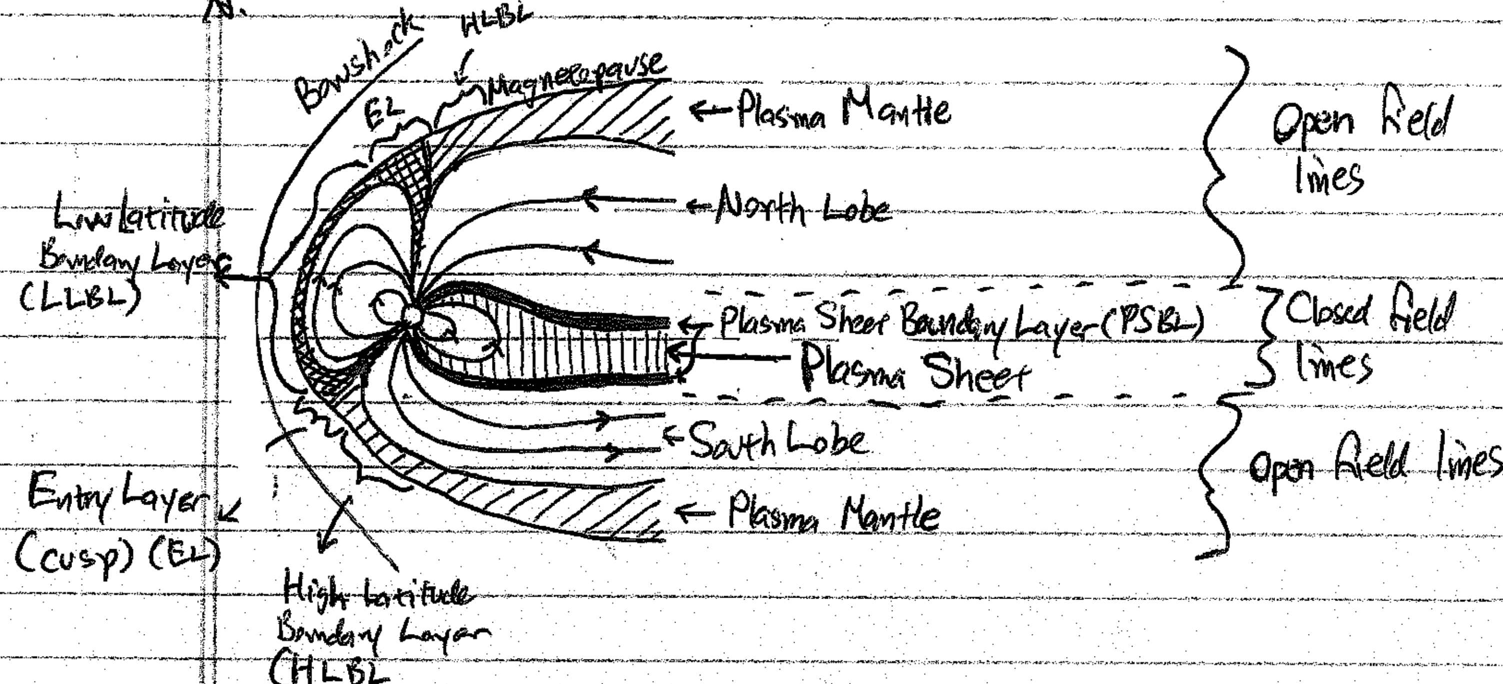
Lecture #10 Boundary Layers, Magnetopause Current, and Magnetotail Current

I. Boundary Layers at the Magnetopause

A.



1. The magnetopause is not observed to be an impenetrable boundary (tangential discontinuity)
   ⇒ Boundary Layers at and within the magnetopause carry a mixture of magnetosheath plasma and magnetospheric plasma.

2. Low Latitude Boundary Layer (LLBL): Mixture of magnetosheath & magnetospheric plasma, about 0.5 R_E thick.

3. Entry Layer (EL): At the polar cusp, these finite-sized regions have open magnetic field lines that allow magnetosphere plasma to penetrate down to the polar atmosphere.

4. High Latitude Boundary Layer (HLBL): Tailward of the cusp, contains the plasma mantle, a mixture of magnetosheath plasma and polar ionospheric plasma, flowing tailward.

I. A. (Continued)

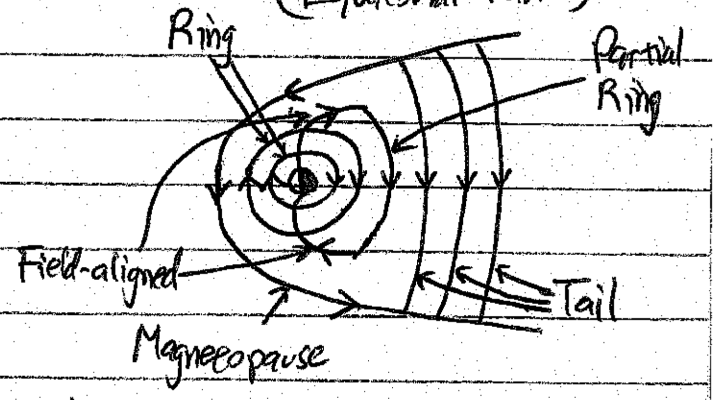
5. Plasma Mantle gradually transitions from magnetosheath plasma to low density lobe conditions. (where magnetic pressure dominates)
6. Plasma Sheet Boundary Layer (PBSL): Contains field aligned ion and electron flows both earthward and tailward, actively connecting reconnection regions in the magnetotail to the high-latitude (polar) Ionosphere
7. Plasma Sheet: Heated and compressed plasma along the equatorial plane of the magnetotail.

II. Magnetospheric Current Systems

A. Currents shape the Geomagnetic Field

1. Currents within the Earth's core generate a nearly perfect dipole field beyond 2 R_E
2. The highly elongated magnetosphere therefore requires external, magnetospheric currents to generate the observed field.
3. Five principal current systems:
   a. Magnetopause current
   b. Tail current
   c. Ring current
   d. Field-aligned currents
   e. Ionospheric currents

(Equatorial Plane)



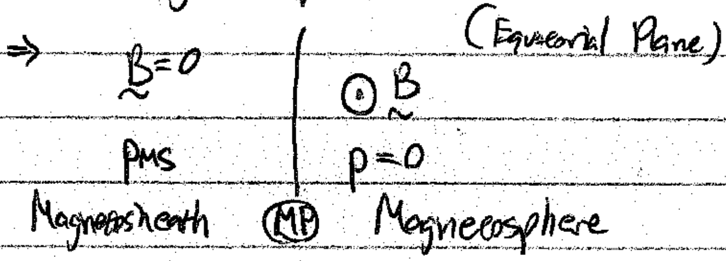
4. Regions where these currents contribute:
   a. Inner Magnetosphere: Ring
   b. Middle Magnetosphere: Partial Ring, Field-aligned
   c. Outer Magnetosphere: Magnetopause, Tail, Field-aligned (not shown)

I. B. The Magnetopause (Chapman-Ferraro) Current

1. Why is a current generated at the magnetopause?

a. An MHD picture of the magnetopause is a tangential discontinuity, with total pressure balance $P_{tot} = p + \frac{B^2}{2\mu_0}$ across the boundary, where we neglect thermal pressure inside and magnetic pressure outside (and $B_n = 0$).

$\Rightarrow$



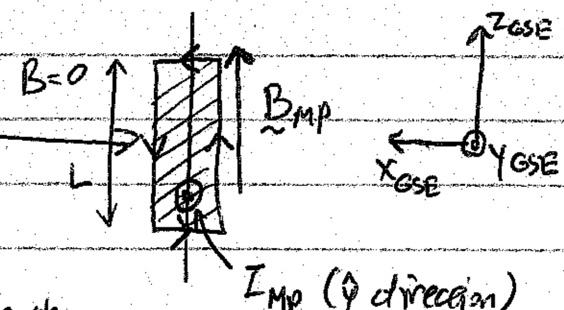
b. To have a discontinuous jump in the tangential $\underset{\sim}{B}_t$ requires a current.

c. From Ampere's Law (Jackson E&M),

1. $\nabla \times \underset{\sim}{B} = \mu_0 j$

(Meridional Plane)

2. Integrate over surface A of Amperian Loop



$$\int d\underset{\sim}{A} \cdot \nabla \times \underset{\sim}{B} = \mu_0 \underbrace{\int d\underset{\sim}{A} \cdot j}_{\text{Total current through loop, } I_{MP}}$$

3. By Stoke's Theorem,

$$\int d\underset{\sim}{A} \cdot \nabla \times \underset{\sim}{B} = \oint d\underset{\sim}{l} \cdot \underset{\sim}{B} = B_{MP}(L) + 0(L) = L\, B_{MP}$$

$\nwarrow$ Magnitude of Magnetospheric $\underset{\sim}{B}$ at the magnetopause.

4. Thus $B_{MP} = \mu_0 \left(\frac{I_{MP}}{L}\right)$ $\leftarrow$ current per unit length along MP.

5. Thus, a current I_{MP} flows along the magnetopause surface in the $\hat{y}$ direction, supporting a discontinuous jump in the $\hat{z}$ component of the magnetic field.

I. B. (Continued)

2. The magnetopause current:

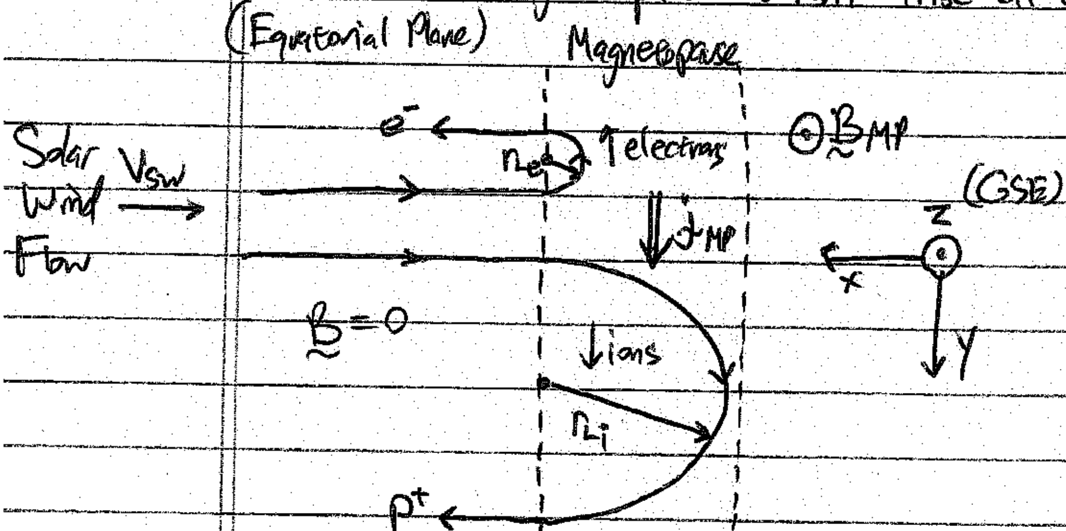
a) Cancels Earth's dipole field outside magnetopause, and doubles the field within the magnetopause (Compression $\mathcal{F} \simeq 2$).

b) Provides force density through $\underset{\sim}{j} \times \underset{\sim}{B}$ force in MHD Momentum Equation to change the momentum of the solar wind.

$$\rho \frac{\partial \underset{\sim}{U}}{\partial t} + \rho (\underset{\sim}{U} \cdot \nabla) \underset{\sim}{U} = -\nabla \left(p + \frac{B^2}{2\mu_0}\right) + \frac{(\underset{\sim}{B} \cdot \nabla)\underset{\sim}{B}}{\mu_0} = -\nabla p + \underset{\sim}{j} \times \underset{\sim}{B}$$

3. How does magnetopause current arise at a single particle motion level?

(Equatorial Plane)



Simple Estimate

a. Ions dominate current due to $r_{Li} \gg r_{Le}$

b. Current passing through $y=0$ plane due to ions:

$\left(\frac{I}{L}\right) = 2 r_{Li} n_{sw} V_{sw} q_i$ per unit length L in z.

c. Since $r_{Li} = \frac{V_{sw}}{\Omega_i} = \frac{V_{sw} m_p}{q_i B_{MP}}$, we obtain $\left(\frac{I}{L}\right) = 2 \frac{n_{sw} m_p V_{sw}^2}{B_{MP}}$

d. But, from Ampere's Law, $\frac{I_{MP}}{L} = \frac{B_{MP}}{\mu_0}$, so we obtain

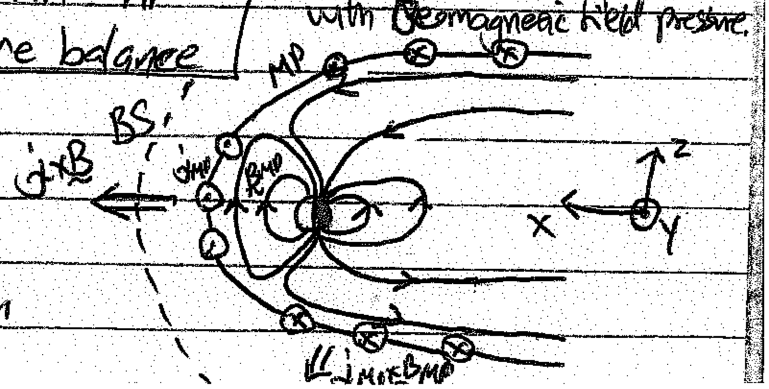
$$\frac{B_{MP}^2}{2\mu_0} = n_{sw} m_p V_{sw}^2 \xrightarrow{\text{Same result}} \text{as Lect \#9, IIC.5.b.}$$

$$\underbrace{\rho_{sw} U_{sw}^2 = \frac{B_{MP}^2}{2\mu_0}}_{\text{Derived from Pressure Balance of SW Ram Pressure with Geomagnetic field pressure.}}$$

$\boxed{\text{e. Therefore the magnetopause current } I_{MP} \text{ is consistent with MHD pressure balance}}$

4. MHD $\underset{\sim}{j} \times \underset{\sim}{B}$ force:

a. At nose of Magnetopause, $\underset{\sim}{j}_{MP} \times \underset{\sim}{B}_{MP}$ is in $\hat{x}$ direction



I. B. 4. (Continued)

b. Consider Force Balance in MHD Steady State

1. $\cancel{\frac{\partial \rho}{\partial t}}^{0} + \nabla\cdot(\rho \underset{\sim}{U}) = 0$ Continuity Eq.

2. $\rho \cancel{\frac{\partial U}{\partial t}}^{0} + \rho(\underset{\sim}{U}\cdot\nabla)\underset{\sim}{U} = -\nabla p + \underset{\sim}{j} \times \underset{\sim}{B}$ Momentum Eq.

3. Using Continuity equation, we obtain $\underbrace{\underset{\sim}{j}\times \underset{\sim}{B}}_{\text{Force of Magnetopause Current \& Magnetospheric field}} = \nabla(\underbrace{\rho U^2 + p}_{\text{Ram + thermal pressure in Magnetosheath}})$

4. Solving for j_{MP}:

$$\boxed{j_{MP} = \frac{B_{MP}}{B_{MP}^2} \times \nabla(\rho U^2 + p)}$$

5. Estimate: a. Magnetosheath ram + thermal pressure = SW ram pressure

$$\Rightarrow \rho U^2 + p = \rho_{sw} U_{sw}^2$$

b. At MP, Magnitude of compressed magnetospheric field is

$$B_{MP} \simeq \frac{\mathcal{F} M}{r^3} \qquad \mathcal{F} = 2 \qquad M = 30.4\,\mu T\, R_E^3 \qquad r = 10 R_E \Rightarrow B_{MP} \simeq 60\,nT$$

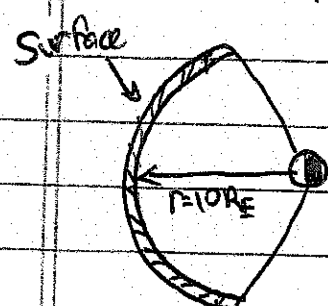
c. Letting $\Delta \equiv$ thickness of Magnetopause, we estimate

$$j_{MP} \simeq \frac{\rho_{sw} U_{sw}^2}{B_{MP}\,\Delta} \qquad n_{sw} = 7\,cm^{-3} = 7\times10^6\,m^{-3} \qquad v_{sw} = 450\,\frac{km}{s} = 4.5\times10^5\,\frac{m}{s} \qquad \rho_{sw} = n_{sw} m_p$$

d. Take $\Delta \simeq 500$ Km (a typical value for magnetopause thickness)

$$j_{MP} \simeq \frac{(7\times10^6\,m^{-3})(1.67\times10^{-27}\,kg)(4.5\times10^5\,\frac{m}{s})^2}{(60\times10^{-9}\,T)(5.0\times10^5\,m)} \simeq \boxed{10^{-7}\,\frac{A}{m^2} = j_{MP}}$$

6. Estimate Total Magnetopause current $I_{MP} \doteq \int d\underset{\sim}{s}\cdot j_{MP} \simeq A \cdot j_{MP}$ (area)



width $= \Delta = 500$ km

length $= \frac{1}{2} C = \frac{2\pi r}{2} = \pi r$

$A = \Delta \pi r$

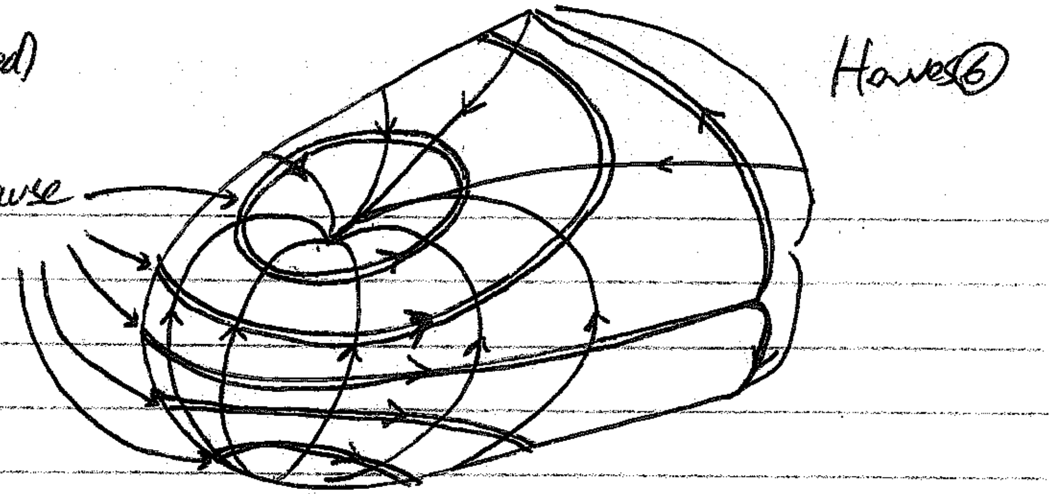
$$\Rightarrow I_{MP} = \overbrace{\Delta \pi\, 10 R_E}^{A}\, j_{MP} = (5\times10^5\,m)\,\pi\,10(6.4\times10^6\,m)(10^{-7}\,\tfrac{A}{m^2})$$

$$\boxed{I_{MP} \simeq 10^7\,A}$$

Lecture #10 (Continued)

I. B. (Continued)

7. Magnetopause current

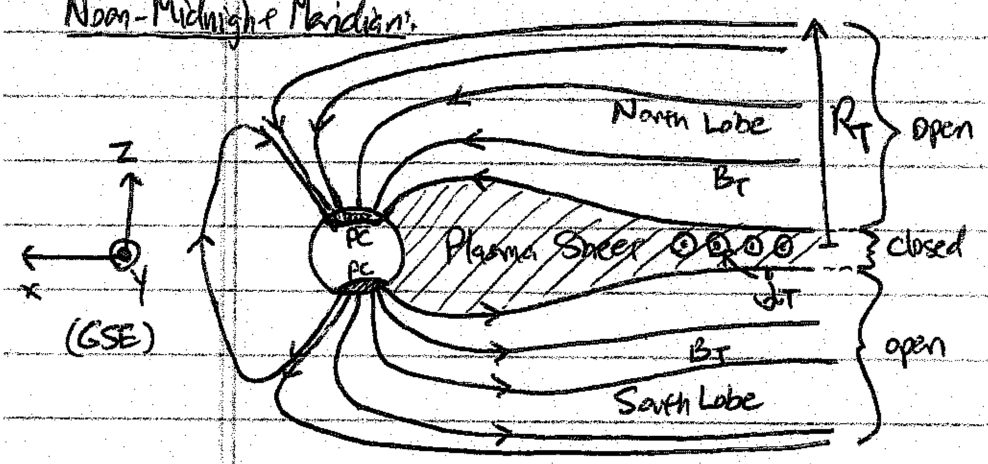


8. a. The effect of the magnetopause current is to increase the current at the Earth's surface.

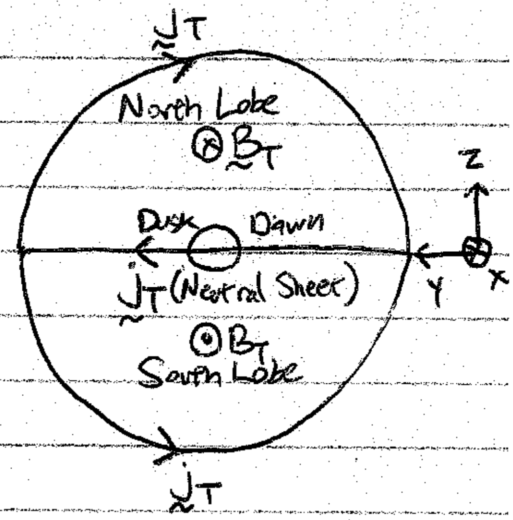
b. When the solar wind dynamic pressure increases (due to interaction with a Coronal Mass Ejection, for example), surface $\underset{\sim}{B}$ field increases by 10's nT.

C. The Tail Current:

Noon-Midnight Meridian:



Dawn-Dusk Meridian:



1. The Tail current is comprised of:
   a. A dawn to dusk neutral sheet current
   b. Return currents around north and south lobes from dusk to dawn on the magnetopause.

 – Theta "Θ" configuration

I. C. (Continued)

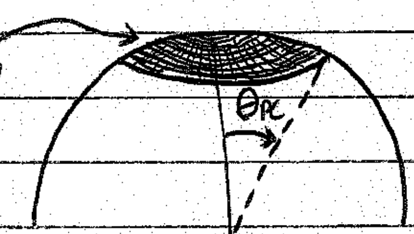
2. The open magnetic field lines of the North Lobe emerge from the North Polar Cap.

a. Because magnetic flux is conserved, we can estimate the tail radius R_T by equating the polar cap magnetic flux Φ_{PC} to the magnetic flux in the North lobe, Φ_T

b. <u>Flux through the North Polar Cap:</u>

1. $\Phi_B \equiv \int d\underset{\sim}{A} \cdot \underset{\sim}{B} = \int dA\, B_r$

Polar cap



$\theta_{PC} \simeq 15°$ (Colatitude)

2. $\Phi_{PC} = \int_0^{2\pi} r\,d\phi \int_0^{\theta_{PC}} r\sin\theta\, d\theta\, B_r(R_E, \theta)$ where $r = R_E$.

3. From lecture #8, $\underset{\sim}{B}(r,\theta) = \frac{2M}{r^3}\cos\theta\, \hat{\underset{\sim}{r}} + \frac{M}{r^3}\sin\theta\, \hat{\underset{\sim}{\theta}}$

where $M = \frac{B_E R_E^3}{r^3}$ and $B_E = -30.4\,\mu T$ is the equatorial field at $r = R_E$,

So, $B_r(R_E, \theta) = \frac{2 B_E R_E^3}{R_E^3}\cos\theta = 2 B_E \cos\theta$

4. $\Phi_{PC} = 2\pi R_E^2 \int_0^{\theta_{PC}} d\theta\, \sin\theta\, [2 B_E \cos\theta] = 4\pi R_E^2 B_E \left. \frac{\sin^2\theta}{2} \right|_0^{\theta_{PC}}$

$$\boxed{\Phi_{PC} = 2\pi R_E^2 B_E \sin^2\theta_{PC}}$$

c. Flux through North Lobe of Magnetotail:

1. $\boxed{\Phi_T = \frac{1}{2}(\pi R_T^2) B_T}$

d. Setting $\Phi_{PC} = \Phi_T$ and solving for R_T, we obtain

$$\boxed{R_T = \left(\frac{4 B_E}{B_T}\right)^{\frac{1}{2}} \sin\theta_{PC}\ R_E}$$

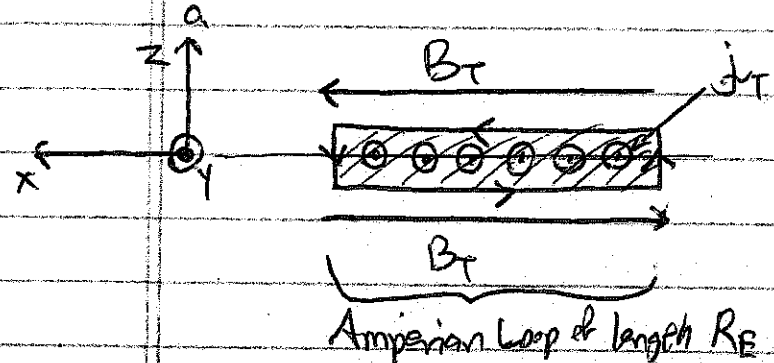
e. Measurements of $B_T = 20\,nT = 20\times10^{-9}\,T$, so

$R_T = \left(\frac{4(30\times10^{-6}T)}{20\times10^{-9}T}\right)^{\frac{1}{2}} \sin 15°\, R_E \simeq 20\, R_E \Rightarrow \boxed{R_T \simeq 20\, R_E}$

Lecture #10 (Continued) Howes (8)

I. C. (Continued)

3. Estimate Magnitude of Tail Current, I_T



b. Again, from Ampere's Law (see I.B.i.c.),

$$\underbrace{\oint d\underset{\sim}{l} \cdot \underset{\sim}{B} = \mu_0 \int d\underset{\sim}{A} \cdot j_T}$$

Total Tail current per length R_E, I_T

c. $2 B_T R_E = \mu_0 I_T \Rightarrow I_T = \frac{2 B_T R_E}{\mu_0} = \frac{(2)(20\times10^{-9}T)(6.4\times10^6 m)}{(4\pi\times10^{-7}\frac{H}{m})} = 2\times10^5 A$

d. Thus, each length R_E of the tail carries $I_T = 2\times10^5 A$.

e. Since the length of the magnetotail $L \sim 100 R_E$, we obtain

$\boxed{I_T \simeq 2\times10^7 A}$ comparable to magnetopause current.

4. The Tail current is carried within the plasma sheet:

a. Plasma sheet:

$n \simeq 0.3\ cm^{-3}$

$T_i \simeq 4$ keV

$T_e \simeq 0.6$ keV

Plasma beta $\beta \gtrsim 1$

Current sheet Thickness $\Delta \sim 1-2\ R_E$

$B \sim 1-5 nT$ } Little magnetic pressure

b. Tail lobes:

$n \sim 0.03\ cm^{-3}$

$T_i \sim 3$ keV

$T_e \sim 0.9$ keV

} Little thermal pressure

$B \sim 20$ nT

Plasma beta: $\beta \ll 1$