29:194 Homework #4

Due at the beginning of class, Thursday, September 23, 2010.

1. A particle is trapped in a magnetic mirror field given by

$$B_z = B_0 \left[1 + \left(\frac{z}{L} \right)^2 \right]$$

Initially, the mirror points of the particle are located at $z = \pm L$.

   (a) B_0 is now slowly increased to $2B_0$. Using the 2nd adiabatic invariant, find the new mirror point locations and the new mirror field B_t.

   (b) L is then slowly decreased to $L/2$, while holding $2B_0$ constant. Using the 2nd adiabatic invariant, find the new mirror point locations and the new mirror field B_t.

2. NUMERICAL: Numerical test of order of convergence
In this problem, we will test the order of convergence of Euler's method and the leapfrog method. The routines for this are available on the course website `http:www.physics.uiowa.edu/ ghowes/teach/phys194/index.html` under the numerical link. You will need Matlab m-files `lorentz.m`, `magnetic.m`, `electric.m`, `euler1.m`, `leapfrog2.m`, and `spm.m`. We will determine the error in our numerical method by calculating the error by which a single Larmor orbit (with $\mathbf{E} = 0$ and $\mathbf{B}$ uniform) does not close.

   Specify $q/m = 1$; let $\mathbf{E} = 0$ and $\mathbf{B} = (0, 0, 1)$. Set the initial conditions to be $\mathbf{x}_0 = (0, 0, 0)$ and $\mathbf{v}_0 = (1, 0, 0)$.

   (a) Calculate the error in position (`terr` in `spm.m`) after a single Larmor period for number of steps `nn`=32 using Euler's method. Successively double the number of steps taken (effectively halving Δt each time) up to `nn`=16384, calculating `terr` for each case. Present a table of this data.

   (b) Perform the same routine using the leapfrog method, and present a table of these results.

   (c) Plot log(`terr`) vs. log(`nn`) on a plot and print out the results. Be sure that you include axis labels and a title.

   (d) What are the slopes of log(`terr`) vs. log(`nn`) for each method?

   (e) Perform the same test using the adaptive RK45 method with a specified tolerance RelTol= 1.0×10^{-5}. How many steps does this adaptive algorithm require, and what is the resulting `terr`? How does this method compare to Euler's method and the leapfrog method in terms of error vs. number of timesteps taken?

3. NUMERICAL: $\mathbf{E} \times \mathbf{B}$ Drift

   Use the same routines as the previous problem with the adaptive RK45 method with a specified tolerance RelTol= 1.0×10^{-5}. Specify $q/m = 1$, and let $\mathbf{B} = (0, 0, 1)$. Set the initial conditions to be $\mathbf{x}_0 = (0, 0, 0)$ and $\mathbf{v}_0 = (1, 0, 0)$. For this problem we will keep $E_z = 0$.

   (a) What electric field is required such that the guiding center of the particle drifts the distance of a single Larmor radius at an angle of 30° off the x-axis (towards the y-axis) over 10 Larmor orbits. Please give your answer to three significant figures.

   (b) Plot the resulting trajectory. Do not forget axis labels and a title.

4. NUMERICAL: ∇B Drift

Use the same routines as the previous problem with the adaptive RK45 method with a specified tolerance RelTol= 1.0×10^{-5}. Specify $q/m = 1$; let $\mathbf{E} = 0$. Set the initial conditions to be $\mathbf{x}_0 = (1, 1, 0)$ and $\mathbf{v}_0 = (1, 0, 0)$.

(a) Specify a magnetic field $\mathbf{B} = (0, 0, 1 - \alpha r)$, where $r = \sqrt{x^2 + y^2}$ is the cylindrical radius. In what direction is the ∇B Drift?

(b) What is the value of α such that the guiding center of the motion will return to its original azimuthal position in 100 Larmor orbits?

(c) Plot the resulting trajectory. Do not forget axis labels and a title.

(d) Now change $\mathbf{x}_0 = (0, 0, 0)$ and $\mathbf{B} = (0, 0, x)$ and compute the motion of the particle. Please explain qualitatively the appearance of this trajectory.

(e) Is the magnetic moment μ invariant for this particle's motion in part (d)? Give justification for your response.