PHYS:5905 Homework #4b

Please submit your solutions as a single PDF file with answers to the questions asked.
Please complete required problems before lecture on Thursday, February 14, 2019.

1. (Required) **Implementation of Adaptive Runge-Kutta (RK45)**

   (a) Use the Matlab function `ode45` to implement an RK45 integration scheme

   (b) Note: The format of the `ode45` function is
   `[t,y] = ode45(odefun,tspan,y0)`
   where

   i. `t` is a column vector of all the times at which the solution computes the values

   ii. For this application, `y` is a matrix of positions and velocities, where each row corresponds to `[ x y z vx vy vz]` and the columns are these values at each time

   iii. `odefun` is a function handle to your function that contain the time derivatives of the values in each column of `y` (in other words, `x`, `vx`, etc.). This function handle is set using the syntax
   `lorhandle=@lorentz;`
   where `lorentz.m` is the file that contains the function with the lorentz force calculation

   iv. `tspan` is a column vector with the initial and final times of integration

   v. `y0` is a row vector with the initial conditions `[x0 y0 z0 vx0 vy0 vz0]`.

   (c) Note that details of the function `ode45` can be found in the documentation

   (d) To specify a specific relative tolerance, you can set the options using
   `options = odeset('RelTol',1.0e-5);`
   and then pass the variable `options` into the function `ode45`
   `[t,y]=ode45(lorhandle,tspan,y0,options);`

   (e) Note that `ode45` implements an embedded Runge-Kutta (4,5) scheme as formulated by Dormand and Prince.

   (f) Test the RK45 routine by comparing the AB3 scheme with the RK45 scheme for the simple case of $\mathbf{E} \times \mathbf{B}$ drift (same problem as HW#2, prob 2). Here take parameters: constant uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$ (normalized $B'_0 = 1$) and electric field $\mathbf{E} = E_0\hat{\mathbf{y}}$ with $E_0 = 0.1 v_\perp B_0$, simulation time interval $0 \leq \Omega t \leq 20\pi$, initial position to be $\mathbf{x}'_0 = (0, 1, 0)$ and the initial velocity to be $\mathbf{v}'_0 = (1, 0, 0)$.

   (g) (Return) Run AB3 with $N = 10,000$ steps and compute the error (difference from the analytical solution) at $\Omega t = 20\pi$. Now, set the relative tolerance for the RK45 scheme to the same value (using `options = odeset('RelTol',1.0e-5);`, where the error here is `1.0e-5`) and determine how many timesteps it requires.

   (h) (Return) Plot the Trajectory of the particle in the (x, y) plane for RK45 with a relative tolerance of 4×10^{-3}. Be sure to plot the analytical solution as well. How many RK45 steps does this require? What is the total error relative to the analytical solution?

   (i) (Return) If the relative tolerance is changed to 1×10^{-6}, how many RK45 steps does this require? What is the total error relative to the analytical solution? Why is the total error larger than the specified relative tolerance?

2. (Optional) **Magnetic Mirror Integration with RK45**

   (a) Implement energy and adiabatic invariant diagnostics for your RK45 code.

   (b) Using the magnetic mirror configuration from HW#4a problem 1, choose the following mirror parameters: mirror length $L' = 10$, minimum axial field value $B'_{00} = 10$ and mirror ratio $R_m = 20$.

   (c) (Return) Using conservation of energy as a measure of accuracy, how many AB3 steps do you need using AB3 to maintain conservation of energy to less than 0.1%? Setting the RK45 relative tolerance to 10^{-3}, how many steps does the RK45 algorithm require, and what is the total resulting error in the energy?