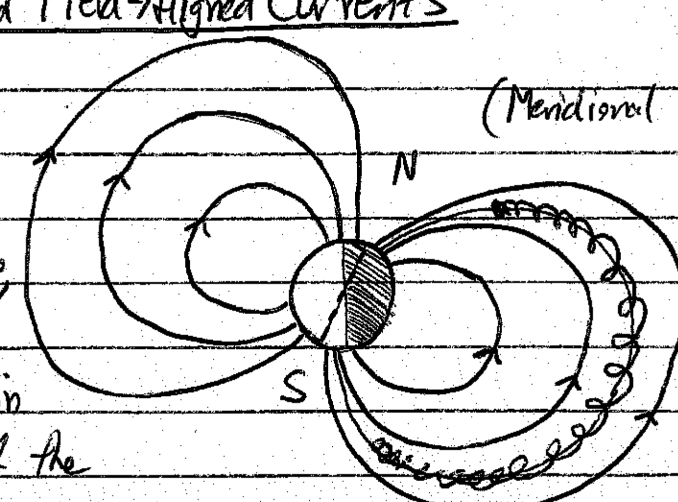
Howes ①

Lecture #11: Ring Current and Field-Aligned Currents

(Meridional plane)

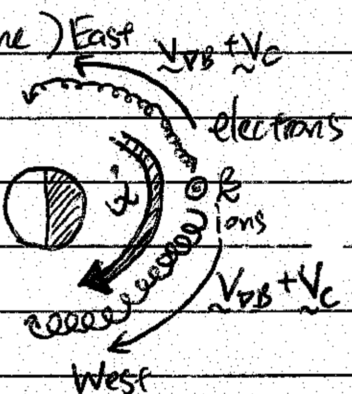


I. Ring Current:

A. General Picture:

1. In the inner magnetosphere, over $3R_E \lesssim r \lesssim 6R_E$, particles can be trapped in the mirror configuration of the dipole magnetic field.
2. Mirror Force: Leads to parallel bounce motion of ions and electrons from north polar to south polar regions and back.
3. Azimuthal Drift: ∇B and curvature drifts lead to drift of ions westward and electrons eastward:

(Equatorial Plane)



a. From lecture #2:

$$\underset{\sim}{V}_{\nabla B} = -\frac{v_\perp^2}{2\Omega} \frac{\nabla B \times \underset{\sim}{B}}{B^2}$$

$$\underset{\sim}{V}_c = \frac{v_\parallel^2}{\Omega B} \frac{\underset{\sim}{R}_c \times \underset{\sim}{B}}{R_c^2}$$

4. Ions dominate ring current

a. Consider ratio of $V_{\nabla B}$ for ions to $\underset{\sim}{V}_{\nabla B}$ for electrons:

$$\frac{|\underset{\sim}{V}_{\nabla B i}|}{|\underset{\sim}{V}_{\nabla B e}|} = \frac{\left(-\frac{v_{\perp i}^2}{\cancel{2}\Omega_i} \cancel{\frac{\nabla B \times B}{B^2}}\right)}{\left(-\frac{v_{\perp e}^2}{\cancel{2}\Omega_e} \cancel{\frac{\nabla B \times B}{B^2}}\right)} = \frac{v_{\perp i}^2 \Omega_e}{v_{\perp e}^2 \Omega_i} = \frac{m_i v_{\perp i}^2}{m_e v_{\perp e}^2} = \frac{\left(\frac{1}{2} m_i v_{\perp i}^2\right)}{\left(\frac{1}{2} m_e v_{\perp e}^2\right)} = \frac{W_i}{W_e}$$

b. Typical ion energies in the ring current are $3\,\text{keV} \lesssim W_i \lesssim 300\,\text{keV}$ while electrons have energies $0.3\,\text{keV} \lesssim W_e \lesssim 30\,\text{keV}$.

$\boxed{\text{Thus ions tend to dominate the ring current.}}$

I. A. (Continued)

5. <u>Dominant Ring Current Ion Species:</u>

Energy	Dominant Species
$E_i < 10$ keV	O^+
$10 \text{ keV} < E_i < 50 \text{ keV}$	O^+ and H^+
$E_i > 50$ keV	H^+

B. Magnetic Field due to a Current Loop:

1. Biot-Savart Law (Jackson, E&M)

$$\underset{\sim}{B}(\underset{\sim}{r}) = \frac{\mu_0}{4\pi}\int d^3\underset{\sim}{r}' \; \frac{\underset{\sim}{j}(\underset{\sim}{r}') \times (\underset{\sim}{r}-\underset{\sim}{r}')}{|\underset{\sim}{r}-\underset{\sim}{r}'|^3}$$

2. For a loop of current I_0 at radius R_0,

$$\underset{\sim}{j} = -I_0 \,\delta(r-R_0)\,\delta(z)\,\hat{\underset{\sim}{\phi}} \qquad (-\hat{\underset{\sim}{\phi}} \text{ is westward})$$

3. Computing the magnetic field at $r=0$ (center of the Earth),

$$\underset{\sim}{B}(0) = -\frac{\mu_0 I_0}{2R_0}\hat{\underset{\sim}{z}}$$

a. Field is generally in $-\hat{\underset{\sim}{z}}$ direction, opposing dipole field at the Earth's surface.

C. Simple Estimate of Ring Current and Magnetic Field Perturbation

1. Consider a single particle with pitch angle $\alpha = 90°$ and energy $W_\perp = \frac{1}{2}mv_\perp^2$ in the equatorial plane.

a. Since $\alpha = 90°$, $V = V_\perp$ and $V_\parallel = 0$, so $\underset{\sim}{V}_c = 0$ and westward drift is due solely to $\nabla \underset{\sim}{B}$ drift.

2. From Lecture #2 V.C.4, $\underset{\sim}{V}_{\nabla B} = -\frac{V_\perp^2}{2\Omega}\frac{\nabla B \times \underset{\sim}{B}}{B^2}$

where

a. Earth's dipole field in equatorial plane, $B = \frac{B_E R_E^3}{r^3}\hat{\underset{\sim}{\theta}}$ with $B_E = -30.4\,\mu T$

So $\nabla B = \frac{\partial}{\partial r}\frac{B_E R_E^3}{r^3}\hat{\underset{\sim}{r}} = \frac{-3B_E R_E^3}{r^4}\hat{\underset{\sim}{r}} = \frac{-3B}{r}\hat{\underset{\sim}{r}}$

I.C. (Continued)

Northward field

$\hat{r}\times\hat{\theta}=\hat{\phi}$

3. Thus, $\mathbf{V}_{\nabla B} = -\frac{m v_\perp^2}{2qB}\,\frac{\left(-\frac{3B}{r}\hat{r}\right)\times\left(-B\hat{\theta}\right)}{\cancel{B^2}} = \frac{-3W_\perp}{qBr}\hat{\phi}$

$$= \frac{-3W_\perp r^{\cancel{3}\,2}}{q\cancel{r}\,B_E R_E^3}\hat{\phi} = -\frac{3W_\perp r^2}{qB_E R_E^3}\hat{\phi}$$

4. Current from the ∇B drift of this single particle, I_1:

$$\mathbf{I}_1 = \frac{Q}{T} = \frac{q}{\left(\frac{2\pi r}{V_{\nabla B}}\right)} = \frac{q\mathbf{V}_{\nabla B}}{2\pi r} = \frac{\cancel{q}}{2\pi\cancel{r}}\left(\frac{3W_\perp r^{\cancel{2}}}{\cancel{q}B_E R_E^3}\right)\hat{\phi} = \boxed{\frac{-3W_\perp r}{2\pi B_E R_E^3}\hat{\phi} = \mathbf{I}_1}$$

Westward

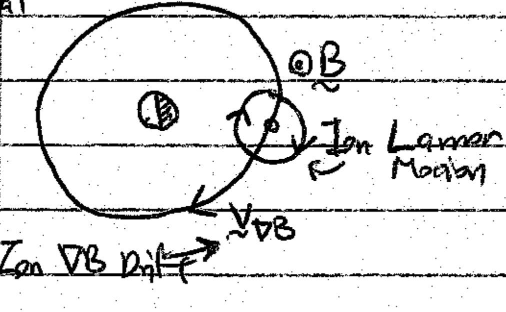
5. Magnetic Perturbation at $r=0$: δB

$$\delta\mathbf{B}(0)_{\nabla B_1} = -\frac{\mu_0 I_1}{2r}\hat{z} = -\frac{\mu_0}{2\cancel{r}}\left(\frac{3W_\perp \cancel{r}}{2\pi B_E R_E^3}\right)\hat{z} = \underbrace{-\frac{3\mu_0}{4\pi}\frac{W_\perp}{B_E R_E^3}}\hat{z}$$

Independent of distance to ring current! (no r dependence)

D. Contribution Due to Larmor Motion:

Equatorial Plane:



1. Field due to Larmor motion at Earth's center $r=0$ is in $+\hat{z}$ direction.

2. Dipole Moment due to Larmor Motion:

$\boldsymbol{\mu} = \frac{-m v_\perp^2}{2B}\hat{z}$ (See Lecture #3.I.A.)

3. Far from the Dipole, the magnetic field is given by

a. $\mathbf{B} = \frac{-\mu_0}{4\pi}\frac{\boldsymbol{\mu}}{r^3}$ ~~in the equatorial plane~~

b. Note, for $r = 5R_E$ and $W_\perp = 50\,\text{keV}$, $r_{L_i} \simeq 100\,\text{km} \ll r$, so the far-field approximation above is valid.

c. Note, the Earth's field has $\mathbf{M}_E = \frac{\mu_0}{4\pi}\boldsymbol{\mu}_E$, where $\mathbf{M}_E = B_E R_E^3$ and $B_E = -30.4\,\mu T$.

Dipole Moment

I. D. (Continued)

4. Therefore, the contribution to the field at $\mathbf{r}=0$ due to dipole moment,

$$\delta \mathbf{B}_{d_1}(0) = \frac{-\mu_0}{4\pi}\left(\frac{-m v_\perp^2}{2B}\hat{\mathbf{z}}\right)\frac{1}{r^3} = \frac{\mu_0}{4\pi}\frac{W_1 r^3}{B_E R_E^3\, r^3}\hat{\mathbf{z}} = \underbrace{+\frac{\mu_0}{4\pi}\frac{W_1}{B_E R_E^3}}_{\text{Again, independent of } r!}\hat{\mathbf{z}}$$

E. Total Magnetic Field Perturbation due to Ring Current, $\Delta \mathbf{B}_{RC}$:

1. For a single particle,

$$\Delta \mathbf{B}_{RC_1}(0) = \delta \mathbf{B}_{\nabla B_1}(0) + \delta \mathbf{B}_{d_1}(0) = -\frac{\mu_0}{2\pi}\frac{W_1}{B_E R_E^3}\hat{\mathbf{z}}$$

a. This field perturbation weakens Northward field at equatorial surface of the earth.

2. For N total particles with energy W_1, $W_{tot} = N W_1$, and total field perturbation is

$$\boxed{\Delta \mathbf{B}_{RC} = -\frac{\mu_0}{2\pi}\frac{W_{tot}}{B_E R_E^3}\hat{\mathbf{z}}} \quad \text{eq (1)}$$

3. This result has made several simplifying assumptions:

a. Distance to ring current r (or L value) is assumed the same for all particles $\Rightarrow$ But r (or L) drops out, so OK!

b. N particles with energy W_1

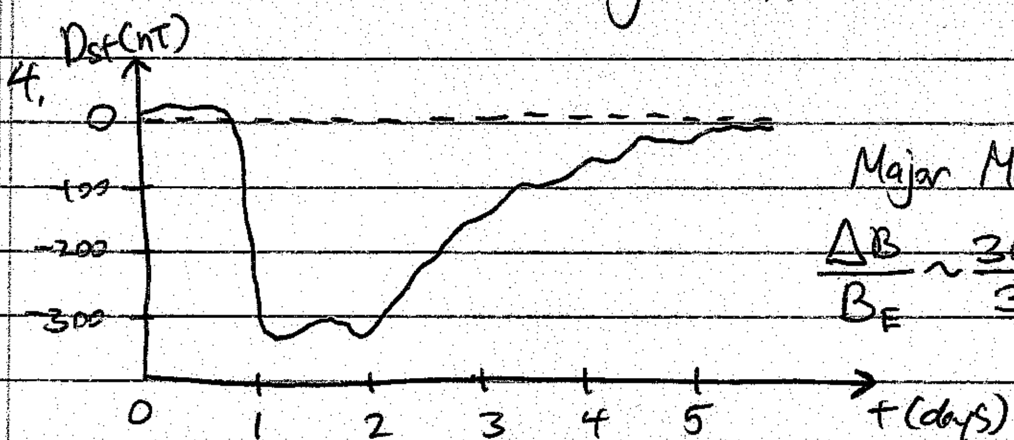
c. All particles have pitch angle $\alpha = 90°$ (equatorially trapped)

4. A more detailed analysis shows eq (1) holds for arbitrary pitch angle distributions. (not just $\alpha = 90°$)

I. (Continued)

F. Magnetic Storms and the Ring Current Index, D_{st}

1. Magnetic Storms lead to an increased injection of particles from the plasma sheet.
2. This increases the total energy W_{tot} of the ring current particles and leads to an enhancement of magnetic ΔB_{RC}.
3. Ring Current Index, D_{st}:
   a. Average of $\Delta \underline{B}$ measured near the equator (hourly)
   b. Stations: Honolulu (USA), San Juan (Puerto Rico), Hermanus (South Africa) and Kakioka (Japan).
   c. Although 30% of D_{st} is due to other sources (magnetopause current, partial ring current, etc.), primarily measures effect of the ring current.

4. 

Major Magnetic Storm:

$$\frac{\Delta B}{B_E} \sim \frac{300\,nT}{30\,\mu T} \sim \underline{1\%}!$$

5. If we assume ΔB_{RC} is responsible for $\frac{1}{2} D_{st}$,

   a. $$\frac{1}{2} D_{st} \simeq \frac{\mu_0}{2\pi} \frac{W_{tot}}{B_E R_E^3} \Rightarrow W_{tot} = \frac{\pi}{\mu_0} B_E R_E^3 D_{st}$$

   b. $$I_{RC} = \frac{3 W_{tot} r}{2\pi B_E R_E^3} = \frac{3r}{2\pi B_E R_E^3}\left(\frac{\pi}{\mu_0} B_E R_E^3 D_{st}\right) = \frac{3 D_{st}}{2\mu_0} r$$

6. For $D_{st} = -300\,nT$ and $r = 5 R_E$, we obtain

$$\boxed{W_{tot} \sim 6\times10^{15}\,J} \quad \text{and} \quad \boxed{I_{RC} \sim 10^7\,A} \leftarrow \text{comparable to } I_{MP} \,\&\, I_T$$

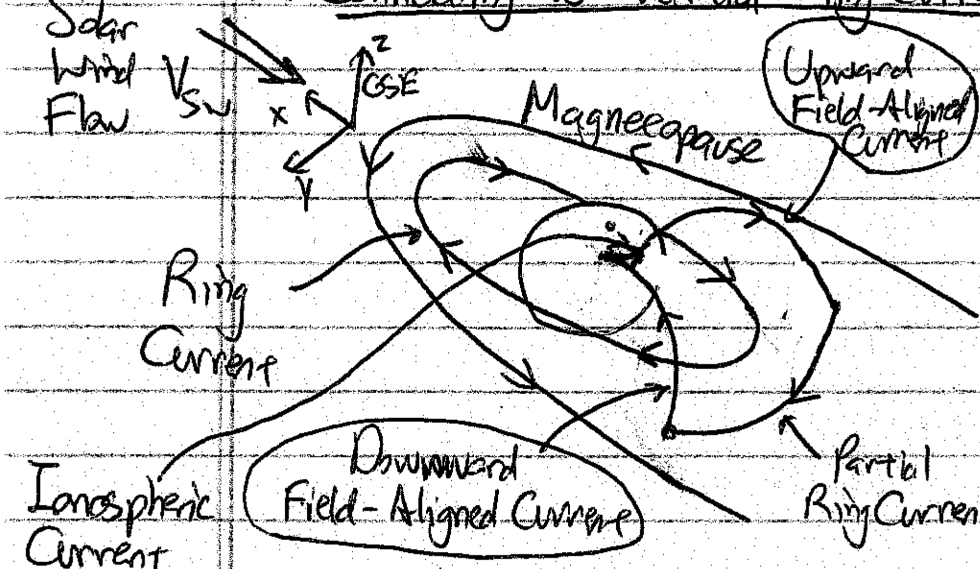
7. Since ring current is closer to Earth, has a bigger effect than I_{MP} & I_T.

Lecture #11 (Continued) Hawes ⑥

II. Field-Aligned (Birkeland) Currents

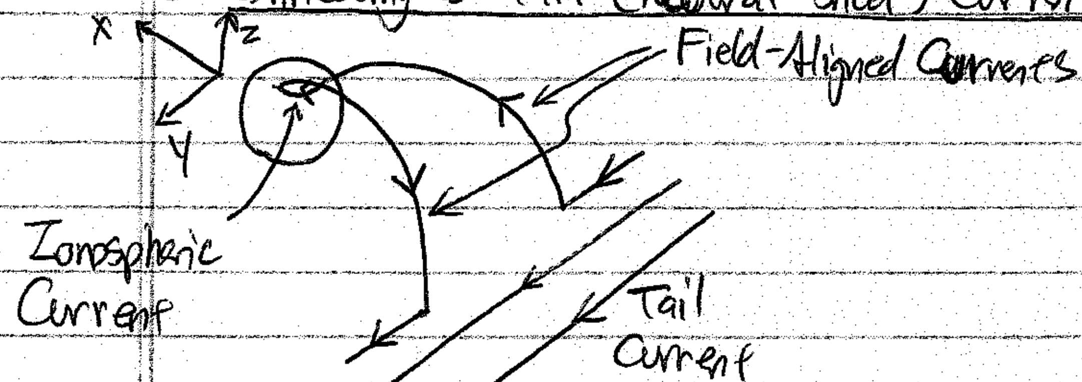
A. Magnetosphere-Ionospheric Coupling (M-I coupling)

1. Field-aligned currents play an important role in coupling the magnetosphere to the auroral ionosphere.
2. Connecting to Partial Ring Current:



   a. Partial Ring Current cannot completely encircle the Earth due to Magnetopause
   b. To complete circuit, Field-Aligned Currents connect to the ionosphere
   c. Ionospheric currents complete the circuit.

3. Connecting to Tail (Neutral Sheet) Current:



   a. Also called the substorm current wedge, enables magnetic reconnection in the tail to dynamically connect to the auroral ionosphere.

4. We'll discuss Ionospheric Currents when we cover the ionosphere in a subsequent lecture.

II. Global Picture of Magnetospheric Current Systems