - d. Galaxy Clusters
    - i. Virial radius,  $L \sim 1 \text{ Mpc}$
    - ii. Central core,  $L \sim 100 \text{ kpc}$

NOTE: What matters for plasma physics is ~~not~~ the absolute scale, but the scale relative to characteristic plasma length scales.

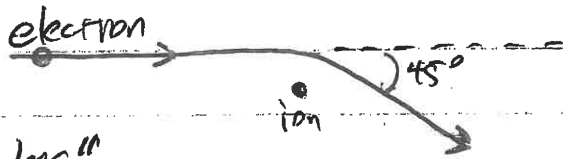
# Lecture #1 (Continued)

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## II. C. (Continued)

2. Mean Free Path,  $\lambda_m$ : Distance between coulomb collisions between charged particles.

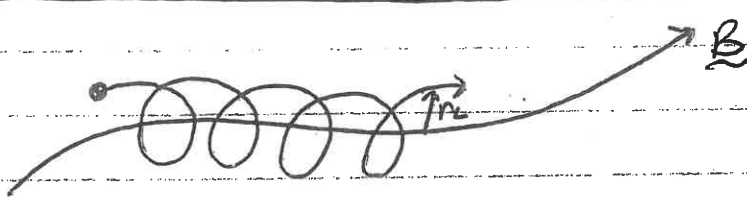
b. A collision is typically defined as a electron deflection of  $\geq 45^\circ$



c. i. IF  $\lambda_m \gg L$ , system is "collisionless".

ii. IF  $\lambda_m \ll L$ , system is collisional.

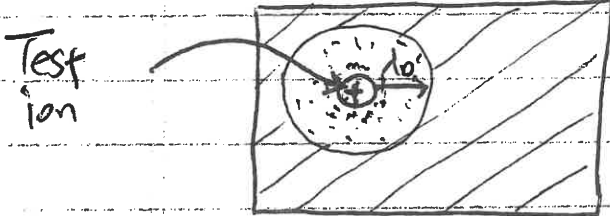
3. Thermal Larmor radius (gyroradius),  $r_L$ :



$$r_L = \frac{v_{Ts}}{\Omega_s}$$

Species thermal velocity  
Species cyclotron frequency

4. Debye Length,  $\lambda_D$ : i. Length scale over which charge imbalance may occur.



ii. Electrons adjust to shield Coulomb field of "test ion"  
 $\Rightarrow$  Debye Shielding (see notes from PHYS 4731, Lec #2)

iii. Net charge inside sphere of radius  $\lambda_D$  is zero.

5. Particle Separation,  $n_0^{-1/3}$ : i. Typical distance between charged particles in plasma.

## D. Velocities:

1. Thermal Velocity,  $v_{Ts}$ : i. Defined

$$v_{Ts} = \sqrt{\frac{2T_s}{m_s}}$$

ii. NOTE! Boltzmann constant  $k = 1.38 \times 10^{-23} \frac{J}{K}$  is absorbed to give  $T$  in energy units.

## Lecture #1 (Continued)

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### III. D.1. (Continued)

iii. For protons  $\frac{m_i}{m_e} = 1836$ , so  $\frac{v_{Te}}{v_{Ti}} = \sqrt{\frac{m_i}{m_e}} \approx 43$  (when  $T_i = T_e$ )

2. Alfven velocity,  $v_A$ : i.  $v_A^2 = \frac{B_0^2}{\mu_0 \rho}$  (SI)

(or)  $v_A^2 = \frac{B_0^2}{4\pi \rho}$  (cgs)

ii. Here,  $\rho$  is mass density  $\left[\frac{M}{L^3}\right]$

iii. This is the characteristic speed of large scale motions ( $L \gg r_{Li}$ ) in a magnetized plasma

3. Sound speed,  $c_s$ : i.  $c_s^2 \equiv \frac{\gamma p}{\rho}$

ii. Here,  $\gamma$  is the adiabatic index ( $\gamma = \frac{5}{3}$  for monatomic gas)

iii.  $p$  is thermal pressure  $\uparrow$  determined by equation of state (fluid closure)

### E. Frequencies (Timescales):

1. Observation Time:  $\tau$ : i. Associated angular frequency  $\omega \sim \frac{2\pi}{\tau}$

ii. If we observe a system for a time  $\tau$ , we are most sensitive to frequencies  $\gtrsim \omega \sim \frac{2\pi}{\tau}$

iii. Dynamics on a slower timescale (lower frequency) will not be apparent.

2. Crossing Time/Frequency,  $\tau_c/\omega_c$ : i.  $\tau_c \sim \frac{L}{v_A}$  or  $\omega_c \sim \frac{v_A}{L}$

ii. The time it takes for a signal to cross the system at characteristic velocity (here we take Alfven velocity,  $v_A$ )

## III, E. (Continued)

3. Collision Frequency,  $\nu$ : i.

$$\nu \equiv \frac{V_{Ts}}{\lambda_m}$$

ii. Typical velocity of particles is  $V_{Ts}$ iii. Distance between collisions is  $\lambda_m$ 4. Cyclotron Frequency,  $\Omega_s$ : i.  $\Omega_s = \frac{q_s B}{m_s}$  (SI)

(Angular)

$$\text{(or)} \quad \Omega_s = \frac{q_s B}{m_s c} \quad (\text{cgs})$$

ii. Characteristic frequency of gyration of charged particle about magnetic field (non-relativistic)

5. Plasma Frequency,  $\omega_p$ : i.  $\omega_p^2 = \frac{n_0 q_e^2}{\epsilon_0 m_e}$  (SI)

(or)

$$\omega_p^2 = \frac{4\pi n_0 q_e^2}{m_e} \quad (\text{cgs})$$

- ii. Typical frequency of charge imbalance oscillations in a plasma (very rapid, much faster than most space or astrophysical plasma time scales)
- iii. An applied electric field with  $\omega < \omega_p$  will be shorted out by rapid electron response in plasma

F. Dimensionless Parameters of a Plasma:1. Plasma Parameter,  $N_D$ : i. Number of particles in a Debye sphere

$$N_D = \frac{4\pi}{3} n \lambda_D^3$$

ii. For nearly all space & astrophysical plasmas of interest,  $N_D \gg 1$ .

⇒ Many particles within a Debye sphere

iii. Often used to define plasma behavior (collective behavior).



# Lecture #1 (Continued)

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## III. F. (Continued)

2. Plasma Beta,  $\beta$ :

$$\beta \equiv \frac{\text{Thermal Pressure}}{\text{Magnetic Pressure}} = \frac{2n_0 n_0 (T_i + T_e)}{B_0^2} \text{ (SI)}$$

or 
$$\beta = \frac{8\pi n_0 (T_i + T_e)}{B_0^2} \text{ (CGS)}$$

iii. Most important parameter affecting plasma behavior

iv. In kinetics,

$$\beta_i \equiv \frac{v_{Ti}^2}{v_A^2}$$

In MHD

$$\beta \equiv \frac{c_s^2}{v_A^2}$$

v. Low beta plasmas,  $\beta \ll 1$ , are magnetically dominated (fusion plasmas, solar corona)

vi. High beta plasmas,  $\beta \gg 1$ , have a magnetic field that can be highly deformed by plasma motions (black hole accretion disks)

3. Magnetization:

- i.  $r_{Li}/L \ll 1$  Magnetized
- ii.  $r_{Li}/L \gg 1$  Unmagnetized

4. Collisionality:

- i.  $\lambda_m/L \gg 1$  "Collisionless"
- ii.  $\lambda_m/L \ll 1$  collisional

## G. Summary

1. Length

Particle spacing,  $n_0^{-1/3}$

Debye length,  $\lambda_D$

Larmor radius,  $r_L$

Mean free path,  $\lambda_m$

System size,  $L$

Time/Frequency

Plasma Frequency,  $\omega_{up}$

Cyclotron Frequency,  $\Omega_i$

Collision Frequency,  $\nu$

Observation "Frequency",  $\frac{1}{T}$

} very small scale  
⇒ "microscopic"

} of most interest for space & astrophysical plasmas

2. Typical Conditions of Space & Astrophysical Plasmas:

a. Dynamics are quasi-neutral (no net charge imbalance)  $L \gg \lambda_D$ ,  $\frac{1}{T} \ll \omega_{up}$

b. Generally magnetized:  $r_{Li}/L \ll 1$

c. Both plasma beta  $\beta$  and collisionality can be large, unity, or small.