PHYS:5905 Homework #8b

Please submit your solutions as a single PDF file with answers to the questions asked.
Please complete required problems before lecture on Tuesday, March 26, 2019.

As usual, you are welcome to do these exercises in any other language of your choice (Fortran, C, Python, Matlab, etc.)

1. (Required) **Phase Errors from Leapfrog Method**

   (a) We saw in HW#7 Prob 1(h) that the Lax Method leads to significant amplitude errors when the ratio $c\Delta t/\Delta x < 1$. The von Neumann stability analysis explains this decay in the wave amplitude due to the numerical algorithm.

   (b) Note that, although the Lax Method performs extremely well when $c\Delta t/\Delta x = 1$, under the more realistic conditions of a typical plasma simulation, the sound velocity c may change as a function of position, making it impossible to maintain the optimal value of $c\Delta t/\Delta x = 1$ throughout the domain.

   (c) Let's investigate the properties of the leapfrog timestepping scheme (with centered spatial derivatives) as the timestep is changed. Initialize a single sinusoidal wave with a normalized wavevector $k_0' = k_0 L = 2\pi$, normalized amplitude $u_0' = 0.1$, and direction $\sigma = +1$. The eigenvector for the Fourier components is $(\hat{\rho}', \hat{u}', \hat{p}') = (\sigma \hat{u_0}', \hat{u_0}', \sigma \hat{u_0}')$ (same setup at HW#7a).

   (d) (Return) Using $n_x = 128$, plot $u'(x')$ vs. x' at time $t' = 100$ for the timestep values $\Delta t' = 1/128, 1/192, 1/256, 1/512, 1/1024, 1/2048$.

   (e) (Return) Compute the phase difference by fitting the form of $u'(x')$ at $t' = 100$ to a curve of form $u_0' \sin(k_0' x' + \delta)$, where $u_0' = 0.1$ and $k_0' = 2\pi$. You can use curve fitting, or just do the fit by adjusting the value of δ to obtain a good fit by eye. Plot the value of the phase error δ/π vs. the normalized timestep $c\Delta t/\Delta x$.

   (f) (Return) Repeat this temporal resolution convergence test with a lower spatial resolution $n_x = 16$. In this case, you will discover both phase and amplitude errors, so fit the resulting $u'(x')$ at $t' = 10$ using a form $u_f' \sin(k_0' x' + \delta)$, where you adjust u_f' and δ to obtain the best fit. Plot the value of the phase error δ/π and vs. the normalized timestep $c\Delta t/\Delta x$ for timestep values $\Delta t' = 1/16, 1/24, 1/32, 1/64, 1/128, 1/256$.

   (g) (Return) Plot the value of the amplitude error u_f'/u_0' and vs. the normalized timestep $c\Delta t/\Delta x$ for the same timestep values $\Delta t' = 1/16, 1/24, 1/32, 1/64, 1/128, 1/256$.