

PHYS:5905 Homework #9b

Please submit your solutions as a single PDF file with answers to the questions asked.

Please complete required problems before lecture on Tuesday, April 2, 2019.

As usual, you are welcome to do these exercises in any other language of your choice (Fortran, C, Python, Matlab, etc.)

1. (Optional) Landau Damping of Langmuir Wave using VP

- Download the tar archive of VP, The 1D-1V Vlasov-Poisson Nonlinear Solver, from the course website under the Codes section, `vp3_190327.tar`
- Create a directory `vp` on the local computer and extract the tar archive inside the directory using `tar -xvf vp3_190327.tar`
- Edit the `Makefile` as needed to compile the code on the platform you are using. Compile the code using the `Makefile`
`make`
If all goes smoothly, this should create the executable file `vp.e`. Note that, on Argon, you can use the Intel Fortran compiler `ifort`.
- Run the input file `landaudamp1.in` using the command
`vp.e landaudamp1.in`
- The output file `landaudamp1.vp.phi.32` outputs the following columns:

Column	Value
1	Time, \hat{t}
2	Position, \hat{x}
3	Electrostatic potential, $\hat{\phi}$
4	Charge density, $\hat{\rho}$
5	Electric Field, $\partial\hat{\phi}/\partial\hat{x}$

Note that the normalized electric field is given by

$$\hat{E} = -\frac{q_e \lambda_{de}}{T_e} E = \frac{\partial \hat{\phi}}{\partial \hat{x}} \quad (1)$$

- (Return) Create a plot of the normalized electric field $-q_e \lambda_{de} E / T_e$ at position $x / \lambda_{de} = 0$ as a function of $\omega_{pe} t$. This plot shows the collisionless Landau damping of the standing Langmuir wave pattern generated by the initial electron density perturbation.
- (Return) Plot $\log(|\hat{E}|)$ vs. $\omega_{pe} t$ and fit the upper envelope of the resulting curve to an equation of the form $\hat{E} = \hat{E}_0 e^{-\hat{\gamma} \hat{t}}$ (this fit can simply be done by eye, where adjusting \hat{E}_0 shifts the curve up and down, and $\hat{\gamma}$ changes its slope). Be sure to plot the fit line along with the curve of the data. What is the resulting normalized damping rate $\hat{\gamma}$?

2. (Optional) Exact Riemann Eigenfunction of 1-D Nonlinear Sound Wave

- The Riemann invariants are given by

$$J_{\pm} = U \pm \frac{2}{\gamma - 1} c_s \quad (2)$$

where γ is the adiabatic index and $c_s^2 = \gamma p / \rho$ is the local sound speed squared.

- (b) We want to initialize a state in which one of the two Riemann invariants are exactly constant as a function of x , so that the nonlinear sound wave will propagate in only one direction.
- (c) Since J_{\pm} is constant along characteristics, we can set it to any constant value, so we choose its value at equilibrium conditions ρ_0 , U_0 , and p_0 . Therefore, the constant is given by

$$J_{\pm} = U \pm \frac{2}{\gamma - 1} c_s = U_0 \pm \frac{2}{\gamma - 1} c_{s0}. \quad (3)$$

Normalizing the velocities $U' = U/c_{s0}$ by the equilibrium sound speed $c_{s0}^2 = \gamma p_0/\rho_0$, we obtain

$$J'_{\pm} = U' \pm \frac{2}{\gamma - 1} c'_s = U'_0 \pm \frac{2}{\gamma - 1}. \quad (4)$$

- (d) Solving for c'_s in terms of the U' , we obtain

$$c'_s = 1 \mp \frac{\gamma - 1}{2} (U' - U'_0) \quad (5)$$

- (e) Note that the pressure-density relation is given by

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0} \right)^{\gamma} \quad (6)$$

Using the normalizations $p' = p/\gamma p_0$ and $\rho' = \rho/\rho_0$, we obtain

$$p' = \rho'^{\gamma}/\gamma \quad (7)$$

- (f) The adiabatic sound speed is fundamentally defined as the partial derivative of the pressure with respect to the density at constant entropy s ,

$$c_s^2 \equiv \left(\frac{\partial p}{\partial \rho} \right)_s = \frac{\gamma p_0}{\rho_0} \left(\frac{\rho}{\rho_0} \right)^{(\gamma-1)} \quad (8)$$

Applying the normalization and solving for c'_s , we obtain

$$c'_s = \rho'^{(\gamma-1)/2} \quad (9)$$

- (g) Using (9) to substitute into (5) for c'_s in terms of ρ' , we can solve for ρ' in terms of U' to obtain

$$\rho' = \left\{ 1 \mp \frac{\gamma - 1}{2} (U' - U'_0) \right\}^{2/(\gamma-1)} \quad (10)$$

- (h) To initialize an exact Riemann invariant in your nonlinear hydrodynamics code, we specify the wave direction we want by σ : (i) $\sigma = +1$ yields a wave moving in the $+x$ direction and requires initializing J_- constant in x ; or (ii) $\sigma = -1$ yields a wave moving in the $-x$ direction and requires initializing J_+ constant in x .

- (i) We specify a nonlinear sound wave with normalized amplitude U'_1 with direction $\sigma = \pm 1$ by

$$U'(x') = U'_0 + U'_1 \sin(k'x') \quad (11)$$

$$\rho'(x') = \left\{ 1 + \sigma \frac{\gamma - 1}{2} (U'(x') - U'_0) \right\}^{2/(\gamma-1)} \quad (12)$$

$$p'(x') = \rho'(x')^{\gamma}/\gamma \quad (13)$$

- (j) Using $n_x = 128$, initialize a nonlinear sound wave using the exact Riemann invariant with $\sigma = +1$, $\gamma = 5/3$, $\rho'_0 = 1$, $U'_1 = 0.08$, and $p'_0 = 1/\gamma$.
- (k) (Return) To verify that the Riemann invariant initialization is correct, plot $J'_{\pm} = J_{\pm}/c_{s0}$ as a function of x' at times $t' = 0$ and $t' = 1.0$. You should see that one of the invariants remains constant at all times, while the other invariant undergoes nonlinear steepening as expected.
- (l) (Return) Plot the normalized velocity U' as a function of x' at times $t' = 0$ and $t' = 1.0$ for the nonlinear simulation along with the Method of Characteristics analytical solution using the exact Riemann invariant.