29:293 Homework #1

Due at the beginning of class, Thursday, February 5, 2015.

1. Electrostatic Drift Waves
   Consider a cylindrical column of plasma of radius a with an equilibrium number density that varies radially as

$$n_0(r) = \overline{n}\sqrt{1 - \frac{r^2}{4a^2}}$$

for $r \leq a$, where $\overline{n}$ is a constant. The plasma is confined by a uniform axial magnetic field, $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$. The low beta plasma has $m_e/m_i \ll \beta_e \ll 1$ and may be treated as a two fluid plasma with cold, singly-charged ions and isothermal electrons. Consider the plasma in the low frequency $\omega \ll \omega_{ci}$ and long wavelength $kC_i \ll \omega_{pi}$ limits, where the ion acoustic speed is defined as $C_i^2 = T_e/m_i$ (and Boltzmann's constant has been absorbed into the temperature).

   (a) Find the magnitude of the equilibrium electron drift velocity U_d as a function of C_i, ω_{ci}, r, a, and physical constants. (Hint: Use the electron momentum equation in the small electron mass limit to obtain a constraint on the equilibrium. Then solve for the electron drift required to satisfy that constraint.)

   (b) In terms of the cylindrical coordinates (r, θ, z), in what direction is this equilibrium drift?

   (c) Does this equilibrium drift correspond to differential or solid body rotation of the plasma? (Note that solid body rotation requires a linear variation of the velocity with radius, $U(r) \propto r$.)

   (d) To investigate the wave behavior of the plasma, a frequently used approximation is to treat the dynamics in the cylindrical plasma locally by a Cartesian coordinate system (taking $r \to x$, $\theta \to y$, and $z \to z$). For a fluctuating wave that varies in this local Cartesian coordinate system as $\exp(ik_y y + ik_\parallel z - i\omega t)$, find the *two* roots of the drift wave frequency in the limit $k_\parallel C_i \ll k_y U_d$. Express your answers in terms of k_y, $k_\parallel$, C_i, ω_{ci}, r, a, and physical constants. [Note that you do *not* simply assume that $k_\parallel C_i = 0$, but expand in a small parameter $\epsilon = k_\parallel C_i/(k_y U_d) \ll 1$.)