

29:194 Homework #3

Due at the beginning of class, Tuesday, September 21, 2010.

1. Show that the curvature drift

$$\mathbf{V}_c = \frac{v_{\parallel}^2}{\omega_c B} \frac{\mathbf{R}_c \times \mathbf{B}}{R_c^2}$$

can be written as

$$\mathbf{V}_c = \frac{v_{\parallel}^2}{\omega_c} \hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \nabla) \hat{\mathbf{b}}$$

You may take the magnetic field to be purely azimuthal $\mathbf{B} = B \hat{\phi}$.

HINT: Convert from cylindrical to Cartesian coordinates.

2. A 20 keV deuteron in a large mirror fusion device has a pitch angle of 45° at the midplane of the machine, where the magnetic field $B = 0.7$ T. Compute its Larmor radius.
3. The equation for a dipole magnetic field in spherical coordinates is given by

$$\mathbf{B} = \frac{\mu_0 M}{4\pi} \frac{1}{r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

where M is the magnetic moment.

- (a) Show that the equation for a magnetic field line is $r = R \sin^2 \theta$, where R is the radius of the magnetic field line at the equator ($\theta = \pi/2$).
- (b) Show that the curvature of the magnetic field line at the equator is $R_C = R/3$.
- (c) Compute the curvature drift of a particle with charge q and parallel velocity v_{\parallel} at a radial distance R at the equator.
- (d) Compute the ∇B drift of a particle with charge q and perpendicular velocity v_{\perp} at a radial distance R at the equator.
- (e) Compare the equations for the curvature drift and the ∇B drift at the equator.
4. A particle is trapped in a magnetic mirror field given by

$$B_z = B_0 \left[1 + \left(\frac{z}{L} \right)^2 \right]$$

and has a total kinetic energy $w = mv^2/2$ and pitch angle α at $z = 0$. Find the oscillation frequency in terms of L , w , and α .