PHYS:5905 Homework #4a

Please submit your solutions as a single PDF file with answers to the questions asked.
Please complete required problems before lecture on Thursday, February 14, 2019.

1. (Required) **Magnetic Mirror Confinement**

   (a) A simple (but unrealistic) magnetic mirror configuration can by a magnetic field in cylindrical coordinates (r, ϕ, z) given by

   $$\frac{\mathbf{B}}{B_0} = \frac{-\pi r}{L}\left(\frac{\delta B_z}{2B_0}\right)\sin\left(\frac{2\pi z}{L}\right)\hat{\mathbf{r}} + \left\{\frac{B_{00}}{B_0} + \left(\frac{\delta B_z}{2B_0}\right)\left[1 - \cos\left(\frac{2\pi z}{L}\right)\right]\right\}\hat{\mathbf{z}} \tag{1}$$

   (b) Taking the normalization of the length L of the magnetic mirror (between positions of maximum field strength along the axis) to be $L' = L/r_L$, the normalization of the axial magnetic field perturbation to be $\delta B'_z = \delta B_z/B_0$, the normalization of the field strength at the center of the magnetic mirror to be $B'_{00} = B_{00}/B_0$, we obtain normalized equations for the magnetic mirror field in Cartesian coordinates

   $$B'_x = \frac{-\pi x'}{2L'}\delta B'_z \sin\left(\frac{2\pi z'}{L'}\right) \tag{2}$$

   $$B'_y = \frac{-\pi y'}{2L'}\delta B'_z \sin\left(\frac{2\pi z'}{L'}\right) \tag{3}$$

   $$B'_z = B'_{00} + \frac{\delta B'_z}{2}\left[1 - \cos\left(\frac{2\pi z'}{L'}\right)\right] \tag{4}$$

   (c) (Return) Verify that $\nabla \cdot \mathbf{B} = 0$ for this magnetic mirror field.

   (d) The magnetic mirror ratio measured along the axis for this configuration is given by $R_m \equiv B_{max}/B_{min} = 1 + \delta B'_z/B'_{00}$.

   (e) Write a function that computes the first adiabatic invariant (magnetic moment), magnetic field magnitude, parallel kinetic energy, total kinetic energy, and perpendicular and parallel components of velocity at a given timestep. Implement this function to compute these quantities as a function of time.

   (f) Set up a magnetic mirror with the parameters: mirror length $L' = 10$, minimum axial field value $B'_{00} = 10$ and mirror ratio $R_m = 4$.

   (g) (Return) For an initial position $\mathbf{x}_0/r_L = (0.1, 0, 4)$, and an initial velocity $\mathbf{v}_0/v_\perp = (0, 1, 0)$, plot the trajectory of the particle on the (z, x) plane over a simulation time $\Omega T = 5\pi$.

   (h) (Optional) Plot the 3D trajectory of the particle of the same simulation over plotted region defined by $-0.5 \le x \le 0.5$, $-0.5 \le y \le 0.5$, and $-10 \le z \le 10$. [Use `axis` function to define plot limits.]

   (i) (Return) Plot the evolution of the magnetic moment $\mu = mv_\perp^2/(2B)$, normalized by the magnetic moment at $t = 0$, $\mu_0 \equiv \mu(t = 0)$. On the same plot, to compare how well the magnetic moment is conserved, plot $v_\perp^2(t)$ normalized by its value at $t = 0$ and the magnetic field magnitude B at the particle position normalized by its value at $t = 0$. How well (in terms of percentage change) is the magnetic moment conserved over the course of the simulation? Does decreasing the timestep improve this lack of conservation? If not, what could be causing any lack of conservation?

   (j) (Return) Plot the evolution of the total particle kinetic energy normalized to its value at $t = 0$. On the same plot, plot the parallel kinetic energy and μB, both normalized to the initial *total particle kinetic energy*. How well is energy conserved?

   (k) (Optional) Try changing just the mirror ratio R_m and observe the effect on the 3D particle trajectory and the evolution of the magnetic moment and total energy. When there is a lack of conservation, how can you distinguish if it is due to numerical effects or physical effects?