

## 29:225 Example of Annotated Bibliography

1. Landau, L., *On the Vibrations of the Electronic Plasma*, Journal of Physics **10**, 25 (1946). [5]

- (a) **Science Question:** What is the evolution of an initial non-equilibrium distribution in a collisionless plasma? What is the penetration of an imposed external electric field on a collisionless plasma?
- (b) **Method:** An analytical solution of the evolution of linear, high frequency (comparable to the plasma frequency) oscillations in response to a non-equilibrium distribution is computed, taking care to handle correctly the analytic continuation of the solution over the entire solution space. A Laplace transform is used to solve for the time dependence. Simplified analytical solutions are computed in the limits weak damping and strong damping.

Also, using a similar analytical approach, the penetration of an imposed external electric field into a plasma is calculated, solving both for cases when the frequency of the applied electric field is far from resonance with the plasma frequency, as well as resonant cases in which the applied frequency is above or below the plasma frequency.

The approximations used in this analytical calculation are:

- i. Collisions are negligible.  $\omega \gg \nu$
- ii. Ions are immobile
- iii. Equilibrium distribution function  $f_0$  is Maxwellian
- iv. Linearized: Small perturbations of distribution function,  $f \ll f_0$
- v. No magnetic field,  $\mathbf{B} = 0$
- vi. Electrostatic:  $\mathbf{E} = -\nabla\phi$
- vii. Fourier transform in space,  $f_{\mathbf{k}}(\mathbf{v}, t)e^{i\mathbf{k}\cdot\mathbf{r}}$ , and focus on the evolution of just a single Fourier component
- viii. One-dimensional: Variation only in  $x$  direction,  $\mathbf{k} = k\hat{\mathbf{x}}$ .
- ix. A Laplace transform in time is used to solve the initial value problem.
- x. Small growth rate approximation,  $|\gamma| \ll \omega$  is used to compute the Landau damping in the long-wavelength limit,  $k\lambda_e \ll 1$

(c) **Citeable Results:**

- i. The main result of this paper is that, in a collisionless plasma, the electric field fluctuations, arising from the initial non-equilibrium distribution, are damped in time. This has come to be known as Landau damping.
- ii. Frequency:

$$\omega = \omega_p \left( 1 + \frac{3}{2} k^2 \lambda_e^2 \right)$$

Damping Rate:

$$\gamma = \omega_p \sqrt{\frac{\pi}{8}} \frac{1}{(k\lambda_e)^3} e^{-1/2(k\lambda_e)^2}$$

(d) **Other Comments:**

- i. Pulls no punches in criticizing Vlasov's incorrect solution of the problem.
- ii. The prediction of Landau damping was very controversial. With a very elegant and rigorous mathematical treatment of the complex contour integration, Landau predicted a profound qualitative effect—the collisionless damping of fluctuations in a kinetic plasma—that had not been intuitively anticipated.
- iii. Landau damping was not experimentally verified until 1964–1966 in experiments by Malmberg and Wharton.
- iv. Similar to what is done in section 2, damping in space (rather than time) can be computed by assuming as real value of  $\omega$  and allowing for a complex value of  $k$ . Thus, the fluctuation damps in space rather than time (a boundary value problem, as opposed to an initial value problem).

2. Van Allen, J. A. and Frank, L. A., *Radiation around the Earth to a Radial Distance of 107,400 km*, Nature **183**, 430 (1959). [13]

- (a) **Science Question:** What is the nature and spatial distribution of high energy particles trapped in the Earth's dipolar magnetic field? These regions constitute the "radiation belts" of the Earth.
- (b) **Method:** The *Pioneer III* spacecraft was launched to sample the space environment to a radial distance of 107,400 km. Van Allen's group at the University of Iowa used two Geiger-Muller tubes as detectors of high energy radiation as part of the scientific payload of the mission.

The approximations used in this analytical calculation are:

- i. Uses the theory of the motion of charged particles in the (approximately known) geomagnetic field to plot the contours of the radiation belts using measurements along a single radial slice.
  - ii. The experimental apparatus measures only particles with sufficient energy to penetrate the shielding of the detector. No information about protons with  $E < 30$  MeV, electrons with  $E < 2.2$  MeV or x-rays with  $E < 250$  keV is known from this experiment.
- (c) **Citeable Results:**
- i. Discovery of the Radiation Belts: Two distinct peaks of high-energy radiation were measured. Figure 5 shows a diagram of the inner and outer radiation belts, trapped in banana-shaped regions of the Earth's dipole magnetic field. Although this diagram was highly conjectural, it captures the essential qualitative details of the electron radiation belts that persist today.
  - ii. Dependence on Solar Activity: The paper conjectured that the intensity of the trapped radiation would be strongly dependent on solar activity.
  - iii. Interplanetary Cosmic Ray Intensity: Direct measurement of the interplanetary cosmic ray intensity:

$$J_0 = 3.6 \pm 0.8 (\text{cm}^2/\text{s})^{-1}$$

- iv. Possible Connection to Aurorae: The outer radiation zone was thought to be associated with the high-latitude aurorae. Magnetic field lines at the geomagnetic equator at  $r = 4.8R_E$  map to the Earth's surface  $63^\circ$  latitude; similarly, at  $r = 10R_E$  map to the Earth's surface  $71.5^\circ$  latitude. These latitudes signify the approximate lower and upper latitude extent of auroral activity.
- (d) **Other Comments:**
- i. Based on this work alone, it was not known if the high-energy radiation was due to protons or electrons (as it turns out, it was dominated by electrons).
  - ii. Future work: This work motivated the desire to measure the energy spectra of protons and electrons in the trapped region.

3. Parker, E. N. *Dynamics of the Interplanetary Gas and Magnetic Fields*, *Astrophysical Journal* **128**, 664 (1958). [6]

- (a) **Science Question:** What mechanism is responsible for gas streaming outward from the sun in all directions at 500–1500 km/s? What configuration of the general solar dipole magnetic field do we expect if the gas is streaming radially outward?
- (b) **Method:** An analytical calculation of the steady-state solution of a spherically symmetric flow driven by a high coronal temperature, where the solution must conserve mass, momentum, and energy.

The approximations used in this analytical calculation are:

- i. Spherical symmetry, reducing problem to one dimension in heliocentric radius,  $r$
- ii. Magnetic Field is neglected
- iii. Outward surface from which solar wind flows is assumed to be at  $r = a$ , where  $a = 10^6$  km to represent the outer corona. Note that  $R_{\odot} = 7 \times 10^5$  km, so this corresponds to  $a = 1.4R_{\odot}$ .
- iv. Assumes conditions of density, temperature, and velocity at  $r = a$ . Uses  $N_0 = 3 \times 10^7 \text{ cm}^{-3}$ , and  $v_0$  is small in the corona.
- v. It is supposed that the heating of the coronal gas to  $T \sim 10^6$  K is the basic process, and the outflow of gas is a secondary effect.
- vi. Corona is fully ionized
- vii. Assume that the temperature as a function of radius in the solar corona  $T(r)$  is given, and do not concern ourselves with the heating mechanisms necessary to sustain this temperature.
- viii. Steady state solution in time.
- ix. Conservation of mass (equation of continuity), momentum (equation of motion), and energy (pressure equation)
- x. **IMPORTANT:** Isothermal approximation: Temperature is maintained at a uniform value (by some unspecified heating mechanism) from  $r = a$  to  $r = b$ .
- xi. **Frozen-In Flux:** The gas flowing out from the sun is not field free, but carries with it the sun's magnetic field.

(c) **Citeable Results:**

- i. The solar corona cannot be in hydrostatic equilibrium out to large distances because this implies pressures that are many orders of magnitude larger than the pressure of the interstellar medium.
- ii. The Solar Wind (Fig 1): Solutions to the equations require that, for coronal temperatures of  $T \sim 10^6$  K, flow velocities asymptote to values in the range 500–1500 km/s. This is a natural consequence of the solution for the spherical expansion. A coronal temperature of  $T \sim 2\text{--}3 \times 10^6$  K over an extended region around the sun is the simplest explanation for the origin of outflowing gas.
- iii. The Parker Spiral: (Fig 6) The steady-state magnetic field due to the spherically symmetric outflow of gas from a rotating star yields an Archimedean spiral structure of the magnetic field.
- iv. The torque exerted on the sun by the interplanetary magnetic field is negligible.

(d) **Other Comments:**

- i. Note that this calculation was inspired by Biermann's pointing out that observations of comet tails would seem to require gas streaming outward from the sun at 500–1500 km/s, assuming an interplanetary density of  $n = 500 \text{ cm}^{-3}$  at 1 AU.
- ii. Parker transforms to a somewhat abstract form of the governing equations for this system. He then uses the mathematical properties to solve this system, and eventually transforms back to see what are the physical consequences. But the transformation to abstract dimensionless variables somewhat obscures the physical meaning of his equations.

- iii. It is hard to see from his derivation, but the main point is that, at the critical point of the solution (the radius where the flow velocity is equal to the local sound speed), both sides of the solution must be equal to zero. This essentially selects a single physical solution out of a family of possible solutions.

4. Armstrong, J. W., Cordes, J. M., and Rickett, B. J. *Density power spectrum in the local interstellar medium*, Nature **291**, 561 (1981). [1]

- (a) **Science Question:** What is the nature of the turbulent energy spectrum in the interstellar medium as observed via electron density fluctuations?
- (b) **Method:** This study uses eight independent methods to probe the power in electron density fluctuations within various ranges of length scales. The methods used are:
  - i. **Angular broadening:** Angular broadening of unresolved radio sources, such as pulsars, and interplanetary scintillation of these sources. ( $L \sim 10^6$  to  $10^7$  m)
  - ii. **Visibility phase scintillation:** Phase scintillation of Very Long Baseline Interferometry (VLBI) measurements. ( $L \sim 10^6$  to  $10^7$  m)
  - iii. **Interstellar scintillation (ISS) decorrelation bandwidth:** ( $L \sim 10^8$  to  $10^9$  m)
  - iv. **Pulsar timing noise:** ( $L \sim 10^{12}$  to  $10^{13}$  m)
  - v. **Frequency-time drift:** Formulated for two discrete scales ( $L \sim 10^{10}$  to  $10^{12}$  m)
  - vi. **Velocity structure functions:** Direct measurements of velocity of stars and neutral gas ( $L \sim 10^{16}$  to  $10^{18}$  m)
  - vii. **Density outer scale:** Argument for density fluctuation at outer scale. ( $L \sim 10^{18}$  m)
  - viii. **ISM clouds:** Density inhomogeneities predicted by a multi-phase ISM model consisting of warm, partially ionized cloud shells. ( $L \sim 10^{17}$  m)

The approximations used in this analytical calculation are:

- i. The electron density fluctuations are assumed to be distributed isotropically (the power depends only on the magnitude of the wavenumber),  $E^{(3)}_k \propto k^{-\alpha}$ .
- ii. The turbulence is homogeneously distributed throughout the ISM.
- iii. Multi-phase ISM model is assumed to relate measured velocity structure functions at large scales to electron density fluctuations: cool ( $T \sim 10^2$  K) dense neutral clouds, warm ( $T \sim 10^4$  K) partially ionized gas, and hot coronal ( $T \sim 10^6$  K) plasma. Low wavenumber measurements are thought to be associated with the warm phase. Takes  $\delta\rho/\rho_0 \sim C\delta v/v_A$ , where  $C = 10^{0\pm 1}$ .

(c) **Citeable Results:**

- i. The electron density spectrum in the interstellar medium is most simply interpreted as a single power-law spectrum scaling as  $E^{(3)}_k \propto k^{-3.6\pm 0.2}$  over 12 orders of magnitude from an outer turbulent length scale of  $100 \text{ pc} \sim 10^{18} \text{ m}$  down to small-scales of  $10^7 \text{ m}$ .
- ii. A Kolmogorov slope of  $\alpha = -3.67$  is consistent with the entire data set; a slope of  $\alpha = -3.5$  is also consistent, however a slope of  $\alpha = -4$  is not consistent with the data.

(d) **Other Comments:**

- i. Radio data alone is consistent with  $E^{(3)}_k \propto k^{-3.7\pm 0.6}$  over  $10^{-11} \lesssim k \lesssim 10^{-6} \text{ m}^{-1}$ , consistent with either  $\alpha = -3.67$  or  $\alpha = -3.5$ . But radio data alone cannot exclude  $\alpha = -4$ .
- ii. A random assembly of density discontinuities would generate a  $\alpha = -4$  spectrum.
- iii. Sometimes called the ‘‘Great Power Law in the Sky’’
- iv. Velocity data at the largest scales may also be associated with motions of individual clouds in the ISM. The lack of measurements over  $10^{12} \text{ m} \lesssim L \lesssim 10^{16} \text{ m}$  does not rule out this possibility, but the spectrum of the velocity structure function appears to be a continuous spectrum over  $10^{16} \text{ m} \lesssim L \lesssim 10^{18} \text{ m}$ .
- v. Potential theoretical problems with a single turbulent energy spectrum: coronal phase could only support waves over  $10^9 \text{ m} \lesssim L \lesssim 10^{17} \text{ m}$ , with smaller scale modes damped; warm phase can only support waves with  $L \gtrsim 10^{13} \text{ m}$ ; and cool phase only  $L \lesssim 10^9 \text{ m}$ .
- vi. Future work required to reconcile turbulent spectrum with models of multi-phase ISM and to explore the transfer of energy from large scales to small scales.

5. Goldreich, P. and Sridhar, S., *Toward a Theory of Interstellar Turbulence II. Strong Alfvénic Turbulence*, *Astrophysical Journal* **438**, 763 (1995). [4]

(a) **Science Question:** What is the nature of strong turbulence due to nonlinear interactions among shear Alfvén waves in an incompressible magnetized fluid?

(b) **Method:**

The approximations used in this analytical calculation are:

- i. This work assumes the properties of weak Alfvénic turbulence from Paper I [9]:
  - A. Resonant 3-wave interactions are empty in weak MHD turbulence (this result turned out to be incorrect in the context of astrophysical turbulence due to the fact that field lines tend to wander).
  - B. In weak MHD turbulence, energy is not transferred to smaller parallel scales, only to smaller perpendicular scales
  - C. As the energy cascades to higher perpendicular wavenumbers, the nonlinear interactions strengthen, ultimately leading to a violation of the assumptions of weak turbulence theory. The turbulence becomes strong.
- ii. Incompressible MHD
- iii. Dissipative terms due to magnetic diffusivity and viscosity are dropped.
- iv. Uniform magnetic field in equilibrium
- v. Consider only shear Alfvén waves
- vi. For a finite but small nonlinearity parameter,  $\zeta_k = k_{\perp} v_{\perp k} / k_z v_A$ , a finite cascade time can be handled by a nonlinear renormalization of frequencies. (This can be thought of as a “frequency-time uncertainty relationship”). As  $\zeta_k \rightarrow 1$  from below, it’s growth ceases because the growth rate of  $/k_z v_A$  approaches that of  $k_{\perp} v_{\perp k}$ .
- vii. From Kraichnan, nonlinear interactions occur only between wave packets moving in opposite directions along the magnetic field.
- viii. Defines a kinetic equation for the evolution of energy per mode in wavevector space, assuming the EDQNM (Eddy Damped Quasi-Normal Markovian) approximation. This includes a linear damping term in the evolution equations of the third-order correlations.
- ix. Stationary solutions are sought (no time dependence)
  - x. Symmetric solutions (corresponding to zero cross helicity, or equal energy fluxes up and down the field)
  - xi. Damping

(c) **Citeable Results:**

- i. Conjecture: Shear Alfvénic turbulence reaches a state of critical balance,  $\zeta_k \sim 1$ , a balance between the linear wave period and the nonlinear timescale at which energy is transferred to shorter scales.
- ii. Scale-dependent anisotropy (parallel and perpendicular scales of turbulence are correlated),  $k_z \sim k_{\perp}^{2/3} L^{-1/3}$ .
- iii. One-dimensional energy spectrum  $E_k \sim k_{\perp}^{-5/3}$ .
- iv. Critically balanced cascade is most vulnerable to damping by ion-neutral collisions in a partially ionized medium when  $k_{\perp} l_0 \sim 1$ , or perpendicular scales of the turbulence  $k_{\perp}^{-1}$  are of order the ion-neutral collision mean free path  $l_0$ . Since diffractive scintillation of pulsars occurs on much smaller scales than  $l_0$ , the electron density fluctuations must arise in highly ionized components of the ISM.
- v. Viscous damping between protons is negligible above the proton gyroradius. Unless ion-neutral collisions are significant, the inner scale of the shear Alfvénic turbulence should not be much larger than the ion gyroradius,  $k_{\perp} \rho_i \sim 1$ .
- vi. Negligible energy is lost to the generation of pseudo-Alfvén waves due to the different polarizations of the modes in a critically balanced cascade with  $k_{\perp} \gg k_z$ . Similarly, it is expected that negligible energy will also be lost to fast waves.

- vii. Electron density fluctuations in the ISM reflect fluctuations of specific entropy at constant plasma pressure, and the specific entropy is mixed as a passive contaminant by the shear Alfvénic turbulence. Thus, the power spectrum of the electron density fluctuations assumes the form of the energy spectrum of the turbulence.

(d) **Other Comments:**

- i. When wave amplitudes are small (and thus energy transfer is weak), kinetic equation (29) describes resonant 3-wave interactions. In this case, the 3-wave interactions are null. When interactions are strong, the equation describes nonresonant 3-mode couplings, by choosing the eddy damping rate of order the eddy turn over rate. In strong turbulence, interactions of all orders contribute, so 3-wave interactions are an adequate proxy for those of all orders.
- ii. Note that this result, an electron density spectrum scaling as  $k_{\perp}^{-5/3}$  is consistent with the observations of the spectrum of electron density fluctuations in the interstellar medium [1], as well as observed anisotropy in some scatter-broadened radio images.
- iii. They cannot explain why shear Alfvén waves are the dominant mode in the turbulence, but it may be due to damp of compressive modes.
- iv. Comparison to related work implies that the related work does not provide any detailed calculations, or even predictions of the energy spectrum. This appears to be a section that was added in response to a referee report. Anecdotal story about Matthaeus.

6. Taylor, G. *The formation of a blast wave by a very intense explosion I. Theoretical discussion*, Royal Society of London Proceedings Series A **201**, 159 (1950). [11]

- (a) **Science Question:** What is the effect of the sudden generation of energy in a highly concentrated form (unaccompanied by the generation of gas, as in conventional explosives)?
- (b) **Method:** The problem is solved by finding a self-similar solution for the evolution of the blast-wave.

The approximations used in this analytical calculation are:

- i. A finite amount of energy is released in an infinitely concentrated form.
- ii. Assume a self-similar solution, in which the disturbance is similar at all times, merely increasing its linear dimensions with increasing time from initiation.
- iii. Spherical symmetry is assumed.
- iv. Self-similar variables: Properly scaled, all important variables (pressure, density, and radial velocity) depend on a single dimensionless coordinate,  $\eta = r/R$ , such that

$$\frac{p}{p_0} = y = R^{-3} f_1(\eta)$$

$$\frac{\rho}{\rho_0} = \psi(\eta)$$

$$u = R^{-3/2} \phi_1(\eta)$$

It is assumed that the solutions have this form, and then we can find equations for the functions  $f_1(\eta)$ ,  $\psi(\eta)$ , and  $\phi_1(\eta)$  using the equations of motion and continuity and the equation of state for a perfect gas,

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} &= -\frac{p_0}{\rho} \frac{\partial y}{\partial r} \\ \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \left( \frac{\partial u}{\partial r} + \frac{2u}{r} \right) &= 0 \\ \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right) (p \rho^{-\gamma}) &= 0 \end{aligned}$$

- v. Note that the self-similar solution is a function of only  $\gamma$ , the ratio of specific heats.
  - vi. The strong shock limit of the Rankin-Hugoniot relations is assumed, limiting the applicability of the solution to  $y_1 = p_1/p_0 \geq 10$ . Therefore, the analysis ceases to be accurate when the maximum pressure (in the thin shell just behind the shock) decreases to about 10 atm.
- (c) **Citeable Results:**

- i. A spherical shock wave propagates outward whose radius  $R$  is given by

$$R = S(\gamma) t^{2/5} E^{1/5} \rho_0^{-1/5}$$

where  $\rho_0$  is the atmospheric density,  $E$  is the energy released, and  $S(\gamma)$  is a function of  $\gamma$ , the ratio of specific heats of air.

- ii. An atomic bomb is only half as efficient, as a blast-producer, as a high explosive releasing the same amount of energy.
- iii. The maximum pressure in the ideal problem  $p_{\max} \propto R^{-3}$ , and this compares well to measurements of pressures near high explosives.
- iv. Although temperatures are very high near the center of the explosion, the density is very low, so the energy per unit volume of the gas [the enthalpy  $p/(\gamma - 1)$ ] is uniform since the pressure is uniform.
- v. For  $\gamma = 1.4$ ,

$$t = 0.926 R^{5/2} \rho_0^{1/2} E^{-1/2}$$



(d) **Other Comments:**

- i. Taylor was one of the first people to use numerical integration of a set of Ordinary Differential Equations (ODEs) to solve a problem. Using values of  $f$ ,  $\phi$ , and  $\psi$ , at a particular value of  $\eta$ , he was able to compute  $f'$  from eq. (14),  $\phi'$ , from eq. (7a), and  $\psi'$  from eq. (9a).
- ii. His method for integrating the set of ODEs was a predictor-corrector type of algorithm.
- iii. Note the derivation of approximate formulae is very useful for the days before computers were widely available. One may apply the results to a particular problem using these simple formulae to get an approximate solution very quickly. This is still a very useful thing to do, but is rarely done, with the exception of occasional work in astrophysics.
- iv. What will happen *after* the blast wave goes by? The high temperatures in the center of the blast may cool radiatively, rapidly reducing the pressure at the center. Atmospheric pressure will then force gas to flow back toward the center rapidly, leading to severe *blow back*.

7. Taylor, G. *The formation of a blast wave by a very intense explosion II. The atomic explosion of 1945*, Royal Society of London Proceedings Series A **201**, 175 (1950). [12]

- (a) **Science Question:** What was the yield of the 1945 Trinity atomic explosion? Does the scaling behavior of the resulting blast wave agree with the self-similar blast-wave solution derived in Paper I,

$$t = 0.926R^{5/2}\rho_0^{1/2}E^{-1/2}$$

valid for  $\gamma = 1.4$ ?

- (b) **Method:** Photographs of the 1945 Trinity atomic explosion are measured to determine the radius of the shock wave  $R$  as a function of time. The measurements are then compared to the self-similar blast-wave solution derived in Paper I.

The approximations used in this calculation are:

- i. It is assumed that air behaves as though  $\gamma$ , the ratio of specific heats, is constant at all temperatures, an assumption which is certainly not true.
- ii. The energy of the explosion can be expressed as

$$E = K\rho_0R^5t^{-2}$$

where  $K$  is a function of  $\gamma$  only, with values in Figure 3 ranging from  $K \simeq 1.7$  at  $\gamma = 1.2$  to  $K \simeq 0.6$  at  $\gamma = 1.6$ . Assuming an effective value of  $\gamma = 1.4$ , the yield of the 1945 Trinity atomic explosion was 16,800 tons of TNT.

- iii. For the temperature of the gas behind the shock wave at  $R = 100$  m, a value of  $\gamma = 1.29$  is appropriate. Neglecting the effects of radiation, a second estimate of the atomic yield is 23,700 tons of TNT.
- iv. The vertical velocity  $U$  of a large bubble in water is related to  $a$ , the radius of curvature at the top of the bubble, by

$$U = \frac{2}{3}\sqrt{ga}$$

- (c) **Citeable Results:**

- i. The measurements taken from photographs of the 1945 Trinity atomic explosion verified the predicted scaling relationship for a blast wave that  $R^{5/2} \propto t$  over the range  $20 \text{ m} \leq R \leq 185 \text{ m}$ .
- ii. Two estimates of the energy of the atomic blast derived from fitting the proportionality constant, assuming a specific value of  $\gamma$ , are 16,800 tons of TNT and 23,700 tons of TNT.
- iii. The measurements rise of the superheated bubble of air give a value of 35 m/s, in remarkable agreement with the predicted vertical velocity of 35.7 m/s.

- (d) **Other Comments:**

- i. Figure 1 shows the remarkable agreement of the evolution of the atomic shock wave radius vs. time with the theoretical solution to the problem derived by Taylor.
- ii. The effects on the adiabatic index  $\gamma$  due to vibrational degrees of freedom for air at high temperature, to the dissociation of air molecules, and to intense radiation from the center and absorption in the outer regions may somewhat neutralize each other, leading the whole system to behave as though  $\gamma$  has an effective value of 1.4.

8. Takeuchi, S. *Field-aligned accelerations by plasma shocks propagating through interstellar magnetic fields*, Physics of Plasmas, **19**, 070703 (2012). [10]

- (a) **Science Question:** What is the mechanism for the acceleration of high-energy cosmic rays associated with interstellar plasma shocks?
- (b) **Method:** The paper proposes a new mechanism, *field-aligned acceleration*, by positing a magnetic field configuration associated with interstellar plasma shocks and estimates the energy gain by particles trapped in a neutral sheet.

The approximations used in this analytical calculation are:

- i. Assumes two plasma clouds, one of which is a shock propagating with velocity  $v_s$  and containing magnetic field  $B_z$ . The second is the interstellar plasma with zero velocity and  $\mathbf{B}_0 = B_0 \sin \theta \hat{\mathbf{x}} + B_0 \cos \theta \hat{\mathbf{z}}$ , where  $B_0 \ll B_z$ . The cross angle is  $\theta$ .
- ii. The electric field in a particular frame of reference obeys the ideal Ohm's law,  $E + \mathbf{v}/c \times \mathbf{B} = 0$ .
- iii. The shock is assumed to carry its frozen-in magnetic field.
- iv. Charged particles are assumed to be accelerated by the motional electric field due to the moving plasma in the direction of the magnetic field in the stationary plasma.
- v. The principle of superposition of the electromagnetic fields is assumed in the transition region near the shock front.
- vi. The transformed velocity  $V_y$  is assumed to be zero initially and to remain zero at all times in equations (7) and (8).

(c) **Citeable Results:**

- i. None, the paper contains serious mathematical flaws that invalidate the results.

(d) **Other Comments:**

- i. This calculation is seriously flawed for the following reasons:
  - A. The proposed magnetic field configuration violates the Rankine-Hugoniot jump conditions for MHD shocks. In particular, for strong shocks,  $B_{t2} \simeq 4B_{t1}$ . His setup assumes tangential fields  $B_0 \ll B_z$ , with his numerical solution taking  $B_0/B_z = 10^{-2}$ .
  - B. The constraint of coplanarity only allows the tangential component of the magnetic field through a shock to change magnitude across the shock, but not direction. Therefore, an MHD shock will always have  $\theta = 0$ , a condition violated by the proposed magnetic configuration.
  - C. A strong (fast-mode) shock has flow through the shock plane, meaning that the magnetic field cannot be frozen into the shock. In the shock frame in Fig 1, plasma  $A$  still has motion in the  $y$ -direction.
  - D. The condition imposed by the magnetic field configuration proposed could be established as a tangential discontinuity, but this requires total pressure balance  $p + B_t^2/2\mu_0 = \text{const}$  across the discontinuity, also not satisfied by the setup.
- ii. The principle of superposition cannot be arbitrarily used to combine the electromagnetic fields near the shock—the electromagnetic fields must be generated self-consistently and satisfy the constraints imposed by Maxwell's equations (which are incorporated into the Rankine-Hugoniot jump conditions for MHD shocks).
- iii. The acceleration of the particles employs the electric field on one side of the shock and the magnetic field on the other side of the shock, apparently a serious inconsistency.
- iv. In solving for equations (7) and (8), the transformed velocity  $V_y$  is assumed to be zero initially and to remain zero at all times, but the Lorentz force will lead to non-zero  $V_y$  immediately according to eq (5).
- v. The energy gain given by (9) does not depend on angle, but the energy gain from the numerical integration indicates an angular dependence, signaling an internal inconsistency.
- vi. The prediction of indefinite energy gain for  $\pi/2 < \theta < \pi$  must be incorrect due to conservation of energy—the fields must necessarily lose energy to the accelerated particles.

- vii. IN SUMMARY: One is not free to choose any field and plasma configuration that you wish. The electromagnetic fields must satisfy the Rankine-Hugoniot jump conditions for MHD shocks and discontinuities. Once a consistent field configuration is determined, one needs to compute the particle acceleration in that field (assuming a single particle motion description), accounting for the correct relativistic Larmor motion. Then, a self-consistent description must be computed to show that the particle motion does not negatively feedback on the electromagnetic fields.

9. Daughton, W. D., Roytershteyn, V., Karimabadi, H., Yin, L., Albright, B. J., Bergen, B., and Bowers, K. J. *Role of electron physics in the development of turbulent magnetic reconnection in collisionless plasmas*, Nature Physics, **7**, 539 (2011). [3]

- (a) **Science Question:** What is the evolution of collisionless magnetic reconnection in three dimensions? What are the observational signatures of such a process?
- (b) **Method:** Particle in Cell (PIC) simulations on petascale supercomputers follow the three dimensional evolution of a Harris current sheet from tens of ion inertial lengths down to Debye length scales.

The approximations used in this analytical calculation are:

- i. Reduced mass ratio  $m_i/m_e = 100$ .
- ii.  $T_i = T_e$
- iii.  $\omega_{pe}/\Omega_e = 2$
- iv. Plasma  $\beta_i$  is unclear.
- v. Guide field geometry with  $B_{y0} = B_{x0}$ .
- vi. Harris current sheet initial conditions,

$$\mathbf{B} = B_{x0} \tanh(z/\lambda) \hat{\mathbf{x}} + B_{y0} \hat{\mathbf{y}}$$

where the half-thickness of the current sheet is equal to the ion inertial length,  $\lambda = d_i$ .

- vii. Open boundary conditions in the  $x$  and  $z$  dimensions and periodic boundary conditions in the  $y$  (guide-field) direction.
- viii. Open boundary are driven with a 4% large-scale flow pattern.
- ix. Density profile is given by

$$n(z) = n_0 \text{sech}^2(z/\lambda) + n_b$$

where  $n_0$  is central Harris sheet density and  $n_b$  is uniform background density.

(c) **Citeable Results:**

- i. Three-dimensional collisionless reconnection evolves in a turbulent manner and the tearing instability naturally forms helical magnetic structures, or flux ropes.
- ii. Magnetic islands that form in 2D simulations correspond to extended flux ropes that form with a particular angle with respect to the guide field. Dominant modes correspond to  $k_x d_e = 0.08$ .
- iii. Traditional asymptotic methods (which assume  $\lambda \gg \rho_i$ ) fail to estimate the development of oblique tearing modes. Exact numerical solutions of the linear tearing instabilities shows that at oblique angles the asymptotic solution greatly overestimates the growth rates. The reduced mass ratio  $m_i/m_e = 100$  is very similar to the realistic mass ratio case  $m_i/m_e = 1836$ , providing justification.
- iv. The narrow range of oblique tearing modes generated differs from previous expectations, suggesting generation of turbulence will not develop as previously thought.
- v. Electron scale current layers are stable in 2D geometries, but highly unstable to the formation of flux ropes (by tearing instabilities) over a range of oblique angles. Growth time and wavelength of the developing structures is consistent with tearing instabilities in electron current layers.
- vi. At long times, the simulation is dominated by the interaction of highly anisotropic structures across multiple scales, including electron-scale current sheets that continually reform and break up into filaments, along with flux ropes generated at these scales and quickly growing well above ion scales.
- vii. Observations of reconnection in space plasmas are based on the idealized 2D geometry, but need to be re-evaluated based on the more complicated reconnection geometry and flux rope evolution observed in the simulation. In particular, many flux ropes may not have been properly recognized in previous studies.

10. Schmidt-Bleker, A., Gassen, W., and Kull, H.-J. *Nonlinear plasma waves and wavebreaking in quantum plasmas*, *Europhysics Letters*, **95**, 55003 (2011). [7]

- (a) **Science Question:** What is the nonlinear evolution of plasma (Langmuir) waves up to and beyond the wavebreaking limit in a quantum plasma?
- (b) **Method:** Numerical method for the simulation of nonlinear quantum plasma dynamics using the carrier-envelope wave (CEW) method.

The approximations used in this analytical calculation are:

- i. Collisionless (quantum Vlasov equation)
- ii. Electrostatic approximation (Vlasov-Poisson system).
- iii. Results are compared to the Lindhard dispersion relation for ideal quantum plasmas.
- iv. Only electron dynamics are computed—ions are considered a neutralizing background  $n_0$ .
- v. Statistical operator  $\rho$  is expressed as an ensemble of representative quantum states  $|\psi\rangle$ ,

$$\rho = \sum_s w_s |\psi\rangle \langle \psi|$$

- vi. In non-equilibrium, the plane waves can be generalized to carrier-envelope waves

$$\Psi_s(\mathbf{r}, t) = \psi_s(\mathbf{r}, t) e^{i(\mathbf{p}_s \cdot \mathbf{r} - \epsilon_s t)/\hbar}$$

These describe the equilibrium properties in accordance with the Pauli principle.

- vii. The time-dependent Schrodinger equation is transformed to the rest frame of the carrier-envelope wave.
  - viii. A finite number (50–100) of carrier-envelope waves are used in the calculation.
  - ix. Waterbag model employs a uniform velocity distribution  $-v_{max} < v < v_{max}$ , where  $v_{max} = v_F$ .
  - x. CEW calculations are performed of 1D Fermi distribution with zero temperature and Fermi velocity  $v_F$ . Thus, this is a cold-plasma calculation.
  - xi. Single-stream model: single CE wave with zero carrier momentum, corresponding to a zero temperature with Fermi energy of order the plasmon energy.
- (c) **Citeable Results:**
- i. The carrier-envelope wave (CEW) method for the nonlinear evolution of quantum kinetic plasmas is comparable in efficiency to PIC simulation of classical plasmas.
  - ii. Classic waterbag model with  $v_{max} = v_F$  accounts very well for the wavebreaking amplitudes in quantum plasmas.
  - iii. Difference from classical wave-breaking: cancellation of positive and negative parts of Wigner distribution lead to broad peak with side maxima instead of split double peak.
  - iv. Quantum effects can be explained by a nonlinear coupling of plasmon to free particle modes in the wavebreaking regime. Wavebreaking amplitudes at negligible fluid pressure are limited quantum-mechanically by nonlinearly coupled plasmon and free-particle modes. In single-stream model, large-phase velocity limit leads to a free-particle (high-frequency) mode and a plasmon (low-frequency) mode. Density develops interference fringes in phase space.
  - v. Damping of small amplitude plasma waves is computed with the parameters  $E_F = 1.1\hbar\omega_p$  and  $T = \hbar\omega_p$ . The degeneracy parameter is given by  $\chi E_F/T$ . At increasing temperature the results converge to the Maxwellian distribution.

(d) **Other Comments:**

- i. In the water bag model the distribution function is a constant inside a certain region of phase space and zero outside [2].
- ii. For Fermi energy exceeding both average interaction energy and temperature, a weakly coupled degenerate electron gas is obtained.

- iii. Plasmon is a quantum of a plasma oscillation. The plasmon is a quasiparticle resulting from the quantization of plasma oscillations just as photons and phonons are quantizations of electromagnetic and mechanical vibrations.
- iv. Wavebreaking is a fundamental threshold for the nonlinear propagation of plasma waves. Wavebreaking occurs when the phase-space trajectory  $v = v(x(x))$  develops a vertical slope. This is important for wave energy dissipation and associated electron heating in collisionless plasmas. Wavebreaking in quantum kinetic theory is of interest for violation of both fluid and classical approximations.
- v. Quantum Vlasov equation is equivalent to the propagation of quantum states by the time-dependent Schrodinger equation,

$$i\hbar\partial_t|\psi\rangle = H|\psi\rangle$$

- vi. Expectation value for any observable  $A$  is  $\langle A \rangle = \text{Tr}(\rho A)$

11. Schwabe, M. *et al.* *Direct measurement of the speed of sound in a complex plasma under microgravity conditions*, Europhysics Letters, **96**, 55001 (2011). [8]

- (a) **Science Question:** What is the speed of sound in a three-dimensional complex (dusty) plasma?  
 (b) **Method:** Measurement of the Mach cone angle arising from a supersonic probe traveling at  $1 < M < 3$  is used, along with the Mach cone relation

$$\sin \theta = c/v = 1/M$$

to determine the speed of sound  $c$ . In addition, a molecular dynamics simulation is performed to compare with and aid in interpretation of the experimental results.

The approximations, experimental setup, or other simplifications used in this paper are:

- i. Experiment is performed on the PK-3 Plus laboratory on board the International Space Station.
- ii. Capacitively coupled plasma chamber with circular electrodes of 6 cm diameter and 3 cm apart, with microparticles injected. Particles form a cloud around the center of the chamber with a central void caused by ions streaming outwards.
- iii. Microparticles of 2.55  $\mu\text{m}$  diameter,  $m_d = 1.31 \times 10^{-14}$  kg.
- iv. Subtraction of two subsequent frames shows double Mach cone structure.
- v. Velocity of probe particle decreases from 80 mm/s to 37 mm/s, determined by manually computing the position of the probe particle as a function of time.
- vi. Mach cone angles are determined by measuring by eye and by correlating the image with a rotated rectangle (this seems like a weird choice).
- vii. Bigger particles of unknown origin are used as natural supersonic probes of the microparticle cloud.
- viii. Three-dimensional molecular dynamics simulation. Forces on the microparticles are
  - A. Force due to the potential field of the probe particle
  - B. Shielded Coulomb interaction from other microparticles,

$$\phi(r_{ij}) = (eZ_d/r_{ij}) \exp(-r_{ij}/\lambda)$$

where  $Z_d = 2400$  is the charge carried by the microparticle and  $\lambda = 38\mu\text{m}$  is the Debye length.

- C. Neutral gas frictional damping, with frequency  $\gamma = 49\text{mbos}^{-1}$
- D. Stochastic force due to random kicks from surrounding molecules, modeled by Gaussian white noise.
- E. Note that friction and random force are related to the complex plasma temperature  $T = 0.025$  eV through a fluctuation-dissipation theorem.
- ix. Various values of average particle distances,  $\Delta = 165 \mu\text{m}$ – $180 \mu\text{m}$ .
- x. Electron density depletion due to the high microparticle density is neglected, characterized by Havnes parameter,  $H = Z_d n_d / n_e$ . How strong of an effect would this have? This is not discussed.
- xi. For the simulation, a particle spacing  $\Delta = 165 \mu\text{m}$  is assumed.
- xii. Speed of sound is predicted using the equation

$$c = \sqrt{\frac{Z_d k_B T_i}{m_d} \frac{H}{1+H}}$$

where ion temperature  $T_i$  is assumed to be room temperature,  $T_i = 300$  K.

(c) **Citeable Results:**

- i. The speed of sound in the complex plasma is 28 mm/s  $\pm 20\%$ , determined by a fit to the Mach cone relation using both experimental measurements and simulation results.



- ii. A double Mach cone structure is seen in both experiment and simulation.
- iii. Using the measured speed of sound and the equation for  $c$ , they infer  $Z_d = 2400$ . Drift motion limited theory prediction for charging is closest to this value (modified orbital motion theory is also close).

(d) **Other Comments:**

- i. What is one of the key unknowns in these complex plasmas? The microparticle charge,  $Z_d$ , which influences the complex plasma properties.
- ii. For the simulation, a particle spacing  $\Delta = 165 \mu\text{m}$  is assumed. But this does not agree with the estimated microparticle density from the experiment,  $300 \text{ mm}^{-3}$  to  $400 \text{ mm}^{-3}$ , which give the values  $\Delta = 149 \mu\text{m}$  to  $\Delta = 136 \mu\text{m}$ . Why this discrepancy is not addressed is unclear.
- iii. Why does this need to be done under microgravity conditions? This prevents the complex plasma from being strongly compressed by gravity to a more two-dimensional configuration. Under microgravity conditions, the dynamics of a three-dimensional complex plasma can be studied.
- iv. It is interesting that they use bigger particles of unknown origin, which would generally be considered a nuisance in this experiment, as a supersonic probe of the complex plasma. “If you are given lemons, make lemonade.”
- v. Double Mach cone structure is attributed to the restoring force on the damped oscillating background microparticles or to the dispersive nature of the sound waves in strongly coupled complex plasmas.
- vi. They mention a number of different theories, but don’t give the specific results from each of these theories, so it is hard to understand the variations with respect to these different theories.

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