

Electron Cyclotron Waves

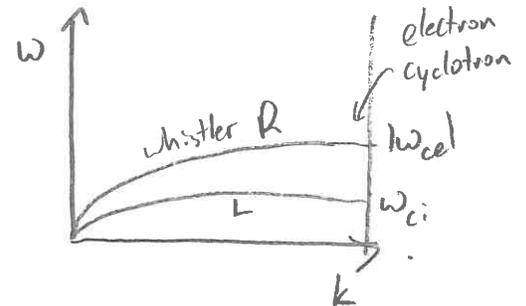
1. Limits of Electron Cyclotron Waves ($\underline{k} \parallel \underline{B}$) ($n^2 = R$) \rightarrow Transverse

A. Take $\underline{k} = k \hat{z}$, $\underline{B}_0 = B_0 \hat{z}$

B. R mode: $\underline{E}_1 = (E_0, iE_0, 0)$

C. $\omega = \omega_{ce}$

D. Limit $k \rightarrow \infty$



2. Ion Current

A. Eq. of motion $\underline{U}_i = \frac{iq_i}{\omega m_i} \underline{E}_1 + \frac{i\omega_{ci}}{\omega} \underline{U}_i \times \hat{b}$

B. Compare mag. $\frac{\text{③}}{\text{①}}$ as $\omega \approx \omega_{ce}$

$$\Rightarrow \frac{\omega_{ci}}{\omega_{ce}} = \frac{q_i B_0}{m_i q_e B_0} \frac{m_e}{m_i} = \frac{m_e}{m_i} \ll 1$$

Lorentz term ③ is negligible.

Ions are unmagnetized.

C. Ion Current

$$\begin{aligned} \underline{j}_i &= n_i q_i \underline{U}_i = \frac{i}{\omega} \frac{n_i q_i^2}{m_i} \underline{E}_1 \\ &= \frac{i}{\omega} \frac{\epsilon_0 n_i q_i^2}{\epsilon_0 m_i} \underline{E}_1 = \frac{i \epsilon_0 \omega_{pi}^2}{\omega} \underline{E}_1 = \underline{j}_i \end{aligned}$$

3. Electron Current

A. Eq. of motion $\underline{U}_e = \frac{iq_e}{\omega m_e} \underline{E}_1 + \frac{i\omega_{ce}}{\omega} \underline{U}_e \times \hat{b}$

B. Compare $\frac{\text{③}}{\text{①}} \rightarrow \frac{\omega_{ce}}{\omega_{ce}} = 1$ (as $\omega \approx \omega_{ce}$)

Electrons are magnetized, can't neglect any terms

C. Solve for \underline{U}_e (Matrix algebra) Def: $A = \frac{iq_e}{\omega m_e}$ $B = \frac{i\omega_{ce}}{\omega}$ $B^2 = -\left(\frac{\omega_{ce}}{\omega}\right)^2$

$$U_{ex} = \left(\frac{A}{1+B^2} + \frac{iAB}{1+B^2} \right) E_0$$

$$U_{ey} = \left(\frac{iA}{1+B^2} - \frac{AB}{1+B^2} \right) E_0$$

$$U_{ez} = 0 \text{ (as } E_{1z} = 0)$$

3. (cont.)

D. Electron Current

$$\vec{j}_e = n_e q_e \vec{u}_e$$

$$\vec{j}_{x,e} = \frac{i n_e q_e^2}{m_e (\omega + \omega_{ce})} E_0 = \frac{i \epsilon_0 \omega_{pe}^2}{(\omega + \omega_{ce})} E_0$$

$$\vec{j}_{y,e} = \frac{-n_e q_e^2}{m_e (\omega + \omega_{ce})} E_0 = \frac{-\epsilon_0 \omega_{pe}^2}{(\omega + \omega_{ce})} E_0$$

$$S_0 \left(\vec{j}_e = \frac{\epsilon_0 \omega_{pe}^2}{(\omega + \omega_{ce})} (i E_0, -E_0, 0) \right) \quad (\text{Note } \vec{E}_1 = (E_0, i E_0, 0))$$

So electron current is $\perp \vec{E}_1$

4. Limiting Behavior and Mode Frequency (after reversing FT)

$$\vec{k} \times \vec{E}_1 \neq 0 \Rightarrow \text{Electromagnetic}$$

$$n^2 = R = \frac{(\omega + \omega_L)(\omega + \omega_R)}{(\omega - \omega_{ce})(\omega + \omega_{ci})}$$

$$\approx \frac{-\omega_L \omega_R}{(\omega - \omega_{ce}) \omega}$$

as $k \rightarrow \infty, n^2 \rightarrow \infty$ (resonance)

We know $\omega \ll \omega_R, \omega \ll \omega_L$
 and $\omega \gg \omega_{ci}$

For $n^2 \rightarrow \infty$, given above, denominator must be small and negative.

Thus, $\omega - \omega_{ce}$ must be small. This happens as $\omega \approx \omega_{ce}$.

5. Physical Description

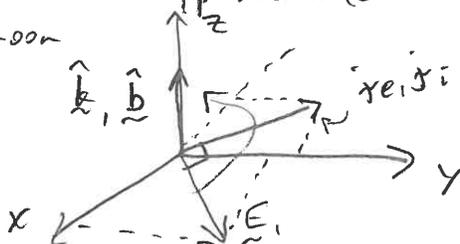
Boyd + Sanderson
 p. 213

- E field rotates about B field in right circular motion.
- As $\omega \rightarrow \omega_{ce}$, the electrons experience a near constant field and are continuously accelerated, resulting in absorption of the wave energy by the particles.

$$V_g = \frac{d\omega}{dk} = \frac{d|\omega_{ce}|}{dk} = 0 \quad \text{as } k \rightarrow \infty$$

Ions don't experience same resonance interaction as they rotate in opposite direction as E field.

6. Cartoon



\vec{E}_1 and \vec{j} rotate around in RCP fashion.

Ion Cyclotron Waves

A. Dewald

1 Limits

The ion cyclotron wave stems from the $n^2 = L$ root of the dispersion relation. We take the wavevector $\mathbf{k} \parallel \mathbf{B}$ (with $\mathbf{B} = B_0 \hat{\mathbf{z}}$ as usual) in the limit of $k \rightarrow \infty$ and $\omega \approx \omega_{ci}$. Since it is an L -mode wave, it has the eigenfunction $\mathbf{E}_1 = (E_0, -iE_0, 0)$.

2 Currents

The current \mathbf{j} of a given species s is given by

$$\mathbf{j}_s = n_s q_s \mathbf{u}_s, \quad (1)$$

where the velocity \mathbf{u}_s is

$$\mathbf{u}_s = \frac{q_s}{-i\omega m_s} \mathbf{E}_1 + \frac{q_s B_0}{-i\omega m_s} \mathbf{u}_s \times \hat{\mathbf{b}}. \quad (2)$$

Using $\omega_{cs} = q_s B_0 / m_s$ and comparing the order of the L.H.S. with the second term of the R.H.S.,

$$\mathcal{O} \left(\frac{\text{R.H.S. \#2}}{\text{L.H.S.}} \right) = \frac{\omega_{cs}}{\omega}, \quad (3)$$

which at $\omega = \omega_{ci}$ gives

$$\frac{\omega_{ci}}{\omega_{ci}} = 1 \quad (4)$$

for the ions and

$$\frac{\omega_{ce}}{\omega_{ci}} \propto \frac{m_i}{m_e} \gg 1 \quad (5)$$

for the electrons. Hence, both the ions and the electrons are magnetized.

Looking back at the equations for single-particle-motion drifts, the velocity can be written as

$$\mathbf{u}_s = \frac{iq_s}{\omega m_s} \mathbf{E}_1 + \frac{\mathbf{E}_1 \times \hat{\mathbf{b}}}{B_0} - \frac{i\omega}{\omega_{cs} B_0} \mathbf{E}_1, \quad (6)$$

where the second term on the R.H.S. is the $\mathbf{E} \times \mathbf{B}$ drift and the last term is the polarization drift. For the electrons,

$$\frac{\omega}{\omega_{ce}} = \frac{\omega_{ci}}{\omega_{ce}} \ll 1 \quad (7)$$

and so the last term can be neglected. Furthermore, for the ions,

$$\frac{iq_i}{\omega m_i} \mathbf{E}_1 - \frac{i\omega}{\omega_{ci} B_0} \mathbf{E}_1 = \frac{iq_i}{m_i} \frac{m_i}{q_i B_0} \mathbf{E}_1 - \frac{i\omega_{ci}}{\omega_{ci} B_0} \mathbf{E}_1 \quad (8)$$

$$= 0. \quad (9)$$

Hence,

$$\mathbf{j}_i = \frac{n_i q_i}{B_0} \mathbf{E}_1 \times \hat{\mathbf{b}} \quad (10)$$

$$\mathbf{j}_e = i\epsilon_0 \frac{\omega_{pe}^2}{\omega_{ci}} \mathbf{E}_1 + \frac{n_e q_e}{B_0} \mathbf{E}_1 \times \hat{\mathbf{b}}. \quad (11)$$

The ion and electron currents are perpendicular to the wavevector \mathbf{k} .

3 Limiting Behavior

Given $\mathbf{E}_1 = (E_0, -iE_0, 0)$, Faraday's law gives

$$\mathbf{k} \times \mathbf{E}_1 = \omega \mathbf{B}_1 \neq 0. \quad (12)$$

Thus, the ion cyclotron wave is a transverse electromagnetic wave.

4 Solution of the Mode Frequency

Starting with

$$n^2 = L = 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega - \omega_{cs})} \quad (13)$$

$$\frac{k^2 c^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega(\omega - \omega_{ce})} - \frac{\omega_{pi}^2}{\omega(\omega - \omega_{ci})}, \quad (14)$$

use the fact that $\omega_{ci} \ll \omega_{pe}$ to drop the first two terms. Then, taking $\omega \approx \omega_{ci}$ and using the relations $v_A = c/n_A$ and $n_A^2 = \omega_{pi}^2/\omega_{ci}^2$, this becomes

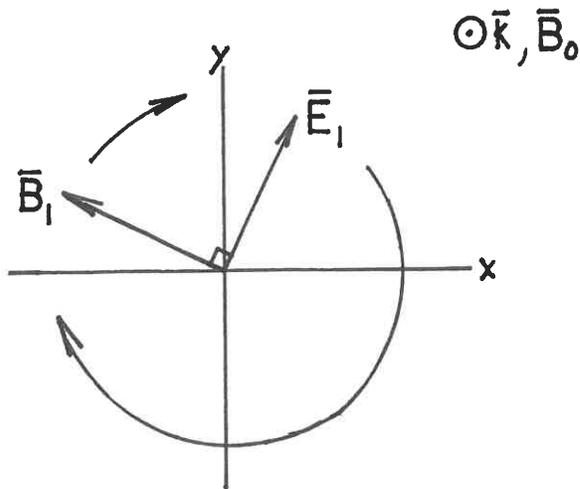
$$\omega - \omega_{ci} = \frac{-\omega_{ci}^3}{k^2 v_A^2}. \quad (15)$$

As $k \rightarrow \infty$, $\omega \rightarrow \omega_{ci}$ as expected.

5 Physical Description

The ion cyclotron wave is a left-circularly-polarized transverse electromagnetic wave. Thus, the fields \mathbf{E}_1 and \mathbf{B}_1 rotate in a clockwise fashion perpendicular to \mathbf{k} . The ions also rotate clockwise.

6 Cartoon



Plasma HW #1

Jodie

Upper Hybrid Waves:

Limits:

$$1. \underline{k} = k \hat{x} \quad \underline{B}_0 = B_0 \hat{z}$$

$$2. \omega_{UH}^2 \approx \omega_{pe}^2 + \omega_{ce}^2$$

$$3. \text{Extraordinary mode: } \underline{E}_1 = (E_0, -i \frac{S}{\Omega} E_0, 0)$$

4. Investigating wave behavior in $k \rightarrow \infty$ limit.

Ion current:

Equation of motion for ions

$$\underline{u}_i = \frac{q_i}{-i\omega m_i} \underline{E}_1 + \frac{q_i B_0}{-i\omega m_i} \underline{u}_i \times \hat{b}$$

$$\underline{u}_i = i \frac{q_i}{\omega m_i} \underline{E}_1 + i \frac{\omega_{ci}}{\omega} \underline{u}_i \times \hat{b}$$

$$\omega_{ci} = \frac{q_i B_0}{m_i}$$

$$\omega_{pe} = \left(\frac{n_e q_e^2}{m_e \epsilon_0} \right)^{1/2}$$

$$\omega_{ce} = \frac{q_e B_0}{m_e}$$

Order of $\frac{\omega_{ci}}{\omega} \ll 1$

$$\frac{\omega_{ci}}{\omega} = \mathcal{O}\left(\frac{i \frac{\omega_{ci}}{\omega} \underline{u}_i}{\underline{u}_i}\right) = \mathcal{O}\left(i \frac{\omega_{ci}}{\omega}\right) = \mathcal{O}\left(\frac{\omega_{ci}}{\omega_{UH}}\right)$$

$$\frac{\omega_{ci}}{\omega_{UH}} = \frac{q_i B_0}{m_i (\omega_{pe}^2 + \omega_{ce}^2)^{1/2}} = \frac{q_i B_0}{m_i \left(\frac{n_e q_e^2}{m_e \epsilon_0} + \frac{q_e^2 B_0^2}{m_e^2} \right)^{1/2}} = \frac{m_e}{m_i} \frac{B_0}{\left(\frac{n_e m_e}{\epsilon_0} + B_0^2 \right)^{1/2}}$$

$$\Rightarrow \mathcal{O}\left(\frac{m_e}{m_i} \frac{B_0}{\left(\frac{n_e m_e}{\epsilon_0} + B_0^2 \right)^{1/2}}\right) \Rightarrow \frac{m_e}{m_i} \ll 1$$

$\ll 1 \quad \sim 1 \rightarrow \text{since } m_e \text{ is small}$

Ions are unmagnetized, Lorentz force is negligible

Electron current continued:

$$U_{ex} = \frac{i q_e}{\omega m_e} E_x + \frac{i \omega_{ce}}{\omega} U_y$$

$$U_{ey} = \frac{i q_e}{\omega m_e} E_y - \frac{i \omega_{ce}}{\omega} U_x$$

into matrix form

$$\Rightarrow 0 = \begin{pmatrix} 1 & -\frac{i \omega_{ce}}{\omega} \\ \frac{i \omega_{ce}}{\omega} & 1 \end{pmatrix} \begin{pmatrix} U_x \\ U_y \end{pmatrix} - \frac{i q_e}{\omega m_e} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{\omega m_e}{i q_e} \begin{pmatrix} 1 & -\frac{i \omega_{ce}}{\omega} \\ \frac{i \omega_{ce}}{\omega} & 1 \end{pmatrix} \begin{pmatrix} U_x \\ U_y \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} U_x \\ U_y \end{pmatrix} = \begin{pmatrix} 1 & -\frac{i \omega_{ce}}{\omega} \\ \frac{i \omega_{ce}}{\omega} & 1 \end{pmatrix}^{-1} \frac{i q_e}{\omega m_e} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$= \frac{1}{1 - \left(\frac{\omega_{ce}}{\omega}\right)^2} \begin{pmatrix} 1 & \frac{i \omega_{ce}}{\omega} \\ -\frac{i \omega_{ce}}{\omega} & 1 \end{pmatrix}$$

$$\frac{1}{1 - \left(\frac{\omega_{ce}}{\omega}\right)^2} = \frac{1}{1 - \frac{\omega_{ce}^2}{\omega^2}} = \frac{\omega^2 + \omega_{pe}^2}{\omega^2 + \omega_{pe}^2 - \omega_{ce}^2}$$

$$= \frac{\omega_{ce}^2 + \omega_{pe}^2}{\omega_{pe}^2} = \frac{\omega_{ce}^2}{\omega_{pe}^2} + 1$$

$$U_x = \frac{i q_e}{\omega m_e} \left(\left(\frac{\omega_{ce}}{\omega_{pe}}\right)^2 + 1 \right) \left[E_x + \frac{i \omega_{ce}}{\omega} E_y \right]$$

$$U_y = \frac{i q_e}{\omega m_e} \left(\left(\frac{\omega_{ce}}{\omega_{pe}}\right)^2 + 1 \right) \left[-\frac{i \omega_{ce}}{\omega} E_x + E_y \right]$$

electron current cont:

$$j_x = n e q_0 u_x$$

$$= \frac{i n e q_0^2}{m_e \omega} \left(\left(\frac{\omega_{ce}}{\omega_{pe}} \right)^2 + 1 \right) \left[E_x + i \frac{\omega_{ce}}{\omega} E_y \right]$$

$$= i \epsilon_0 \frac{\omega_{pe}^2}{\omega} \left(\left(\frac{\omega_{ce}}{\omega_{pe}} \right)^2 + 1 \right) \left[E_x + i \frac{\omega_{ce}}{\omega} E_y \right]$$

plug in $\underline{E}_1 = (E_0, -\frac{iS}{D} E_0, 0)$

$$j_x = i \epsilon_0 \frac{\omega_{pe}^2}{\omega} \left(\left(\frac{\omega_{ce}}{\omega_{pe}} \right)^2 + 1 \right) \left[E_0 + \frac{\omega_{ce}}{\omega} \frac{S}{D} E_0 \right]$$

$$= i \epsilon_0 \left(\frac{\omega_{ce}^2}{\omega} + \frac{\omega_{pe}^2}{\omega} \right) \left[E_0 \left(1 + \frac{\omega_{ce}}{\omega} \frac{S}{D} \right) \right]$$

$$= i \epsilon_0 \frac{\omega^2}{\omega} E_0 \left[1 + \frac{\omega_{ce}}{\omega} \frac{S}{D} \right]$$

$$j_x = i \epsilon_0 \omega E_0 \left(1 + \frac{\omega_{ce}}{\omega} \frac{S}{D} \right)$$

$$j_y = n e q_0 u_y$$

$$= \frac{i q_0^2 n e}{m_e \omega} \left(\left(\frac{\omega_{ce}}{\omega_{pe}} \right)^2 + 1 \right) \left[-i \frac{\omega_{ce}}{\omega} E_x + E_y \right]$$

$$= i \epsilon_0 \frac{\omega_{pe}^2}{\omega} \left(\frac{\omega_{ce}^2}{\omega_{pe}^2} + 1 \right) \left[i \frac{\omega_{ce}}{\omega} E_0 - i \frac{S}{D} E_0 \right]$$

$$= i \epsilon_0 (\omega_{ce}^2 + \omega_{pe}^2) \left[i \frac{\omega_{ce}}{\omega^2} E_0 - \frac{iS}{\omega D} E_0 \right]$$

$$j_y = \epsilon_0 E_0 \omega \left[-\frac{\omega_{ce}}{\omega} + \frac{S}{D} \right]$$

Limiting behavior as $k \rightarrow \infty$:

Faraday's: $\vec{k} \times \vec{E}_1 = \omega \vec{B}_1$

$\vec{k} \times \vec{E}_{1(\omega)} = 0$

since as $k \rightarrow \infty$, $S \rightarrow 0$, $E_y \rightarrow 0$

$E_{1(\omega)} = E_0 \hat{x} \rightarrow \text{Electrostatic}$

Amp / Maxwell's: $\vec{k} \times \vec{B}_1 = -i\mu_0 j - \frac{\omega}{c^2} \vec{E}_1$ no \hat{x} term

$\vec{k} \times \vec{B}_1 = -i\mu_0 \left[i\epsilon_0 \frac{\omega p_i}{\omega} \vec{E}_1 + \left(i\epsilon_0 \omega E_0 \left(1 + \frac{\omega_{ce} S}{\omega D} \right) \hat{x} + \epsilon_0 E_0 \omega \left[\frac{-\omega_{ce}}{\omega}, \frac{S}{D} \right] \hat{y} \right) \right] - \frac{\omega}{c^2} \vec{E}_1$

$\vec{k} \times \vec{B}_{1(\omega)} = \mu_0 \epsilon_0 \frac{\omega p_i}{\omega} E_0 \hat{k} - \frac{\omega}{c^2} E_0 + \mu_0 \epsilon_0 E_0 \omega \left(\frac{-\omega_{ce}}{\omega} \right)$

$c^2 \vec{k} \times \vec{B}_{1(\omega)} = \frac{\omega p_i}{\omega} E_0 + \omega \left(\frac{\omega_{ce}}{\omega} \right) E_0 = \omega = 0$

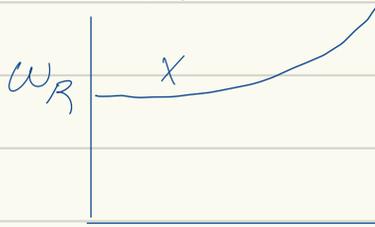
→ something wrong here, should have an extra ω_{ce} term and one less ω in $\omega \left(\frac{\omega_{ce}}{\omega} \right)$

Extraordinary mode light waves

- Perpendicular $\theta = \frac{\pi}{2}$

- $n^2 = \frac{R_L}{S} \Rightarrow$ Longitudinal & transverse

$\rightarrow \vec{k} \times \vec{E} = 0 \quad \wedge \quad \vec{k} \cdot \vec{E}_1 = 0$



how get $\omega^2 = k^2 c^2 + \omega_R^2$ in limit
want

- $\omega^2 \approx k^2 c^2 + \omega_R^2$ limit $\omega > \omega_R$
- set $\vec{k} = k_x \hat{x} + \vec{B}_0 = B_0 \hat{z}$

\therefore X-traordinary mode $\Rightarrow \vec{E} = (E_0, \frac{-iS}{D} E_0, 0)$

Discerning the current

momentum equation: $\vec{U}_s = \frac{q_s}{-i\omega m_s} \vec{E}_1 + \frac{q_s B_0}{-i\omega m_s} \vec{U}_s \times \hat{b} = \frac{i q_s}{\omega m_s} \vec{E}_1 + \frac{i \omega_{cs}}{\omega} \vec{U}_s \times \hat{b}$

Note for X-treme light waves
 $\omega^2 = k^2 c^2 + \omega_R^2$

$$\omega_R = \frac{\omega_{ce}}{2} + \frac{1}{2} \sqrt{\omega_{ce}^2 + 4(\omega_{pe}^2 + |\omega_{ce}| \omega_{ci})}$$

$$\omega_R = \frac{|\omega_{ce}|}{2} + \omega_{pe}$$

from parallel chart L3P5

for species $s = \text{ion}$

$\vec{U}_i = \frac{i q_i}{\omega m_i} \vec{E}_1 + \frac{i \omega_{ci}}{\omega} \vec{U}_i \times \hat{b}$

① $\frac{\omega_{ci}}{\omega} \vec{U}_i = \frac{\omega_{ci}}{\omega} - \frac{\omega_{ci}}{k^2 c^2 + \frac{\omega_{ce}^2}{2} + \omega_{pe}^2} \ll 1$

further thought $\frac{\omega_{ci}}{k^2 c^2 + \omega_R^2} \ll 1$ from chart on lec 3 pg 5

this the case \Rightarrow protons are unmagnetized

for species $s = \text{electron}$

$\vec{U}_e = \frac{i q_e}{\omega m_e} \vec{E}_1 + \frac{i \omega_{ce}}{\omega} \vec{U}_e \times \hat{b}$ $\frac{\omega_{ce}}{k^2 c^2 + \omega_R^2} < 1 \Rightarrow$ but the Ω_e is

$\vec{j} = \sum_s n_s q_s \vec{U}_s = \frac{\sum_{s=\text{ions}} n_s q_s \vec{U}_s}{\sum_{s=\text{electrons}} n_s q_s \vec{U}_s} = \frac{i q_s \vec{E}_1 \frac{E_0}{\omega m_s E_0}}{\sum_s \frac{i \epsilon_0 \vec{E}_1 \frac{n_s q_s^2}{\epsilon_0 m_s}}{\omega}} = \frac{i \epsilon_0 \vec{E}_1}{\omega} (\omega_{pe}^2 + \omega_{pi}^2) + \frac{i \omega_{ce}}{\omega} \vec{U}_e \times \hat{b}$

the minor yet appreciable effect from the $\vec{U}_e \perp \hat{b}$ generates this mode's dual transverse and longitudinal comps.

$$\frac{i\omega_{ce}}{\omega} \vec{u}_e \times \hat{b} \Rightarrow \begin{cases} \vec{u}_{ex} = \frac{iq_e}{\omega m_e} E_x + \frac{i\omega_{ce}}{\omega} u_{ey} \\ \vec{u}_{ey} = \frac{iq_e}{\omega m_e} E_y - \frac{i\omega_{ce}}{\omega} u_{ex} \end{cases} \quad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 1 & -\frac{i\omega_{ce}}{\omega} \\ \frac{i\omega_{ce}}{\omega} & 1 \end{pmatrix} \begin{pmatrix} u_{ex} \\ u_{ey} \end{pmatrix}$$

$$\frac{iq_e}{\omega m_e} \frac{1}{2 - \frac{\omega_{ce}^2}{\omega^2}} \begin{pmatrix} 1 & \frac{i\omega_{ce}}{\omega} \\ -\frac{i\omega_{ce}}{\omega} & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} u_{ex} \\ u_{ey} \end{pmatrix}$$

$$\vec{u}_{ey} = \frac{iq_e}{\omega m_e} E_y - \frac{i\omega_{ce}}{\omega} \left(\frac{iq_e}{\omega m_e} E_x + \frac{i\omega_{ce}}{\omega} u_{ey} \right)$$

$$= \frac{iq_e}{\omega m_e} E_y + \frac{\omega_{ce} q_e}{\omega^2 m_e} E_x - \frac{\omega_{ce}^2}{\omega^2} u_{ey}$$

$$\Rightarrow \vec{u}_{ey} = \frac{\frac{iq_e}{\omega m_e} E_y + \frac{\omega_{ce} q_e}{\omega^2 m_e} E_x}{1 - \frac{\omega_{ce}^2}{\omega^2}}$$

$$\vec{u}_{ex} = \frac{iq_e}{\omega m_e} E_x + \frac{i\omega_{ce}}{\omega} \left[\frac{iq_e}{\omega m_e} E_y - \frac{i\omega_{ce}}{\omega} u_{ex} \right]$$

$$= \frac{iq_e}{\omega m_e} E_x - \frac{\omega_{ce} q_e}{\omega^2 m_e} E_y + \frac{\omega_{ce}^2}{\omega^2} u_{ex}$$

$$\vec{u}_{ex} = \frac{\frac{iq_e}{\omega m_e} E_x - \frac{\omega_{ce} q_e}{\omega^2 m_e} E_y}{1 - \frac{\omega_{ce}^2}{\omega^2}}$$

$$\vec{j} = \frac{i\epsilon_0 \vec{E}_1}{\omega} (\omega_{pe}^2 + \omega_{pi}^2) + i\frac{\omega_{ce}}{\omega} \vec{u}_e \times \hat{b} = \frac{i\epsilon_0 \vec{E}_1}{\omega} (\omega_{pe}^2 + \omega_{pi}^2) + \underbrace{\left[\frac{iq_e \omega E_x - \omega_{ce} q_e E_y}{\omega^2 - \omega_{ce}^2} \right] \hat{x} + \left[\frac{iq_e \omega E_y + \omega_{ce} q_e E_x}{\omega^2 - \omega_{ce}^2} \right] \hat{y}}_{\frac{q}{m_e (\omega^2 - \omega_{ce}^2)} \left[iE_x - \omega_{ce} E_y \right] \hat{x} + \left[iE_y + \omega_{ce} E_x \right] \hat{y}}$$

limiting behavior

Faradays law $\vec{k} \times \vec{E}_1 = \omega \vec{B}_1 \Rightarrow \vec{k} \times \vec{E}_{(a)} = \vec{B}_{(a)} = \hat{Q} + E_{1(a)} = E_0 \hat{k}$

Ampere/Maxwell law $\vec{k} \times \vec{B} = -i\mu_0 \vec{j} - \frac{\omega}{c^2} \vec{E}_1$

from earlier $\vec{j} = \frac{i\epsilon_0 \vec{E}_1}{\omega} (\omega_{pe}^2 + \omega_{pi}^2)$

$$\Rightarrow \vec{k} \times \vec{B} = \underbrace{\mu_0 \frac{\epsilon_0}{\omega} (\omega_{pe}^2 + \omega_{pi}^2)}_{\text{1}} \vec{E}_1 - \underbrace{\frac{\omega}{c^2} \vec{E}_1}_{\text{2}} - \underbrace{\frac{i\mu_0 q}{m_e (\omega^2 - \omega_{ce}^2)} \left[iE_x - \omega_{ce} E_y \right] \hat{x} + \left[iE_y + \omega_{ce} E_x \right] \hat{y}}_{\text{3}}$$

recall $\frac{1}{\mu_0 \epsilon_0} = c^2$

recall $\vec{E} = (E_0, \frac{-iS}{D} E_0, 0)$

$$\omega^2 - \omega_R^2 = c^2 k^2$$

first limit $k \rightarrow 0 \quad \omega \rightarrow \omega_R \quad (\omega \rightarrow \text{smaller})$

term 1+3 scale $\frac{1}{\omega} + \frac{1}{\omega^2} \rightarrow$ more important as ω shrinks

$$S + D \rightarrow 1, \quad \vec{k} \times \vec{B} \rightarrow \frac{1}{c^2 \omega} (\omega_{pe}^2) + \frac{\mu_0 q \epsilon_0}{m_e (\omega_R^2 - \omega_{ce}^2)} \left[1 - \omega_{ce} \right] \hat{x} + \left[1 + \omega_{ce} \right] \hat{y}$$

$$\Rightarrow c^2 k^2 = \omega_{pe}^2 + \frac{\omega_{ce}}{2}$$

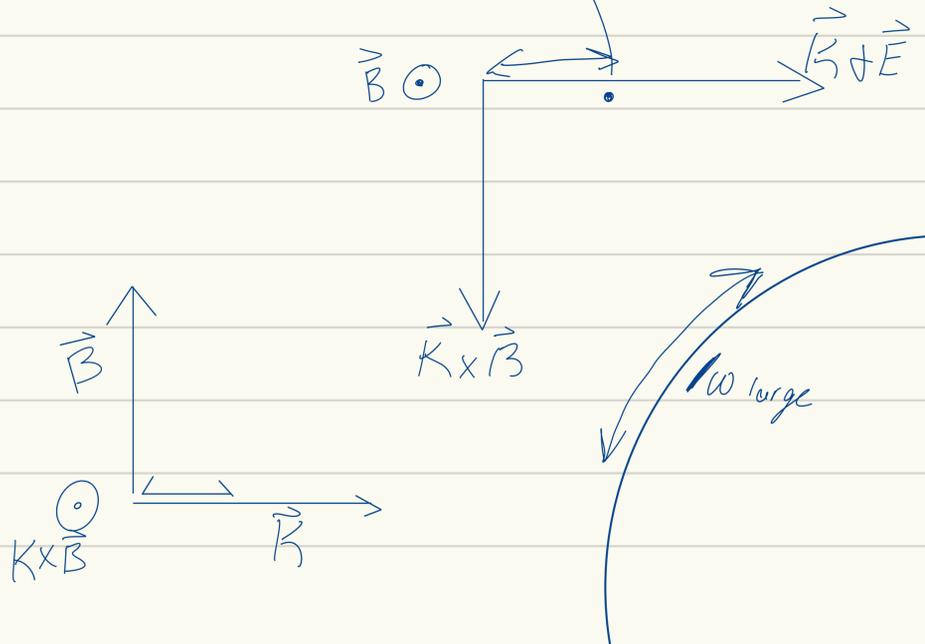
removing term 3 we approach ω_{pe}

second limit $k \rightarrow \infty \quad \omega \rightarrow \infty \quad \vec{k} \times \vec{B} \Rightarrow kc = \omega \because \frac{1}{\omega} + \frac{1}{\omega^2} \rightarrow 0$

$$kB = \frac{\omega}{c^2} E \Rightarrow kc^2 \frac{B}{E} = \omega \Rightarrow kc^2 \frac{1}{c} = \omega$$

physical desc

- ions unmagnetized
- electrons slightly magnetized
- electro static
- $u_e \times \vec{B}$ introduces transverse component



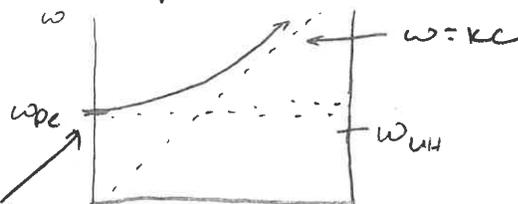
Ordinary Light Waves

① $\underline{k} \perp \underline{B}$ $\underline{k} = k \hat{x}$ $\underline{B} = B_0 \hat{z}$ $n^2 = P$
 (Lap7) $\underline{E}_1 = (0, 0, E_0)$ → $\underline{k} \times \underline{E}_1 \neq 0$ → transverse
 $\omega^2 = k^2 c^2 + \omega_p^2$ $q_i = q_e$ $m_i/m_e = 1836$
 $\omega_{pi} \approx \omega_{ci} \ll \omega_{pe}$ $\omega_{ce} \ll \omega_{pe}$

② Current: $\underline{u} = i \frac{q}{\omega m} \underline{E}_1 + i \frac{q \underline{B}}{\omega m} \underline{u} \times \underline{b}$ where $\omega_{ci} = \frac{qB}{m}$
 no Lorentz force! ω_{ci}/ω from limit → term = 0
 $\underline{u}_i = i \frac{q_i}{\omega m_i} \underline{E}_1$ $\underline{u}_e = i \frac{q_e}{\omega m_e} \underline{E}_1$
 $\underline{j}_i = n_i q_i \underline{u}_i \rightarrow \underline{j}_i = n_i q_i (i \frac{q_i}{\omega m_i} \underline{E}_1)$
 $= i \frac{n_i q_i^2}{\omega m_i} \underline{E}_1 \cdot \frac{\epsilon_0}{\epsilon_0}$ $\omega_{pi}^2 = \frac{n_i q_i^2}{\epsilon_0 m_i}$
 $\underline{j}_i = i \frac{\epsilon_0 \omega_{pi}^2}{\omega} \underline{E}_1$ unmagnetized
 same process for $\underline{j}_e = i \frac{\epsilon_0 \omega_{pe}^2}{\omega} \underline{E}_1$ unmagnetized
 total $\underline{j} = \underline{j}_i + \underline{j}_e = i \frac{\epsilon_0 \omega_{pe}^2}{\omega} \underline{E}_1$
 $\omega_p^2 = \omega_{pi}^2 + \omega_{pe}^2 \rightarrow \underline{j} = i \frac{\epsilon_0 \omega_p^2}{\omega} \underline{E}_1$

③ $\underline{k} \times \underline{E}_1 = \omega \underline{B}_1 \checkmark \leftarrow \neq 0$ (Faradays → since its transverse → EM)
 Ampere's law: $c^2 \underline{k} \times \underline{B} = -i \mu_0 (\frac{\epsilon_0 \omega_p^2}{\omega} \underline{E}_1) - \omega \underline{E}_1$ displacement current
 $\hookrightarrow c^2 \frac{\underline{k} \times \underline{B}}{\omega} = \mu_0 \epsilon_0 (\frac{\omega_p^2}{\omega} \underline{E}_1) - \underline{E}_1$ plug in ω
 $= \mu_0 \epsilon_0 (\frac{\omega_p^2}{k^2 c^2 + \omega_p^2} \underline{E}_1) - \underline{E}_1$
 $\underline{k} \rightarrow 0$: $= \frac{\omega_p^2}{0 + \omega_p^2} \underline{E}_1 - \underline{E}_1 = 0$ don't worry about constants cutoff (since $\omega_{pi} \ll \omega_{pe}, \omega_{pe} \approx \omega_{pe}$)
 $\underline{k} \rightarrow \infty$: $= \frac{\omega_p^2}{\infty + \omega_p^2} \underline{E}_1 - \underline{E}_1 = -\underline{E}_1$ independent on displacement current resonance

as shown on graph from lecture



← as $k \rightarrow \infty$ the frequency → that of light

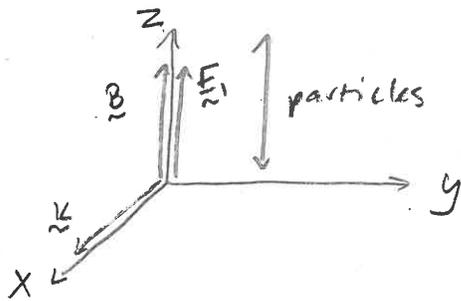
start @ ω_{pe} for $k=0$ since $\omega_p \propto \omega_{pe}$



(4)

physical description

- $L^+ \approx e^-$ unmagnetized
- $\underline{E} \parallel \underline{B}$ $\underline{k} \perp \underline{B}$
- \underline{B} has no effect on polarization (no Lorentz force)
- linearly polarized wave



particles will oscillate
along B field