

II. How do we describe plasmas?

A. Basic Parameters:

1. A plasma consists of one, or more, ion species and electrons.

2. Intensive variables:
- a. Density n_s
 - b. Temperature T_s
 - c. Magnetic Field B_0

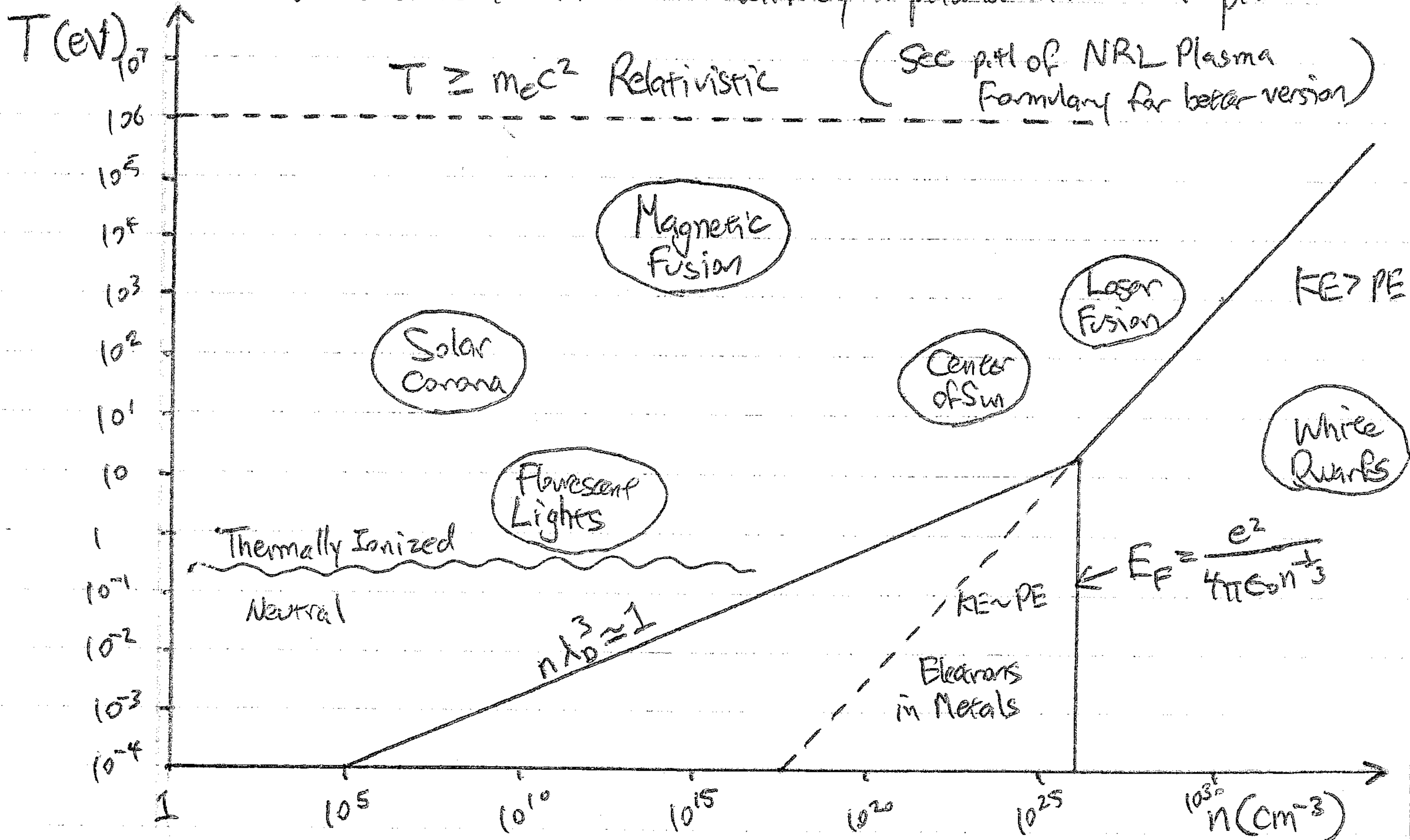
3. Physical properties
- a. mass m_s
 - b. charge q_s

NOTE: We often use $m_e/m_i \ll 1$ to simplify a problem.

4. Most plasmas have magnetic fields (although possibly no mean magnetic field) due to currents from charged particles moving within the plasma

(Ampere's Law: $\nabla \times \underline{B} = \mu_0 \underline{j} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$)

5. We can describe the incredible variety of plasma on the $n-T$ plane.



Lect #1 (Continued)

II. (Continued)

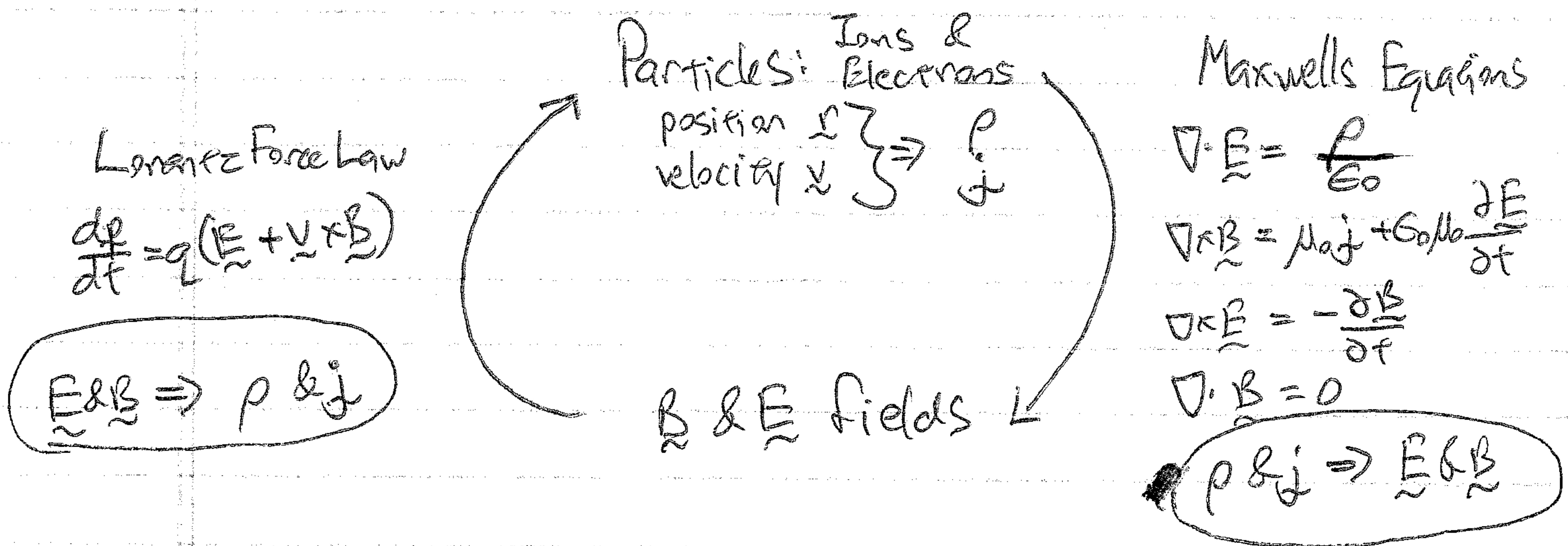
B. Units:

1. Two major systems:
 - a. SI Units (mks)
 - b. Gaussian Units (cgs)
2. We'll be using SI to agree with text (Chen)

III. How do we study the physics of plasmas?

A. The Basic Picture:

1. Plasma physics is really nothing more than the study of fluctuating magnetic and electric fields and the motion of charged particles in those fields.



2. This coupling is precisely what presents the challenge of plasma physics!

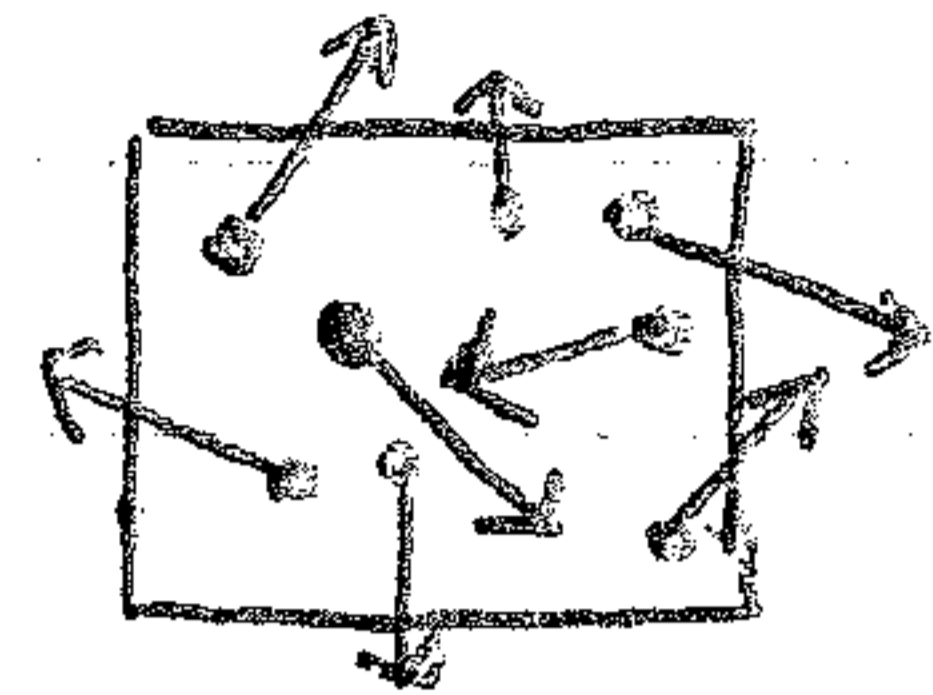
B. Inconsistent Models: Single Particle Motion

1. Assume we know \underline{E} & \underline{B} fields
2. Determine motion of a particle in these fields
3. Good approach is develop intuition
4. Does not account for the collective of other particles' motion (and the resulting \underline{E} & \underline{B} fields) on that single particle.

III. (Continued)

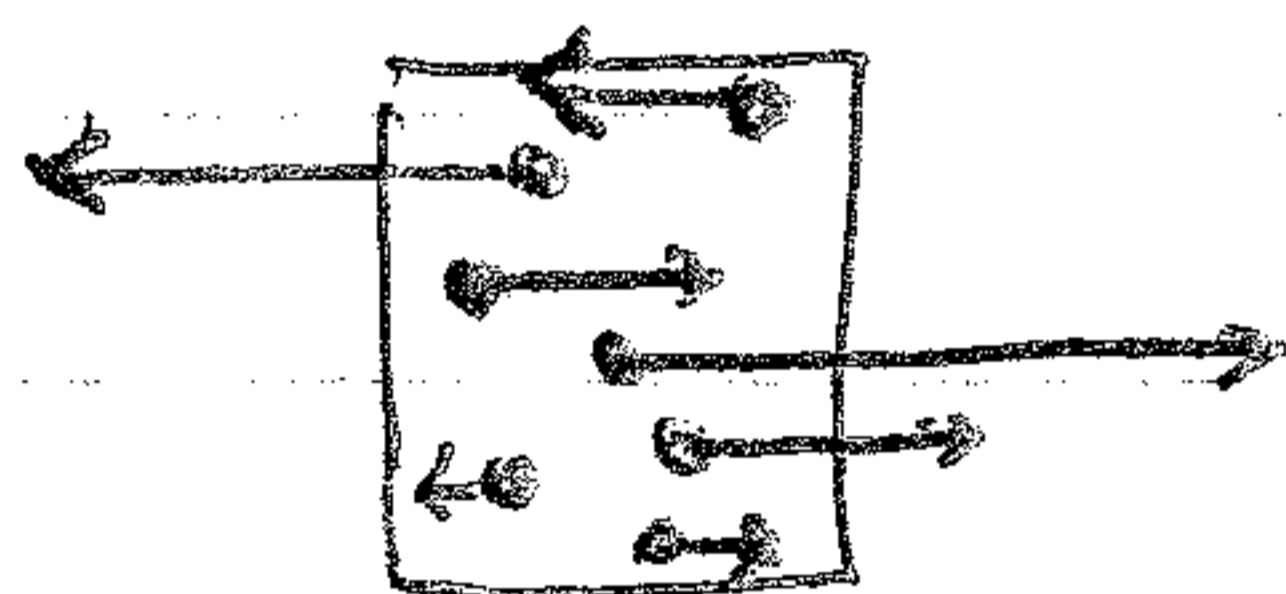
C. Consistent Models:1. Kinetic Theory:

a. Statistical theory averages over the motion of many particles

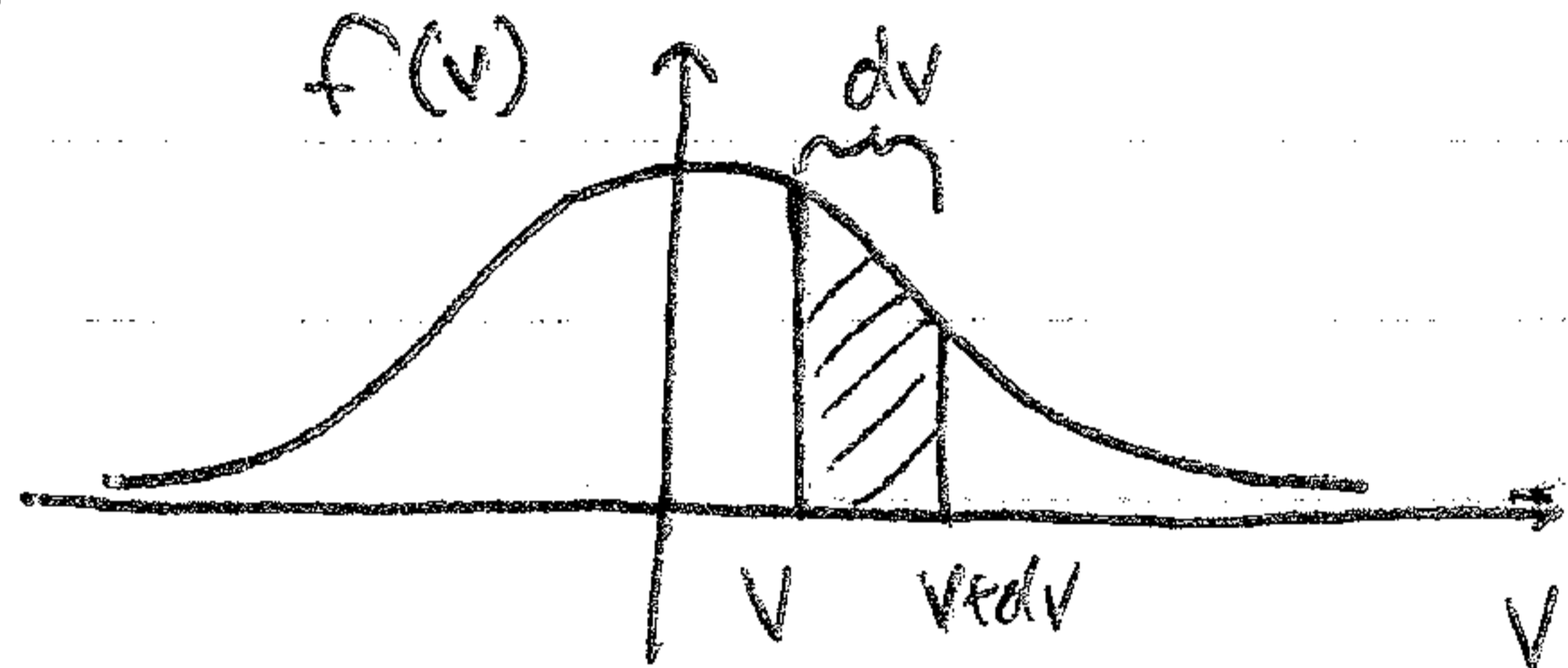


⇒ Distribution function $f(\underline{x}, \underline{v}, t)$

Ex: For a 1-D system, at a single point in space, particle velocities may be distributed as follows:



⇒



Here $f(v)$ is number of particles with velocity between v & $v+dv$.

Well discuss this more later.

- b. Assuming a Maxwellian distribution (thermal equilibrium) allows some closed form results to be derived.
- c. Any more detailed treatment quickly becomes cumbersome, not possible in closed form.

2. Fluid Theory:

a. Assumes all particles at a given point move similarly

b. One may take moments of the distribution function:

i) Density $n = \int d^3\underline{v} f(\underline{v})$

ii) Fluid velocity $\underline{u} = \frac{\int d^3\underline{v} \underline{v} f(\underline{v})}{\int d^3\underline{v} f(\underline{v})}$

etc. . . .

c. One must assume some closure to obtain a closed set of fluid equations.

III. C. 2.

d. Two Fluid Theory allows for different behavior between ions and electrons

e. If you assume that ions and electrons move together, you get a single fluid theory,

Magnetohydrodynamics or MHD.

i) This is the most simple, consistent description of a plasma.

ii) Probably the most widely used system, especially by "non-plasma physicists"

f. Even fluid theory is difficult in the complicated, nonlinear situations of most practical interest.

D. Computational Studies:

1. Numerical Solution: Solving a system of equations using the computer.

Ex: a. Boundary value/eigenvalue problems.

b. May use ODE integrators, shooting methods, etc.

2. Simulation: (Initial Value codes) Big business/big codes

a. Particle-in-Cell (particle method)

b. Fokker-Planck (Continuum method)

c. Gyrokinetic

d. Two Fluid / MHD

IV. Why do we study plasma physics?

A. Plasma physics and the methods used in plasma physics contribute significantly to a wide variety of fields:

1. Stellar and Galactic Astrophysics
2. Earth's magnetosphere and the heliosphere
3. Fusion Energy research
 - a. Magnetic Confinement
 - b. Inertial confinement (laser fusion)
4. Advanced Accelerators
5. Laser-plasma interactions
6. Industrial processing (especially computer chips)
7. Solid state, electrons in a metal

V. The NRL Plasma Formulary

Meet your new best friend!

Most Frequently Used Sections:

1. Vector Identities p. 4-5
2. Differential Operators in Curvilinear Coordinates p. 6-9
3. Physical Constants (SI) p. 14-15
4. Fundamental Plasma Parameters p. 28-29

NOTE: I will refer to the Plasma Formulary in the lecture notes as NRL.

Ex: Use $\underline{A} \times (\underline{B} \times \underline{C}) = \underline{B}(\underline{A} \cdot \underline{C}) - \underline{C}(\underline{A} \cdot \underline{B})$ [NRL, p. 4, (2)]

VI. Vector Notation Review & Vector Calculus

- A. Why? 1. Vector notation simplifies the mathematical notation
 2. You will get lots of practice with vector algebra & calculus.

B. Notation:

1. Under-tilde denotes vector quantity \underline{B}

a. In cartesian coordinates \hat{x} , \hat{y} , & \hat{z} ,

$$\underline{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

2. Unit vectors: $\hat{b} \equiv \frac{\underline{B}}{|\underline{B}|}$

3. Magnitude: $|\underline{B}| = \sqrt{\underline{B} \cdot \underline{B}} = \sqrt{B_x^2 + B_y^2 + B_z^2}$

4. Tensor: Denoted by double under-tilde $\underline{\underline{E}} = \begin{pmatrix} E_{xx} & E_{xy} & E_{xz} \\ E_{yx} & E_{yy} & E_{yz} \\ E_{zx} & E_{zy} & E_{zz} \end{pmatrix}$

Vector Calculus:

5. $\underline{\nabla} f \equiv \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}$

a. $\underline{\nabla} f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}$

b. Thus $\underline{\nabla} = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}$

C. Vector Algebra and Calculus Review:

1. Dot Product: $\underline{A} \cdot \underline{B} = A_x B_x + A_y B_y + A_z B_z$

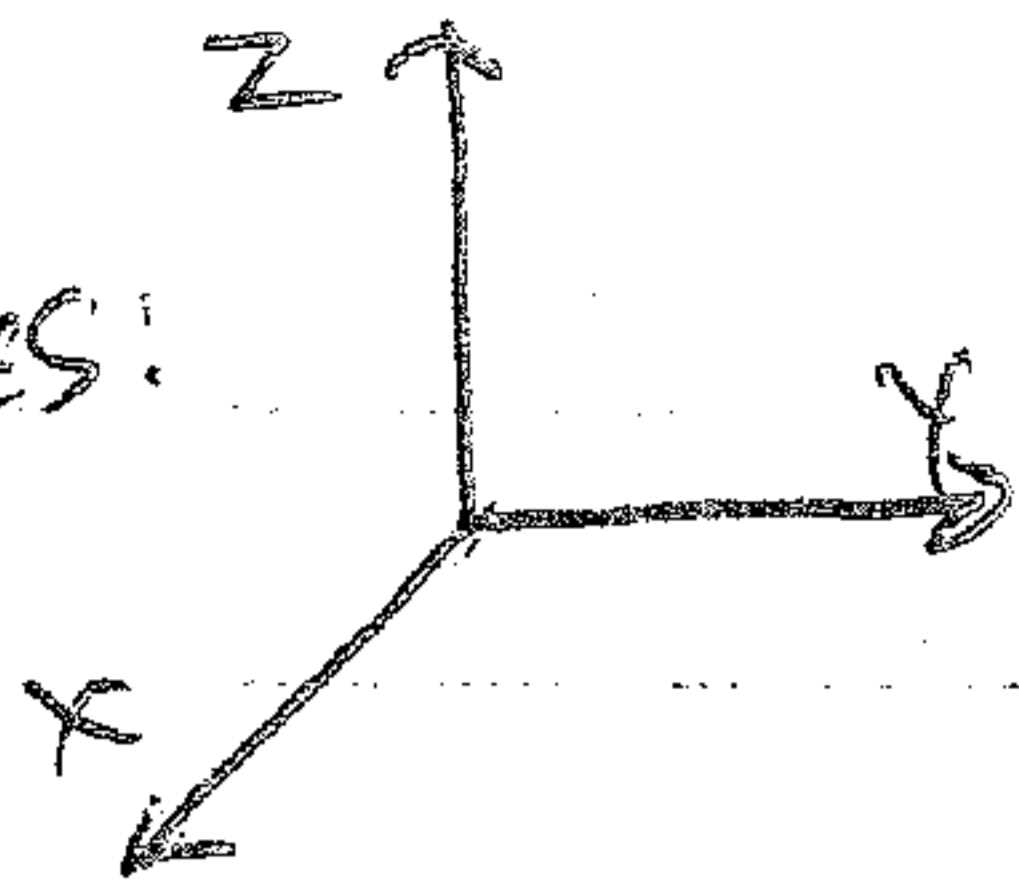
2. Cross Product: $\underline{A} \times \underline{B} = (A_y B_z - A_z B_y) \hat{x}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + (A_x B_z - A_z B_x) \hat{y} + (A_x B_y - A_y B_x) \hat{z}$$

Lecture #1 (Continued)
 VI C. (Continued)

Howes (8)

3. Right-handed coordinates:



$$\begin{aligned}\hat{x} \times \hat{y} &= \hat{z} \\ \hat{y} \times \hat{z} &= \hat{x} \\ \hat{z} \times \hat{x} &= \hat{y}\end{aligned}$$

4. Integration: $\int d^3\underline{v} f(\underline{v}) \equiv \int dv_x \int dv_y \int dv_z f(\underline{v})$

5. $\underline{v} \cdot \nabla = (v_x \hat{x} + v_y \hat{y} + v_z \hat{z}) \cdot \left(\hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \right) = v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}$

6. $\underline{v} \times \nabla = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = (v_y \frac{\partial}{\partial z} - v_z \frac{\partial}{\partial y}) \hat{x} + (v_z \frac{\partial}{\partial x} - v_x \frac{\partial}{\partial z}) \hat{y} + (v_x \frac{\partial}{\partial y} - v_y \frac{\partial}{\partial x}) \hat{z}$

D. Examples:

1. MHD continuity: $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0$

a. By NRL p.4 (7), $\nabla \cdot (\rho \underline{v}) = \rho \nabla \cdot \underline{v} + (\underline{v} \cdot \nabla) \rho$, so

$$\boxed{\frac{\partial \rho}{\partial t} + (\underline{v} \cdot \nabla) \rho = -\rho \nabla \cdot \underline{v}}$$

2. MHD momentum: $\rho \frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \nabla) \underline{v} = -\nabla p + \underline{j} \times \underline{B}$

a. We also have $\underline{j} = \frac{1}{\mu_0} \nabla \times \underline{B}$ (Ampere's Law, displacement current dropped)

b. Rewrite $\underline{j} \times \underline{B}$ term in terms of only \underline{B} :

$$\underline{j} \times \underline{B} = \frac{1}{\mu_0} (\nabla \times \underline{B}) \times \underline{B} = -\frac{1}{\mu_0} \underline{B} \times (\nabla \times \underline{B})$$

From NRL p.4 (12) with $\underline{B} = \underline{A} \Rightarrow \nabla(\underline{B} \cdot \underline{B}) = 2 \underline{B} \times (\nabla \times \underline{B}) + 2(\underline{B} \cdot \nabla) \underline{B}$

So $\underline{B} \times (\nabla \times \underline{B}) = \frac{1}{2} \nabla(\underline{B} \cdot \underline{B}) - (\underline{B} \cdot \nabla) \underline{B}$

Thus $\boxed{\underline{j} \times \underline{B} = -\frac{\nabla B^2}{2\mu_0} + \frac{(\underline{B} \cdot \nabla) \underline{B}}{\mu_0}}$