Howes (1)

Numerical Lecture #4 Invariants

I. Invariants

A. ~~Use~~ Determining Numerical Integration Issues

1. Invariants of a given system are extremely useful as diagnostics, often providing a simple way to diagnose numerical accuracy problems.

2. Exact vs. Asymptotic Invariants
   a. Some invariants are exact (e.g., energy) and should be conserved to high precision by the numerical scheme.
   b. Other invariants are conserved under more restrictive conditions, with violation asymptoting to zero in the limit of validity. (e.g., adiabatic invariants)

B. Violation of Invariants
   a. Non-conservation of an adiabatic invariant may not indicate a numerical problem, but rather violation of the conditions for invariance.

e.g., the first adiabatic invariant (magnetic moment) requires (1) smooth spatial variation relative to the Larmor radius $\left(\frac{\nabla B}{B} \sim \frac{1}{L_{grad}} \Rightarrow \frac{r_L}{L_{grad}} = \frac{r_L |\nabla B|}{B} \ll 1 \right)$

and (2) slow variation relative to gyromotion, $\left(\frac{\frac{\partial B}{\partial t}}{B} \sim \frac{1}{T} \rightarrow \Omega T \gg 1 \right)$

I. A. (Continued)

4. Conservative Integration Methods
   a. When using a method that conserves a particular quantity exactly (e.g. energy or flux), the quantity will always be conserved unless there is a bug.
   b. ~~But, as~~ the integration may not be accurate even if the quantity is conserved.
   c. One cannot use an exactly conserved quantity (one that is exactly conserved by the algorithm) to diagnose accuracy!

B. Plasma Simulations and Invariants

1. Many plasma systems have one or more invariants.
2. Plotting the evolution of these invariants vs. time is a useful diagnostic of a code.

3. We'll try diagnosing total particle kinetic energy

$$\mathcal{E} = \frac{1}{2} m v^2 = \frac{1}{2} m v_{\parallel}^2 + \frac{1}{2} m v_{\perp}^2$$

and magnetic moment (first adiabatic invariant)

$$\mu = \frac{m v_{\perp}^2}{2B}$$

to test particle motion simulation in a magnetic mirror configuration.