29:235 Homework #9

Suggested Reading: Read KR95 Chapter 4 (p.91–127)

Due at the beginning of class, Thursday, March 29, 2012.

1. Given an average solar wind, estimate how much (a) mass, (b) momentum, and (c) energy is lost by the Sun per unit time. If the solar wind were constant in time, (d) how long would it take to lose all the solar mass at this rate?

2. The deflection of cometary tails was one of the first pieces of evidence pointing towards the existence of a continuous solar wind flow. A comet's ion tail is composed of cometary ions embedded in the solar wind. Consider an ion tail that points 4° away from the radial direction. Estimate the solar wind speed if the azimuthal component of the comet's orbital velocity is 30 km/s.

3. Above what coronal temperature would a transonic solution for an isothermal solar wind be impossible? HINT: Consider the form of the equation for the critical radius for the isothermal solar wind model.

4. Determining how the plasma parameters of a given system depend upon the independent variables is a powerful technique called scaling theory. Here we will construct a scaling theory for the near-earth solar wind, determining how the temperature varies with the heliocentric radius, $T \propto r^\alpha$, where we want to determine the exponent α.

   A simple steady-state model of the near-earth solar wind specifies a solar wind with a constant radial velocity $\mathbf{v} = v_0\hat{\mathbf{r}}$ and a density that scales with radial distance from the sun as $n = n_0(r_0/r)^2$ where $r_0 = 1$ AU is the heliocentric distance of the earth.

   (a) Assuming the solar wind is a fully ionized plasma of protons and electrons, but that the effects of the magnetic field are negligible, calculate the predicted scaling of the plasma temperature as a function of radius using the Adiabatic Equation of State. You may make use of the ideal gas law, $p = nT$.

   (b) The solar wind, however, is a weakly collisional magnetized plasma. Therefore, the temperatures parallel and perpendicular to the magnetic field will evolve independently for adiabatic changes. Near the earth, the interplanetary magnetic field forms the Parker spiral, with the radial and azimuthal components of the magnetic field related by

   $$\frac{B_\phi}{B_r} \simeq \frac{-r\Omega_0}{v_r},$$

   where $\Omega_0 = 2.85 \times 10^{-6}$ rad/s is the equatorial angular velocity of the solar surface and we assume $B_\theta = 0$. The conservation of radial magnetic flux in a spherically symmetric system implies that the radial magnetic field scales as $B_r = B_0(r_0/r)^2$. Using the Double Adiabatic Equations of State, find the predicted scaling of the perpendicular and parallel temperatures $T_\perp$ and $T_\parallel$ with the radius *in the limit* that $r\Omega_0/v_0 \ll 1$. You may use the relations $p_\perp = nkT_\perp$ and $p_\parallel = nkT_\parallel$.

   (c) Find the scaling of $T_\perp$ and $T_\parallel$ with r in the opposite limit $r\Omega_0/v_0 \gg 1$.