1. Show that for an electric field of the form

\[ E(x, \tau, t) = E_0(x, \tau) \cos(\omega t - k \cdot x) \]

the magnetic field is given by

\[ B(x, \tau, t) = -\frac{1}{\omega} \{ [\nabla \times E_0(x, \tau)] \sin(\omega t - k \cdot x) - [k \times E_0(x, \tau)] \cos(\omega t - k \cdot x) \} \]

2. A mirror machine has a mirror ratio \( R_m = 2 \). A group of electrons with an isotropic velocity distribution (Maxwellian) is released at the center of the machine. In the absence of collisions, what fraction of these electrons is confined?

3. A singly ionized, 20 eV Argon plasma is confined in a magnetic cusp field such that throughout the volume of the plasma, except for the edges, the magnetic field is zero. The plasma diameter is 1.0 m, and its density is \( n = 10^{11} \text{ m}^{-3} \). A small, 0.5 cm diameter spherical probe is placed at the center of the plasma and set to a potential of 100 V above the wall potential. At what distance from the probe surface will the measured potential be 1 V?

4. An electron of charge \( q_e = -e \) and mass \( m_e \) and an proton of charge \( q_e = e \) and mass \( m_i = m_p \) are initially at rest at \( x = (0, 0, 0) \) in a magnetic field \( B = B_0 \hat{z} \). An electric field is then turned on at \( t = 0 \) and increased linearly until time \( t_1 = \frac{2\pi m_i}{eB_0} \), at which point the electric field is held constant,

\[ E(t) = \begin{cases} 
0 & t < 0 \\
E_0(t/t_1)\hat{y} & 0 \leq t \leq t_1 \\
E_0\hat{y} & t > t_1 
\end{cases} \]

Find the total current density as a function of time \( j(t) \) due to the drifts of the two particles (neglect the current due to the fast Larmor oscillation).

5. NUMERICAL: Polarization Drift

Use the same Matlab m-files \texttt{lorentz.m}, \texttt{magnetic.m}, \texttt{electric.m}, \texttt{euler1.m}, \texttt{leapfrog2.m}, and \texttt{spm.m} as used in HW#4 and the adaptive RK45 method with a specified tolerance RelTol= \( 1.10 \times 10^{-5} \). Specify \( B = (0, 0, 1) \), \( q_i/m_i = 1 \), \( q_e = -q_i \) and an artificial mass ratio \( m_i/m_e = 10 \). Specify an electric field that increases with time \( E(t) = E_0 t/t_f \hat{y} \) with \( E_0 = 0.5 \) and \( t_f \) equal to 10 ion cyclotron periods. NOTE: The \texttt{hold on} command can be used to plot a second trace on the same plot; \texttt{hold off} turns this off.

(a) Plot the trajectories over \( t = [0, t_f] \), on the same plot, of both the ion and electron each with an initial position \( x_0 = (0, 0, 0) \) and initial velocity \( v_0 = (-1, 0, 0) \).

(b) Why do we not use a realistic mass ratio (for protons) of \( m_i/m_e = 1836 \) to do this calculation? HINT: Try using \( m_i/m_e = 40 \).
6. Laser Trapping: A charged particle can be trapped by a spatially varying intense laser field. Using interference of several lasers, the electric field near a charged particle is given by

\[ E(x, t) = E_0[1 + (x/x_0)^2] \sin(\omega t - k_y y) \hat{x}. \]

Calculate the velocity of the oscillation center \( U \) as a function of position \( x \) for a particle initially at rest at \( t = 0 \) at position \( x = (x_0, 0, 0) \). You may assume that the particle velocity \( v \) and laser frequency \( \omega \) satisfy \( v \ll \omega/k_y \) and \( v/x_0 \ll \omega \).

7. NUMERICAL: Ponderomotive Force

Use the adaptive RK45 method with a specified tolerance \( \text{RelTol} = 1.0 \times 10^{-5} \). Plot the \( x \) position of the particle vs. time \( t \) over a time \( t = [0, 50] \) for the problem above using \( E_0 = 2, \ x_0 = 1, \ \omega = 10, \ k_y = 0.01, \) and \( q/m = 1 \).