PHYS:5905 Homework #2b

Please submit your solutions as a single PDF file with answers to the questions asked.
Please complete required problems before lecture on Thursday, January 24, 2019.

1. (Required) **Second-order timestepping for $\mathbf{E} \times \mathbf{B}$ drift**

   (a) Convert the numerical code for single-particle motion to the dimensionless equations

   (b) Equations:

   $$\frac{d\mathbf{x}'}{dt'} = \mathbf{v}' \quad (1)$$

   $$\frac{d\mathbf{v}'}{dt'} = \mathbf{E}' + \mathbf{v}' \times \mathbf{B}' \quad (2)$$

   where the normalizations are given by

   $$\mathbf{x}' = \frac{\mathbf{x}}{r_L} \quad (3)$$

   $$t' = \Omega t \quad (4)$$

   $$\mathbf{v}' = \frac{\mathbf{v}}{v_\perp} \quad (5)$$

   $$\mathbf{B}' = \frac{\mathbf{B}}{B_0} \quad (6)$$

   $$\mathbf{E}' = \frac{\mathbf{E}}{v_\perp B_0} \quad (7)$$

   and the angular ion cyclotron frequency and Larmor radius are given by

   $$\Omega = \frac{qB_0}{m} \quad (8)$$

   $$r_L = \frac{v_\perp}{\Omega} \quad (9)$$

   (c) Implement a second-order leapfrog timestepping scheme in your previous single particle motion code for a particle motion in constant, uniform magnetic and electric fields.

   (d) I recommend using a `switch-case` programming structure to allow for either Euler timestepping or Leapfrog timestepping in the same code with a single parameter change. You may also wish to write the Lorentz Force computation as a function call to simplify the code.

   (e) Take the same parameters as the $\mathbf{E} \times \mathbf{B}$ drift problem in HW#2, with the same non-zero electric field $\mathbf{E} = E_0 \hat{\mathbf{y}}$ with $E_0 = 0.1 v_\perp B_0$

   (f) Take $N = 1000$ timesteps over the simulation time T.

   (g) (Return) Create two output plots of $\mathbf{x}(t)$:

      i. Plot the Trajectory of the particle in the (x, y) plane.

      ii. Plot the Position x as a function of Time t.

   Note that, for both of these plots, you should plot the numerical solution along with the analytical solution. You should also appropriately normalize the axes to physical quantities r_L and Ω.

   (h) (Return) Compute the error in the position at $t = 20\pi$ from the analytical solution as a function of the number of timesteps taken N and compare to the result for Euler method in HW#2. Plot the error over the a minimum range $1000 \leq N \leq 1000000$ using an appropriate choice to visualize the results. What is the slope of the resulting Leapfrog method error plot?

2. (Required) Second-order Leapfrog

   (a) Use a Taylor expansion to demonstrate that the Leapfrog timestepping scheme is a second-order method.