Lecture #18 The Parker Solar Wind Solution

I. The Solar Wind

A. Importance

1. The effects of solar activity are transmitted to the Earth and other planets via the solar wind. Therefore, the solar wind is a critical element of the coupled Solar-Terrestrial system
2. The physical mechanisms occuring in the solar wind (turbulence, magnetic reconnection, shock acceleration of energetic particles) occur over a very wide range of plasma parameters (plasma beta, collisionality) at macroscopic and microscopic scales. Therefore, given that we can sample these processes in the solar wind directly with spacecraft, we are able to study in great detail the plasma physics processes that are simply inaccessible in other astrophysical plasmas.

II. Static Models of the Solar Atmosphere

A. History

1. Since the discovery of temperatures of $\sim 10^6 K$ in the corona, attempts have been made to understand the coronal structure.

B. Hydrostatic Equilibrium

1. Consider a spherically symmetric, steady-state model.

Assumptions: 1) Spherical symmetry: $\frac{\partial}{\partial \phi}=0$, $\frac{\partial}{\partial \theta}=0$,

$\Rightarrow$ All variables depend only on r!

(3-D system that may vary in 1 dimension).

2) Radial Flow: $\underset{\sim}{U} = U_r \hat{\underset{\sim}{r}}$, $U_\phi = U_\theta = 0$

Lecture #18 (Continued) Howes ③

II. B.1. (Continued)

3) Steady-State: $\frac{\partial}{\partial t} = 0$

4) Ignore Effects of the Magnetic Field.

5) Isothermal Atmosphere, $T =$ constant

2. MHD Equations:

Gravity: $\Phi_G = -\frac{GM}{r}$

Continuity a. $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underset{\sim}{U}) = 0$

Momentum b. $\rho \frac{\partial \underset{\sim}{U}}{\partial t} + \rho (\underset{\sim}{U} \cdot \nabla) \underset{\sim}{U} = -\nabla\left(p + \frac{B^2}{2\mu_0}\right) + \frac{(\underset{\sim}{B} \cdot \nabla)\underset{\sim}{B}}{\mu_0} - \rho \nabla \Phi_G$

Induction c. $\frac{\partial \underset{\sim}{B}}{\partial t} + (\underset{\sim}{U} \cdot \nabla) \underset{\sim}{B} = -\underset{\sim}{B}(\nabla \cdot \underset{\sim}{U}) + (\underset{\sim}{B} \cdot \nabla) \underset{\sim}{U}$

Pressure d. $\frac{\partial p}{\partial t} + (\underset{\sim}{U} \cdot \nabla) p = -\gamma p (\nabla \cdot \underset{\sim}{U})$

3. Steady-state Equilibrium: (Momentum Eq. Balance).

a. In steady-state, we balance

$$\underbrace{\rho (\underset{\sim}{U} \cdot \nabla) \underset{\sim}{U}}_{\substack{\text{inertia} \\ \text{(ram pressure)}}} + \underbrace{\nabla\left(p + \frac{B^2}{2\mu_0}\right)}_{\substack{\text{Thermal and} \\ \text{Magnetic Pressure}}} - \underbrace{\frac{\underset{\sim}{B} \cdot \nabla \underset{\sim}{B}}{\mu_0}}_{\substack{\text{Magnetic} \\ \text{Tension}}} + \underbrace{\rho \nabla \Phi_G}_{\text{Gravity}} = 0$$

b. The presence of the magnetic field introduces some mathematical complexity, so we will neglect it for this simple calculation.

$$\Rightarrow \quad \rho (\underset{\sim}{U} \cdot \nabla) \underset{\sim}{U} + \nabla p + \rho \nabla \Phi_G = 0$$

c. Thus, we may also drop the Induction equation.

4. Applying steady state, 1-D assumption with radial flow,

a) $\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \rho U_r) = 0$

b) $\rho U_r \frac{\partial U_r}{\partial r} = -\frac{\partial p}{\partial r} - \rho \frac{GM}{r^2}$ c) $U_r \frac{\partial p}{\partial r} = -\gamma p \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 U_r)$

III B. (Continued)

5. This system of equations for $U_r(r)$, $\rho(r)$, & $p(r)$ governs any 1-D, steady-state, hydrodynamic model.

6. <u>Static Atmosphere</u> ($U_r=0$)

a. The most simple model is to assume a <u>static atmosphere</u> ($U_r=0$) (Earth has a static atmosphere), which leaves only

$$\boxed{\frac{\partial p}{\partial r} = -\rho \frac{GM}{r^2}}$$ Hydrostatic Equilibrium

7. <u>Isothermal Assumption</u>

a. Early, simple models assumed the corona was isothermal, $T=$ constant.

b. In this case, $p = n_p T_p + n_e T_e = 2nT$ where $n=n_p=n_e$ (quasineutrality) and $T=T_p=T_e$.

c. Since $\rho = n(m_p+m_e) \simeq n m_p$, we obtain

$$p = \frac{2\rho T}{m_p}, \text{ or } \rho = \frac{m_p p}{2T}$$

d. <u>Note</u>: Sound Speed $C_s^2 \equiv \frac{\gamma p}{\rho}$ (see Lec #1)

1) For isothermal gas, $\gamma=1$, so $C_s^2 = \frac{p}{\rho} = \frac{2T}{m_p} =$ constant.

e. Applying isothermal assumption, $\frac{\partial p}{\partial r} = -\frac{GMm_p}{2Tr^2} p$

8. Solving for $p(r)$:

a. $$\int_{p_0}^{p} \frac{\partial p}{p} = -\frac{GMm_p}{2T}\int_{r_0}^{r}\frac{\partial r}{r^2} \Rightarrow \ln\frac{p}{p_0} = +\frac{GMm_p}{2T}\left[\frac{1}{r}-\frac{1}{r_0}\right]$$

where r_0 = radius at base of corona
and p_0 = pressure at base of corona

II. B8. (Continued)

b. Note that altitude at base of corona, $z_0 \simeq 2000\,km = 2\times10^6\,m$.
Thus, since $r_0 = z_0 + R_\odot \simeq R_\odot$ since $R_\odot = 7\times10^8\,m \gg z_0$
So, for numerical estimates, $r_0 \simeq R_\odot$.

9. Gravitational Acceleration at base of corona; g_0

a. Define: $g_0 = \frac{GM}{r_0^2}$

b. Thus $$\boxed{p = p_0\, e^{-\frac{m_p g_0}{2T} r_0 \underbrace{\left(1-\frac{r_0}{r}\right)}}}$$

$1-\frac{r_0}{r} > 0$ for $r > r_0$, so pressure decreases with radius, as expected.

c. Typical "Plane Atmosphere" limit for $z \ll r_0$, where $z = r - r_0$,

1) $r_0\left(1-\frac{r_0}{r}\right) = r_0\left(1-\frac{r_0}{z+r_0}\right) = r_0\left(1-\frac{\cancel{r_0}}{\cancel{r_0}(1+\frac{z}{r_0})}\right) \simeq r_0\left[\cancel{1}-\left(\cancel{1}-\frac{z}{r_0}\right)\right] = \frac{z\,\cancel{r_0}}{\cancel{r_0}} = z$

2) Define Scale Height, $H \equiv \frac{2T}{m_p g_0}$

3) Thus, we obtain $\boxed{p = p_0 e^{-\frac{z}{H}}}$ Barometric Altitude Formula for isothermal atmosphere, Valid when $z \ll r_0$.

10. Check Hydrostatic Solution

a. At $r \to \infty$, $p \to p_\infty = p_0\, e^{-\frac{m_p g_0 r_0}{2T}}$

b. For a coronal base temperature $T_0 = 1.5\times10^6\,K$,

$$\frac{m_p g_0 r_0}{2T_0} = \frac{GMm_p}{2T_0 R_\odot} = \frac{(6.67\times10^{-11}\frac{m^3}{kg\,s^2})(2\times10^{30}kg)(1.67\times10^{-27}kg)}{2(1.38\times10^{-23}\frac{J}{K})(1.5\times10^6K)(7\times10^8m)} \simeq 8,$$

So $\frac{p_\infty}{p_0} = e^{-8} \simeq 3\times10^{-4}$

II. B. 10. (Continued)

c. Coronal base pressure: $p_0 = n_p T_p + n_e T_e = 2 n_0 T_0$
where $n_0 \simeq 10^{15}\ m^{-3}$ and $T_0 = 1.5 \times 10^6 K$ (Lec #16),
$p_0 = 0.04$ Pa

d. Pressure at 1AU: $p_1 = 2 n_1 T_1$ where $n_1 = 10^7\ m^{-3}$, $T_1 = 10^5 K$ (Lec #7)
$p_1 = 3 \times 10^{-11}$ Pa

e. Thus
$$\frac{p_1}{p_0} = 10^{-9} \gg 3 \times 10^{-4}$$

f. In the distant ISM, $p_\infty \simeq 10^{-13}$ to 10^{-14} Pa, so even worse disagreement!

11. What is wrong with Hydrostatic Atmosphere Model?

a. A static isothermal atmosphere cannot support the very large drop in pressure observed.

b. Dropping isothermal assumption or including the effect of magnetic fields does not solve this fundamental problem.

c. The problem is that a hydrostatic solution cannot represent an equilibrium between the hot corona and distant interstellar medium

$\Rightarrow$ $\boxed{\text{We must consider nonzero flow speeds, } U_r \neq 0!}$

III. The Parker Solution for the Solar Wind

Ref: Parker, E. N., Astrophys. J. 128, 664 (1958).

A. Parker Solution

1. We maintain all of the above simplifications (1-D, radial flow, steady-state, neglect magnetic fields, isothermal), but allow for non-zero U_r.

$\Rightarrow$ (Begin with equations in II. B.4.)

III. A. (Continued)

2. Momentum Eq: $\rho U_r \frac{\partial U_r}{\partial r} = -\frac{\partial p}{\partial r} - \rho \frac{GM}{r^2}$ ①

a. From pressure equation, $U_r \frac{\partial p}{\partial r} = -\gamma p \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 U_r)$ ②

b. Substitute ② into $\frac{U_r ①}{\rho}$:

$$U_r^2 \frac{\partial U_r}{\partial r} = +\left(\frac{\gamma p}{\rho}\right)\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 U_r) - \frac{GM}{r^2}U_r$$

c. Note that $C_s^2 = \frac{\gamma p}{\rho}$ = const (due to isothermal approximation).

$$U_r^2 \frac{\partial U_r}{\partial r} = C_s^2\left[\frac{\partial U_r}{\partial r} + \frac{2U_r}{r}\right] - \frac{GM}{r^2}U_r$$

d. Finally, we obtain

$$\boxed{\left[\frac{U_r^2 - C_s^2}{U_r}\right]\frac{\partial U_r}{\partial r} = \frac{2C_s^2}{r} - \frac{GM}{r^2}} \quad ③$$

3. This can be integrated using separation of variables to obtain,

$$\frac{U_r^2}{2} - C_s^2 \ln U_r = 2C_s^2 \ln r + \frac{GM}{r} + C$$ ← Integration constant.

a. There are a number of mathematically admissible classes of solutions to this equation that depend on the constant, C

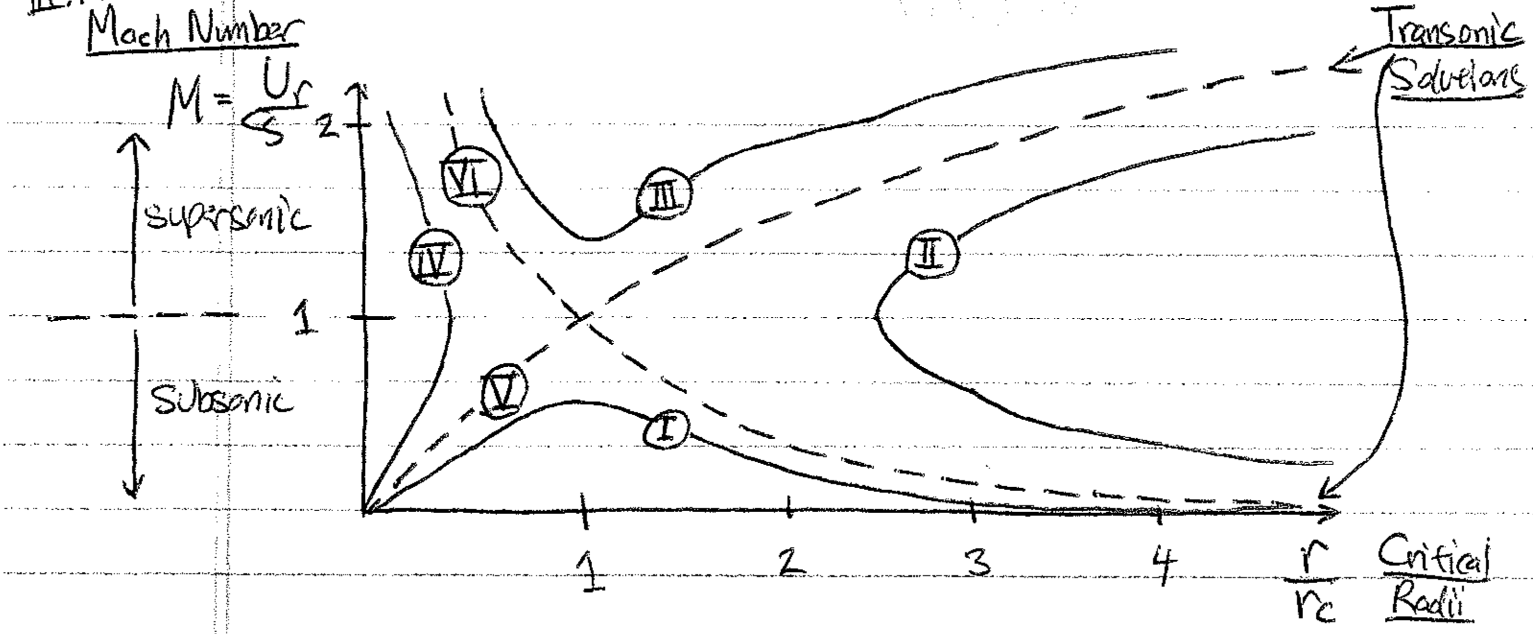
4. We plot these solutions below, and we can learn a lot about their properties looking at equation ③ above.

a. Eq ③ has an apparent singularity where RHS = 0, at $r = r_c$

$$\Rightarrow \frac{2C_s^2}{r_c} - \frac{GM}{r_c^2} = 0 \Rightarrow \boxed{r_c = \frac{GM}{2C_s^2}}$$ $r_c \simeq 4R_\odot$ for $T = 1.5\times10^6$ K

b. The equation can only be solved if $\boxed{U_r^2 - C_s^2 = 0}$ or $\boxed{\frac{\partial U_r}{\partial r} = 0}$.

III. A. **Lecture #18 (Continued)**



5. Above are contours of the function

$$f(u_r, r) = \frac{u_r^2}{2} - c_s^2 \ln u_r - 2c_s^2 \ln r - \frac{GM}{r} = C$$ for different values of C.

6. Four general types of solutions:

(1) Unphysical, double-valued solutions, II & IV — No solution at $r = r_c$.

(2) Purely supersonic solutions, III — $\frac{\partial u_r}{\partial r} = 0$ at $r = r_c$

(3) Purely subsonic solutions, I — $\frac{\partial u_r}{\partial r} = 0$ at $r = r_c$

(4) Transonic Solutions, V & VI — $u_r^2 - c_s^2 = 0$ at $r = r_c$

7. Solutions II & IV: a. double-valued, and thus not physically meaningful.

b. Also, these solutions have no values at $r = r_c$

8. Solution III: a. Supersonic at all radii, including coronal base

b. Inconsistent with small Doppler shifts of atomic emission lines

9. Solution I: The Solar Breeze

a. Subsonic at all radii, with a velocity $u_r \simeq 10$ km/s at 1 AU.

b. Before measurements of supersonic solar wind, considered seriously.

c. Has the same problem of finite pressure at $r \to \infty$ as static solution.

Lecture #18 (Continued) Howes (8)

III. A. (Continued)

10. Solutions (V) & (VI): Transonic Solutions

a. Solution (VI) has the same problem of supersonic speeds at coronal base.

b. Parker favored (V) as the correct solution for the solar atmosphere ⇒ Predicted the existence of a supersonic solar wind expanding out from the corona in steady state.

c. The idea was meet with incredible skepticism, driving mainly by the apparent singularity in the equation at $r = r_c$.

d. i. Predicted by Parker in 1958

ii. First sporadically detected by Soviet Lunik 2 & 3 in 1960.

iii. First continuous observation by Mariner 2 in 1966.

e. Solution has $p \to 0$ as $r \to \infty$.

i) Since (V) has $\frac{\partial U_r}{\partial r} > 0$ for all r, continuity equation tells us that $r^2 \rho U_r = \text{constant}$, thus $\rho \propto \frac{1}{U_r r^2}$ and density decreases faster than r^{-2}.

ii) Thus $p = 2nT \to 0$ since $n \to 0$ as $r \to \infty$

iii) So, the solar wind solution (V) can connect to the low interstellar medium pressures.

11. Why does solution V work?

a. $$\left(\frac{U_r^2 - c_s^2}{U_r}\right)\frac{\partial U_r}{\partial r} = \frac{2c_s^2}{r} - \frac{GM}{r^2}$$

pressure drive, gravity braking

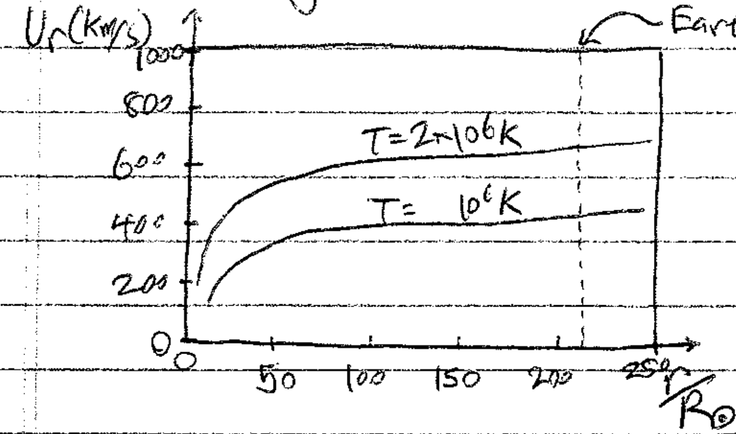
pressure & gravity terms balance (cancel) at $r = r_c$.

b. At $r < r_c$, gravity term dominates, so solution will be have similar characteristics to the exponential scale height dependence of the hydrostatic solution.

c. At $r > r_c$, pressure term dominates, so flow continues to accelerate, enabling density to keep dropping, making connection to $p \to 0$ as $r \to \infty$.

III. A. (Continued)

12. Resulting Solar Wind Velocity Profile



a. Isothermal model predicts $\boxed{V_{SW} \sim 400-600 \text{ km/s}}$ for reasonable coronal temperatures.

b. Critical radius (beyond which flow is supersonic) is $\boxed{r_c \sim 6-10 R_\odot}$

B. Relaxing Isothermal & Hydrodynamic ($B=0$) Assumptions

1. Solar wind temperature is not isothermal ($\gamma = 1$), but decreases with radius $\propto r^{-\alpha}$, where $0 < \alpha < 4/3$
2. An adiabatic model ($\gamma = 5/3$) leads to $T \propto r^{-4/3}$
   a. General qualitative characteristics of solution are similar.
3. Observed $T(r)$ has $\alpha \sim 0.3$ to 0.4, so temperature decrease is not adiabatic requiring some heating.

4. The introduction of a magnetic field complicates the model, but leads to no detrimental qualitative differences.
   a. Not one sonic point, but three for fast, Alfven, & slow waves.
   b. Typical solutions have Alfven critical point $r_A \simeq 10 R_\odot$.
   c. Even in magnetized case (where dynamics quickly become collisionless), solutions imply some unknown heating mechanism ⇒ Coronal heating problem
   d. Dissipation of Alfvenic turbulence or reconnection heating are key ideas.