Lecture #19  The Structure of the Heliosphere

I. Solar and Stellar Winds:

1. Consider a model for the solar wind in spherical coords \((r, \theta, \phi)\)

\[
U = U_r (r) \hat{r} + U_\theta (r) \hat{\theta} \\
B = B_r (r) \hat{r} + B_\theta (r) \hat{\theta}
\]

2. By symmetry on equatorial plane, \(U_\theta = 0\) and \(B_\theta = 0\)
3. Assume steady state conditions, \(\frac{\partial U}{\partial t} = 0\)
4. From MHD, lets determine the global structure of the Interplanetary Magnetic Field (IMF) emerging from the rotating sun with the solar wind flow.

5. Conservation of Mass:
   a. Continuity Equation: \(\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho U) = 0\)
   b. In spherical coordinates, \(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho U_r) = 0\)
   c. Thus: \(r^2 \rho U_r = \text{constant} \quad 0 \leq r \leq R_0\)

6. Zero Magnetic Divergence:
   a. \(\nabla \cdot B = 0\)
   b. Thus, \(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 B_r) = 0\)
   c. So, \(r^2 B_r = \text{constant} \Rightarrow B_r = B_0 \left(\frac{R_0}{r}\right)^2\)
7. Magnetic Induction Equations:

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) \]

b. Consider the \( \phi \)-component of curl: \( \frac{1}{r} \frac{\partial}{\partial r} \left[ r (U_r B_\phi - U_\phi B_r) \right] = 0 \)

c. Thus, \( \frac{1}{r} \frac{\partial}{\partial r} \left[ r (U_r B_\phi - U_\phi B_r) \right] \) is constant.

At solar surface, \( U_\phi \gg U_r \), and \( U_\phi = R_0 \omega_0 \), so

\[ \text{constant} = - \frac{R_0^2 \omega_0 B_0}{5} \]

where \( \omega_0 = \frac{285 \times 10^6 \text{ rad s}^{-1}}{5} \)

d. Since \( r^2 B_\phi = R_0^2 B_0 \), we obtain constant = \( -r^2 \frac{R_0^2 \omega_0}{5} \)

Substituting this result above and simplifying,

\[ \frac{B_\phi}{B_r} = \frac{U_\phi - r \frac{R_0^2 \omega_0}{5}}{U_r} \]

8. Conservation of Angular Momentum:

\[ \rho \frac{D U}{D t} + \rho \cdot \nabla U = -\nabla (\rho + \frac{U^2}{2}) + \frac{B \cdot \nabla B}{\mu_0} - \rho \frac{GM}{r^2} \]

b. The azimuthal component gives, in steady spheroidal corotating, \( \rho U_r \frac{D (r U_r)}{D r} = \frac{B_\phi}{\mu_0} \frac{D}{D r} (r B_\phi) \)

c. \( \frac{D (r U_r)}{D r} = \frac{B_\phi}{\mu_0 U_r} \frac{D}{D r} (r B_\phi) = 0 \)

From DE \( r \), \[ \frac{B_\phi}{\mu_0 U_r} = \left( \frac{x^2}{a} \right) \left( \frac{B_0 R_0^2}{a^2} \right) = \frac{B_0 R_0^2}{M} = \text{constant} \]

d. \( \frac{D}{D r} (r U_\phi - \frac{r B_\phi B_\theta}{\mu_0 U_r}) = 0 \)
Lecture 19 (Continued)

Z A. P. (Continued)

\[ L = r U_\phi - \frac{r B_r B_\phi}{M \rho U_r} \]

\[ \text{Conservation of Angular Momentum,} \]
\[ \text{where } L = \frac{\text{Angular Momentum}}{\text{unit mass}} \]

9. Therefore, we have

a. \[ r^2 \rho U_r = M \]

b. If \( U_r(r) \) is known, we can solve for \( \rho(r) \), \( B_r(r) \), \( B_\phi(r) \), and \( U_\phi(r) \)

\[ B_r = B_0 \left( \frac{R_0}{r} \right)^2 \]

\[ \frac{B_\phi}{U_r} = \frac{U_\phi - r B_r B_\phi}{U_r} \]

\[ L = r U_\phi - \frac{r B_r B_\phi}{M \rho U_r} \]

c. We can continue, using Conservation of Energy and Radial Momentum Equation to obtain a fully self-consistent (and complicated) system, but for now well focus on the structure of \( B_r \).

B. The Parker Spiral Magnetic Field

1. At distances \( r \gg R_0 \), \( U_\phi \ll U_r \), so (3) can be simplified to

\[ \frac{B_\phi}{B_r} = -\frac{r U_\phi}{U_r} \]

2. Substituting (2),

\[ B_\phi = -\frac{r \Omega_0 B_0 R_0^2}{U_r r^2} = -B_0 \frac{R_0^2 \Omega_0}{r U_r} \]

3. Therefore, we see that \( B_r \propto \frac{1}{r^2} \) and \( B_\phi \propto \frac{1}{r} \), so the magnetic begins primarily radial and eventually becomes more and more azimuthal.
4. One may write a function $\phi(r)$ that describes a magnetic field line, \[ \phi(r) = \phi_0 - \frac{\Omega \sigma_0}{V_{sw}} (r - R_A) \] \[ \text{Archimedean Spiral} \]

where $\phi_0$ is angle at "foot" of field line at $r = R_A$ and $R_A$ is the critical radius for Alfvén waves, $R_A = 10 R_\odot$.

4. Magnetic field magnitude, $B = (B_r^2 + B_\theta^2)^{\frac{1}{2}}$.

\[ B = B_\odot \frac{(R_\odot)}{r} \sqrt{1 + \frac{r^2 \Omega_\odot^2}{U_r^2}} \]

C. Loss of Angular Momentum

1. Weber & Davis (1967) first demonstrated that the solar wind magnetic field significantly enhanced the loss of angular momentum (spindown) of the Sun, (in addition spin down by mass loss alone).

2. "Effective corotation" of solar wind with the Sun cut to the Alfvén radius $R_A = 10 R_\odot$. 

\[ r \sim \frac{4c}{c_\odot} = 1 \text{AU} \]
II. The Heliospheric Current Sheet

A. 1. Priestman & Kopp (1978) computed a self-consistent steady-state solution of MHD equations with Parker transonic solution along each flow line.

2. Open field lines from the poles eventually spread out to cover 4π solid angle beyond 3R⊙.

3. Along the equator, oppositely directed field lines from opposite poles come close together, leading to the Heliospheric Current Sheet required to support the jump in IMF field direction from sunward to anti-sunward.

4. Because the magnetic and rotation axes of the sun do not coincide, this current sheet becomes rippled, like a billowing skirt.

a. When Earth's orbit passes through a magnetic sector of sunward IMF to a sector of anti-sunward IMF,
II. B. Coronal Holes and Coronal Helmet Streamer Belt

1. The simplified picture above is reasonably accurate for solar minimum.

2. Coronal Holes:
   - Low density regions of open magnetic field lines that appear dark on coronographs.

3. Coronal Streamer Belt:
   - Bright teardrop shaped region (coronal helmet streamers) that appear to straddle field line reversals. Generally presumed to create a belt around the sun where the heliographic current sheet meets the corona, but may be more geometrically complicated during solar maximum.

4. Connection to Solar Wind Speed:

   a. Ulysses spacecraft went to high heliographic latitude, and at solar minimum found fast solar wind (2500-5000 km/s) at high latitudes, and slow solar wind (<500 km/s) at low latitudes ≤ 30°N/S.

   b. Fast solar wind is assumed to originate at coronal holes.

   c. Slow solar wind is believed to originate at the outer regions of coronal streamers.
II. Corotating Intersecion Regions

A.1. Because the Sun is rotating, regions of fast solar wind can emerge from the Sun at a position radially behind previously emitted slow solar wind streams.

2. The fast radial flow in fast streams catches up with and compresses the slow solar wind radially beyond it.

3. Since the regions of fast & slow solar wind persist for several solar rotations, this pattern of compression and rarefaction repeats at each solar rotation.

4. The region of compression are known as corotating interaction regions, or CZRs.

III. Global Structure of the Heliosphere

A. The Termination Shock

1. The supersonic solar wind at large heliocentric radius connects to a boundary condition \( p \to 0 \) at \( r \to \infty \) (Lec #18)

2. But, if a finite pressure exists in the surrounding Local Interstellar Medium (LISM), the coronal expansion must eventually stop when solar wind pressure becomes smaller than LISM pressure.
3. A **terminator shock** must develop to transition from supersonic flow to subsonic flow to enable the solar wind flow to stop.

4. Beyond this is the heliopause, the tangential discontinuity separating plasma and magnetic field of solar origin from that of the local ISM.

5. Neptune's orbit is at \( R \approx 30 \text{ AU} \)

6a. Voyager I crossed the terminator shock in the Northern heliospheric hemisphere at \( R \approx 94 \text{ AU} \) in December 2004

6b. Voyager II crossed the terminator shock in the Southern hemisphere at \( R \approx 84 \text{ AU} \) in August 2007.

7. The **Interstellar Boundary Explorer (IBEX)** mission, launched in October 2008, uses measurements of energetic neutral atoms to map the heliospheric boundaries, showing that the direction of the local ISM magnetic field \( B_{\text{ISM}} \) may influence the shape of the heliosphere.