29:293 Homework #7

Reading:	Required:	Read GB Chapter 11, Sec 11.1–11.2 (p.415–420)
	Optional:	Read BS Chapter 8, Sec 8.4–8.5 (p.307–317)

Due at the beginning of class, Thursday, April 9, 2013.

1. Show that for an electric field of the form

$$\mathbf{E}(\mathbf{x},\tau,t) = \mathbf{E}_0(\mathbf{x},\tau)\cos(\omega t - \mathbf{k}\cdot\mathbf{x})$$

the magnetic field is given by

$$\mathbf{B}(\mathbf{x},\tau,t) = -\frac{1}{\omega}\left\{[\nabla\times\mathbf{E}_0(\mathbf{x},\tau)]\sin(\omega t - \mathbf{k}\cdot\mathbf{x}) - [\mathbf{k}\times\mathbf{E}_0(\mathbf{x},\tau)]\cos(\omega t - \mathbf{k}\cdot\mathbf{x})\right\}$$

2. An electron of charge $q_e = -e$ and mass m_e and an proton of charge $q_e = e$ and mass $m_i = m_p$ are initially at rest at $\mathbf{x} = (0,0,0)$ in a magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. An electric field is then turned on at $t = 0$ and increased linearly until time $t_1 = \frac{20\pi m_i}{eB_0}$, at which point the electric field is held constant,

$$\mathbf{E}(t) = \begin{cases} 0 & t < 0 \\ E_0(t/t_1)\hat{\mathbf{y}} & 0 \le t \le t_1 \\ E_0\hat{\mathbf{y}} & t > t_1 \end{cases}$$

Find the total current density as a function of time $\mathbf{j}(t)$ due to the drifts of the two particles (neglect the current due to the fast Larmor oscillation).

3. Laser Trapping: A charged particle in an unmagnetized plasma can be trapped by a spatially varying intense laser field. Using intereference of several lasers, the electric field near a charged particle is given by

$$\mathbf{E}(\mathbf{x},t) = E_0[1 + (x/x_0)^2]\sin(\omega t - k_y y)\hat{\mathbf{x}}.$$

Calculate the velocity of the oscillation center U as a function of position x for a particle initially at rest at $t = 0$ at position $\mathbf{x} = (x_0, 0, 0)$. You may assume that the particle velocity v and laser frequency ω satisfy $v \ll \omega/k_y$ and $v/x_0 \ll \omega$.

4. Calculate the numerical values of $\ln\Lambda$ for hydrogen plasmas in the range of density 1–10^{12} cm^{-3} and temperature 10^2–10^8 K. How sensitive is $\ln\Lambda$ to such a wide range of density and temperature?

5. How does the mean free path for electron-ion collisions $\lambda_{m(e-i)}$ depend on the electron temperature T_e?