Lecture #5: Kinetic Plasma Physics

I. Kinetic Description of a Plasma

A. Overall Framework of Plasma Physics

Particles: Ions & Electrons

$\underset{\sim}{x}$, $\underset{\sim}{v}$ $\Rightarrow$ ρ_q, $\underset{\sim}{j}$

Maxwell's Equations

ρ_q & $\underset{\sim}{j}$ $\Rightarrow$ $\underset{\sim}{E}$ & $\underset{\sim}{B}$

$\underset{\sim}{E}$ & $\underset{\sim}{B}$ fields

Lorentz Force Law

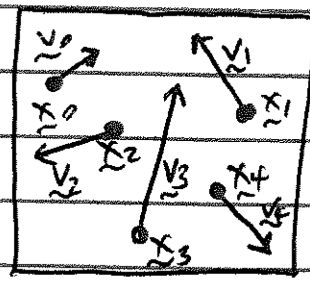
$$m_s \frac{d\underset{\sim}{v}_s}{dt} = q_s(\underset{\sim}{E} + \underset{\sim}{v}_s \times \underset{\sim}{B})$$

$\underset{\sim}{E}$ & $\underset{\sim}{B}$ $\Rightarrow$ $\underset{\sim}{x}$ & $\underset{\sim}{v}$

B. Vlasov-Maxwell Equations for a Kinetic Plasma

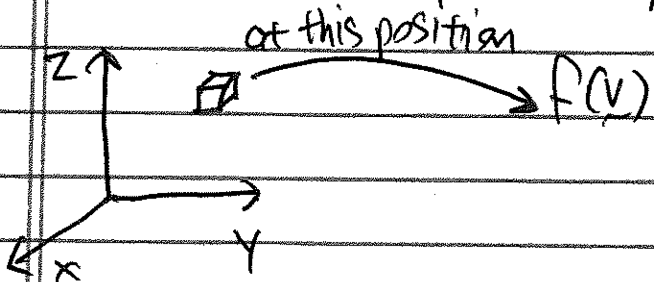
1. The Boltzmann Equation

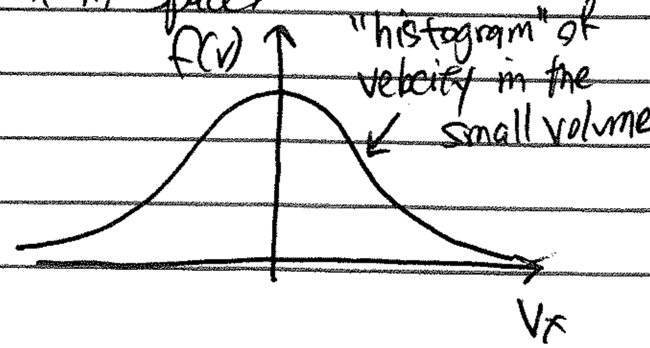
a. Microscopically, a plasma consists of a collection of charged particles, each with a position $\underset{\sim}{x}$ and velocity $\underset{\sim}{v}$.



b. To describe a collection of particles statistically, we use a 6-dimensional (3D-3V) (plus time) distribution function $f_s(\underset{\sim}{x}, \underset{\sim}{v}, t)$ for each species.

c. We can think of this as describing the velocity distribution in a small volume at each point in space.





I. B. 1. (Continued)

d. The evolution of the distribution function $f_s(\mathbf{x}, \mathbf{v}, t)$ is given by

$$\boxed{\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \frac{q_s}{m_s}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \left(\frac{\partial f_s}{\partial t}\right)_{coll}}$$

The Boltzmann Equation

e. Together with Maxwell's Equations, this yields the closed set of <u>Maxwell-Boltzmann Equations</u>

2. In the weakly collisional conditions, relevant to fusion plasmas and many space & astrophysical plasmas, collisions can often be neglected: $\Rightarrow \left(\frac{\partial f}{\partial t}\right)_{coll} = 0$

$\Rightarrow$ This yields the <u>Vlasov equation</u> for collisionless plasmas.

3. <u>Vlasov-Maxwell Equations</u>

a. $$\boxed{\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \frac{q_s}{m_s}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0}$$

Vlasov equation for each species s

b. $$\boxed{\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{\rho_q}{\epsilon_0} \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned}}$$

Maxwell's Equations

c. <u>Sources</u>:

Charge Density

$$\boxed{\rho_q(\mathbf{x}, t) = \sum_s \int d^3\mathbf{v}\, q_s f_s(\mathbf{x}, \mathbf{v}, t)}$$

Current Density

$$\boxed{\mathbf{j}(\mathbf{x}, t) = \sum_s \int d^3\mathbf{v}\, q_s \mathbf{v} f_s(\mathbf{x}, \mathbf{v}, t)}$$

d. This is a closed set of integro-differential equations evolving $f_s(\mathbf{x}, \mathbf{v}, t)$ for each species and $\mathbf{E}(\mathbf{x}, t)$, $\mathbf{B}(\mathbf{x}, t)$

(3D-3V phase space) (Electromagnetic Fields)

I. (Continued)

C. Physics of Kinetic Plasmas

1. Although fluid descriptions of plasmas (3D space) are computationally efficient and can capture many important aspects of plasma dynamics, the kinetic description (3D-3V phase space) is necessary to capture some important physical behaviors:

   a. Collisionless wave-particle interactions: Resonant mechanisms, such as Landau damping and cyclotron damping, interact only with particles in resonance with the phase velocity of waves.

   b. Collisionless magnetic reconnection

   c. Collisionless shocks and particle acceleration

   d. Kinetic instabilities

II. 1D-1V Vlasov-Poisson Equations

A. Simple Kinetic System

1. The 1D-1V Vlasov-Poisson equations are a simple system the resolves kinetic effects and illustrates the challenges of simulating kinetic plasmas.

2. Vlasov-Poisson Equations

a. $$\boxed{\frac{\partial f_s}{\partial t} + v \frac{\partial f_s}{\partial x} - \frac{q_s}{m_s} \frac{\partial \phi}{\partial x} \frac{\partial f_s}{\partial v} = 0}$$ 1D-1V Vlasov Equation

NOTE: cgs units (Not SI) →

b. $$\boxed{\frac{\partial^2 \phi}{\partial x^2} = -4\pi \sum_s \int dv \, q_s f_s}$$ Poisson Equation

c. Closed set of integro-differential equations for
$f_s(x,v,t)$ for each species (1D-1V phase space) $\phi(x,t)$ (Electrostatic potential)

d. NOTE: The electrostatic ~~electric~~ field is $E = -\frac{\partial \phi}{\partial x}$

B. Linear Dispersion Relation

a. $\omega = \omega(k\lambda_{de}, T_i/T_e, m_i/m_e)$

b. Langmuir Waves: $\omega^2 = \omega_{pe}^2(1 + 3k^2\lambda_{de}^2)$, $\frac{\omega}{k} \gg v_{te}$

c. Ion Acoustic Waves $\omega^2 = \frac{k^2 c_s^2}{1 + k^2\lambda_{de}^2}$, $c_s^2 \equiv \frac{T_e}{m_i}$, $v_{ti} \ll \frac{\omega}{k} \ll v_{te}$