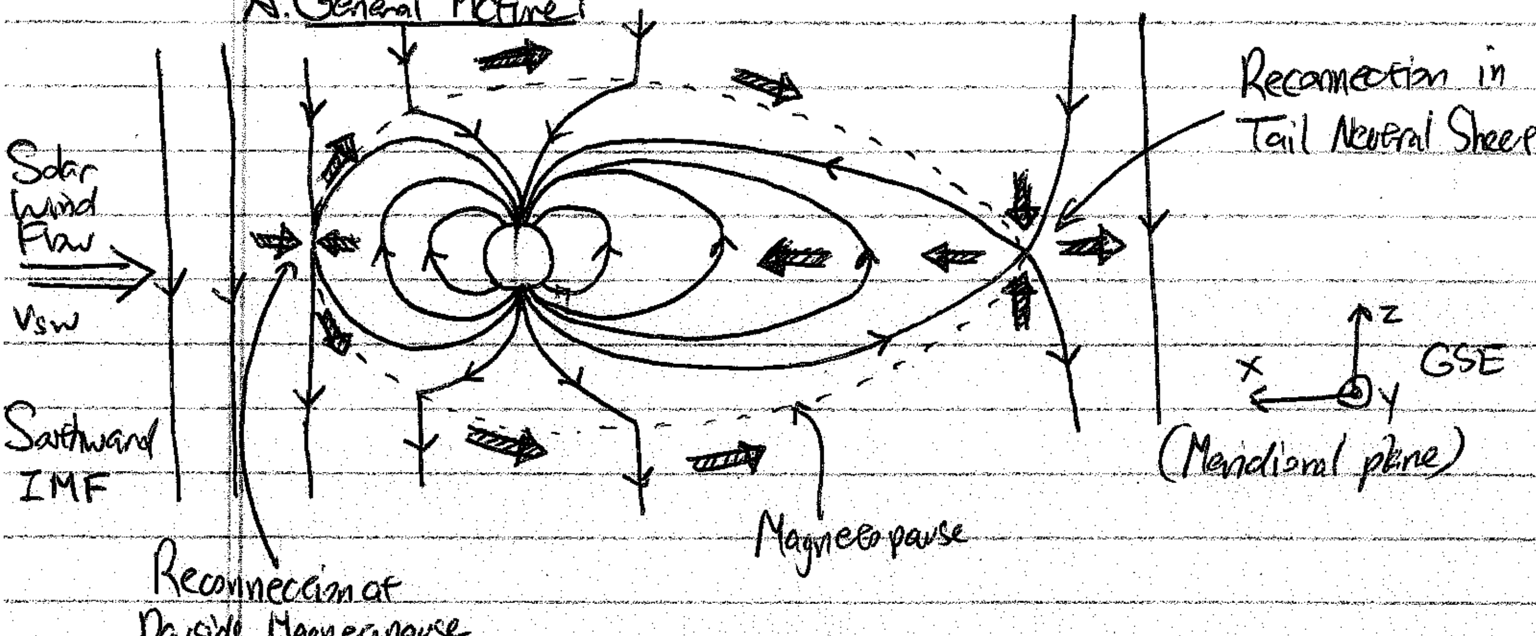
Lecture #12: Magnetospheric Convection, Plasmasphere, and Partial Ring Current

I. Magnetospheric Convection

A. General Picture:



1. During periods of <u>Southward interplanetary magnetic field</u> (IMF), the solar wind flow drives the pattern of convective motions in the magnetosphere sketched above.
2. Although, due to the high conductivity (low resistivity) of the solar wind and magnetosphere plasmas, there is no electric field in the plasma frame, in a fixed (GSE) frame, there arises a <u>convection electric field</u> driving these motions.
3. In this lecture, we hope to the basic properties of the convection.

B. Lorentz Transformation of Electric and Magnetic Fields

1. The transformation from rest frame K to a frame K' moving with respect to K with velocity $\underset{\sim}{V}$,

$$\underset{\sim}{E}' = \gamma(\underset{\sim}{E} + \underset{\sim}{V} \times \underset{\sim}{B}) - \frac{\gamma^2}{\gamma+1}\left(\frac{\underset{\sim}{V}}{c}\cdot\underset{\sim}{E}\right)\frac{\underset{\sim}{V}}{c}$$

where $\gamma = \dfrac{1}{\sqrt{1-\frac{V^2}{c^2}}}$

$$\underset{\sim}{B}' = \gamma\left(\underset{\sim}{B} - \frac{\underset{\sim}{V}}{c^2}\times\underset{\sim}{E}\right) - \frac{\gamma^2}{\gamma+1}\left(\frac{\underset{\sim}{V}}{c}\cdot\underset{\sim}{B}\right)\frac{\underset{\sim}{V}}{c}$$

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I. B. (Continued)

2. Typical solar wind velocity is $\underset{\sim}{V} \sim 500$ km/s, so

$$\frac{V}{c} = \frac{5.0 \times 10^5 \, m/s}{3 \times 10^8 \, m/s} \sim 10^{-3}$$

a. Thus $\gamma = \dfrac{1}{\sqrt{1-\frac{V^2}{c^2}}} \simeq 1$

b. Terms containing $\frac{V}{c}$ may be neglected.

3. Therefore, we find

$$\boxed{\begin{aligned} \underset{\sim}{E}' &= \underset{\sim}{E} + \underset{\sim}{V} \times \underset{\sim}{B} \\ \underset{\sim}{B}' &= \underset{\sim}{B} \end{aligned}}$$

C. Electric Field $\underset{\sim}{E} = 0$ in Plasma Frame

1. Ohm's Law (see Lect #5) $\underset{\sim}{E} + \underset{\sim}{U} \times \underset{\sim}{B} = \eta \underset{\sim}{j}$

a. Note that this equation is valid for any collisionality.

b. In the frame moving with the plasma flow, $\underset{\sim}{U} = 0$

2. In solar wind and magnetospheric plasma, the resistivity is very low due to the low collisionality, $\eta = \dfrac{m_e \nu_{ei}}{e^2 n_0}$

3. Thus $\underset{\sim}{E} + \cancelto{0}{\underset{\sim}{U}} \times \underset{\sim}{B} = \cancelto{0}{\eta} \underset{\sim}{j} \Rightarrow \boxed{\underset{\sim}{E} = 0 \text{ in plasma frame}}$

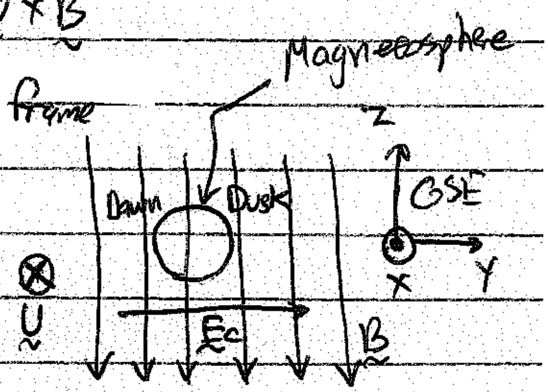
4. But, in the GSE frame, we transform by $-\underset{\sim}{U}$

a. From I.B.3, we find

$$\underbrace{\underset{\sim}{E}'}_{\text{GSE frame}} = \underbrace{\cancelto{0}{\underset{\sim}{E}}}_{\text{plasma frame}} - \underset{\sim}{U} \times \overset{\text{Either frame}}{\underset{\sim}{B}}$$

b. Define: Convection Electric Field, $\underset{\sim}{E}_c$

$$\boxed{\underset{\sim}{E}_c = -\underset{\sim}{U} \times \underset{\sim}{B}}$$



I. (Continued)

D. Potential Is Constant Along $\underset{\sim}{U}$ streamlines & $\underset{\sim}{B}$ field lines

1. In steady state, Faraday's Law $\cancel{\frac{\partial \underset{\sim}{B}}{\partial t}}^{0} = -\nabla \times \underset{\sim}{E}$

a. Since $\nabla \times \underset{\sim}{E} = 0$, we can represent $\underset{\sim}{E}$ using electrostatic potential

Define: Electrostatic Potential, Φ

$$\boxed{\underset{\sim}{E} = -\nabla \Phi}$$

2. $\underset{\sim}{E}$ is perpendicular to both $\underset{\sim}{U}$ & $\underset{\sim}{B}$:

a. From Ohm's Law, $\underset{\sim}{E} = -\underset{\sim}{U} \times \underset{\sim}{B}$

b. $\underset{\sim}{U} \cdot \underset{\sim}{E} = \underset{\sim}{U} \cdot (-\underset{\sim}{U} \times \underset{\sim}{B}) = 0$

c. $\underset{\sim}{B} \cdot \underset{\sim}{E} = \underset{\sim}{B} \cdot (-\underset{\sim}{U} \times \underset{\sim}{B}) = 0$

3. Writing $\underset{\sim}{E} = -\nabla \Phi$

a. $\underset{\sim}{U} \cdot \underset{\sim}{E} = -(\underset{\sim}{U} \cdot \nabla) \Phi = -U \frac{\partial \Phi}{\partial s_U} = 0 \Rightarrow \boxed{\Phi = \text{constant along streamlines}}$

streamline along flow

b. $\underset{\sim}{B} \cdot \underset{\sim}{E} = -(\underset{\sim}{B} \cdot \nabla) \Phi = -B \frac{\partial \Phi}{\partial s} = 0 \Rightarrow \boxed{\Phi = \text{constant along field lines}}$

distance along field

4. In the equatorial plane, the Earth's field is vertical, $\underset{\sim}{B} = B\hat{z}$ (by symmetry, assuming magnetic dipole is aligned with z_{GSE})

a. Therefore, in an equatorial plane, contours of constant potential Φ are the same as the flow streamlines!

E. How does the potential Φ penetrate the magnetosphere to drive convection?

1. If the magnetopause is a tangential discontinuity (see Lect #9), field lines do not penetrate the magnetopause, and the convection potential cannot drive motions within the magnetosphere.

a. For a Northward IMF, this is generally the case, with little convection

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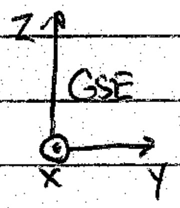
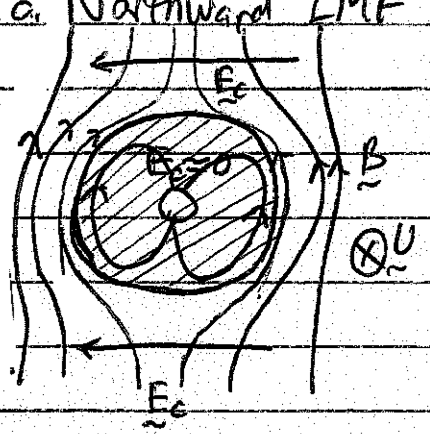
I.E. (Continued)

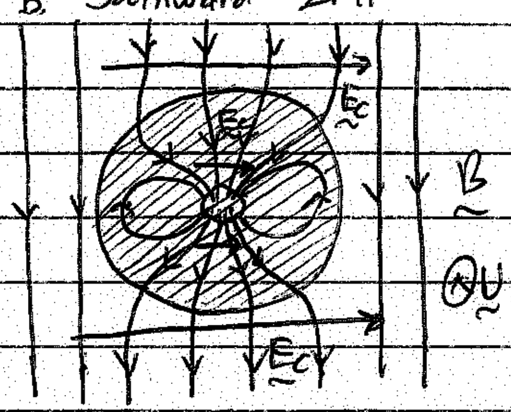
2. But, for the Southward IMF case in I.A., magnetic reconnection leads to field lines that pass through the magnetopause.
   a. Therefore, the solar wind electrostatic potential can enter the magnetosphere, supporting a dawn-to-dusk convection electric field inside



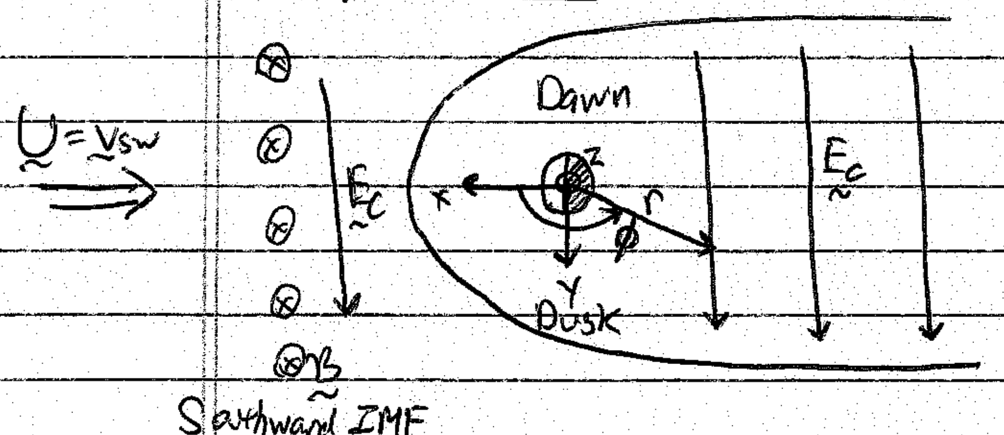
3. a. Northward IMF



b. Southward IMF



4. Equatorial Plane (Southward IMF)



Dawn-to-Dusk Electric Field inside magnetosphere

a. Cylindrical coordinates in equatorial plane (r, ϕ)

5. Convection Electric Field:

a. $\underset{\sim}{E}_c = E_c \hat{y}$ (where $\hat{y} = \sin\phi\, \hat{r} + \cos\phi\, \hat{\phi}$)

b. Setting $\underset{\sim}{E}_c = -\nabla \Phi_c$, one may integrate to obtain

$$\boxed{\Phi_c = -E_c r \sin\phi}$$

I. E. (Continued)

6. <u>Magnitude of E_c</u>: a. In the solar wind, $\underset{\sim}{E}_c = \underset{\sim}{U}\times\underset{\sim}{B} = V_{sw}\hat{\underset{\sim}{x}}\times(-B_{sw}\hat{\underset{\sim}{z}})$ (Southward),

$$= V_{sw}B_{sw}\hat{\underset{\sim}{y}}$$

b. Thus, for $V_{sw} \sim 450$ km/s and $B_{sw} \sim 7$ nT, we obtain

$$E_c = (450\times10^3\ \mathrm{m/s})(7\times10^{-9}\ \mathrm{T}) \simeq 0.003\ \mathrm{V/m} = \boxed{3\ \mathrm{mV/m} = E_{c_{sw}}}$$

c. Within the magnetosphere, measured convection electric fields are

$E_c \sim 0.2-0.5$ mV/m, about a factor of 10 smaller.

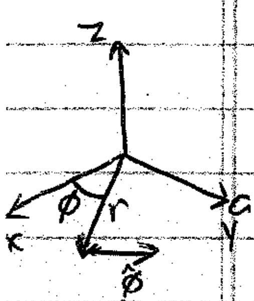
$\Rightarrow$ About 10% of the potential drop penetrates magnetosphere.

F. <u>Corotation Electric Field:</u>

1. The solar wind flow is <u>not</u> the only driver of magnetospheric convection.
   a. The <u>rotation of the Earth</u> also drives convection.
2. The <u>neutral atmosphere corotates</u> with the Earth
3. In the ionosphere, ion-neutral and electron-neutral collisions in the partially ionized plasma enforce <u>corotation of the plasma.</u>
4. In the non-rotating GSE frame, the electric field due to corotation is

$$\underset{\sim}{E}_{CR} = -(\underset{\sim}{\Omega}_E\times\underset{\sim}{r})\times\underset{\sim}{B}$$

where $\underset{\sim}{\Omega}_E = 7.27\times10^{-5}$ rad/s $\hat{z}$ is the Earth's angular velocity in GSE



a. $$\underset{\sim}{E}_{CR} = -Br\Omega_E\underbrace{(\hat{\underset{\sim}{z}}\times\hat{\underset{\sim}{r}})}_{\hat{\phi}}\times\hat{\underset{\sim}{z}} = -Br\Omega_E\hat{\underset{\sim}{r}} = \frac{B_E R_E^3}{r^3}\Omega_E r\,\hat{\underset{\sim}{r}}$$

$$\hat{\phi}\times\hat{\underset{\sim}{z}} = \hat{\underset{\sim}{r}}$$

$$= \frac{-B_E\Omega_E R_E^3}{r^2}\hat{\underset{\sim}{r}}$$

b. $$\underset{\sim}{E}_{CR} = -\nabla\Phi_{CR} = -\frac{\partial\Phi_{CR}}{\partial r}\hat{\underset{\sim}{r}} = \frac{-B_E\Omega_E R_E^3}{r^2}\hat{\underset{\sim}{r}} \Rightarrow \boxed{\Phi_{CR} = \frac{-B_E\Omega_E R_E^3}{r}}$$

G. Azimuthal Drift and the Effective Drift Potential Φ_d

1. Since the $\mathbf{E}\times\mathbf{B}$ drift velocity is represented by $\mathbf{V}_E = \frac{-\nabla\Phi \times \mathbf{B}}{B^2}$,

we can write any drift as due to an effective potential Φ_d,

$$\mathbf{V}_d = \frac{-\nabla\Phi_d \times \mathbf{B}}{B^2}$$

2. Consider, for simplicity, equatorial trapped particles with $\alpha = 90°$.
   a. Therefore, since $v_\parallel = 0$, curvature drift $\mathbf{V}_c$ is zero.
3. Can we write the ∇B drift as an effective drift potential Φ_d?
   a. $\mathbf{V}_{\nabla B} = \frac{-v_\perp^2}{2\Omega} \frac{\nabla B \times \mathbf{B}}{B^2}$
   b. Setting $\mathbf{V}_d = \mathbf{V}_{\nabla B}$ $\Rightarrow$ $-\frac{\nabla\Phi_d \times \mathbf{B}}{\cancel{B^2}} = \frac{-v_\perp^2}{2\Omega}\frac{\nabla B \times \mathbf{B}}{\cancel{B^2}}$
   c. $\nabla\Phi_d \times \mathbf{B} = \frac{1}{q}\underbrace{\left(\frac{m v_\perp^2}{2B}\right)}_{=\mu \text{ (See lecture \#3)}} \nabla B \times B = \frac{\mu}{q}\nabla B \times \mathbf{B}$
   d. But, since the magnetic moment is an adiabatic invariant,

$$\frac{\mu}{q}\nabla B = \nabla\left(\frac{\mu B}{q}\right) \text{, and therefore we find}$$

$$\Phi_d = \frac{\mu B}{q} = \frac{\mu B_E R_E^3}{q r^3} \Rightarrow \boxed{\Phi_d = \frac{\mu B_E R_E^3}{q r^3}}$$

H. Total Effective Potential of Plasma Convection

1. $$\boxed{\Phi_{tot} = \underbrace{-E_c r \sin\phi}_{\text{Convection}} - \underbrace{\frac{\Omega_E B_E R_E^3}{r}}_{\text{Corotation}} + \underbrace{\frac{\mu B_E R_E^3}{q r^3}}_{\text{Azimuthal Drift}}}$$

2. Contours of this potential are the same as the streamlines of the convective flow in the equatorial plane.

I. H. (Continued)

3. Balance of the corotation and azimuthal drift terms depends strongly on the energy $W_\perp = \frac{1}{2} m v_\perp^2$ of plasma particles:

a. $$\frac{\Omega_E B_E R_E^3}{r} \sim \frac{\mu}{q} \underbrace{\left(\frac{B_E R_E^3}{r^3}\right)}_{=B}$$

b. $$\frac{\Omega_E B_E R_E^{\cancel{3}2}}{L \cancel{R_E}} \sim \frac{W_\perp \cancel{B}}{q \cancel{B}} \Rightarrow W_\perp \sim \frac{q \Omega_E B_E R_E^2}{L}$$

c. $$W_\perp \sim \frac{(1.6\times10^{-19} C)(7.27\times10^{-5} \frac{rad}{s})(30.4\times10^{-6} T)(6.4\times10^{6} m)^2}{L}\left(\frac{1 eV}{1.6\times10^{-19} J}\right) \sim \frac{90 keV}{L}$$

d. Therefore: a. Cold plasma $W_\perp \ll 10$ keV $\Rightarrow$ Corotation dominates
$\Rightarrow$ Plasmasphere

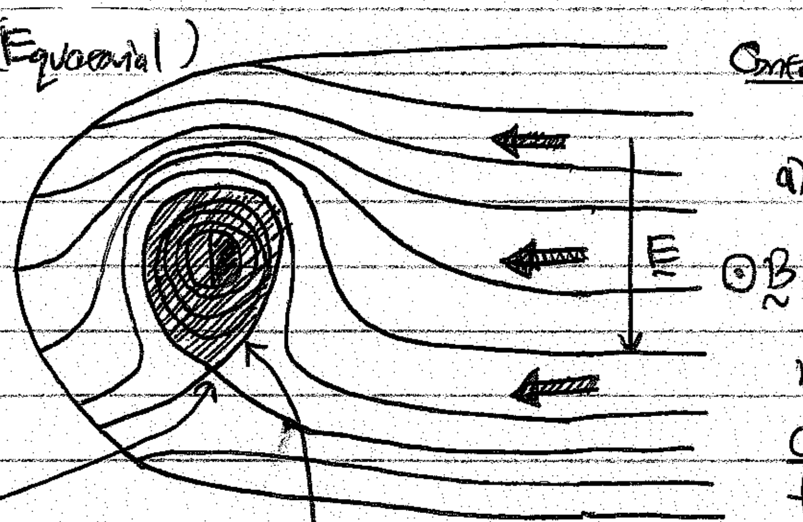
b. Hot plasma $W_\perp \gtrsim 10$ keV $\Rightarrow$ Azimuthal drift dominates.
$\Rightarrow$ Ring Current

J. Plasmasphere

1. For low energy particles with $W \lesssim 1$ keV, the corotation term dominates over the azimuthal drift term, leaving

$$\Phi_{tot} = -E_c r \sin\phi - \frac{\Omega_E B_E R_E^3}{r}$$

2. (Equatorial)



Stagnation Point ($E=0$)

Plasmapause encloses the Plasmasphere

Contours of Potential Φ_{tot}:

a) Convection flow is along equipotential contours

b) In shaded regions, closed trajectories circle the Earth

c) Elsewhere, open trajectories flow toward dayside magnetopause.

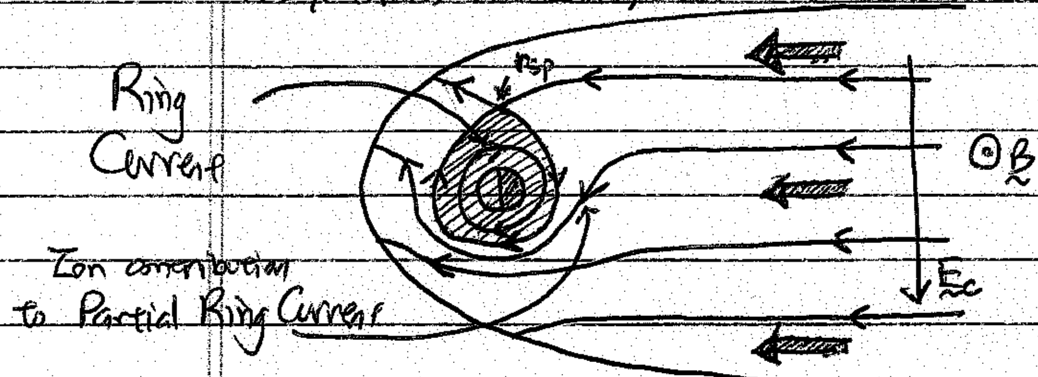
Lecture #12 (Continued)

I. Plasmasphere (Continued)

3. Radius of Stagnation Point: a. Stagnation point where $\mathbf{E} = \nabla \Phi_{tot} = 0$
   is found to be
   $$r_{sp} = \sqrt{\frac{\Omega_E B_E R_E^3}{E_c}} \simeq 3.8 R_E \quad \text{for } E_c \simeq 1 \text{ mV/m}$$
   b. The separatrix containing closed trajectories encompasses the plasmasphere, and is denoted the plasmapause.
4. a. The plasmasphere contains a dense, cold plasma of ionospheric origin, with $n \sim 10^3 \text{ cm}^{-3}$ and $T \sim 1 \text{ eV}$
   b. Outside the plasmapause, plasma is convected from the plasma sheet and has much lower densities $n \lesssim 1 \text{ cm}^{-3}$ and $T \gtrsim 1 \text{ keV}$.

J. Ring Current and Partial Ring Current

1. For hot plasma with $W \gtrsim 10$ keV, the azimuthal drift term dominates over the corotation term, $\Phi_{tot} = -E_c r \sin\phi + \frac{\mu B_E R_E^3}{q r^3}$
2. (Equatorial) For Ions,



3. Setting $\mathbf{E} = -\nabla \Phi_{tot} = 0$, one finds $E_c = \frac{3\mu}{q} \frac{B_E R_E^3}{r_{sp}^4}$
   a. $$r_{sp} = \frac{3 W_\perp}{q E_c} \simeq 4.6 R_E \quad \text{for } W_\perp = 10 \text{ keV and } E_c = 1 \text{ mV/m}$$
4. a. Note that as E_c increases (strength of convection increases), stagnation point moves closer to Earth, allowing plasma sheet particles to penetrate more closely $\Rightarrow$ Storm injection to ring current!
   b. Outside the separatrix, one cannot complete the ring current loop and one obtains the Partial Ring Current.