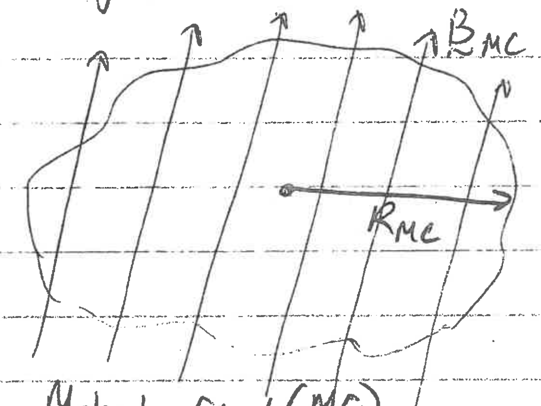
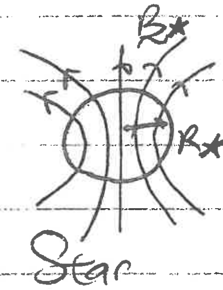
Lecture #25 Magnetized Molecular Clouds and Ambipolar Diffusion

I. The Physics of Magnetized Molecular Clouds

A. The Magnetic Flux Problem

1.





a. Magnetic Flux $\Phi_B \equiv \int_S \underset{\sim}{B} \cdot d\underset{\sim}{A}$ (See Lec #5) $\simeq B\pi R^2$

b. $\Phi_{B_{MC}} >> \Phi_{B\star}$ Magnetic flux threading molecular clouds is much larger than the magnetic flux threading stars.

$\Rightarrow$ $\boxed{\text{Magnetic flux must be expelled from plasma during star formation}}$

c. This necessarily requires some non-ideal effect to break the frozen-in flux condition (Lec #5).

d. Ambipolar diffusion is one possible mechanism to do this.

B. Equilibrium for a self-gravitating, magnetized plasma

Momentum Eq:

1. $$\rho \frac{\partial \underset{\sim}{U}}{\partial t} + \rho \underset{\sim}{U} \cdot \nabla \underset{\sim}{U} = -\nabla p - \nabla \frac{B^2}{8\pi} + \frac{\underset{\sim}{B} \cdot \nabla \underset{\sim}{B}}{4\pi} - \rho \nabla \Phi_G$$

where $\nabla^2 \Phi_G = 4\pi G \rho$

2. In steady state, we take $\frac{\partial}{\partial t} = 0$

3. We also separate rotational motion $\underset{\sim}{U}_R$ from turbulent motion $\underset{\sim}{U}_T$

$\Rightarrow \underset{\sim}{U} = \underset{\sim}{U}_R + \underset{\sim}{U}_T$

I. B. (Continued)

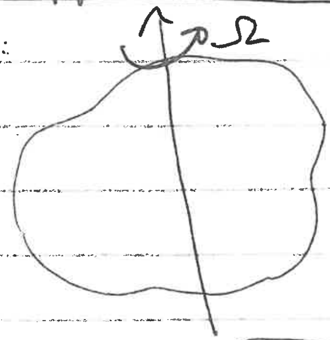
4. Therefore, forces that can balance gravity to prevent collapse are:

a. $$\rho \nabla \Phi_G = \underbrace{-\rho \underset{\sim}{U}_R \cdot \nabla \underset{\sim}{U}_R}_{\text{Centrifugal support}} \underbrace{- \rho \underset{\sim}{U}_t \cdot \nabla \underset{\sim}{U}_t}_{\text{turbulent support}} \underbrace{- \nabla p}_{\text{thermal pressure}} \underbrace{- \nabla \frac{B^2}{8\pi}}_{\text{magnetic pressure}} + \underbrace{\frac{\underset{\sim}{B} \cdot \nabla \underset{\sim}{B}}{4\pi}}_{\text{magnetic tension}}$$

Leading candidates for support of molecular clouds against self-gravitational collapse

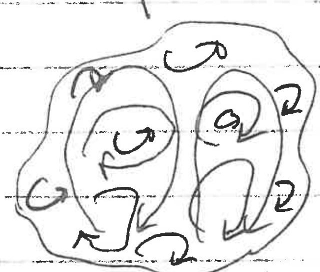
5. Possible support mechanisms:

a. Rotation:



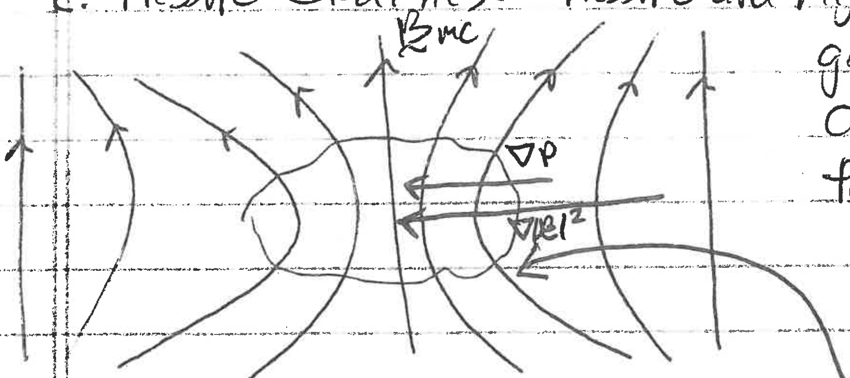
The centrifugal force can support against gravity, but rotation rates are much too low for this to play a role.

b. Turbulence:



Turbulent motions at small scales provide an effective "turbulent pressure" that can inhibit collapse.

c. Pressure Gradients: Pressure and magnetic field magnitude generally decrease with radius, creating pressure gradients that oppose gravitational infall.



d. Magnetic Tension: Bending of magnetic field lines can prevent collapse.

2. (Continued)

C. <u>Mass to Magnetic Flux Ratio</u>, $\frac{M}{\Phi_B}$

1. Key parameter that measures the ability of a static magnetic field to support a cloud against collapse.
2. We can estimate a critical mass-to-flux ratio $(M/\Phi_B)_c$ by balancing magnetic pressure and gravity:

a. $\nabla \frac{B^2}{8\pi} \sim \rho \nabla \Phi_G$

b. Estimating $\Phi_G \simeq \frac{G M_{enc}}{r}$ and $\nabla \sim \frac{1}{r}$, $\frac{B^2}{8\pi r} \sim \frac{G M_{enc}}{r^2} \rho$

c. Using $M = \frac{4\pi}{3} \rho r^3 \Rightarrow \rho = \frac{3 M_{enc}}{4\pi r^3}$, so $\frac{B^2}{8\pi r} \sim \frac{3 G M_{enc}^2}{4\pi r^5}$

d. Thus $\frac{M_{enc}^2}{(\pi r^2 B)^2} \sim \frac{1}{6\pi^2 G}$ where $\Phi_B = \pi r^2 B$

e. Define Critical Mass-to-flux Ratio $\left(\frac{M}{\Phi_B}\right)_c \simeq \frac{1}{\sqrt{6\pi G}}$

3. More detailed analyses give $\left(\frac{M}{\Phi_B}\right)_c = \frac{C_\Phi}{\sqrt{G}}$ with $C_\Phi \simeq 0.12$,

yielding $\left(\frac{M}{\Phi_B}\right)_c \simeq 10^{-20} \frac{N(H_2)}{|B|} \frac{g}{\mu G}$ where $N(H_2)$ is the column density in $\frac{g}{cm^2}$

4. The critical mass may be expressed,

$$M_c \sim 10^3 \left(\frac{B}{30\mu G}\right)\left(\frac{r}{2pc}\right)^2 M_\odot ,$$

So it takes a very large mass to trigger gravitational collapse.

D. <u>Observations of Magnetic Fields in Molecular Clouds</u>

<u>Ref:</u> R.M. Crutcher, <u>Magnetic Fields in Molecular Clouds: Observations Confront Theory</u>, ApJ, 520: 706 (1999).

1. Internal motions are supersonic ($M_s \sim 5$), but typically Alfvénic ($M_A \sim 1$), so <u>turbulence</u> is likely to be MHD waves (Lec#29)

I, D. Observations (Continued)

2. Plasma Beta, $\beta_p \simeq 0.04$, so magnetic fields dominate over thermal pressure

3\. a. $\frac{M}{\Phi_B}$ is about twice the critical value (supercritical → unstable).

Therefore, static magnetic field alone is insufficient to support clouds against collapse. Turbulence is likely to provide half of support.

b. No observation of magnetically subcritical (stable) clouds.

4. Kinetic energy (of turbulence) and magnetic energy are about equal, so magnetic pressure and turbulence provide support roughly equally

5. For $n_n \gtrsim 10^3 - 10^4 \text{ cm}^{-3}$, $|B| \propto \rho_n^{\kappa}$, with $\kappa \simeq 0.47$.

⇒ Therefore, magnetic fields provide an important (and possibly dominant) support of molecular clouds, where the remainder of support is due to Alfvénic turbulence.

E. Simplified Model of Magnetic Cloud Collapse

1. Magnetically supercritical clouds (unstable) will collapse on short timescales (compression during collapse will not stop collapse) and will likely form high mass stars.
2. Subcritical clouds (stable) will become unstable in the core due to ambipolar diffusion (slip of neutrals with respect to magnetic field in a partially ionized plasma). This allows an increase of core mass without an increase of magnetic flux.
3. Core eventually becomes supercritical, and core undergoes collapse, leaving the surrounding envelope in place.
4. Magnetic field lines can efficiently transfer angular momentum from collapsing core to surrounding envelope (magnetic braking)

II. Ambipolar Diffusion

A. Partially Ionized Plasma

1. Cosmic Ray Ionization: Cosmic rays support a mean rate of ionization in molecular clouds, $\gamma_i \sim 10^{-17}\ s^{-1}$.
2. Recombination Rate: For gas phase ion-electron recombination, the rate is proportional to $n_e n_i$, $\gamma_r \propto n_e n_i \propto n_i^2$
3. Setting rates equal:
$$\gamma_r \propto \gamma_i n_n \Rightarrow n_i^2 \propto n_n \rightarrow n_i \propto n_n^{\frac{1}{2}}$$
4. Therefore, $\boxed{\rho_i = C \rho_n^{\frac{1}{2}} \quad \text{where } C = 3\times10^{-16}\ cm^{-3/2} g^{\frac{1}{2}}}$
5. a. For neutral density $n_n \sim 10^4\ cm^{-3}$ (typical for MC core), this predicts fractional ionization $\boxed{f \sim \frac{n_i}{n_n} \sim 10^{-7}}$
   b. Agrees with observational limits $10^{-6} \lesssim f \lesssim 10^{-8}$ for MC cores.
6. The fractional ionization is very small, however, we will discover that nevertheless the neutrals are still coupled to the magnetic field through collisions with ions.

B. Two Fluid Model of Ions and Neutrals in Partially Ionized Plasma

1. Continuity: $\frac{\partial \rho_i}{\partial t} + \nabla\cdot(\rho_i \underset{\sim}{U_i}) = 0 \qquad \frac{\partial \rho_n}{\partial t} + \nabla\cdot(\rho_n \underset{\sim}{U_n}) = 0$
2. Momentum:
$$\rho_i \frac{\partial \underset{\sim}{U_i}}{\partial t} + \rho_i \underset{\sim}{U_i}\cdot\nabla\underset{\sim}{U_i} = -\nabla p_i - \rho_i\nabla\Phi_G \underbrace{-\nabla\frac{B^2}{8\pi} + \frac{\underset{\sim}{B}\cdot\nabla\underset{\sim}{B}}{4\pi}}_{\underset{\sim}{f_L}} \underbrace{-\rho_i \nu_{in}(\underset{\sim}{U_i}-\underset{\sim}{U_n})}_{\underset{\sim}{f_d}}$$
$$\rho_n \frac{\partial \underset{\sim}{U_n}}{\partial t} + \rho_n \underset{\sim}{U_n}\cdot\nabla\underset{\sim}{U_n} = -\nabla p_n - \rho_n\nabla\Phi_G - \rho_n \nu_{ni}(\underset{\sim}{U_n}-\underset{\sim}{U_i})$$
3. Induction: $\frac{\partial \underset{\sim}{B}}{\partial t} = \nabla\times(\underset{\sim}{U_i}\times\underset{\sim}{B})$
4. Self-Gravity: $\nabla^2\Phi_G = 4\pi G\rho$
5. Eq of State: (Complicated).

II. B. (Continued)

5. For ions, Lorentz force $\underset{\sim}{f}_L = \frac{1}{4\pi}(\nabla\times\underset{\sim}{B})\times\underset{\sim}{B} = -\nabla\frac{B^2}{8\pi} + \frac{\underset{\sim}{B}\cdot\nabla\underset{\sim}{B}}{4\pi}$

6. Frictional Drag Force $\underset{\sim}{f}_d$ is due to collisions between ions & neutrals.

a. $$\rho_i \nu_{in} = \rho_n \nu_{ni} = \frac{\rho_i \rho_n}{m_i + m_n}\langle\sigma v\rangle$$

where $\langle\sigma v\rangle$ is rate coefficient for ion-neutral collisions

b. For p–H collisions at $10K \le T \le 1000K$,

$$\langle\sigma v\rangle \simeq (3.3 - 13.6)\times 10^{-10}\ \text{cm}^3\ \text{s}^{-1}$$

C. Ion Momentum Equation Balance:

1. In steady state, $\frac{\partial}{\partial t} = 0$
2. Weak Rotation from observations of MCs, so drop $\rho_i \underset{\sim}{U}_i\cdot\nabla\underset{\sim}{U}_i$ term. (and we will neglect turbulence here)
3. For low β conditions, $\beta \ll 1$, thermal pressure is small compared to Lorentz force $\underset{\sim}{f}_L$.
4. Thus, we are left with

$$0 = \underbrace{-\rho_i\nabla\Phi_G}_{\text{gravity}} + \underbrace{\frac{(\nabla\times\underset{\sim}{B})\times\underset{\sim}{B}}{4\pi}}_{\text{Lorentz force, } \underset{\sim}{f}_L} \underbrace{- \rho_i\nu_{in}(\underset{\sim}{U}_i - \underset{\sim}{U}_n)}_{\text{ion-neutral frictional drag, } \underset{\sim}{f}_d}$$

5. Estimate timescale of frictional drag:

a. $\rho_i\frac{\partial \underset{\sim}{U}_i}{\partial t} \sim \rho_i\nu_{in}(\underset{\sim}{U}_i - \underset{\sim}{U}_n) \Rightarrow \frac{\rho_i U_i}{\tau_{fric}} \sim \cancel{\rho_i U_i}\nu_{in} \rightarrow \tau_{fric} \sim \frac{1}{\nu_{in}}$

b. $\tau_{fric} \sim \frac{1}{\nu_{in}} \sim \frac{m_i + m_n}{\rho_n\langle\sigma v\rangle} \sim \frac{1 + \frac{m_i}{m_n}}{n_n\langle\sigma v\rangle}$

c. $\boxed{\tau_{fric} \sim 10^3\left(\frac{n_n}{10^6\,\text{cm}^{-3}}\right)^{-1}\text{s}}$ for $\frac{m_i}{m_n} \sim 1$ and $\langle\sigma v\rangle \sim 6\times10^{-10}\,\text{cm}^3\,\text{s}^{-1}$

Lecture #25 (Continued)

II. C. (Continued)

6. Estimate dynamical time scale of gravitational collapse:

a. $\rho_i \frac{\partial U_i}{\partial t} \sim \rho_i \nabla \Phi_G \Rightarrow \frac{\rho_i U_i}{\tau_{dyn}} \sim \rho_i \frac{G M_{enc}}{r^2}$

b. $M_{enc} \simeq \frac{4\pi}{3}\rho_n r^3$ $\quad \frac{\cancel{\rho_i} U_i}{\tau_{dyn}} \sim \frac{\cancel{\rho_i} G \frac{4\pi}{3}\rho_n r^{\cancel{3}}}{\cancel{r^2}} \Rightarrow \tau_{dyn} \sim \frac{U_i}{r}\frac{3}{4\pi G\rho}$

c. Note: $\frac{U_i}{r} \sim \frac{1}{\tau_{dyn}}$, so $\tau_{dyn} \sim \frac{1}{\left(\frac{4\pi}{3}G\rho\right)^{\frac{1}{2}}}$

d. $\boxed{\tau_{dyn} \sim 10^{12}\left(\frac{n_n}{10^6\,cm^{-3}}\right)^{-1} s}$

7. Because $\tau_{fric} \ll \tau_{dyn}$, to lowest order we can neglect gravity compared to $\underset{\sim}{f}_d$, and we obtain

$\boxed{\underset{\sim}{f}_L = \underset{\sim}{f}_d}$ $\Rightarrow$ Lorentz force balances frictional drag to lowest order.

8. Thus, we can solve for the ambipolar drift $\underset{\sim}{U}_d$,

$$\boxed{\underset{\sim}{U}_d \equiv \underset{\sim}{U}_i - \underset{\sim}{U}_n = \frac{(\nabla \times \underset{\sim}{B}) \times \underset{\sim}{B}}{4\pi \rho_i \nu_{in}}}$$

9. Typical Values:

a. $U_d \sim 10^4 \left(\frac{n_n}{10^6\,cm^{-3}}\right)^{-2}\left(\frac{f}{10^{-7}}\right)^{-1}\left(\frac{B}{10^{-3}G}\right)^{2}\left(\frac{L}{0.3pc}\right)^{-1} \frac{cm}{s}$

b. $\boxed{\tau_D \sim \frac{L}{U_d} \sim 3\times 10^6 \left(\frac{f}{10^{-7}}\right) \text{ years}}$

Timescale of Ambipolar diffusion

c. For gravitational collapse of $n_n \sim 10^6\,cm^{-3}$, $\boxed{\tau_{Dyn} \sim 10^5 \text{ years}}$

Thus, ambipolar diffusion is a relatively slow process.

Lecture #25 (Continued)

II. D. Ambipolar Diffusion

1. We want to solve for the dynamics of a partially ionized plasma.
   a. Construct single fluid theory → what change does small fractional ionization create?
2. Solve for $\underset{\sim}{U}_i$ and substitute into Induction Equation.
   a. $\underset{\sim}{U}_i = \underset{\sim}{U}_n + \frac{(\nabla\times \underset{\sim}{B})\times \underset{\sim}{B}}{4\pi\rho_i \nu_{in}}$
   b. $\frac{\partial \underset{\sim}{B}}{\partial t} = \nabla\times(\underset{\sim}{U}_n\times \underset{\sim}{B}) + \underbrace{\nabla\times\left[\frac{(\nabla\times \underset{\sim}{B})\times \underset{\sim}{B}}{4\pi\rho_i\nu_{in}}\times \underset{\sim}{B}\right]}_{\text{Diffusion term (two derivatives)}}$
3. a. Consider a mass density weighted flow velocity: $\underset{\sim}{U} = \frac{\rho_i \underset{\sim}{U}_i + \rho_n \underset{\sim}{U}_n}{\rho_i + \rho_n} \simeq \underset{\sim}{U}_n + \underbrace{\frac{\rho_i}{\rho_n}}_{\text{very small}}\underset{\sim}{U}_i \simeq \underset{\sim}{U}_n$
   b. $\rho = \rho_i + \rho_n \simeq \rho_n$
4. a. Discard ion density equation
   b. Ion momentum equation yields $\underset{\sim}{f}_L = \underset{\sim}{f}_d$, substitute into neutral momentum.
5. Single Fluid Equations:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\underset{\sim}{U}) = 0$$

$$\rho\frac{\partial \underset{\sim}{U}}{\partial t} + \rho\underset{\sim}{U}\cdot\nabla\underset{\sim}{U} = -\nabla p - \rho\nabla\Phi_G \overbrace{- \nabla\frac{B^2}{8\pi} + \frac{\underset{\sim}{B}\cdot\nabla\underset{\sim}{B}}{4\pi}}^{\text{Lorentz force arises from substituting } \underset{\sim}{f}_d = \underset{\sim}{f}_L}$$

$$\frac{\partial \underset{\sim}{B}}{\partial t} = \nabla\times(\underset{\sim}{U}\times\underset{\sim}{B}) + \underline{\nabla\times\left[\frac{(\nabla\times B)\times \underset{\sim}{B}}{4\pi\rho\,\nu_{ni}}\times\underset{\sim}{B}\right]}$$

only new term!

$$\nabla^2\Phi_G = 4\pi G\rho$$

$$\frac{\partial p}{\partial t} + (\underset{\sim}{U}\cdot\nabla)p = -\gamma p\nabla\cdot\underset{\sim}{U}$$

a. NOTE, the only real change is a diffusive term in the induction equation, allowing the mass density to slip with respect to $\underset{\sim}{B}$ → Not frozen-in.

b. Form is $\frac{\partial \underset{\sim}{B}}{\partial t} \sim D_{AD}\nabla^2\underset{\sim}{B}$, where $D_{AD} \sim \frac{V_A^2}{\nu_{ni}}$ is diffusion coefficient.