Lecture #27 Blast Waves and Supernova Remnants

I. Taylor-Sedov Blast Wave Solution

A. History

1. Atomic weapons research after WWII saw the development of the theory of the release of a tremendous amount of energy at a single point in an ambient, uniform medium — the blast wave
2. Taylor in the West, and Sedov in the East, independently this theory of blast waves.
3. Later, Shklovskii applied the theory to describe the early stages of evolution of a supernova remnant.

B. G. I. Taylor's Similarity Analysis

1. Using his formidable intuition (inspired guesses), Taylor surmised the blast wave solution for nuclear explosions could depend on the five physical quantities:

		Dimensions
(1) Radius of spherical wave front	R	L
(2) Time	t	T
(3) Ambient pressure	p_0	M/LT^2
(4) Ambient density	ρ_0	M/L^3
(5) Energy released	E	$M\frac{L^2}{T^2}$

2. Number of physical quantities: $n=5$
Number of primary quantities: $k=3$ (L, M, T)
Number of nondimensional parameters $m=n-k = 2$

3. Dimensionless parameters:

a. Energy E: $\frac{t^2 E}{\rho_0 R^5} = \left(\frac{\cancel{M}\cancel{L^2}}{\cancel{T^2}}\right)\left(\frac{\cancel{L^3}}{\cancel{M}}\right)\left(\frac{1}{\cancel{L^5}}\right)\left(\cancel{T^2}\right) = \#$

b. Pressure p_0: $\frac{R^3 p_0}{E} = \left(\frac{\cancel{M}}{\cancel{L}\cancel{T^2}}\right)\left(\frac{\cancel{T^2}}{\cancel{M}\cancel{L^2}}\right)\left(\cancel{L^3}\right) = \#$

I. B3. (Continued)

c. Therefore the solution possibly depends on $\pi_1 = \frac{\rho_0 R^5}{E t^2}$, $\pi_2 = \frac{p_0 R^3}{E}$

4. Assuming the existence of a similarity solution, he asserted that a function relationship exists between π_1 and π_2,

$$\pi_1 = f(\pi_2)$$

where the form of f must be determined empirically.

5. Scale Analysis: a. Taylor estimated some first guesses for the value of each dimensional physical quantity:

R	10^2 m	
t	10^{-2} s	
p_0	10^5 Pa	(ambient atmospheric pressure)
ρ_0	1 kg/m^3	(ambient atmospheric density)
E	10^{14} J	(guess)

NOTE: 1 megaton TNT = 4×10^{15} J
10^3 kg TNT = 1 ton TNT

b. Therefore i) $\mathcal{O}\left(\frac{\rho_0 R^5}{E t^2}\right) = \frac{(1 \frac{kg}{m^3})(10^2 m)^5}{(10^{14} J)(10^{-2} s)^2} = \frac{10^{10}}{10^{10}} = 1$

ii) $\mathcal{O}\left(\frac{p_0 R^3}{E}\right) = \frac{(10^5 Pa)(10^2 m)^3}{(10^{14} J)} = \frac{10^{11}}{10^{14}} = 10^{-3}$

c. Since $\mathcal{O}\left(\frac{p_0 R^3}{E}\right) \ll \mathcal{O}\left(\frac{\rho_0 R^5}{E t^2}\right)$, Taylor concluded this dimensionless parameter was physically irrelevant

d. Physically, ambient atmospheric pressure is much smaller than the internal pressure of the nuclear fireball, so the solution doesn't depend on atmospheric pressure $\Rightarrow$ This is essentially a requirement for the blast wave solution to apply.

6. We are left with $\pi_1 = \frac{\rho_0 R^5}{E t^2} = A =$ constant, where A is expected to be $\mathcal{O}(1)$.

Lecture #27 (Continued)

7. B. (Continued)

7. a. The similarity solution by Taylor has the form

$$R^5 = \underbrace{\left(\frac{AE}{\rho_0}\right)}_{\text{Independent of time}} t^2$$

b. Taylor was able to confirm that R^5 increased as t^2 using published (unclassified) photos of the nuclear explosion.

c. Then assuming $A = 1$, Taylor used the formula

$$E = \frac{R^5 \rho_0}{t^2 A}$$ to estimate the yield of the first atomic explosion at about **20,000** tons of TNT.

Ref: Taylor, G.I., 1950 The Formation of a blast wave by a very intense explosion II; The atomic explosion of 1945, Proc Roy Soc A, 201 175-186.

d. Taylor's estimate was accurate, embarrassing the government, which had not declassified the energy released in the explosion.

C. Taylor-Sedov Solution for the Interior of a Blast Wave

1. a. Taking $R = \left(\frac{AE}{\rho_0}\right)^{\frac{1}{5}} t^{2/5}$, one may find $U_{sh} = \frac{dR}{dt}$

b. $U_{sh} = \frac{2}{5}\left(\frac{AE}{\rho_0 t^3}\right)^{\frac{1}{5}}$, so $U_{sh} \propto t^{-\frac{3}{5}}$ (also $U_{sh} \propto R^{-\frac{3}{2}}$)

2. One can use this estimate of the shock front velocity U_{sh}, along with the Rankine-Hugoniot jump conditions for strong shocks, to write the hydrodynamic equations for mass, momentum, and energy conservation (in spherical symmetry) in terms of the similarity dimensionless variable $\boxed{\xi = R\left(\frac{\rho_0}{Et^2}\right)^{\frac{1}{5}}}$

See Lect 8 for Rankine-Hugoniot Jump Conditions

I.C. (Continued)

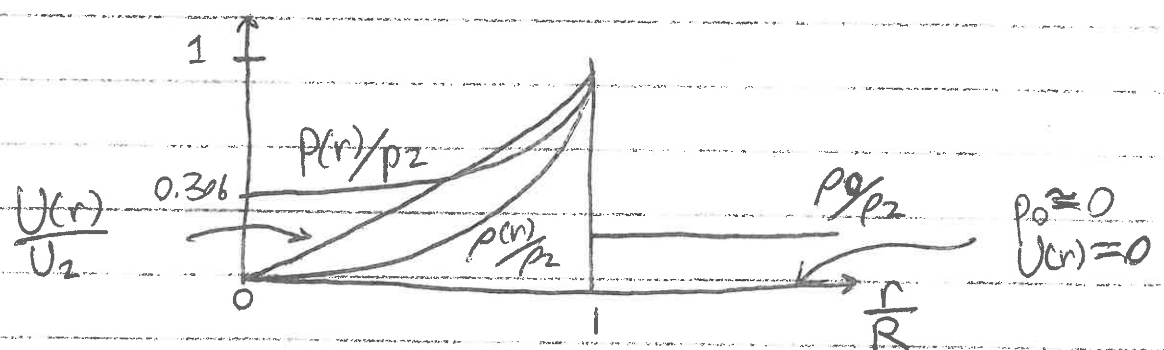
3. This yields a closed set of ordinary differential equations in ξ for the density $\rho(\xi)$, radial velocity $U(\xi)$, and pressure $p(\xi)$.

4. The set of ODE's needs to be integrated from 0 to ξ_0, subject to a normalization for the total energy E.

5. Taylor carried out one of the first numerical integrations on a modern electronic computer, obtaining the complete solution to the blast wave problem.

6. Sedov, without access to a powerful computer in the Soviet Union, found an analytical solution to the equations in closed form.

7. Blast Wave Solution



a. Values p_2, U_2, and ρ_2 are the values immediately inside the shock (determined by Rankine-Hugoniot jump conditions)

8. Validity of Blast Wave Solution:
   a. Valid for all times until: i) Total Radiated energy begins to approach E,
   or ii) Ram pressure at shock $\rho_0 U_{sh}^2$ ceases to be much greater than p_0.
   b. This is the energy-conserving phase of the blast wave.
   c. Also, valid only after influence of initial conditions is minimal

II. Application to Supernova Remnants:

A. Order of Magnitude Estimates for Supernovae

1. Energy: Estimate energy released to be equivalent to energy needed to eject 1 $M_\odot$ at 10^4 km/s

$\Rightarrow$ $E \sim 10^{51}$ ergs (Compared to 10^{21} ergs for atomic bomb).

2. Density: $n \sim 1$ cm^{-3} $\Rightarrow$ $\rho_0 \sim 2 \times 10^{-24} \frac{g}{cm^2}$

3. For $U_{sh} = \frac{2}{5}\left(\frac{E}{\rho_0 t^3}\right)^{\frac{1}{5}}$ and $R = \left(\frac{Et^2}{\rho_0}\right)^{1/5}$, we obtain

t (yr)	1	10	100	1,000	10,000	100,000
R (pc)	0.315	0.791	1.99	4.99	12.5	31.5
U_{sh} (km/s)	124,000	31,000	7,820	1,970	494	124

$\longleftrightarrow$ Validity of energy-conserving blast wave

4. Self-similar solution is not valid at initial times $t \lesssim 100$ y
   a. Shock wave cannot move much faster than original ejecta
   b. Swept up mass in shock $m = \rho_0 \left(\frac{4\pi}{3}\right) R^3$ equals the mass of the original ejecta 1 $M_\odot$, at $R \simeq 2$ pc, or $t = 100$ y.
   c. At this point, the blast enters the self-similar stage of evolution.

5. Self-similar solution breaks down at long times $t \gtrsim 10^5$ y.
   a. Shocked material has temperature $T_2 \gtrsim 10^6$ K, and accordingly should radiate strongly in x-rays.
   b. At $t = 10^4$ y, $T_2 \sim 3 \times 10^6$ K and $R \simeq 10$ pc. Direct observations of supernova remnants of size 10 pc verifies strong x-ray emission.
   c. For $t \gtrsim 10^5$ y, total radiated energy approaches E, and solution breaks down.

Lecture #27 (Continued)

VI. (Continued)

B. Properties of Supernova Remnants

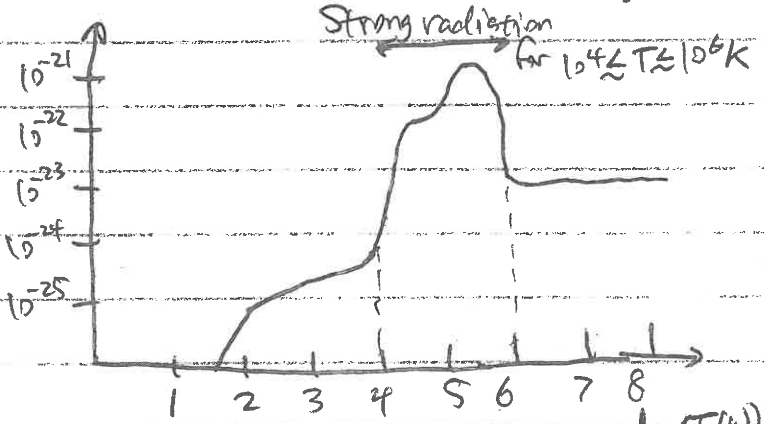
1. Temperature profile: a. Ideal gas law $pV = nT \Rightarrow p = \rho T$.
   b. $T = \frac{p}{\rho}$, so looking a numerical solution for interior of blast wave, the temperature is at its highest value at $r=0$, and reaches a minimum at shock edge.
   c. This occurs because material near the center passed through the shock front earlier, when the shock velocity U_{sh} was higher.
   d. Rankine-Hugoniot conditions for a strong shock give $T_2 \propto \frac{p_2}{\rho_2} \propto U_{sh}^2$, so $T_2 \propto t^{-6/5}$

2. Radiative Cooling Function:

Cooling Function $\Lambda(x, T)$ (erg $\frac{cm^3}{s}$)

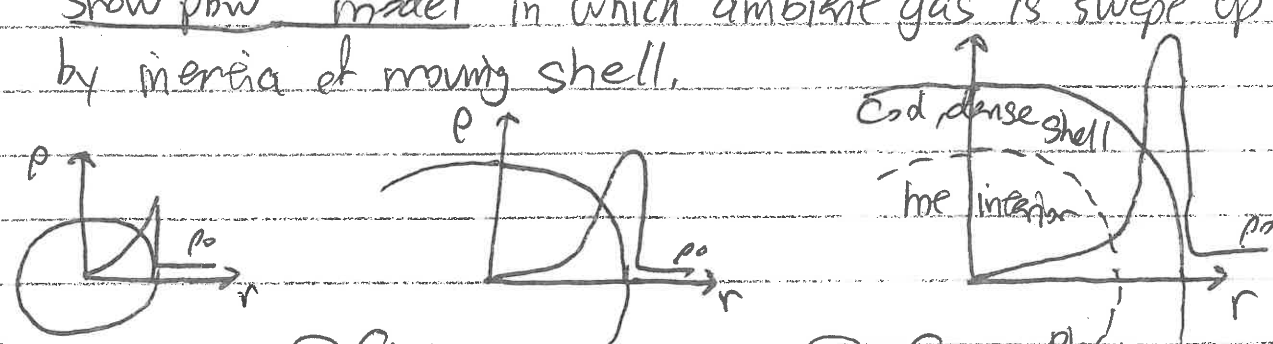
x = electron fraction

T = gas temperature



3. When T_2 drops to 10^6 K (around 10^4 y), radiative losses inside shock become severe.
   $\Rightarrow$ Supernova remnant enters phase of dense shell formation.

4. After this, supernova no longer conserves energy (due to radiative losses), but must conserve momentum. Thus, one obtains a "Snow plow" model in which ambient gas is swept up by inertia of moving shell.



① Blast Wave (energy conserving)

② Shell Formation (radiative losses $\lesssim E$)

③ Snow Plow (momentum conserving)