Lecture #9 MHD Shock Properties, the Bowshock and Magnetopause Howes ①

I. Types of MHD Discontinuities and Shocks

A. Rankine-Hugoniot Jump Conditions

$$\boxed{\begin{aligned}
&[\rho U_n] = 0 \\
&\left[\rho U_n \underset{\sim}{U_t} - \frac{B_n \underset{\sim}{B_t}}{\mu_0}\right] = 0 \\
&\left[\rho U_n^2 + p + \frac{B_t^2 - B_n^2}{2\mu_0}\right] = 0 \\
&\left[\frac{1}{2}\rho U^2 U_n + \frac{\gamma p}{\gamma - 1} U_n + \frac{B_t^2}{\mu_0} U_n - \frac{B_n}{\mu_0}(\underset{\sim}{B_t} \cdot \underset{\sim}{U_t})\right] = 0 \\
&[U_n \underset{\sim}{B_t} - B_n \underset{\sim}{U_t}] = 0 \\
&[B_n] = 0
\end{aligned}}$$

1. Mass flux ρU_n across shock is constant.

2. Normal component of magnetic field B_n is constant.

3. Three types: a. Discontinuities
   b. Noncompressive shocks
   c. Compressive shocks.

B. Discontinuities ($U_{n_1} = U_{n_2} = 0$) → No plasma flow across discontinuity

1. When there is no transport across discontinuity: $U_{n_1} = U_{n_2} = 0$, if $B_n = 0$, the only remaining condition is

$$\left[p + \frac{B_t^2}{2\mu_0}\right] = 0 \Leftarrow \text{Total pressure balance}$$

2. This is a tangential discontinuity (only tangential components, normal components are zero)

I. B. Continued)

3. Neither plasma flow nor magnetic field crosses tangential discontinuity.

$\Rightarrow$ This is a good model for the $\boxed{\text{Magnetopause}}$.

4. Note, when $B_n \neq 0$, the only field that can discontinuously change is the density, $[\rho] \neq 0$. This is called a <u>contact discontinuity</u>.

C. <u>Noncompressive Shocks</u> ($[U_n]=0$, $[\rho]=0$)

1. In this case, normal continuous, $[U_n]=0$, but non-zero, $U_n \neq 0$.
   a. By conservation of mass flux, $[\rho]=0$.
   b. Thus, the shock is <u>noncompressive</u>
2. This is an <u>Intermediate Shock</u> (Alfvén Shock).
3. In this case, $[p]=0 \Rightarrow$ no pressure jump
   $[B_t^2]=0 \Rightarrow$ no jump of <u>magnitude</u> of $\underset{\sim}{B}_t$

   $\Rightarrow$ Thus, total pressure is constant across the shock.
4. Tangential magnetic field $\underset{\sim}{B}_t$ and velocity $\underset{\sim}{U}_t$ vectors <u>rotate</u> across shock by some angle, but have constant magnitude.

   $\Rightarrow$ Also called a <u>rotational discontinuity</u>.
5. Plasma velocity across shock

$$U_n = \pm \frac{B_n}{\sqrt{\mu_0 \rho}} = V_{A_n} \Leftarrow \text{Normal component of Alfvén velocity.}$$

$\Rightarrow$ This shock corresponds to the <u>Alfvén wave</u> in nonlinear limit.

D. <u>Compressive Shocks</u> ($[U_n]<0$, $[\rho]>0$)

1. Because the <u>bowshock</u> leads to a discontinuous decrease of solar wind velocity, it corresponds to this type of <u>compressive MHD shock</u>.

I. D. (Continued)

2. In general, at a compressive shock the flow velocity decreases $[U_n] < 0$ and the density increases $[\rho] > 0$.

3. Coplanarity:
   a. Rankine-Hugoniot jumps conditions can be used to show that the tangential components of the magnetic field $\underline{B}_t$ and velocity $\underline{U}_t$ may change magnitude but not direction.
   b. Thus, the magnetic field vector on either side of the shock are in the same plane normal to the shock.
   c. This simplifies the shock geometry for $\underline{U}$ and $\underline{B}$ to two-dimensions (although the planes for each may be different).

4. Two types of compressive shocks

Fast Shock		Slow shock	
U_n decreases		U_n decreases	
ρ increases		ρ increases	
p, T increase		p, T increase	
S (entropy) increases		S (entropy) increases	← irreversible
B_t increases	↔ difference	B_t decreases	
Propagates at fast wave speed	↔ difference.	Propagates at slow wave speed	

5. Strong Shocks: For hypersonic shocks ($U_n \gg C_s, V_A$) such as the solar wind, simplification for fast shock:

$$U_2 \simeq U_1/4$$
$$\rho_2 \simeq 4\rho_1$$
$$B_{t_2} \simeq 4B_{t_1}$$

} For hydrodynamic shocks, this can be proven. For MHD, it is experimentally observed.

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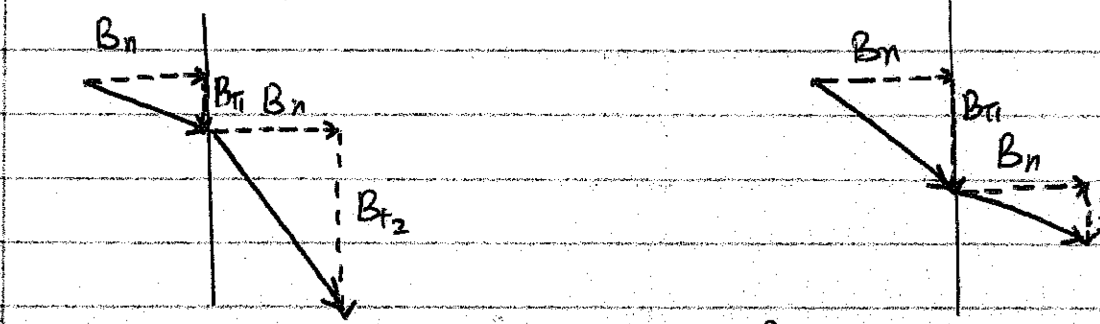
I. D. (Continued)

6. Bending of Magnetic field through fast or slow shocks:

a. Fast shock ($B_{T_2} > B_{T_1}$) b. Slow shock ($B_{T_2} < B_{T_1}$)

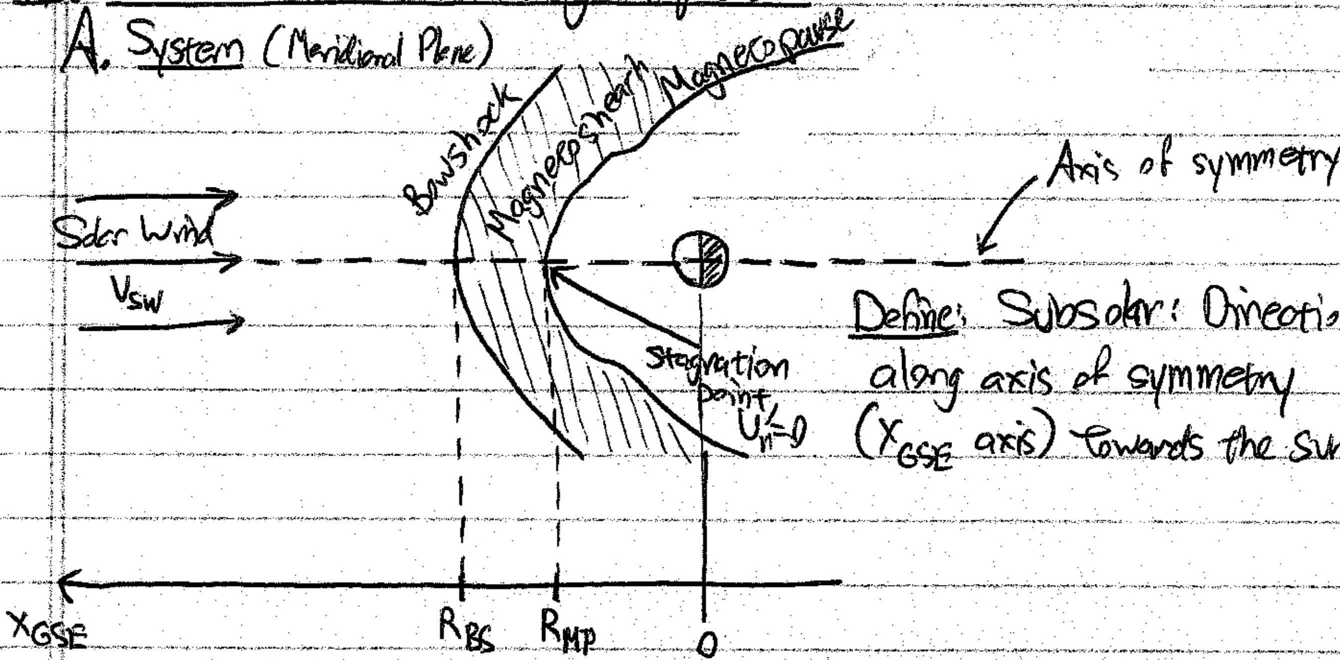


Bends towards shock front Bends towards shock normal

7. Due to the high velocity of the solar wind flow, the bowshock is a fast MHD shock.

II. The Bowshock and Magnetopause

A. System (Meridional Plane)



Define: Subsolar: Direction along axis of symmetry (X_{GSE} axis) towards the sun.

1. We want to compute the standoff distance to the magnetopause, R_{MP}.
2. The symmetry implies a stagnation point in the magnetosheath flow at the subsolar point on the magnetopause, $U_n = 0$.
3. Along axis of symmetry, $\underset{\sim}{U}_t = 0$

Lecture #9

II. (Continued)

4. The subsolar point is also called the nose of the magnetopause.

B. Pressure Balance (Conservation of Momentum)

1. Momentum Equation in Conservative Form

$$\frac{\partial(\rho \underset{\sim}{U})}{\partial t} + \nabla \cdot \left(\rho \underset{\sim}{U}\underset{\sim}{U} + p \underset{\approx}{I} + \frac{B^2}{2\mu_0}\underset{\approx}{I} - \frac{\underset{\sim}{B}\underset{\sim}{B}}{\mu_0} \right) = 0$$

2. Along the axis of symmetry, we allow variation only along x.
   a. Thus $\nabla \rightarrow \frac{\partial}{\partial x}\hat{x}$ (Note that $\hat{x}$ is normal to bowshock)

3. For a steady steady solar wind flow, in the GSE frame, $\frac{\partial}{\partial t} = 0$.

4. Thus, we obtain:
   a. $\frac{\partial}{\partial x}\left(\rho U_n^2 + p + \frac{B_t^2 - B_n^2}{2\mu_0} \right) = 0$

5. From $\nabla \cdot \underset{\sim}{B} = 0$, we obtain $\frac{\partial B_n}{\partial x} = 0 \Rightarrow B_n = \text{const}$, so we may simplify:

$$\boxed{\frac{\partial}{\partial x}\Big(\underbrace{\rho U_n^2}_{\substack{\text{Dynamical Pressure}\\\text{(or ram pressure)}\\\text{of solar wind flow}}} + \underbrace{p}_{\substack{\text{Thermal}\\\text{Pressure}}} + \underbrace{\frac{B_t^2}{2\mu_0}}_{\substack{\text{Magnetic}\\\text{Pressure}}} \Big) = 0}$$

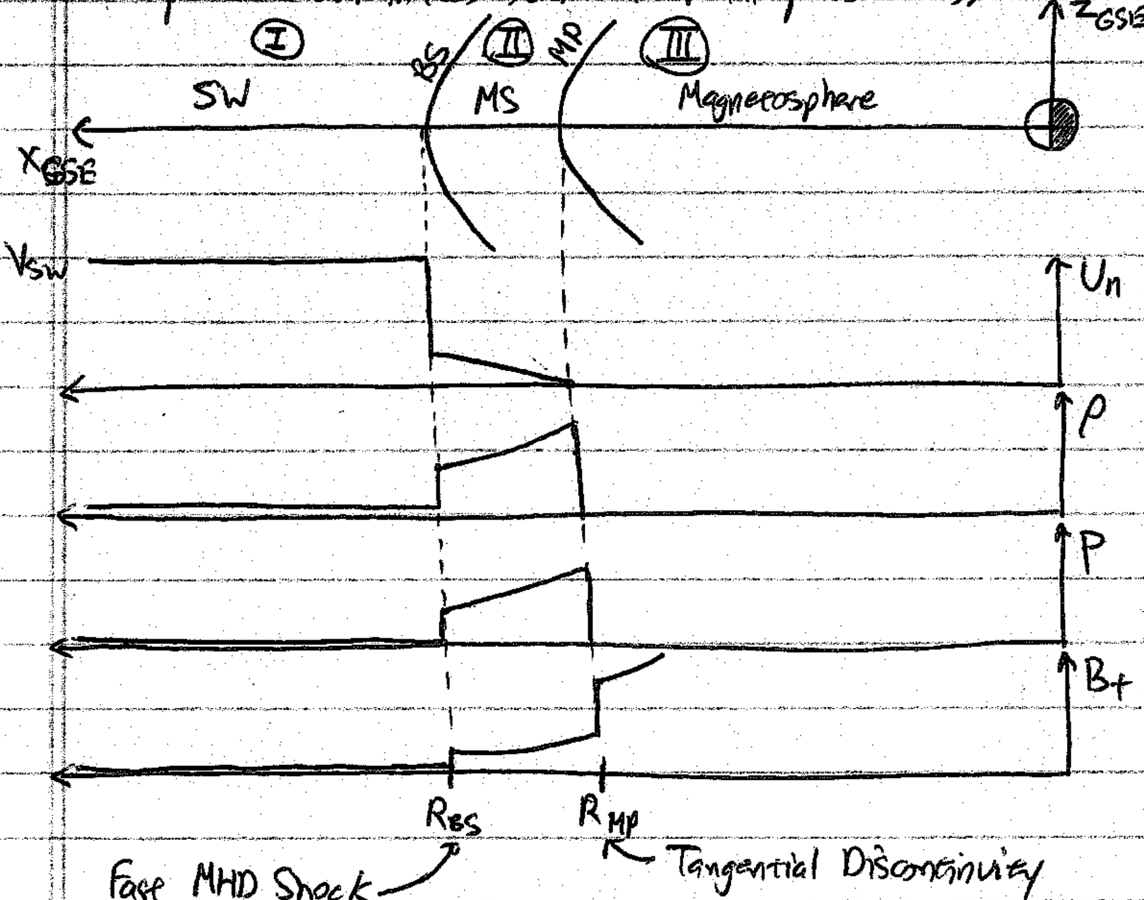
Expression of Pressure Balance along subsolar direction.

a. Using some simplifying assumptions, we will balance the ram pressure of the solar wind with the magnetic pressure of the Earth's dipole field to compute R_{MP}.

II. C. Simplifying Assumptions:

1. In the Solar wind $\rho U_n^2 \gg p$ and $\rho U_n^2 \gg \frac{B_t^2}{2\mu_0}$

2. In the magnetosheath, U_n decreases as ρ, p, B_t increase.
3. At the magnetopause (subsolar point), $U_n = 0$, $B_n = 0$ (because the magnetopause is a <u>tangential discontinuity</u>).
4. In the magnetosphere, $U_n = 0$ and $p \ll \frac{B_t^2}{2\mu_0}$ (magnetic pressure dominates over thermal pressure).



5. <u>Solar Wind</u> (BS) <u>Magnetosheath</u> (MP) Magnetosphere

a. $$\rho_1 U_1^2 + \cancel{p_1}^{0} + \cancel{\frac{B_{t1}^2}{2\mu_0}}^{0} = \underbrace{\rho_2 U_2^2 + p_2 + \frac{B_{t2}^2}{2\mu_0}}_{\text{Requires a somewhat complicated model.}} = \rho_3 \cancel{U_3^2}^{0} + \cancel{p_3}^{0} + \frac{B_{t3}^2}{2\mu_0}$$

b. $$\boxed{\rho_1 U_1^2 = \frac{B_{t3}^2}{2\mu_0}}$$

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II. D. Solve for R_{MP}

1. a. From lecture #8, along the equator ($\theta = 90°$), the (Compressed due to "mirror dipole") magnetosphere field is purely polar (tangential to magnetopause) with magnitude
$$\boxed{B = \frac{\mathcal{F} M}{r^3}}$$
where $\mathcal{F} \simeq 2$ is field compression factor,
and, $M = -7.84 \times 10^{15}\ T\ m^3$

b. We take $r = R_{MP}$

2. From Lecture #7, Solar wind density $n_{sw} \simeq 7\ cm^{-3} = 7 \times 10^6\ m^{-3}$
and velocity $V_{sw} = 450 \frac{km}{s} = 4.5 \times 10^5 \frac{m}{s}$

a. So $\rho_1 = n_{sw} m_p$ and $U_1 = V_{sw}$

3. Therefore, we obtain a. $n_{sw} m_p V_{sw}^2 = \frac{\mathcal{F}^2 M^2}{2 \mu_0 R_{MP}^6}$

b. Solving for R_{MP}:
$$\boxed{R_{MP} = \left[\frac{\mathcal{F}^2 M^2}{2 \mu_0 n_{sw} m_p V_{sw}^2}\right]^{1/6}}$$

4. $$R_{MP} = \left[\frac{(2)^2 (-7.84 \times 10^{15}\ T m^3)^2}{2(4\pi \times 10^{-7} \frac{H}{m})(7 \times 10^6 m^{-3})(1.67 \times 10^{-27} kg)(4.5 \times 10^5 \frac{m}{s})}\right]^{1/6} = 5.9 \times 10^7\ m$$

$$= (5.9 \times 10^7\ m)\left(\frac{1\ R_E}{6.378 \times 10^6\ m}\right) = 9.2\ R_E \simeq 10\ R_E$$

Thus, the magnetopause is located at $\boxed{R_{MP} \simeq 10\ R_E}$ in excellent agreement with observations.

E. The Bowshock:

1. The bowshock distance in front of a blunt, impenetrable body of radius R_b is given by $\boxed{R_{BS} = \left(1 + 1.1 \frac{\rho_{sw}}{\rho_{BS}}\right) R_b}$

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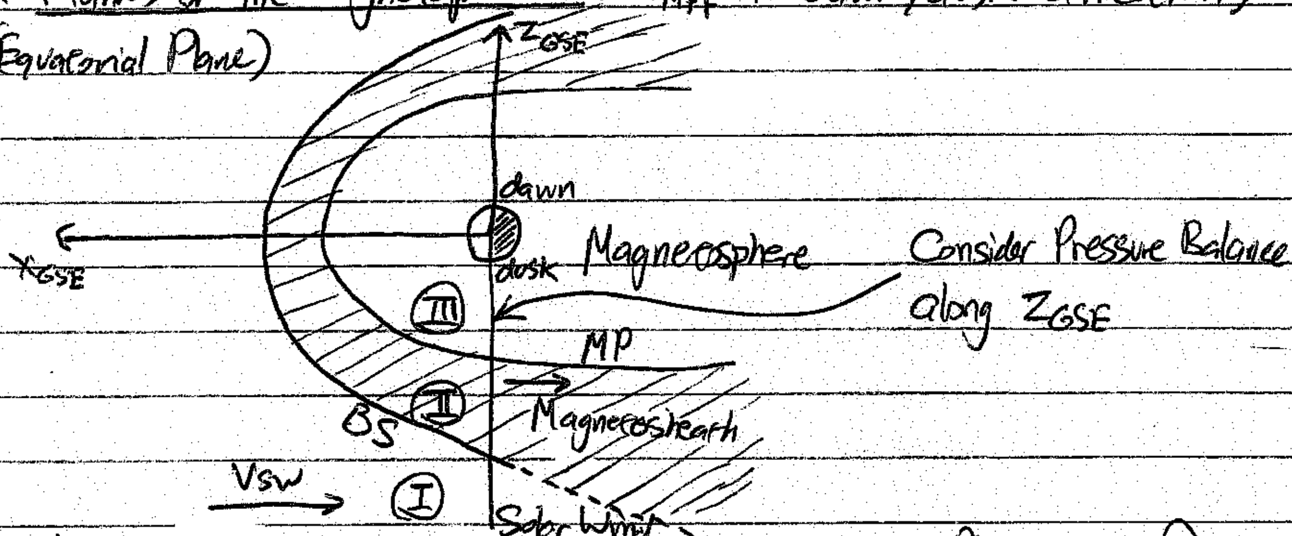
II. E. (Continued)

2. The density jump at the (strong) bowshock is $\frac{\rho_{BS}}{\rho_{SW}} = 4$

3. Thus $R_{BS} = \left[1 + 1.1\left(\frac{\rho_{SW}}{\rho_{BS}}\right)\right] R_{MP} = \left[1 + 1.1\left(\frac{1}{4}\right)\right](10 R_E) \simeq \boxed{13 R_E = R_{BS}}$

F. Flanks of the Magnetopause (R_{MPf} in dawn/dusk direction)

1. (Equatorial Plane)



a. Assume magnetopause is parallel to solar wind flow on the flanks.

b. Therefore, $U_n = 0$ and ram pressure vanishes.

2. Solar wind (BS) Magnetosheath (MP) Magnetosphere

a. $\cancel{\rho_1 U_{n_1}^2} + p_1 + \cancel{\frac{B_{T_1}^2}{2\mu_0}} = \rho_2 U_{n_2}^2 + p_2 + \frac{B_{T_2}^2}{2\mu_0} = \cancel{\rho_3 U_{n_3}^2} + \cancel{p_3} + \frac{B_{T_3}^2}{2\mu_0}$

b. $p_1 = \frac{B_{T_3}^2}{2\mu_0}$

3. Solar wind thermal pressure: $p = n_i T_i + n_e T_e \simeq 2 n_e T_e$

a. $n_e = 7\,cm^{-3} = 7\times10^6\,m^{-3}$, $T_e = 1.4\times10^5\,K$

4. Same estimate for magnetosphere magnetic field, $B_{T_3} = \frac{\mathcal{F} M}{r^3}$

5. Thus,

$$2 n_e T_e = \frac{\mathcal{F}^2 M^2}{2\mu_0 R_{MPf}^6} \Rightarrow \boxed{R_{MPf} = \left(\frac{\mathcal{F}^2 M^2}{4\mu_0 n_e T_e}\right)^{\frac{1}{6}}}$$

$$R_{MPf} = \left(\frac{2^2(7.84\times10^{15}\,T m^3)^2}{4(4\pi\times10^{-7}\,\frac{H}{m})(7\times10^6\,m^{-3})(1.38\times10^{-23}\,\frac{J}{K})(1.4\times10^5\,K)}\right)^{\frac{1}{6}} = \frac{1.2\times10^8\,m}{6.378\times10^6\,m/R_E} \simeq \boxed{19 R_E = R_{MPf}}$$

6a. Measurements give $R_{MPf} \simeq 14 R_E$, slightly smaller. At dawn-dusk meridian, magnetosphere is still expanding $\Rightarrow$