PHYS:5905 Homework #3a

Please submit your solutions as a single PDF file with answers to the questions asked.
Please complete required problems before lecture on Tuesday, February 5, 2019.

1. (Required) **Third-order Adams-Bashforth timestepping**

   (a) Implement a third-order Adams-Bashforth timestepping in your previous single particle motion code for a particle motion in constant, uniform magnetic and electric fields.

   (b) (Return) How have you chosen to initialize the timesteps to initiate third-order Adams-Bashforth timestepping?

   (c) Take the same parameters as the $\mathbf{E} \times \mathbf{B}$ drift problem in HW#2, with the same non-zero electric field $\mathbf{E} = E_0 \hat{\mathbf{y}}$ with $E_0 = 0.1 v_\perp B_0$

   (d) Take $N = 1000$ timesteps over the simulation time T.

   (e) (Do not Return) Check output plots of $\mathbf{x}(t)$ to make sure the results are similar to your previous schemes:

      i. Plot the Trajectory of the particle in the (x, y) plane.

      ii. Plot the Position x as a function of Time t.

   Note that, for both of these plots, you should plot the numerical solution along with the analytical solution. You should also appropriately normalize the axes to physical quantities r_L and Ω.

   (f) (Return) Compute the error in the position at $t = 20\pi$ from the analytical solution as a function of the number of timesteps taken N and compare to the result for Euler method in HW#2. Plot the error over the a minimum range $1000 \leq N \leq 1000000$ using an appropriate choice to visualize the results. What is the slope of the resulting third-order Adams-Bashforth method error plot? (Optional) You may choose to plot the error vs. timestep for all three methods: Euler, Leapfrog, and Adams-Bashforth on the same plot.

2. (Required) ∇B **Drift**

   (a) Modify your single particle motion program to accept magnetic and electric fields that vary in time and space. A sample function to use for the magnetic field is `magfield.m`, available on the course website.

   (b) Specify zero electric field and a dipole magnetic field. A dipolar magnetic field is given in spherical coordinates by the equation

   $$\mathbf{B} = \frac{\mu_0 M}{4\pi} \frac{1}{r^3} \left\{ 2 \cos\theta \hat{\mathbf{r}} + \sin\theta \hat{\boldsymbol{\theta}} \right\} \tag{1}$$

   where M is the magnetic moment. In Cartesian coordinates, the same field can be expressed in dimensionless units by

   $$\frac{\mathbf{B}}{B_0} = \left[\frac{\mu_0 M}{4\pi B_0} \right] \frac{1}{r^3} \left\{ \frac{3xz}{r^2} \hat{\mathbf{x}} + \frac{3yz}{r^2} \hat{\mathbf{y}} + \left(\frac{3z^2}{r^2} - \frac{R^2}{r^2} \right) \hat{\mathbf{z}} \right\} \tag{2}$$

   where $r = \sqrt{x^2 + y^2 + z^2}$ is the spherical radius and $R = \sqrt{x^2 + y^2}$ is the cylindrical radius. Note that the term in brackets simply controls the overall amplitude of the dipolar magnetic field.

   Note that the Matlab function `magfield.m` already has this field coded up.

   (c) (Return) For a normalized magnetic dipole constant of $\mu_0 M / (4\pi B_0) = -100$, an initial position $\mathbf{x}_0 / r_L = (5, 0, 0)$, and an initial velocity $\mathbf{v}_0 / v_\perp = (0, -1, 0)$, plot the trajectory of the particle on the (x, y) plane over a simulation time $\Omega T = 30\pi$.

   (d) (Return) If the simulation time is increased to $\Omega T = 60\pi$, for the timestepping scheme you are using, what is the minimum number of timesteps needed so that the drift properly closes on itself (by visual inspection) and does not appear to become obviously unphysical?

   (e) (Return) We do not have an analytical solution for the trajectory in this problem, so what can we do to ensure that our solution is accurate?

3. (Optional) **Ring Current due to ∇B Drift**

   (a) The ∇B drift, unlike the $\mathbf{E} \times \mathbf{B}$ drift, depends on the sign of the particle charge. Thus, energetic ions trapped in the Earth's dipolar magnetosphere will drift one direction, while the electrons will drift the other direction.

   (b) (Return) Assume that the solution to the ∇B Drift problem above is for the ions. What do you change to obtain the solution for the electrons with a reduced mass ratio of $m_i/m_e = 100$? Plot both ion ∇B drift and electron ∇B drift trajectories on the (x, y) plane. Assume the perpendicular temperatures of the ions and electrons are the same (*i.e.*, $T_{\perp s} = m_s v_\perp^2/2$, where the Boltzmann constant is absorbed into the temperature, giving temperature in units of energy).

   (c) (Return) Which species do you expect dominates the ring current?

   (d) Experiment with different field strengths to observe the effect on the particle trajectories and drifts.

4. (Optional) ∇B **and Curvature Drifts**

   (a) If the particle has a velocity parallel to a curved magnetic field, it will experience a Curvature Drift in addition to the ∇B Drift in a dipolar magnetic field.

   (b) (Return) For a normalized magnetic dipole constant of $\mu_0 M/(4\pi B_0) = -1000$, an initial position $\mathbf{x}_0/r_L = (5, 0, 1)$, and an initial velocity $\mathbf{v}_0/v_\perp = (0, -1, 2)$, plot the 3D trajectory of the particle over a simulation time $\Omega T = 200\pi$.

   The Matlab function `plot3(x,y,z)` will be useful for visualizing the trajectories.

   (c) Experiment with different field strengths and initial z velocities to observe the effect on the particle trajectories and drifts.