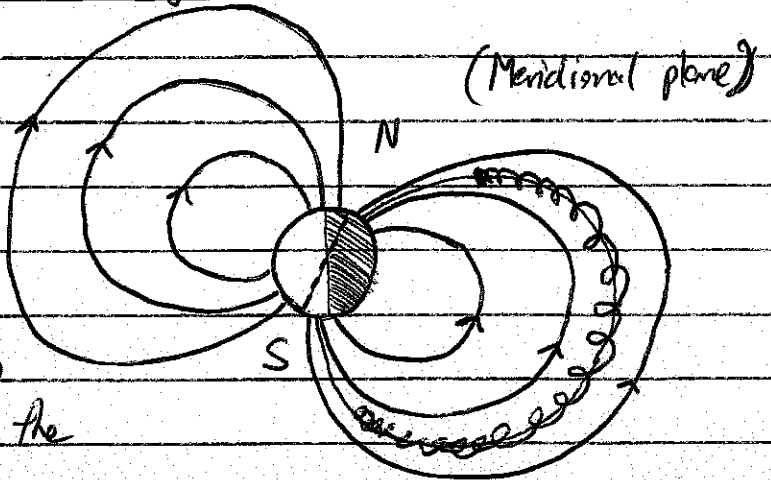


Lecture #11: Ring Current and Field-Aligned Currents

I. Ring Current:

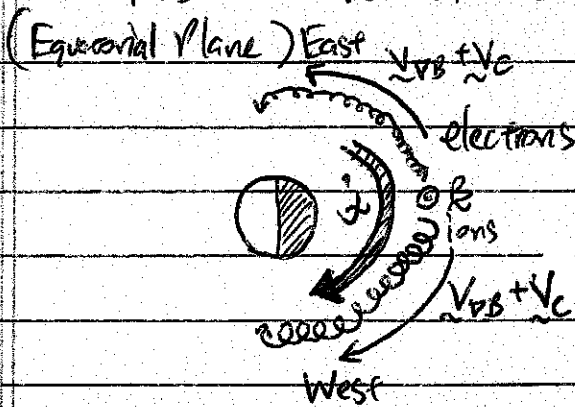
A. General Picture:

1. In the inner magnetosphere, over $3R_E \leq r \leq 6R_E$, particles can be trapped in the mirror configuration and the dipole magnetic field.



2. Mirror Force: Leads to parallel bounce motion of ions and electrons from north polar to south polar regions and back.

3. Azimuthal Drift: ∇B and curvature drifts lead to drift of ions westward and electrons eastward:



a. From lecture #2:

$$v_{VB} = -\frac{v_{\perp}^2}{2\Omega} \frac{\nabla B \times \underline{B}}{B^2}$$

$$v_c = \frac{v_{\perp}^2}{\Omega R} \frac{\underline{R} \times \underline{B}}{R^2}$$

4. Ions dominate ring current

a. Consider ratio of v_{VB} for ions to v_{VB} for electrons:

$$\frac{|v_{VB_i}|}{|v_{VB_e}|} = \frac{\left(-\frac{v_{\perp i}^2}{2\Omega_i} \frac{\nabla B \times B}{B^2} \right)}{\left(-\frac{v_{\perp e}^2}{2\Omega_e} \frac{\nabla B \times B}{B^2} \right)} = \frac{v_{\perp i}^2 \Omega_e}{v_{\perp e}^2 \Omega_i} = \frac{m_i v_{\perp i}^2 \left(\frac{1}{2} m_i v_{\perp i}^2 \right)}{m_e v_{\perp e}^2 \left(\frac{1}{2} m_e v_{\perp e}^2 \right)} = \frac{W_i}{W_e}$$

b. Typical ion energies in the ring current are $3 \text{ keV} \leq W_i \leq 300 \text{ keV}$ while electrons have energies $0.3 \text{ keV} \leq W_e \leq 30 \text{ keV}$.

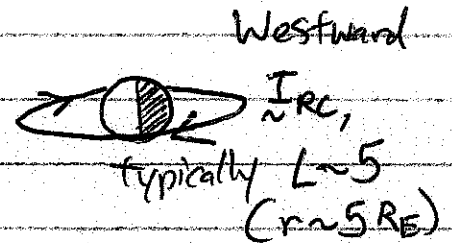
[Thus ions tend to dominate the ring current.]

	Energy	Dominant Species
5. Dominant Ring Current Ion Species:	$E_i < 10 \text{ keV}$	O^+
	$10 \text{ keV} < E_i < 50 \text{ keV}$	O^+ and H^+
	$E_i > 50 \text{ keV}$	H^+

B. Magnetic Field due to a Current Loop:

1. Biot-Savart Law (Jackson, ECM)

$$\underline{B}(\underline{r}) = \frac{\mu_0}{4\pi} \int d^3r' \frac{j(\underline{r}') \times (\underline{r} - \underline{r}')}{|\underline{r} - \underline{r}'|^3}$$



2. For a loop of current I_0 at radius R_0 ,

$$j = -I_0 \delta(r - R_0) \delta(z) \hat{\phi} \quad (-\hat{\phi} \text{ is westward})$$

3. Computing the magnetic field at $\underline{r} = 0$ (center of the Earth),

$$\underline{B}(0) = -\frac{\mu_0 I_0}{2R_0} \hat{z}$$

a. Field is generally in $-\hat{z}$ direction, opposing dipole field of the Earth's surface.

C. Simple Estimate of Ring Current and Magnetic Field Perturbation

1. Consider a single particle with pitch angle $\alpha = 90^\circ$ and energy $W_i = \frac{1}{2} m v^2$ in the equatorial plane.

a. Since $\alpha = 90^\circ$, $v = v_\perp$ and $v_{||} = 0$, so $\underline{v}_c = 0$ and westward drift is due solely to $\nabla \underline{B}$ drift.

2. From Lecture #2, V.C.4, $\underline{v}_{\nabla B} = -\frac{v_\perp^2}{2\Omega} \frac{\nabla B \times \underline{B}}{B^2}$

where

a. Earth's dipole field in equatorial plane, $\underline{B} = \frac{B_E R_E^3}{r^3} \hat{\phi}$ with $B_E = -30.4 \mu T$

$$\text{So } \nabla B = \frac{\partial}{\partial r} \frac{B_E R_E^3}{r^3} \hat{\phi} = -\frac{3 B_E R_E^3}{r^4} \hat{\phi} = -\frac{3 B}{r} \hat{r}$$

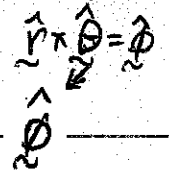
Lecture #1 (Continued)

I.C. (Continued)

Northward Field

Lines 3

3. Thus,
$$\vec{v}_{\perp B} = \frac{m v_{\perp}^2}{2qB} \frac{\left(-\frac{3B}{r} \hat{r}\right) \times (-B \hat{\theta})}{B^2} = \frac{-3W_{\perp}}{qBr} \hat{\phi}$$



$$= \frac{-3W_{\perp} r^2}{q B_E R_E^3} \hat{\phi} = \frac{-3W_{\perp} r^2}{q B_E R_E^3} \hat{\phi}$$

4. Current from the v_B drift of this single particle, I_i :

$$\vec{I}_i = \frac{Q}{T} = \frac{q}{\left(\frac{2\pi r}{v_{\perp B}}\right)} = \frac{q v_{\perp B}}{2\pi r} = \frac{q}{2\pi r} \left(\frac{-3W_{\perp} r^2}{q B_E R_E^3}\right) \hat{\phi} = \frac{-3W_{\perp} r}{2\pi B_E R_E^3} \hat{\phi} = \vec{I}$$

Westward

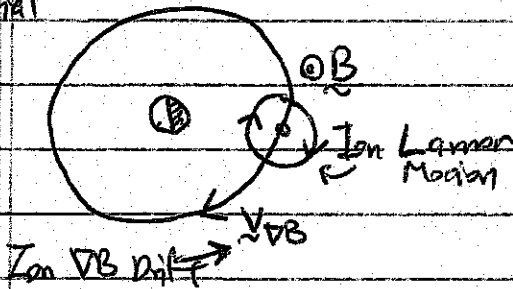
5. Magnetic Perturbation at $r=0$: δB

$$\delta \vec{B}(0) = -\frac{\mu_0 I_i}{2r} \hat{z} = -\frac{\mu_0}{2r} \left(\frac{3W_{\perp} r}{2\pi B_E R_E^3}\right) \hat{z} = -\frac{3\mu_0 W_{\perp}}{4\pi B_E R_E^3} \hat{z}$$

Independent of distance to ring current! (no r dependence)

D. Contribution Due to Larmor Motion:

Equatorial Plane



1. Field due to Larmor motion at Earth's center $r=0$ is in $+\hat{z}$ direction.

2. Dipole Moment due to Larmor Motion:

$$\vec{\mu} = \frac{-m v_{\perp}^2}{2B} \hat{z}$$
 (see Lecture #3, I.A.)

3. Far from the Dipole, the magnetic field is given by

a.
$$\vec{B} = \frac{-\mu_0}{4\pi} \frac{\vec{\mu}}{r^3}$$
 in the equatorial plane

b. Note, for $r = 5R_E$ and $W_{\perp} = 50 \text{ keV}$, $r_L \approx 100 \text{ km} \ll r$, so the far-field approximation above is valid.

c. Note, the Earth's field has $\vec{\mu}_E = \frac{\mu_0}{4\pi} \vec{\mu}_E$, where $\vec{\mu}_E = B_E R_E^3$ and $B_E = -30.4 \mu\text{T}$.

Dipole Moment

Z.D. (Continued)

4. Therefore, the contribution to the field at $r=0$ due to dipole moment,

$$\delta \vec{B}_{d1}(0) = \frac{\mu_0}{4\pi} \left(\frac{-m v^2}{2B} \hat{z} \right) \frac{1}{r^3} = \frac{\mu_0}{4\pi} \frac{W_1}{B_E R_E^3} \frac{1}{r^3} \hat{z} = \frac{\mu_0}{4\pi} \frac{W_1}{B_E R_E^3} \hat{z}$$

Again, independent of r !

E. Total Magnetic Field Perturbation due to Ring Current, $\Delta \vec{B}_{RC}$:

1. For a single particle,

$$\Delta \vec{B}_{RC1}(0) = \delta \vec{B}_{DB1}(0) + \delta \vec{B}_{d1}(0) = -\frac{\mu_0}{2\pi} \frac{W_1}{B_E R_E^3} \hat{z}$$

a. This field perturbation weakens Northward field at equatorial surface of the earth.

2. For N total particles with energy W_1 , $W_{tot} = N W_1$, and total field perturbation is

$$\Delta \vec{B}_{RC} = -\frac{\mu_0}{2\pi} \frac{W_{tot}}{B_E R_E^3} \hat{z} \quad \text{eq. (1)}$$

3. This result has made several simplifying assumptions:

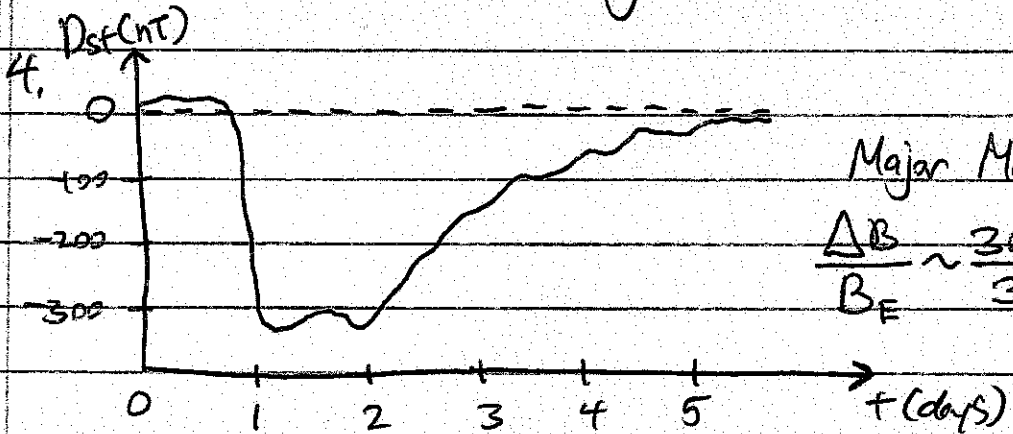
- Distance to ring current r (or L value) is assumed the same for all particles \Rightarrow But r (or L) drops off, so etc!
- N particles with energy W_1
- All particles have pitch angle $\alpha = 90^\circ$ (equatorially trapped)

4. A more detailed analysis shows eq. (1) holds for arbitrary pitch angle distributions. (not just $\alpha = 90^\circ$)

5. The energy distribution of ring current ions has a large impact on the total ring current and therefore total $\Delta \vec{B}_{RC}$.

F. Magnetic Storms and the Ring Current Index, Dst

1. Magnetic Storms lead to an increased injection of particles from the plasma sheet.
2. This increases the total energy W_{tot} of the ring current particles and leads to an enhancement of magnetic ΔB_{RC} .
3. Ring Current Index, Dst:
 - a. Average of ΔB measured near the equator (hourly)
 - b. Stations: Honolulu (USA), San Juan (Puerto Rico), Hermanus (South Africa) and Kakioaka (Japan).
 - c. Although 30% of Dst is due to other sources (magnetopause current, partial ring current, etc.), primarily measures effect of the ring current.



Major Magnetic Storm:
 $\frac{\Delta B}{B_E} \sim \frac{300 \text{ nT}}{30 \mu\text{T}} \sim \underline{\underline{1\%}}$

5. If we assume ΔB_{RC} is responsible for $\frac{1}{2}$ Dst,

a. $\frac{1}{2} Dst \approx \frac{\mu_0}{2\pi} \frac{W_{tot}}{B_E R_E^3} \Rightarrow W_{tot} = \frac{\pi}{\mu_0} B_E R_E^3 Dst$

b. $I_{RC} = \frac{3W_{tot}r}{2\pi B_E R_E^3} = \frac{3r}{2\pi B_E R_E^3} \left(\frac{\pi B_E R_E^3 Dst}{\mu_0} \right) = \frac{3 Dst r}{2 \mu_0}$

6. For $Dst = -300 \text{ nT}$ and $r = 5R_E$, we obtain

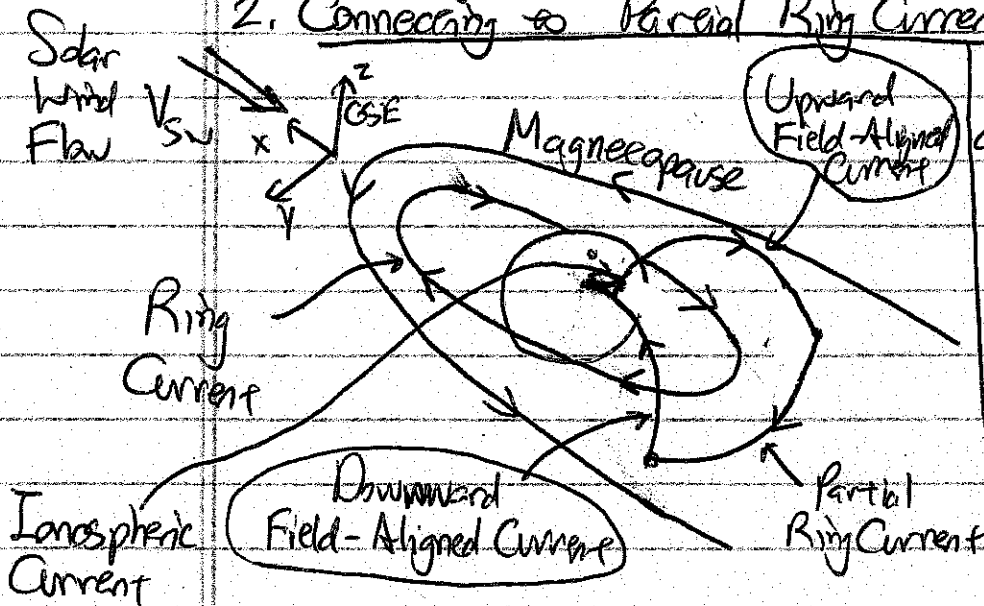
$W_{tot} \sim 6 \times 10^{15} \text{ J}$ and $I_{RC} \sim 10^7 \text{ A}$ ← Comparable to I_{mp} & I_T

7. Since ring current is closer to Earth, has a bigger effect than I_{mp} & I_T .

II. Field-Aligned (Birkeland) Currents

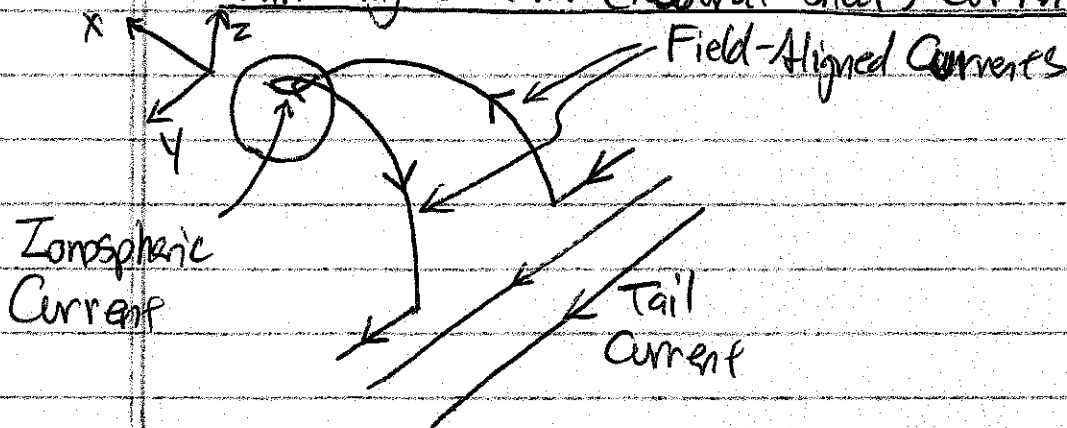
A. Magnetospheric-Ionospheric Coupling (M-I coupling)

1. Field-aligned currents play an important role in coupling the magnetosphere to the auroral ionosphere.
2. Connecting to Partial Ring Currents:



- a. Partial Ring Current cannot completely encircle the Earth due to Magnetopause
- b. To complete circuit, Field-Aligned Currents connect to the ionosphere
- c. Ionospheric currents complete the circuit.

3. Connecting to Tail (Neutral Sheet) Currents:



- a. Also called the substorm current wedge, enables magnetic reconnection in the tail to dynamically connect to the auroral ionosphere.

4. We'll discuss Ionospheric Currents when we cover the ionosphere in a subsequent lecture.

III. Global Picture of Magnetospheric Current Systems

