29:225 Examples of Notes for Class Discussion

1. Landau, L., *On the Vibrations of the Electronic Plasma*, Journal of Physics **10**, 25 (1946).
   (a) **Science Question:** What is the evolution of an initial non-equilibrium distribution in a collisionless plasma? What is the penetration of an imposed external electric field on a collisionless plasma?
   (b) **Method:** An analytical solution of the evolution of linear, high frequency (comparable to the plasma frequency) oscillations in response to a non-equilibrium distribution is computed, taking care to handle correctly the analytic continuation of the solution over the entire solution space. A Laplace transform is used to solve for the time dependence. Simplified analytical solutions are computed in the limits weak damping and strong damping.

       Also, using a similar analytical approach, the penetration of an imposed external electric field into a plasma is calculated, solving both for cases when the frequency of the applied electric field is far from resonance with the plasma frequency, as well as resonant cases in which the applied frequency is above or below the plasma frequency.

       The approximations used in this analytical calculation are:
       i. Collisions are negligible. $\omega \gg \nu$
       ii. Ions are immobile
       iii. Equilibrium distribution function f_0 is Maxwellian
       iv. Linearized: Small perturbations of distribution function, $f \ll f_0$
       v. No magnetic field, $\mathbf{B} = 0$
       vi. Electrostatic: $\mathbf{E} = -\nabla \phi$
       vii. Fourier transform in space, $f_{\mathbf{k}}(\mathbf{v}, t) e^{i\mathbf{k}\cdot\mathbf{r}}$, and focus on the evolution of just a single Fourier component
       viii. One-dimensional: Variation only in x direction, $\mathbf{k} = k\hat{\mathbf{x}}$.
       ix. A Laplace transform in time is used to solve the initial value problem.
       x. Small growth rate approximation, $|\gamma| \ll \omega$ is used to compute the Landau damping in the long-wavelength limit, $k\lambda_e \ll 1$
   (c) **Citeable Results:**
       i. The main result of this paper is that, in a collisionless plasma, the electric field fluctuations, arising from the initial non-equilibrium distribution, are damped in time. This has come to be known as Landau damping.
       ii. Frequency:
       $$\omega = \omega_p \left(1 + \frac{3}{2} k^2 \lambda_e^2\right)$$
       Damping Rate:
       $$\gamma = \omega_p \sqrt{\frac{\pi}{8}} \frac{1}{(k\lambda_e)^3} e^{-1/2(k\lambda_e)^2}$$
   (d) **Other Comments:**
       i. Pulls no punches in criticizing Vlasov's incorrect solution of the problem.
       ii. The prediction of Landau damping was very controversial. With a very elegant and rigorous mathematical treatment of the complex contour integration, Landau predicted a profound qualitative effect—the collisionless damping of fluctuations in a kinetic plasma—that had not been intuitively anticipated.
       iii. Landau damping was not experimentally verified until 1964–1966 in experiments by Malmberg and Wharton.
       iv. Similar to what is done in section 2, damping in space (rather than time) can be computed by assuming as real value of ω and allowing for a complex value of k. Thus, the fluctuation damps in space rather than time (a boundary value problem, as opposed to an initial value problem).

2. Parker, E. N. *Dynamics of the Interplanetary Gas and Magnetic Fields*, Astrophysical Journal **128**, 664 (1958).

   (a) **Science Question:** What mechanism is responsible for gas streaming outward from the sun in all directions at 500–1500 km/s? What configuration of the general solar dipole magnetic field do we expect if the gas is streaming radially outward?

   (b) **Method:** An analytical calculation of the steady-state solution of a spherically symmetric flow driven by a high coronal temperature, where the solution must conserve mass, momentum, and energy.

   The approximations used in this analytical calculation are:

      i. Spherical symmetry, reducing problem to one dimension in heliocentric radius, r
      ii. Magnetic Field is neglected
      iii. Outward surface from with solar wind flows is assumed to be at $r = a$, where $a = 10^6$ km to represent the outer corona. Note that $R_\odot = 7 \times 10^5$ km, so this corresponds to $a = 1.4 R_\odot$.
      iv. Assumes conditions of density, temperature, and velocity at $r = a$. Uses $N_0 = 3 \times 10^7$ cm^{-3}, and v_0 is small in the corona.
      v. It is supposed that the heating of the coronal gas to $T \sim 10^6$ K is the basic process, and the outflow of gas is a secondary effect.
      vi. Corona is fully ionized
      vii. Assume that the temperature as a function of radius in the solar corona $T(r)$ is given, and do not concern ourselves with the heating mechanisms necessary to sustain this temperature.
      viii. Steady state solution in time.
      ix. Conservation of mass (equation of continuity), momentum (equation of motion), and energy (pressure equation)
      x. IMPORTANT: Isothermal approximation: Temperature is maintained at a uniform value (by some unspecified heating mechanism) from $r = a$ to $r = b$.
      xi. Frozen-In Flux: The gas flowing out from the sun is not field free, but carries with it the sun's magnetic field.

   (c) **Citeable Results:**

      i. The solar corona cannot be in hydrostatic equilibrium out to large distances because this implies pressures that are many orders of magnitude larger than the pressure of the interstellar medium.
      ii. The Solar Wind (Fig 1): Solutions to the equations require that, for coronal temperatures of $T \sim 10^6$ K, flow velocities asymptote to values in the range 500–1500 km/s. This is a natural consequence of the solution for the spherical expansion. A coronal temperature of $T \sim 2$–3×10^6 K over an extended region around the sun is the simplest explanation for the origin of outflowing gas.
      iii. The Parker Spiral: (Fig 6) The steady-state magnetic field due to the spherically symmetric outflow of gas from a rotating star yields an Archimedean spiral structure of the magnetic field.
      iv. The torque exerted on the sun by the interplanetary magnetic field is negligible.

   (d) **Other Comments:**

      i. Note that this calculation was inspired by Biermann's pointing out that observations of comet tails would seem to require gas streaming outward from the sun at 500–1500 km/s, assuming an interplanetary density of $n = 500$ cm^{-3} at 1 AU.
      ii. Parker transforms to a somewhat abstract form of the governing equations for this system. He then uses the mathematical properties to solve this system, and eventually transforms back to see what are the physical consequences. But the transformation to abstract dimensionless variables somewhat obscures the physical meaning of his equations.

iii. It is hard to see from his derivation, but the main point is that, at the critical point of the solution (the radius where the flow velocity is equal to the local sound speed), both sides of the solution must be equal to zero. This essentially selects a single physical solution out of a family of possible solutions.