29:195 Homework #2

Reading: Required: Read GB Chapter 6, Section 6.7 (p.219–239)
Optional: Read BS Chapter 4, Sections 4.5–4.7 (p.108–130)

Due at the beginning of class, Thursday, February 17, 2011.

1. Show that the determinant of the matrix

$$\begin{pmatrix} S - n^2\cos^2\theta & -iD & n^2\sin\theta\cos\theta \\ iD & S - n^2 & 0 \\ n^2\sin\theta\cos\theta & 0 & P - n^2\sin^2\theta \end{pmatrix}$$

can be written in the form of the Booker Quartic

$$An^4 - Bn^2 + C = 0$$

where

$$A = S\sin^2\theta + P\cos^2\theta,$$
$$B = RL\sin^2\theta + PS(1 + \cos^2\theta),$$

and

$$C = RLP.$$

2. Prove that the index of refraction for cold plasma waves (the solution of the Booker Quartic above) is either purely real or purely imaginary, but never complex. Hint: Show that the discriminant $B^2 - 4AC$ is positive definite.

3. In the limit $\omega \to 0$, show that

$$R = L = S = 1 + \sum_s \frac{\omega_{ps}^2}{\omega_{cs}^2},$$
$$D = 0,$$

and

$$P = -\sum_s \frac{\omega_{ps}^2}{\omega^2}.$$

4. Assuming that the ions are infinitely massive, derive the equations for the following characteristic frequencies:
   (a) The right-hand cutoff frequency, ω_R
   (b) The left-hand cutoff frequency, ω_L
   (c) The upper hybrid frequency, ω_{UH}

5. Whistler Waves
   (a) Assuming the wave frequency is sufficiently high that the ions do not move, that $\omega \ll \omega_p$, and that $|\omega_{ce}| \ll \omega_p$, show that the index of refraction for whistler waves with a wave vector at an angle θ with respect to the mean magnetic field is approximately

$$n^2 = \frac{\omega_p^2}{\omega(|\omega_{ce}|\cos\theta - \omega)}$$

   (b) Sketch $n(\theta)$ for $\omega \ll |\omega_{ce}|$ as a polar plot.
   (c) Sketch $n(\theta)$ for $\omega = |\omega_{ce}|/4$ as a polar plot.
   (d) Sketch $n(\theta)$ for $\omega = |\omega_{ce}|/2$ as a polar plot.