Lecture #22: The Physics of Accretion

I. The Physics of Accretion

Ref: Frank, King, & Raine, Accretion Power in Astrophysics, Cambridge Univ Press, 2002.

A. History

1. 19th century: Gravity was known source of astrophysical energy, but was inadequate to power the sun → nuclear energy!
2. Present: The most luminous objects in the universe (AGN, quasars, etc.) cannot be powered by nuclear burning → accretion power is the critical driver of these systems!

B. Nuclear vs. Accretion Power

1. Estimate of energy gain in accretion
   a. Gravitational Potential $\Phi_G = \frac{-GM}{r}$
   b. For an object of mass M and radius R_*, the gravitational potential energy released as an object falls from $r=\infty$ is

$$\Delta E_{acc} = -\left(\frac{-GMm}{R_*} - \cancel{\frac{-GMm}{r_\infty}}^{0}\right) = \boxed{\frac{GMm}{R_*} = \Delta E_{acc}}$$

2. Estimates for typical objects:
   a. Sun: $M = M_\odot = 2\times10^{33}\,g$ $\quad R_\odot = 7\times10^{10}\,cm$ $\quad G = 6.67\times10^{-8}\,\frac{dyne\,cm^2}{g^2}$

   i. $\frac{\Delta E_{acc}}{m} = \frac{GM_\odot}{R_\odot} = \frac{(6.67\times10^{-8}\,\frac{dyne\,cm^2}{g^2})(2\times10^{33}\,g)}{(7\times10^{10}\,cm)} = 2\times10^{15}\,\frac{ergs}{g}$

   ii. NOTE: We will adopt cgs units for consistency with astrophysical convention.

   iii. For comparison, $L_\odot = 3.83\times10^{33}\,\frac{erg}{s}$

   – This requires $\dot{M} = \frac{L_\odot}{\Delta E_{acc}/m} = \frac{3.8\times10^{33}\,\frac{erg}{s}}{2\times10^{15}\,\frac{erg}{g}} = 2\times10^{18}\,\frac{g}{s}$

   – Accretion of the entire solar mass would take $\tau = \frac{M}{\dot{M}} = \frac{2\times10^{33}\,g}{2\times10^{18}\,\frac{g}{s}} = 10^{15}\,s$ #
   $= 10^{15}\,s \big/ (\pi\times10^{7}\,\frac{s}{y}) = 3.0\times10^{7}\,y = 30\,My$

Z. B. 2.a.iii (Continued)

- Therefore, since $\tau \sim 30\,My \ll$ lifetime of the sun, accretion is insufficient to power the sun.

b. <u>White Dwarf</u>: $M \sim M_\odot$, $R_* \sim 10^9\,cm$

$$\frac{\Delta E_{acc}}{m} = \frac{GM_\odot}{R_*} = \frac{(6.67\times10^{-8}\,\frac{dyn\,cm^2}{g^2})(2\times10^{33}\,g)}{10^9\,cm} \approx 10^{17}\,\frac{erg}{g}$$

c. <u>Neutron Star</u>: $M \sim M_\odot$, $R_* \sim 10\,km \sim 10^6\,cm$

$$\frac{\Delta E_{acc}}{m} = \frac{GM_\odot}{R_*} = \frac{(6.67\times10^{-8}\,\frac{dyn\,cm^2}{g^2})(2\times10^{33}\,g)}{10^6\,cm} \sim 10^{20}\,\frac{erg}{g}$$

d. <u>Black Hole</u>: $M \sim M_\odot$, $R_* \sim \frac{2GM}{c^2} \sim 3\times10^5\,cm$

$$\frac{\Delta E_{acc}}{m} \sim \frac{GM_\odot}{R_*} \sim \frac{\cancel{G}\cancel{M_\odot}}{\left(\frac{2\cancel{GM}}{c^2}\right)} \sim \frac{c^2}{2} \sim 5\times10^{20}\,\frac{erg}{g} \rightarrow \Delta E_{acc} \sim \frac{1}{2}mc^2$$

Half-rest mass energy.

- Note, however, that energy released from gravitational potential may pass through event horizon and not be emitted (more later).

3. <u>Nuclear Burning Energy Release</u>

a. Fusion of hydrogen to helium releases an energy

$$\Delta E_{nuc} = 0.007\,mc^2$$

b. Thus $\frac{\Delta E_{nuc}}{m} \sim 6\times10^{18}\,\frac{erg}{g}$

4.a. Nuclear Burning is much less efficient than accretion for compact objects such as neutron stars or black holes.

b. $\Delta E_{nuc} \sim 60\,\Delta E_{acc}$ for white dwarfs, but nuclear burning is episodic

c. $\Delta E_{nuc} \gg \Delta E_{acc}$ for the Sun.

d. <u>Compactness of object is important for accretion to be dominant!</u>

I. C. The Eddington Limit

1. Here, we consider the limiting luminosity due to accretion if radiation pressure balances gravitational force.

2. Consider spherically symmetric accretion, and steady flow.

3. For a hydrogenic (proton plus electron) fully ionized plasma, radiation exerts a force mainly on free electrons through Thompson Scattering, $\sigma_T = 6.7 \times 10^{-25}\ cm^2$
   a. For a radiant energy flux $S\ (\frac{erg}{s\,cm^2})$, momentum from scattering is $\frac{\sigma_T S}{c}$
   b. This pushes out on electrons, but plasma wants to maintain quasineutrality, so the electrons drag the protons with them.

4. Gravity: $\frac{GM(m_p + m_e)}{r^2} \simeq \frac{GMm_p}{r^2}$

5. Balance Gravity and radiation pressure force.
   a. $S = \frac{L}{4\pi r^2}$ for an accreting source of Luminosity $L\ (\frac{erg}{s})$
   b. $\frac{GMm_p}{r^2} - \frac{L\sigma_T}{4\pi c r^2} = 0$
   c. $L_{edd} = \frac{4\pi G M m_p c}{\sigma_T} \simeq \boxed{1.3 \times 10^{38} \left(\frac{M}{M_\odot}\right) \frac{erg}{s} = L_{edd}}$
   d. This gives the maximum Luminosity for a steady, spherically symmetric accretion powered object.
   e. Note that unsteady events (supernova explosions) can significantly exceed this limit. ($L \sim 10^{42}$ to 10^{43} erg/s at peak)

I. C. (Continued)

6. Accretion Luminosity:

a. $\boxed{L_{acc} = \frac{GM\dot{M}}{R_*}}$

b. White Dwarfs: $L_{acc} = 1.3\times10^{33}\ \dot{M}_{16}\left(\frac{M}{M_\odot}\right)\left(\frac{10^9\,cm}{R_*}\right)\frac{erg}{s}$

where $\dot{M}_{16} = \frac{\dot{M}}{10^{16}\ g/s}$ ← Typical $\dot{M}$ in close binary systems.

c. Neutron Star: $L_{acc} = 1.3\times10^{36}\ \dot{M}_{16}\left(\frac{M}{M_\odot}\right)\left(\frac{10\,km}{R_*}\right)\frac{erg}{s}$

7. Black Holes:

a. Since black hole can swallow some energy released in accretion, we model this by an efficiency, η.

$$L_{acc} = \frac{2\eta GM\dot{M}}{R_*} = \frac{2\eta GM\dot{M}}{\left(\frac{2GM}{c^2}\right)} = \eta\dot{M}c^2$$

↑ efficiency for converted accreted rest mass to radiation.

b. Typical values from modeling are $\eta \sim 0.1$

D. Spectrum of Accretion Powered Emission

1. Characterize frequency of emitted radiation: $\boxed{h\nu \sim T_{rad}}$

photon energy ~ thermal energy

2. Blackbody radiation:

a. $S = \frac{L}{4\pi r^2} = \sigma T^4$ $\quad \sigma = 5.67\times10^{-5}\ \frac{erg}{cm^2\,s\,K^4}$

b. Thus $\boxed{T_b = \left(\frac{L_{acc}}{4\pi R_*^2\sigma}\right)^{1/4}}$ "Blackbody temperature"

3. Thermal Temperature: a. Potential energy: $\frac{GM(m_p+m_e)}{R_*} \simeq \frac{GMm_p}{R_*}$

b. Thermal energy: $\underbrace{2}_{\text{ions \& electrons}} \times \frac{3}{2}T \simeq 3T$

c. Thus, $\boxed{T_{th} = \frac{GMm_p}{3R_*}}$

I. D. (Continued)

4. Conversion of accretion energy to radiation: Two limits.

a. In optically thick conditions, radiation reaches thermal equilibrium with radiated material before escaping, so $T_{rad} \sim T_b$

b. If accretion energy is directly converted to radiation, $T_{rad} \sim T_{th}$.

c. In general, we have

$$T_b \lesssim T_{rad} \lesssim T_{th}$$

5. a. Neutron Star $\boxed{1\,keV \lesssim h\bar{\nu} \lesssim 50\,MeV}$
(x-rays) (γ-rays)
$L_{acc} \sim 10^{36}\,\frac{erg}{s}$

b. White Dwarf $\boxed{6\,eV \lesssim h\bar{\nu} \lesssim 100\,keV}$
(UV) (x-rays)
$L_{acc} \sim 10^{33}\,\frac{erg}{s}$

6. Electromagnetic Spectrum

Classification	Wavelength (nm)	Energy
γ-rays	$\lesssim 0.01$	$\gtrsim 100$ keV
Hard X-rays	0.01 – 1	1–100 keV
Soft x-rays	1 – 10	100 eV – 1 keV
Extreme UV	10 – 121	10 – 100 eV
UV	122 – 400	3 – 10 eV
Visible	400 – 760	$\lesssim 3$ eV

II. Bondi Accretion:

Ref: Bondi (1952) "On spherically symmetrical accretion", MNRAS 112, 195.

A. Motivation

1. For spherically symmetric accretion onto a star at rest with respect to the interstellar medium (ISM), can we determine the accretion rate $\dot{M}$ and effective radius in terms of stellar mass and ISM properties?

2. We'll see this is essentially the same as the Parker Solar Wind Solution.

II. B. Bondi Accretion Solution

1. Assumptions: a. Spherically Symmetric, $\frac{\partial}{\partial \phi}=0$, $\frac{\partial}{\partial \theta}=0$
   b. Steady-state, $\frac{\partial}{\partial t}=0$
   c. Radial inflow, $\underset{\sim}{U}=U_r(r)\,\hat{\underset{\sim}{r}}$, $U_\phi = U_\theta = 0$
   d. Ignore Magnetic Field, $\underset{\sim}{B}=0$

2. MHD Equations: a. Continuity: $\cancelto{0}{\frac{\partial \rho}{\partial t}} + \nabla\cdot(\rho \underset{\sim}{U}) = 0$
   b. Momentum: $\rho \cancelto{0}{\frac{\partial \underset{\sim}{U}}{\partial t}} + \rho(\underset{\sim}{U}\cdot\nabla)\underset{\sim}{U} = -\nabla\left(p + \cancelto{0}{\frac{B^2}{8\pi}}\right) + \cancelto{0}{\frac{\underset{\sim}{B}\cdot\nabla\underset{\sim}{B}}{4\pi}} - \rho\frac{GM}{r^2}\hat{\underset{\sim}{r}}$
   c. Induction: $\underset{\sim}{B}=0$, so not required
   d. Polytropic Relation: $p = K\rho^\gamma$ $\left[\text{same as } \frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right)=0\right]$
   $\uparrow$ constant K

3. NOTE: Sound speed $C_s^2 \equiv \left(\frac{dp}{d\rho}\right)$
   a. Adiabatic $(\gamma=\frac{5}{3})$: $C_s = \left(\frac{\gamma p}{\rho}\right)^{\frac{1}{2}} \propto \rho^{\frac{1}{3}}$
   b. Isothermal $(\gamma=1)$: $C_s = \left(\frac{p}{\rho}\right)^{\frac{1}{2}} = \text{constant}$

4. Ideal Gas Law: $p = \frac{\rho T}{\mu m_p}$, where μ is mean molecular weight.
   a. For hydrogen $\mu = 1$
   b. For ionized hydrogen (protons + electrons), $\mu = \left(\frac{m_p + m_e}{2}\right)\frac{1}{m_p} \simeq \frac{m_p}{2m_p} = \frac{1}{2}$
   c. Thus, for a fully ionized plasma, $\boxed{p = \frac{2\rho T}{m_p}}$ (same as before, Lect 18)

5. Continuity Equation: $\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho U_r) = 0 \Rightarrow \boxed{4\pi r^2 \rho U_r = \dot{M}}$
   Mass accretion rate

6. Radial Momentum: $\rho U_r \frac{\partial U_r}{\partial r} + \frac{\partial p}{\partial r} + \rho\frac{GM}{r^2} = 0$
   a. $\frac{\partial p}{\partial r} = \left(\frac{\partial p}{\partial \rho}\right)\frac{\partial \rho}{\partial r} = C_s^2 \frac{\partial \rho}{\partial r}$
   b. Thus, $U_r \frac{\partial U_r}{\partial r} + \frac{1}{\rho}\left[C_s^2 \frac{\partial \rho}{\partial r}\right] + \frac{GM}{r^2} = 0$
   c. From continuity eq., $r^2 U_r \frac{\partial \rho}{\partial r} + \rho\frac{\partial}{\partial r}(r^2 U_r) = 0 \Rightarrow \frac{1}{\rho}\frac{\partial \rho}{\partial r} = -\frac{1}{r^2 U_r}\frac{\partial}{\partial r}(r^2 U_r)$

II B. 6. (Continued)

d. Thus $U_r \frac{\partial U_r}{\partial r} + \frac{c_s^2}{r^2 U_r} \underbrace{\frac{\partial}{\partial r}(r^2 U_r)}_{= r^2 \frac{\partial U_r}{\partial r} + 2 r U_r} + \frac{GM}{r^2} = 0$

e. Finally $$\boxed{\frac{1}{2}\left(1 - \frac{c_s^2}{U_r^2}\right)\frac{\partial (U_r^2)}{\partial r} = -\frac{GM}{r^2}\left[1 - \frac{2 c_s^2 r}{GM}\right]}$$

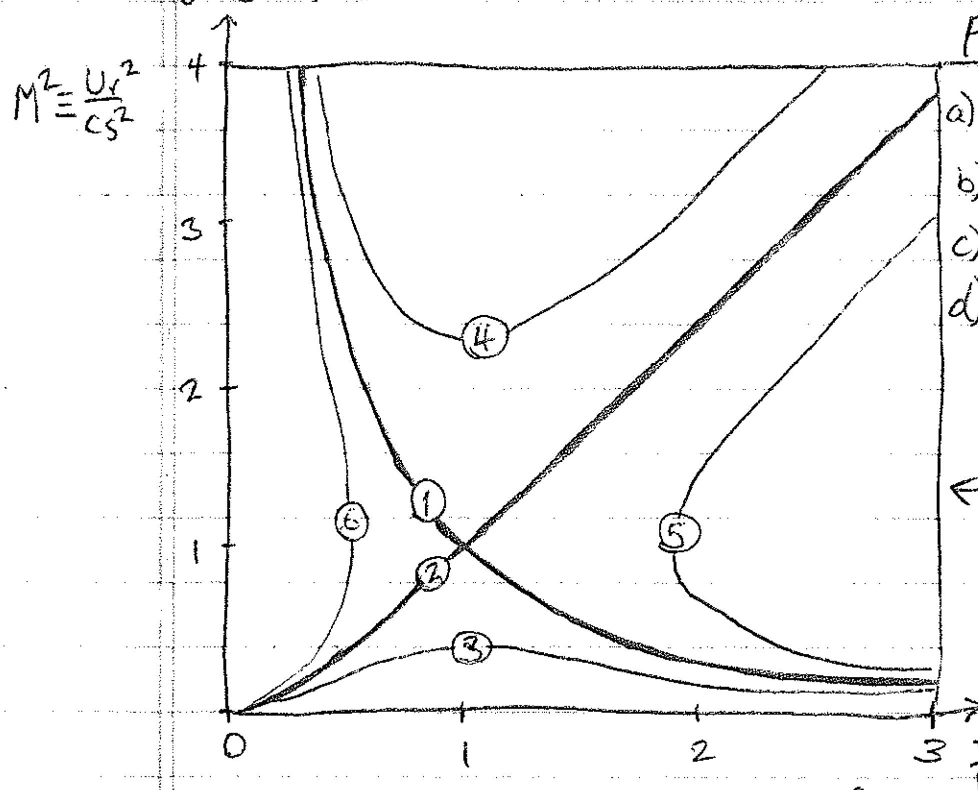
7. This is the same as the Parker Solar wind solution with general γ!

a. Critical radius: (RHS=0) $$\boxed{r_s = \frac{GM}{2c_s^2} \simeq 7.5 \times 10^{13} \left(\frac{T}{10^4 K}\right)^{-1}\left(\frac{M}{M_\odot}\right) \text{ cm}}$$

using $c_s \simeq 10 \left(\frac{T}{10^4 K}\right)^{\frac{1}{2}} \frac{km}{s}$ as sound speed for $1 \lesssim \gamma \lesssim \frac{5}{3}$

b. At $r = r_s$, LHS = 0 requires i) $U_r^2 = c_s^2$ or ii) $\frac{\partial U_r^2}{\partial r} = 0$

8. Solutions:



Four Types of Solutions
a) Unphysical Solutions ⑤ & ⑥
b) Purely supersonic ④
c) Purely subsonic ③
d) Transonic Solutions ① & ②

← Plot for $\gamma = \frac{4}{3}$

9. a) We expect $U_r \to 0$ as $r \to \infty$ for accretion problem → Solutions ① & ③

b) Since RHS < 0 for small $r \to 0$, this implies $U_r > c_s$ as $r \to 0$ ⇒ Solution ①!

II B. 9. c. Solution ① is unique solution for accretion problem!

C. Properties of Bondi Accretion Flow:

1. Integrate $\int dr$ on $U_r \frac{\partial U_r}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{GM}{r^2} = 0$

a. $\frac{U_r^2}{2} + \int \frac{dp}{\rho} - \frac{GM}{r} = \text{const.}$

b. NOTE: $dp = \gamma k \rho^{\gamma-1} d\rho$, so $\int \frac{dp}{\rho} = \gamma k \int \frac{\rho^{\gamma-1}}{\rho} d\rho = \frac{\gamma k}{\gamma-1}\rho^{\gamma-1} =$

$= \frac{\gamma k \rho^{\gamma}}{(\gamma-1)\rho} = \frac{1}{\gamma-1}\left(\frac{\gamma p}{\rho}\right) = \frac{c_s^2}{\gamma-1}$

c. Thus, we obtain the Bernoulli Integral $\boxed{\frac{U_r^2}{2} + \frac{c_s^2}{\gamma-1} - \frac{GM}{r} = \text{const}}$

d. At $r \to \infty$, $U_r \to 0$, so $\cancel{\frac{U_r^2}{2}}^{\to 0} + \frac{c_{s\infty}^2}{\gamma-1} - \cancel{\frac{GM}{r}}^{\to 0} = \text{const} \leftarrow$ Constant determined.

$$\boxed{\frac{U_r^2}{2} + \frac{c_s^2}{\gamma-1} - \frac{GM}{r} = \frac{c_{s\infty}^2}{\gamma-1}}$$

2. At $r = r_s$, Solution ① gives $U_r^2 = c_s^2(r_s)$, so $c_s^2(r_s)\left[\frac{1}{2} + \frac{1}{\gamma-1} - 2\right] = \frac{c_{s\infty}^2}{\gamma-1}$

a. Thus $\boxed{c_s(r_s) = c_{s\infty}\left(\frac{2}{5-3\gamma}\right)^{\frac{1}{2}}}$ Connects sound speed at critical radius to sound speed in ISM, $c_{s\infty}$ (Valid for $1<\gamma<\frac{5}{3}$)

3. Connect $\rho(r_s)$ to ρ_∞:

a. $c_s^2 \propto \frac{\gamma p}{\rho} \propto \rho^{\gamma-1} \Rightarrow \frac{c_s^2(r_s)}{c_{s\infty}^2} = \left(\frac{\rho(r_s)}{\rho_\infty}\right)^{\gamma-1} \to \boxed{\rho(r_s) = \rho_\infty\left[\frac{c_s(r_s)}{c_{s\infty}}\right]^{\frac{2}{\gamma-1}}}$

4. Obtain mass accretion rate $\dot{M}$ in terms of $c_{s\infty}$ & ρ_∞:

a. $\dot{M} = 4\pi r^2 \rho U_r = 4\pi r_s^2 \rho(r_s) c_s(r_s)$

b. Substituting for r_s, $\rho(r_s)$, & $c_s(r_s)$ in terms of $c_{s\infty}$ & ρ_∞, II C. 4. Continued)

c. $$\boxed{\dot{M} = \pi G^2 M^2 \frac{\rho_\infty}{c_{s\infty}^3}\left(\frac{2}{5-3\gamma}\right)^{\frac{5-3\gamma}{2(\gamma-1)}}}$$ for $1<\gamma<\frac{5}{3}$

d. Typical Values $c_{s\infty} = 10\,\frac{km}{s}$ and $\rho_\infty = 10^{-24}\frac{g}{cm^3}$ $(n_\infty \simeq 1\,cm^{-3})$

$$\boxed{\dot{M} \simeq 1.4\times 10^{11}\left(\frac{M}{M_\odot}\right)^2\left(\frac{\rho_\infty}{10^{-24}\frac{g}{cm^3}}\right)\left(\frac{c_{s\infty}}{10\,\frac{km}{s}}\right)^{-3}\frac{g}{s}}$$ for $\gamma = 1.4$

This accretion rate is quite low, and Bondi accretion powered radiation is probably not observable ($L_{acc} \sim 10^{31}\frac{erg}{s}$ for neutron star).

NOTE: For binary mass transfer, typically $\dot{M} \sim 10^{16}\, g/s$ (much larger)

5. Accretion radius, r_{acc}:

a. Bernoulli Integral: $$\boxed{\frac{U_r^2}{2} + \frac{c_s^2 - c_{s\infty}^2}{\gamma - 1} - \frac{GM}{r} = 0}$$

b. At $r\to\infty$, gravity is weak $U_r \to 0$, and $c_s = c_{s\infty}$.

c. As r decreases, inflow velocity reaches $U_r = -c_{s\infty}$ (inflow).
- Only gravitational term can balance this flow term, so

$$\frac{U_r^2}{2} \simeq \frac{GM}{r_{acc}} \Rightarrow \boxed{r_{acc} \simeq \frac{2GM}{c_{s\infty}^2}}$$

d. $$\boxed{r_{acc} = 3\times 10^{14}\left(\frac{M}{M_\odot}\right)\left(\frac{10^4 K}{T_\infty}\right) cm}$$

D. Summary of Bondi Accretion Flow:

1. Accretion Rate: $$\dot{M} = \pi G^2 M^2 \frac{\rho_\infty}{c_{s\infty}^3}\left(\frac{2}{5-3\gamma}\right)^{\frac{5-3\gamma}{2(\gamma-1)}}$$

2. Accretion Radius: $$r_{acc} = \frac{2GM}{c_{s\infty}^2}$$

3. Accretion from ISM to isolated stars is unlikely to be observable
⇒ Must look at binaries to find more powerful accreting systems.