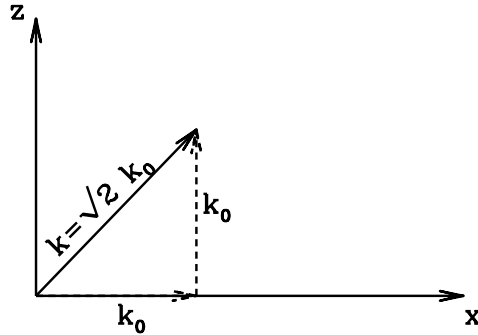


29:195 Homework #3

Due at the beginning of class, Thursday, February 24, 2011.

1. Ray Tracing

Consider a radio wave packet launched from an antenna located at $(x, z) = (0, 0)$ at time $t = 0$ in a plane-parallel atmosphere as depicted in the figure below.



The plasma electron density in the atmosphere increases with height as

$$n_e(z) = n_0 \frac{z^2}{H^2}.$$

The wave vector of the radio wave has components $k_x = k_z = k_0$. Please give all answers in terms of the parameters of the problem $\omega_{pe0}^2 = (n_0 q_e^2)/(\epsilon_0 m_e)$, k_0 , c , and H . You may consider the plasma to be unmagnetized.

- Calculate the frequency of the radio wave ω as a function of time.
- Find the rate of change of the wavevector components k_x and k_z with respect to time in terms of the problem parameters and x and z .
- Determine the motion of the wavepacket in time as the functions $x(t)$ and $z(t)$. Be sure to use initial conditions to solve for any unknown constants in terms of the problem parameters.
- Determine the trajectory in the (x, z) plane in the form $z(x)$.
- What is the total distance traveled in the horizontal direction before the wavepacket returns to the ground?
- What is the maximum height the wavepacket reaches?

2. Electrostatic Drift Waves

Consider a cylindrical column of plasma of radius a with an equilibrium number density that varies radially as

$$n_0(r) = \bar{n} \sqrt{1 - \frac{r^2}{4a^2}}$$

for $r \leq a$, where \bar{n} is a constant. The low beta plasma has $m_e/m_i \ll \beta_e \ll 1$ and may be treated as two fluid plasma with cold, singly-charged ions and isothermal electrons. Consider the plasma in the low frequency $\omega \ll \omega_{ci}$ and long wavelength $kC_i \ll \omega_{pi}$ limits, where the ion acoustic speed is defined as $C_i^2 = T_e/m_i$ (and Boltzmann's constant has been absorbed into the temperature).

- (a) Find the magnitude of the equilibrium electron drift velocity U_d as a function of C_i , ω_{ci} , r , a , and physical constants.
- (b) In terms of the cylindrical coordinates (r, θ, z) , in what direction is this equilibrium drift?
- (c) Does this equilibrium drift correspond to differential or solid body rotation of the plasma?
- (d) To investigate the wave behavior of the plasma, a frequently used approximation is to treat the dynamics in the cylindrical plasma locally by a Cartesian coordinate system (taking $r \rightarrow x$, $\theta \rightarrow y$, and $z \rightarrow z$). For a fluctuating wave that varies in this local Cartesian coordinate system as $\exp(ik_y y + ik_{\parallel} z - i\omega t)$, find the *two* roots of the drift wave frequency in the limit $k_{\parallel} C_i \ll k_y U_d$. Express your answers in terms of k_y , k_{\parallel} , C_i , ω_{ci} , r , a , and physical constants.