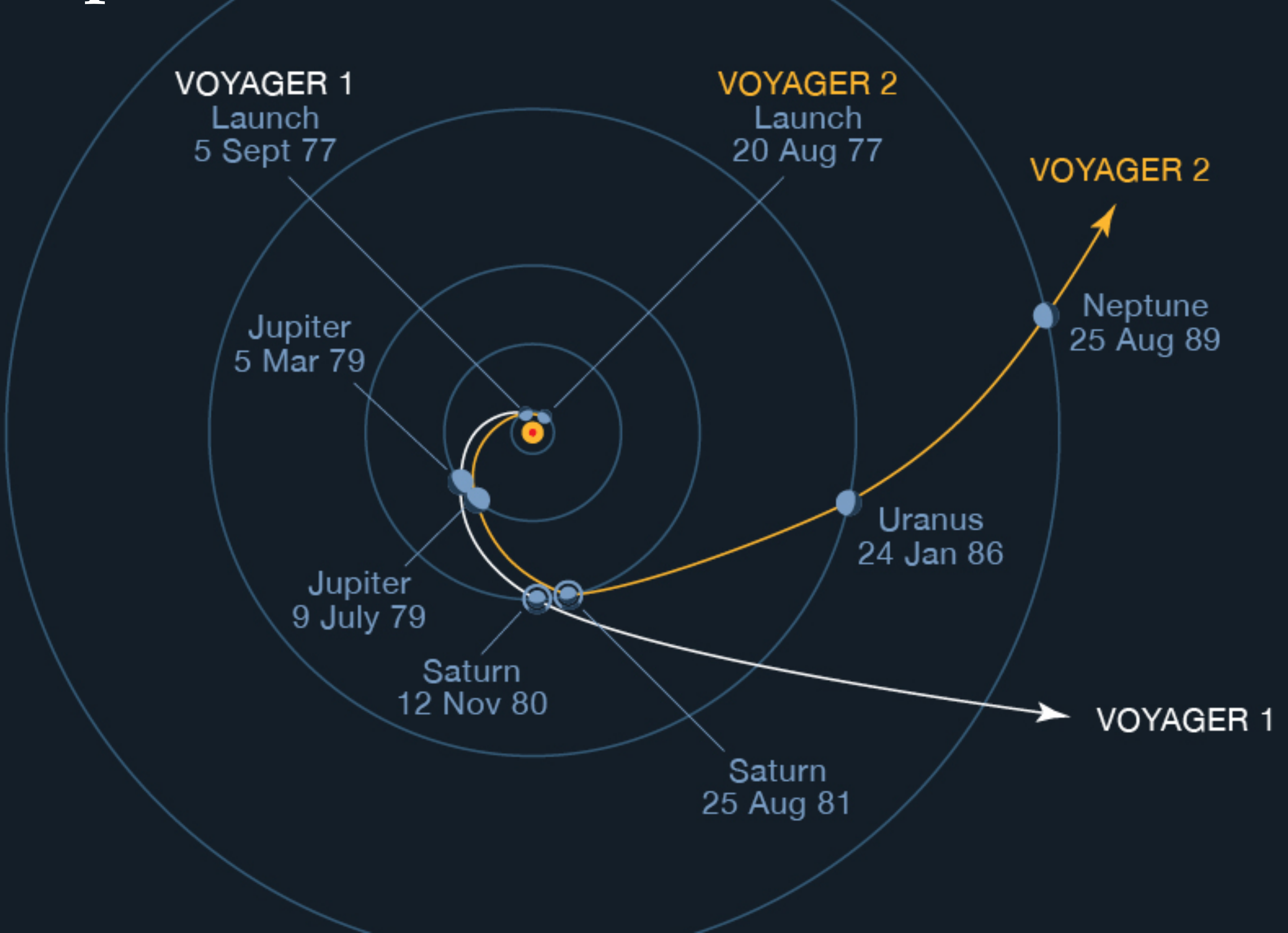
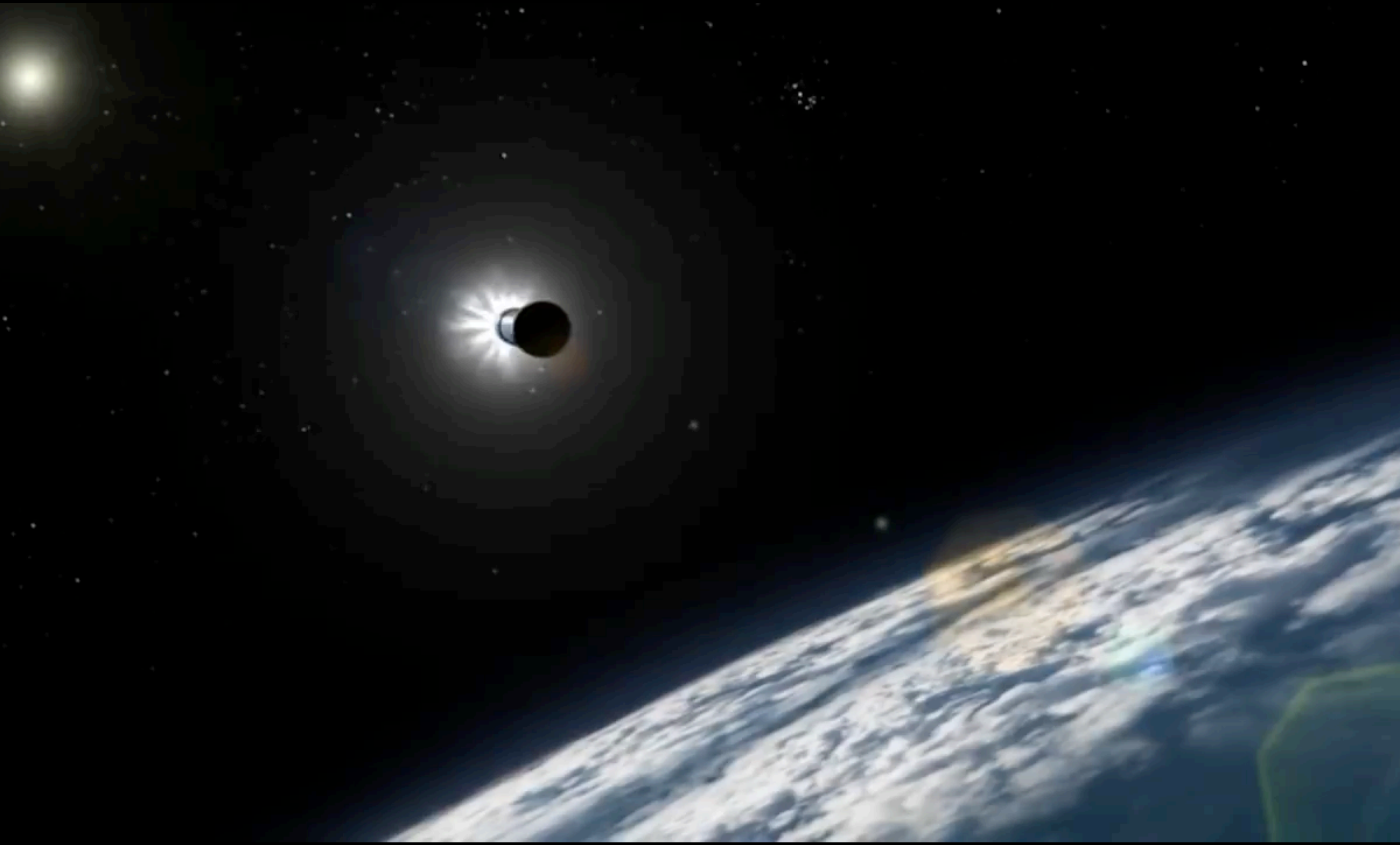
Chap 4: Celestial Mechanics

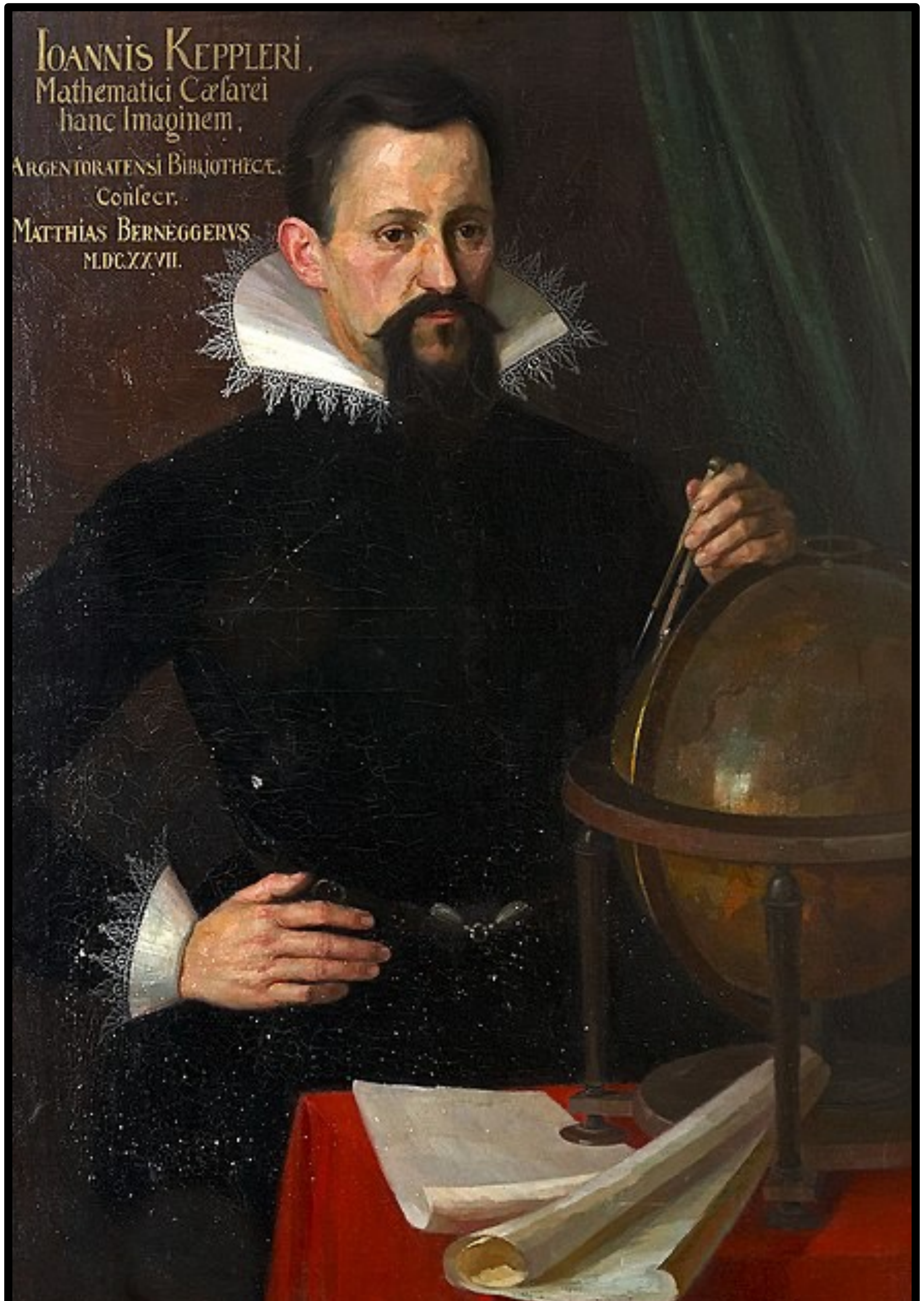


Delivering an SUV to Mars: Curiosity Rover



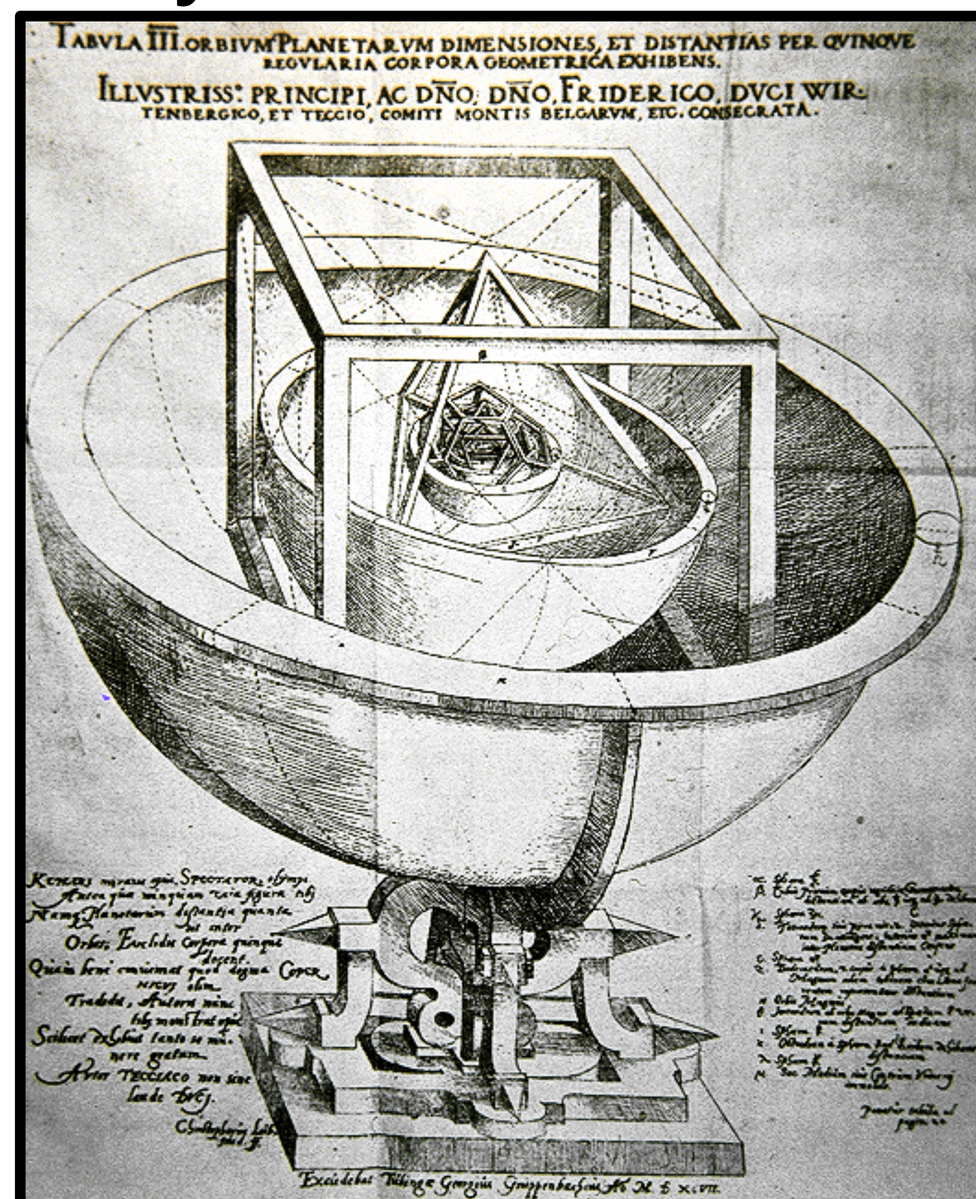
Space exploration in Iowa - James Van Allen

Johannes Kepler (1571-1630), German astronomer



portrait

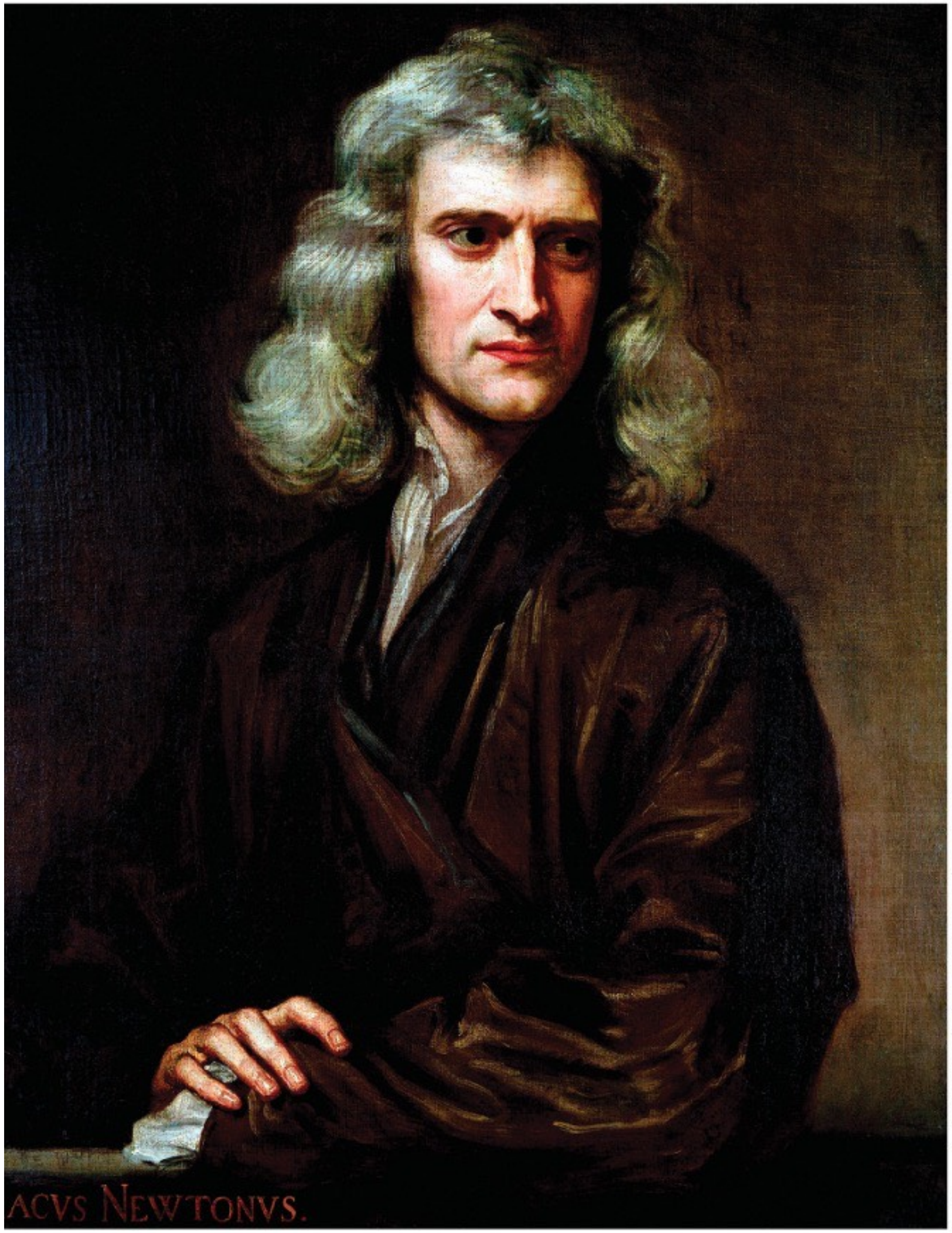
In 1600, Kepler became Tycho's assistant. After Tycho's death in 1601, he used Tycho's data to develop his **three laws of planetary motion**



Kepler's model of the Solar System

Issac Newton (1642-1727), British astrophysicist

- Established the three **laws of motion** and the **law of gravitation**
- Invented **Calculus**
- Conducted experiment to discover the composition of **white light**
- Invented the first reflecting telescopes (**Newtonian telescopes**)



portrait

Newton’s reflecting telescope

Chapter 4: Celestial Mechanics

- Newton's Three Laws of Motion (a quick review)
  - Acceleration along a circular orbit: **centripetal acceleration**
- Newton's law of universal gravitation
  - Derivation of Kepler's 3rd law: **Circular Velocity**
  - Newtons' theorems & **surface gravitational acceleration**
- Energy conservation (**Vis Viva Equation**)
  - **Escape Velocity**
  - **Hohmann transfer orbit**
- Momentum conservation (**Rocket Equation**)
- Tidal forces (differential gravity)
  - **Ocean tides**, **Roche limit**, and tidal tails of galaxies

Newton's Three Laws of Motion

covered in Physics I (PHYS:1701)

1. **An object at rest or in motion remains so unless acted on by a force. And the motion is in a straight line.**
   This defines an ***inertial frame of reference***

2. **Applying a force to any object gives it an acceleration**

$$\overrightarrow{F} = m\vec{a}$$

   vice versa: if an object accelerates as observed from an ***inertial frame of reference****, it must be influenced by an external force*

3. **For every action there is an equal and opposite reaction**
   e.g., *rocket's engine generates a force which drives hot gas out the back and at the same time, propels the rocket forwards*

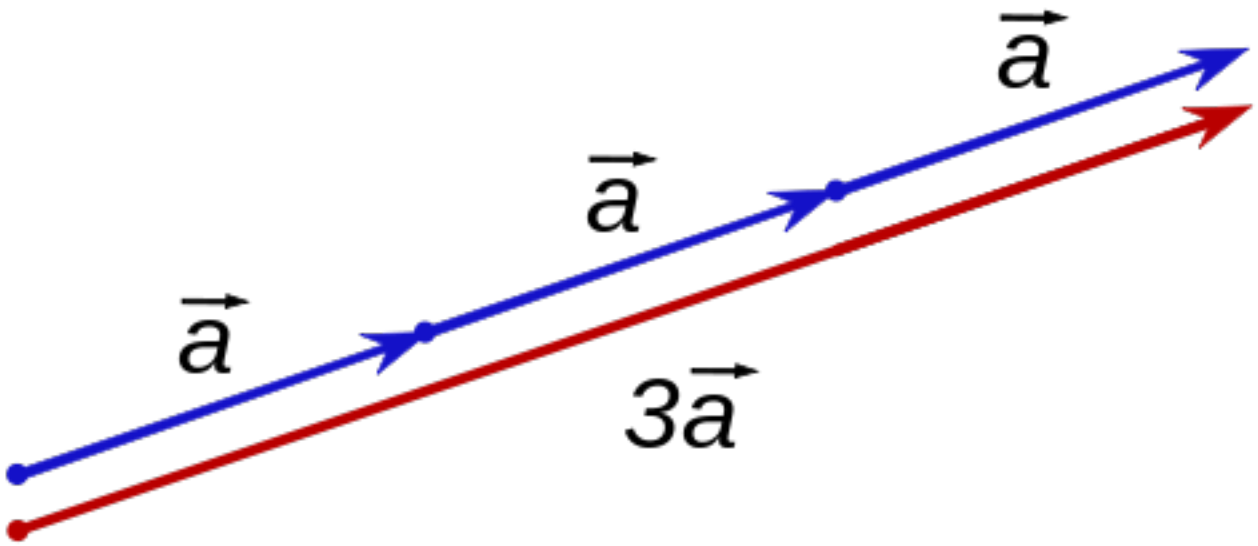
Part I: Newton's Law of Gravitation

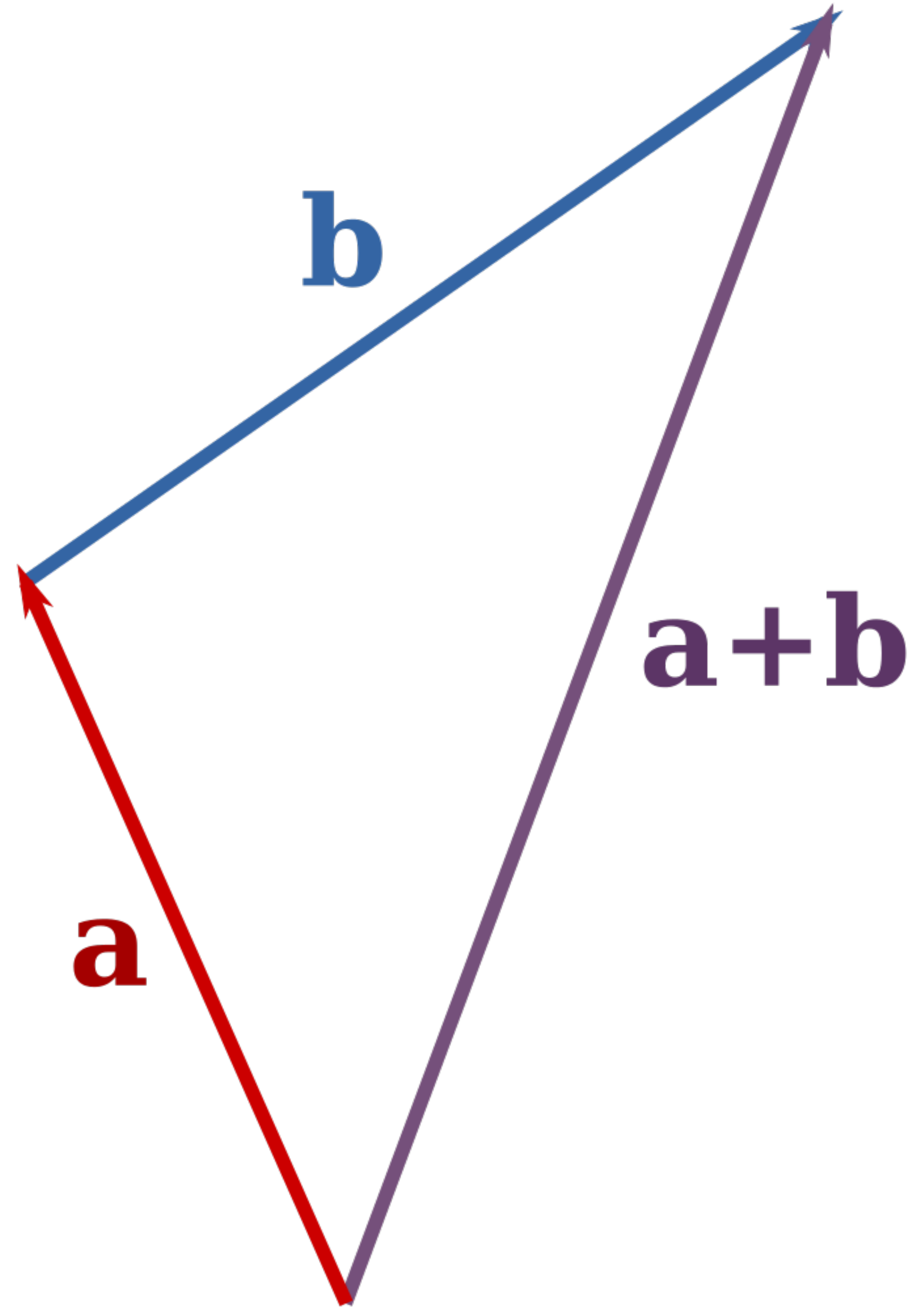
centripetal acceleration, Newton's theorem, surface gravity

Acceleration along a circular orbit

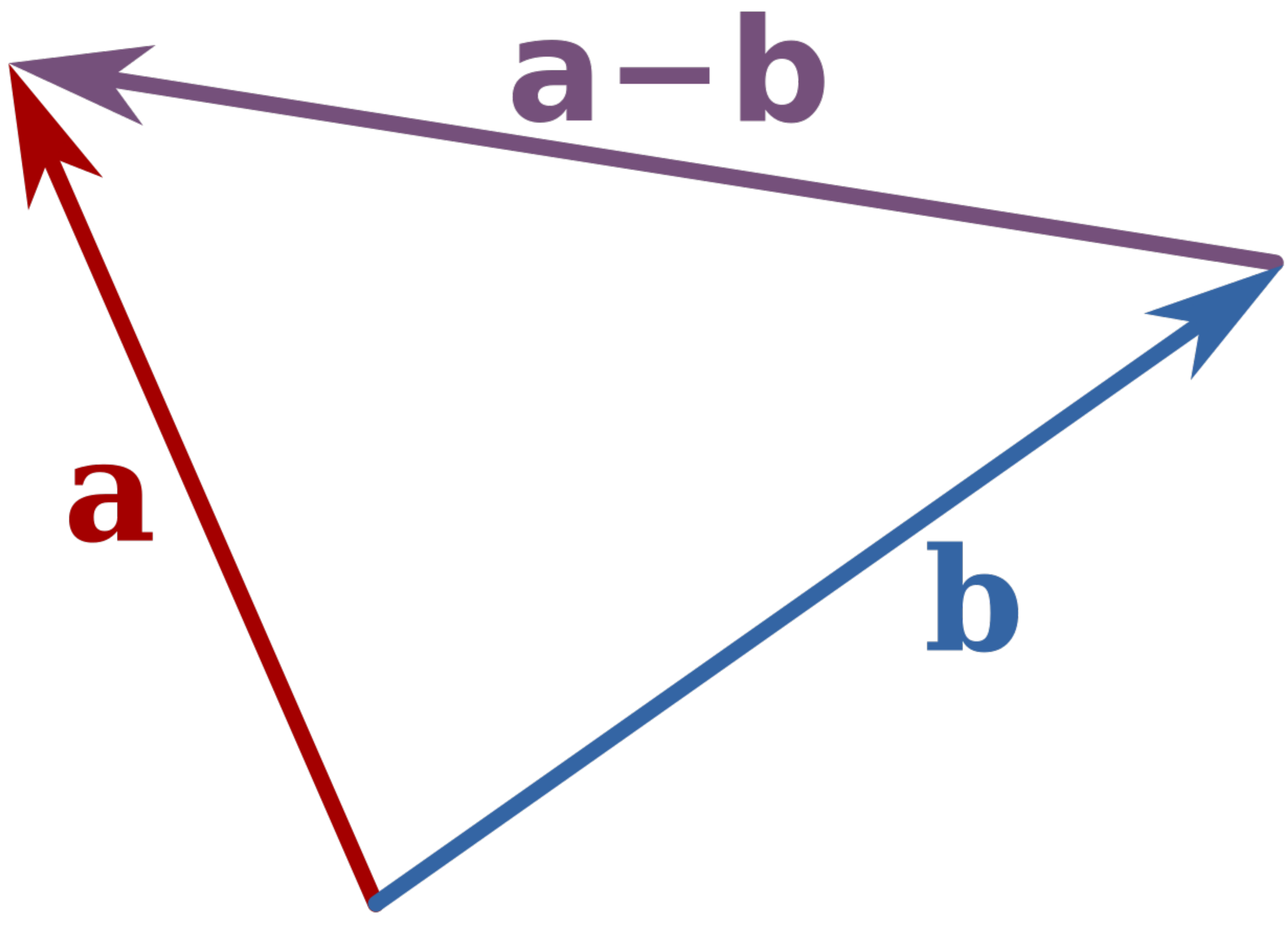
centripetal acceleration

Vector Addition

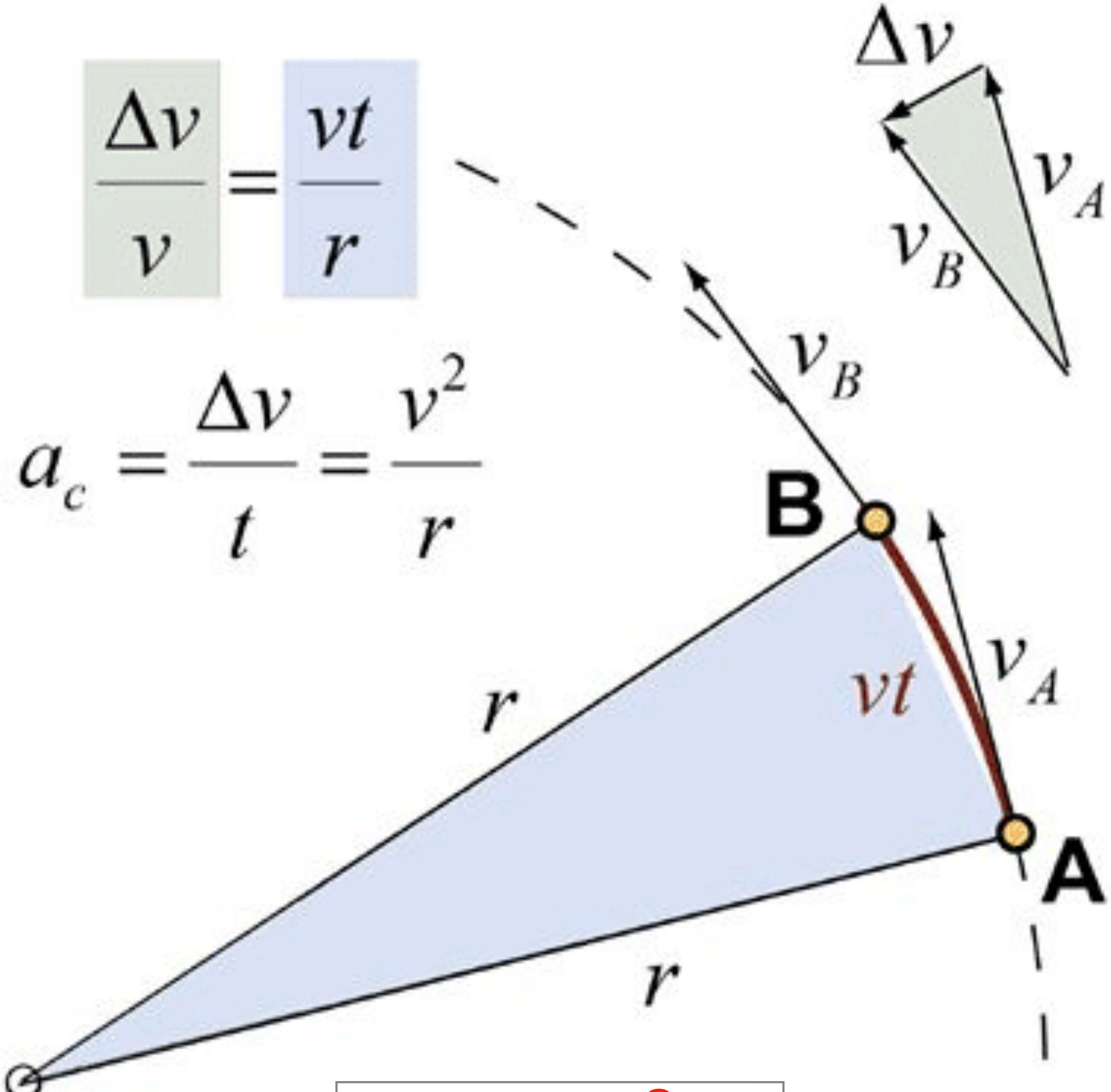




Vector Subtraction



The acceleration of an object moving along a circular trajectory is called the **Centripetal Acceleration. Below shows the derivation.**



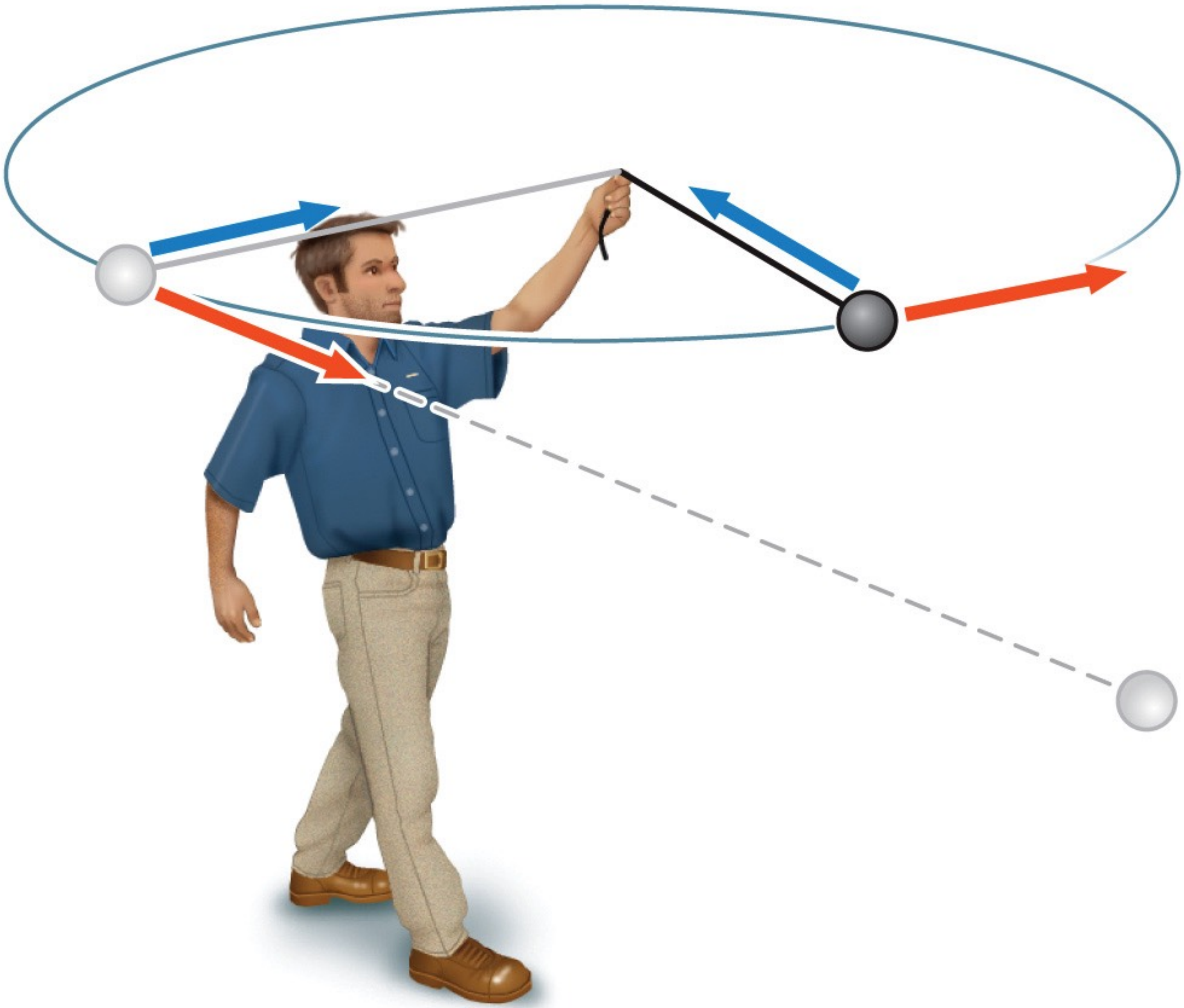
$$a_c = v^2/r$$

$$v = 2\pi r/P,\ \omega = 2\pi/P \Rightarrow v = \omega r, a_c = \omega^2 r$$

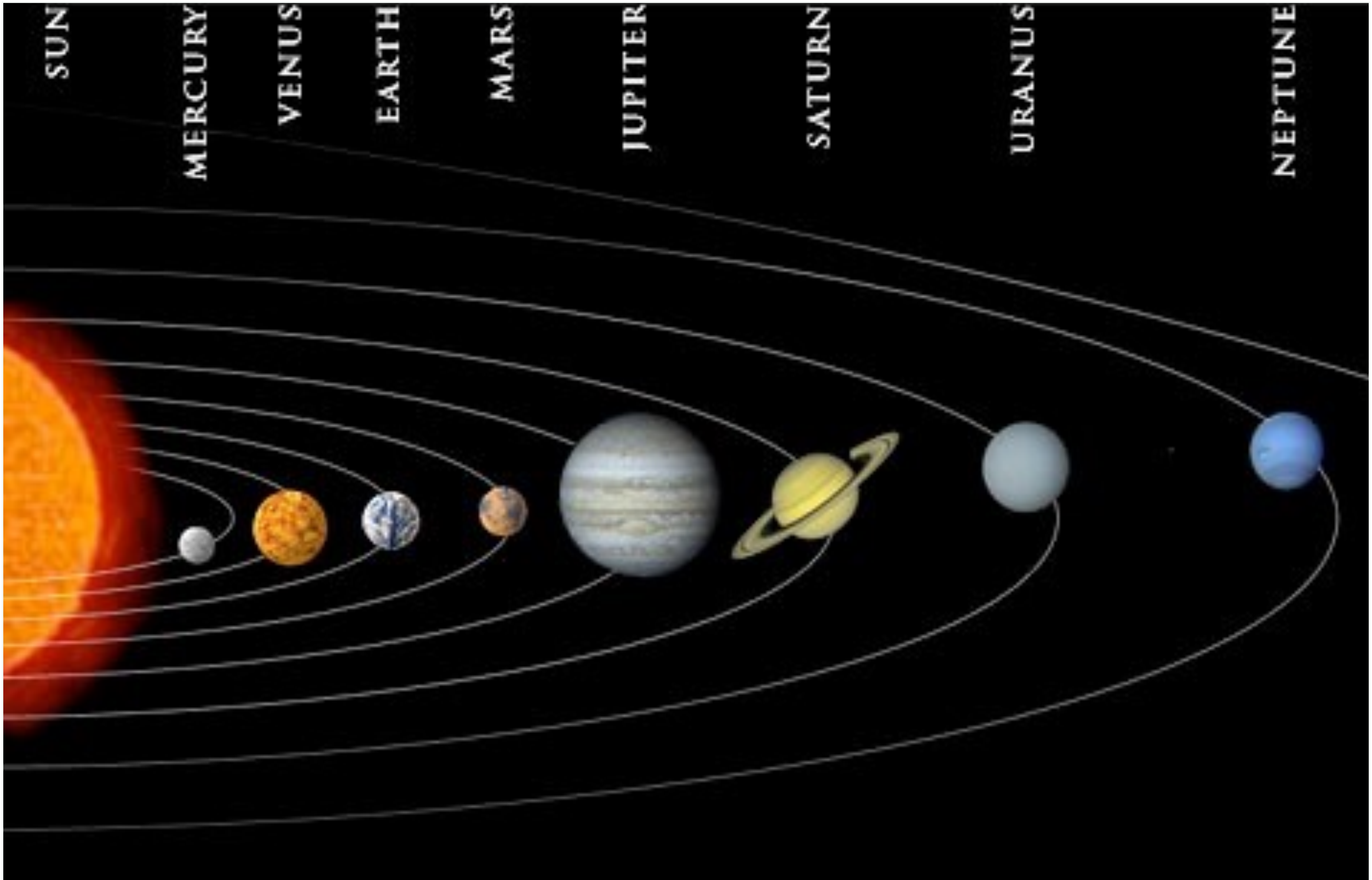
Newton's Law of Universal Gravitation

$$F_G \propto Mm/d^2$$

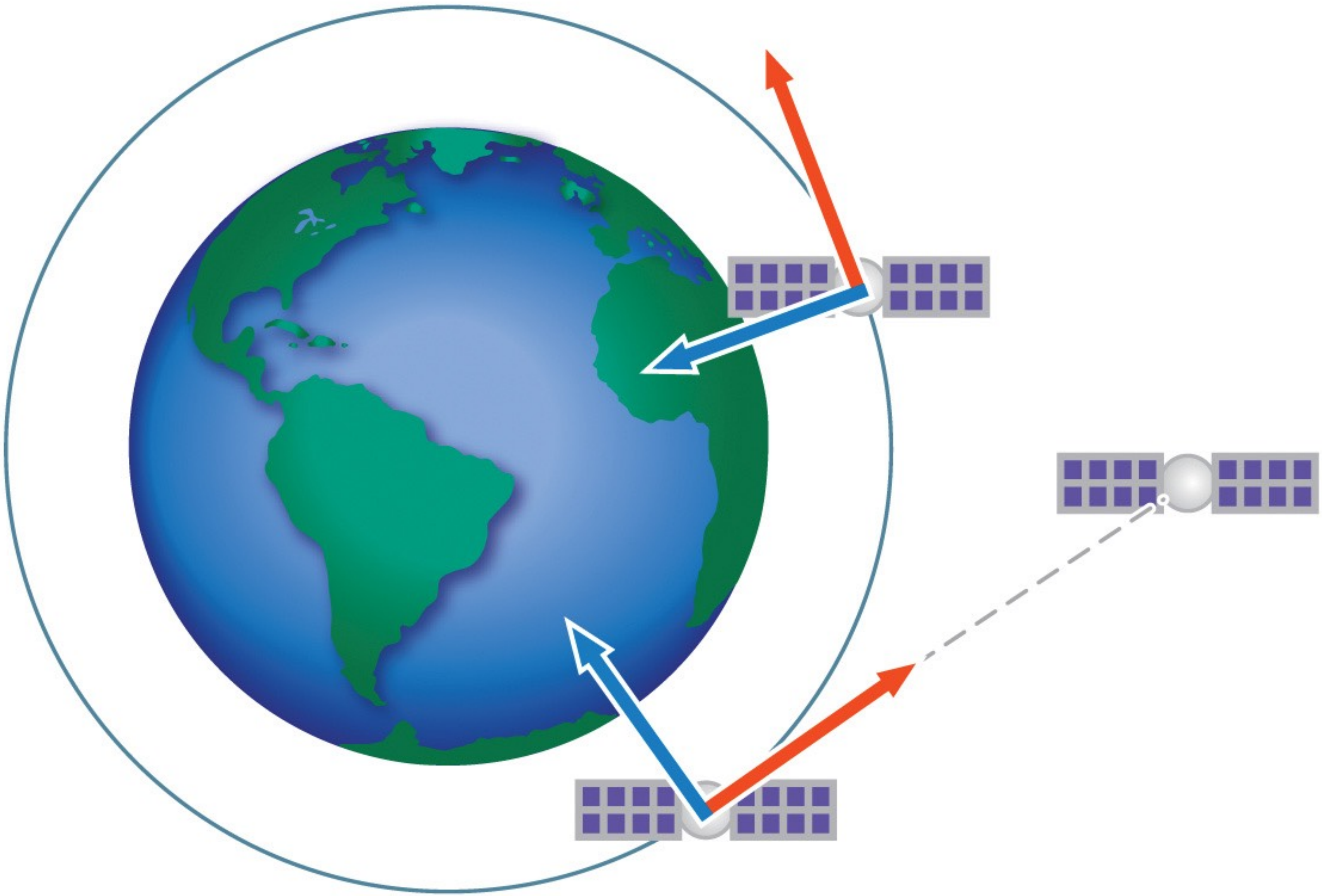
An object moving along a circular orbit is constantly accelerating, implying the exertion of an external force (N2: $F = ma$ & $a = v^2/r$)



This realization implies that *some external force* must be acting on the planets to keep them in elliptical orbits around the Sun



The same applies to satellites and the Moon orbiting around the Earth
The amount of gravitational force should depend on the masses of both objects and their distance, but how do we derive the formula for gravity?



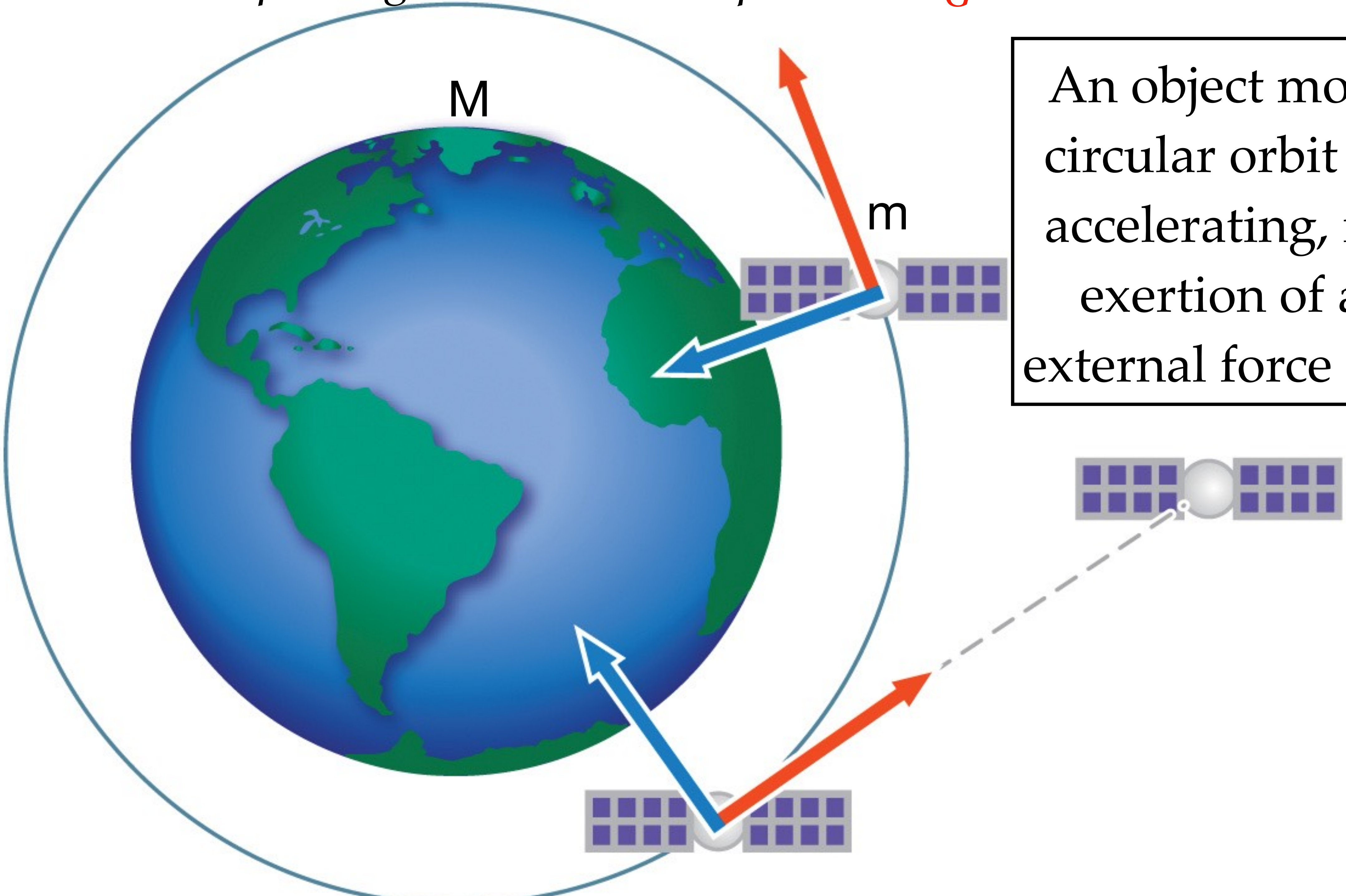
Derivation of Newton's Law of Gravitation - Part I

acceleration: $a = v^2/r = \omega^2 r = 4\pi^2 r/P^2$

required gravitational force: $F_G = ma = 4\pi^2 mr/P^2$

Kepler's 3rd law: $r^3/P^2 = b$ (which is some constant)

Replacing P^2 in the 2nd equation: $F_G = 4\pi^2 bm/r^2 \propto m/r^2$



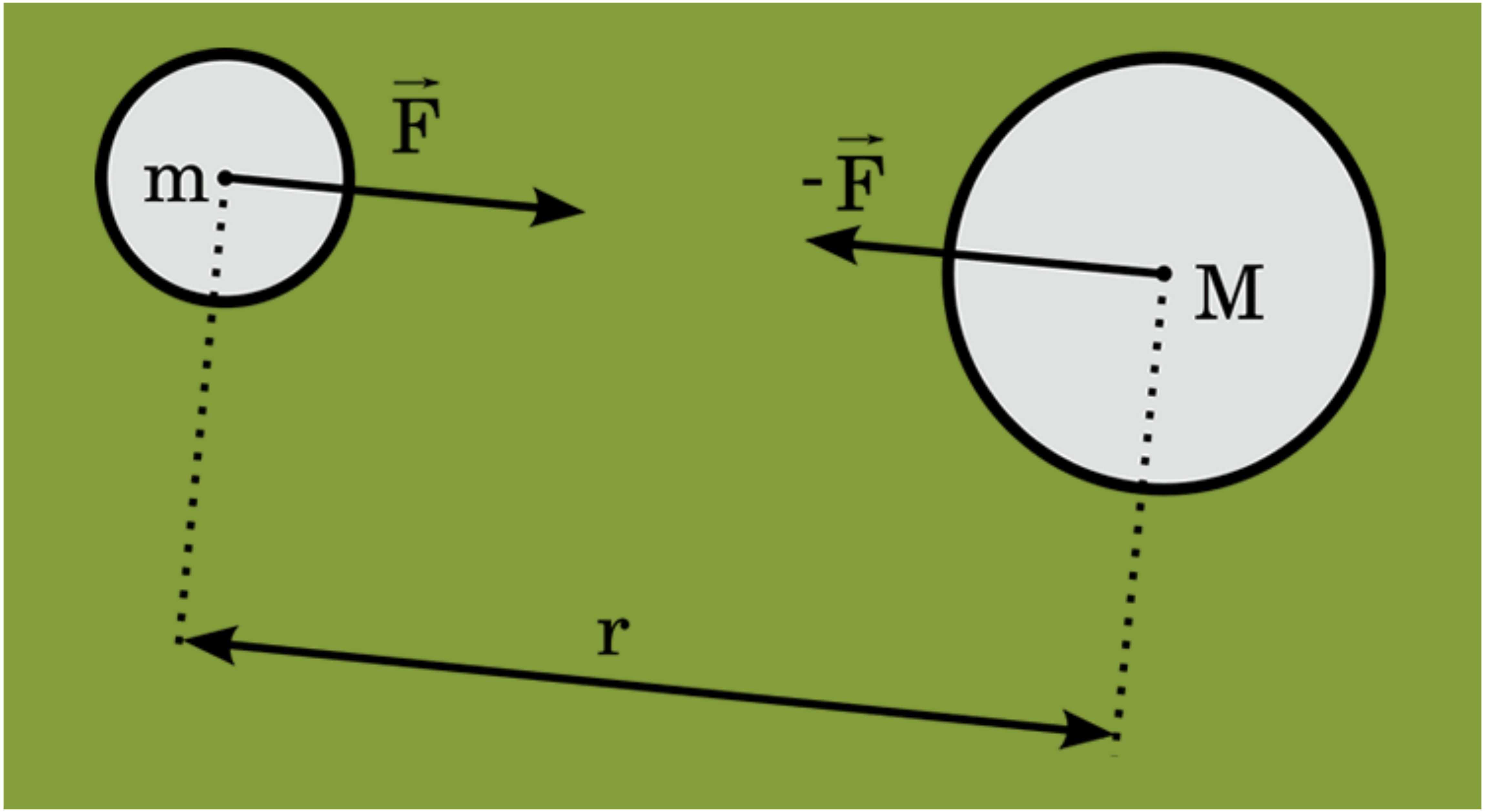
An object moving along a circular orbit is constantly accelerating, implying the exertion of an invisible external force called gravity

Derivation of Newton's Law of Gravitation - Part II

In the previous slide, we got $F_G = 4\pi^2 bm/r^2 \propto m/r^2$

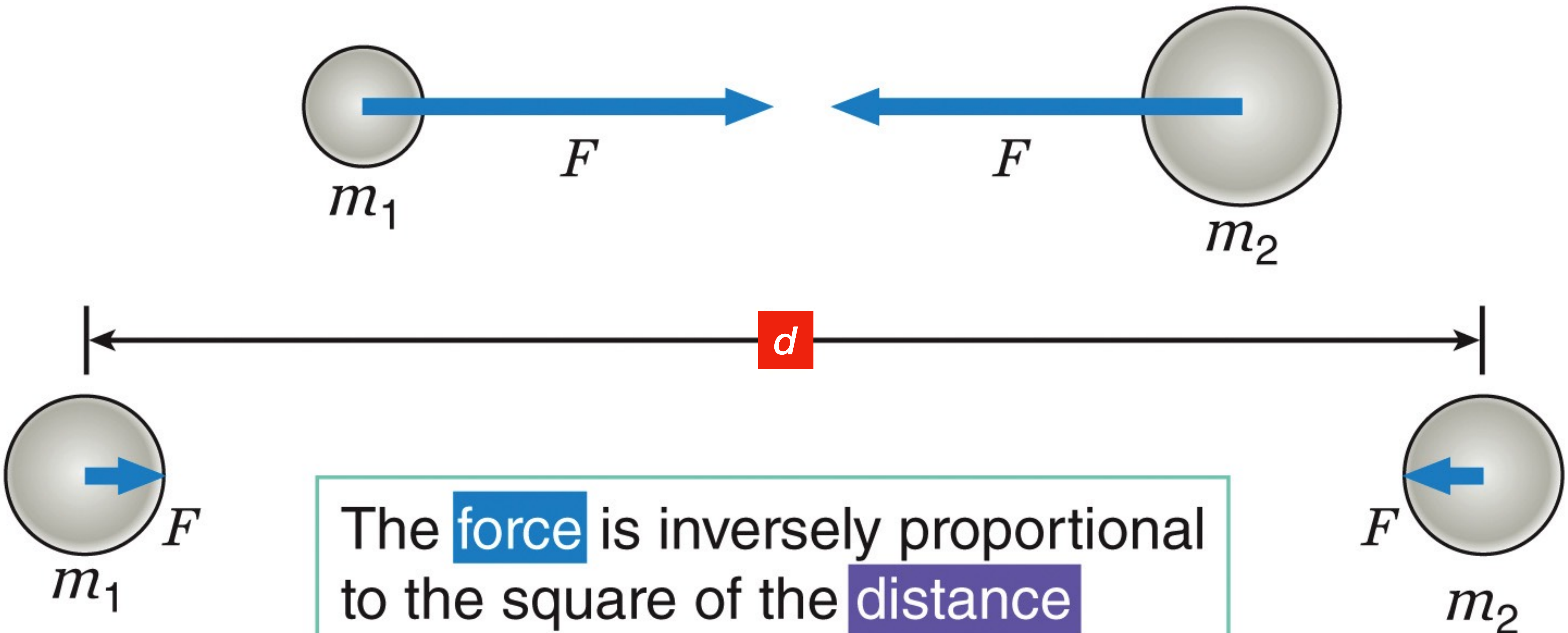
Newton's 3rd Law implies that *the gravitational force from M to m must equal to that from m to M, so F_G must be proportional to the mass product.*

The simplest form is $F_G = G\dfrac{Mm}{r^2}$ *and thus* $b = \dfrac{GM}{4\pi^2} = \dfrac{r^3}{P^2}$



Newton's Universal Law of Gravitation:

$$F = G\frac{m_1 m_2}{d^2}$$



The force is inversely proportional to the square of the distance between the masses. Larger distances produce smaller forces.

What is G exactly? It is the constant that keeps *inertial* mass and *gravitational* mass equal

$$G = 6.67 \times 10^{-11} \mathrm{m}^3/\mathrm{kg}/\mathrm{s}^2$$

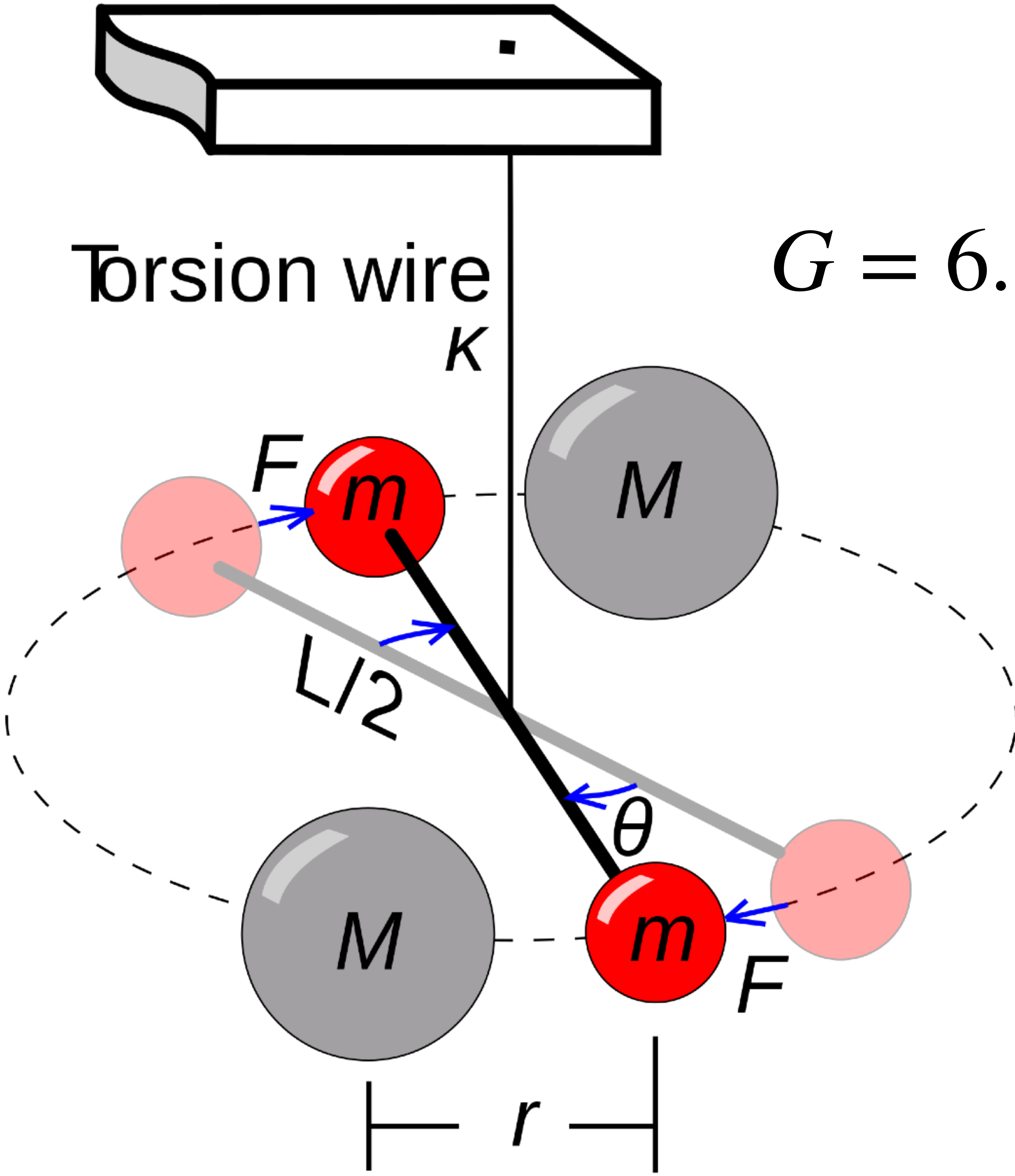
- $\mathrm{m}_{\mathrm{inert}} = F/a$
  here mass is the **inertia** to external force

- $\mathrm{m}_{\mathrm{grav}}{}^2 = F_{grav}\, d^2/G$
  here mass is the **origin of gravity**

- In Newtonian physics, these two masses are assumed to be identical, and G makes sure they are equal.

Cavendish experiment (1798) to measure G

The torsion pendulum consists of a bar suspended from its middle by a thin fiber.



$$G = 6.67 \times 10^{-11} \text{m}^3/\text{kg}/\text{s}^2$$

Astronomical Mass Measurement Example: Solar Mass

For example, using Kepler's third law:

$$\frac{r^3}{P^2} = \frac{GM}{4\pi^2}$$

we can measure the mass of the Sun with the Earth's sidereal period P (365.25 days = 1 yr) and its distance r to the Sun (1 AU):

$$M = \frac{4\pi^2 r^3}{GP^2}$$

plugging the numbers in their convenient units, we have the Solar mass:

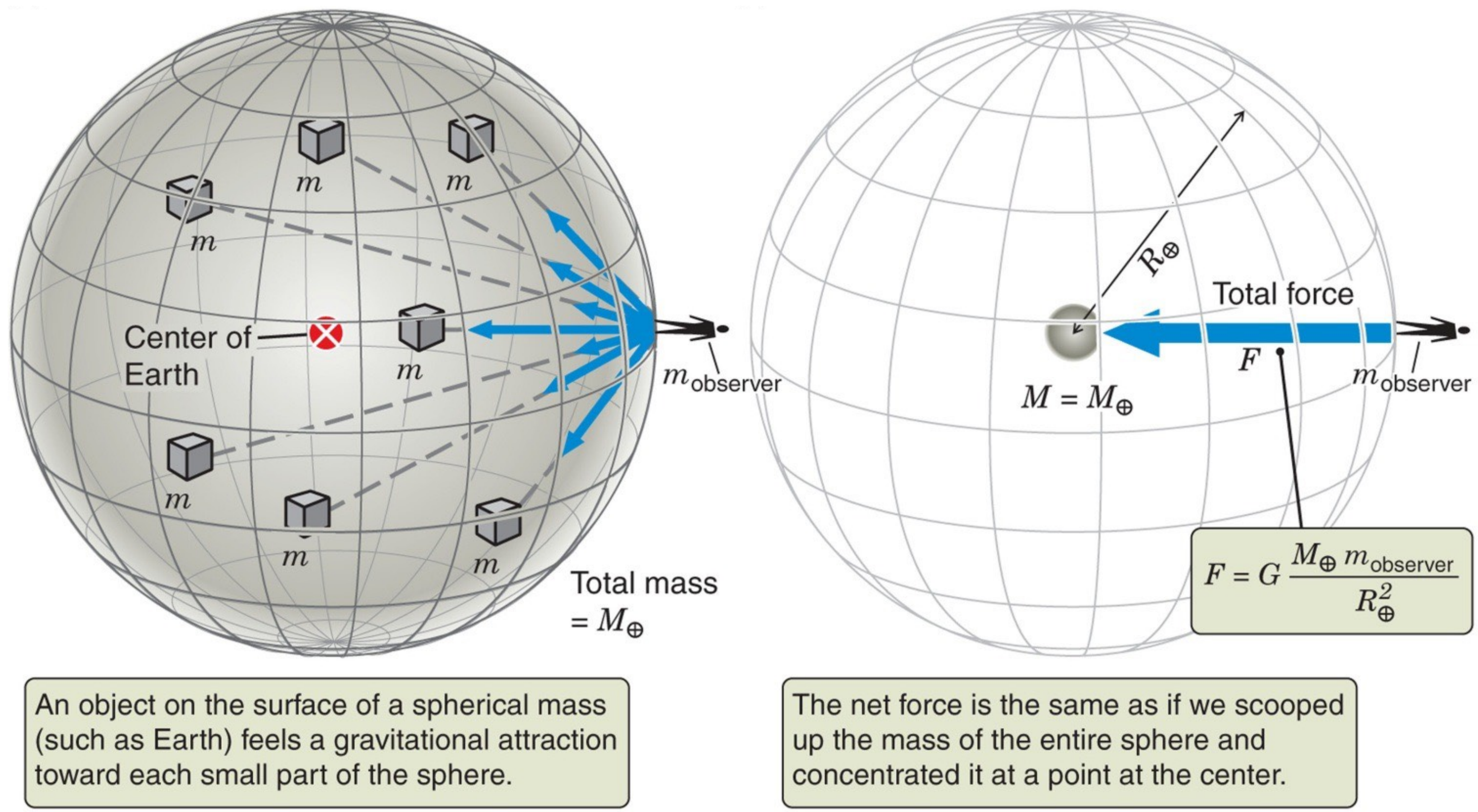
$$M_\odot = \frac{4\pi^2 \times (1\mathrm{AU})^3}{G \times (1\mathrm{yr})^2}$$

Note that without an independent measurement of G, we cannot measure mass, so what astronomical data provide is the product of G and M: ***GM**, which has a unit of* ***length³/time²***

Newton’s Theorems:

extend the law of gravity from point masses to spherical symmetric bodies

Newton derived two theorems for Spherically Symmetric Shells with Calculus



1. ***Outside*** spherically symmetric shells, net gravity is **the same as from a point mass with the same mass** as the shells placed at the center of the shells.
2. ***Inside*** spherically symmetric shells, there is **no net gravity**

Law of Gravity Application:

Surface Gravitational Acceleration

Gravitational Acceleration at a Planet's Surface

- The gravitational acceleration at the surface of Earth, g, can be solved for by using Newton's theorem for spherically symmetric objects:

$$F_G = G\frac{Mm}{R^2}$$

- and Newton's second law:

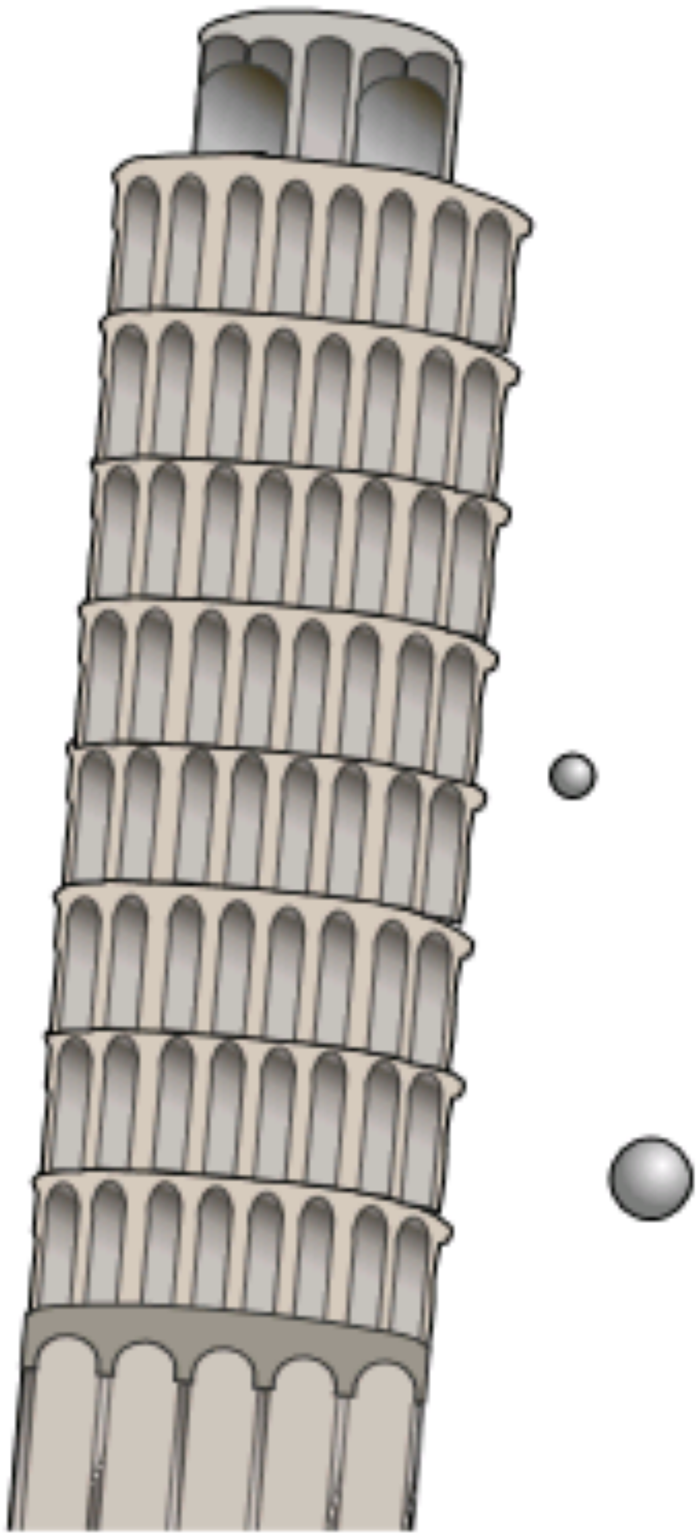
$$F_G = mg$$

- Set these two equations equal to each other, and then the mass m will cancel:

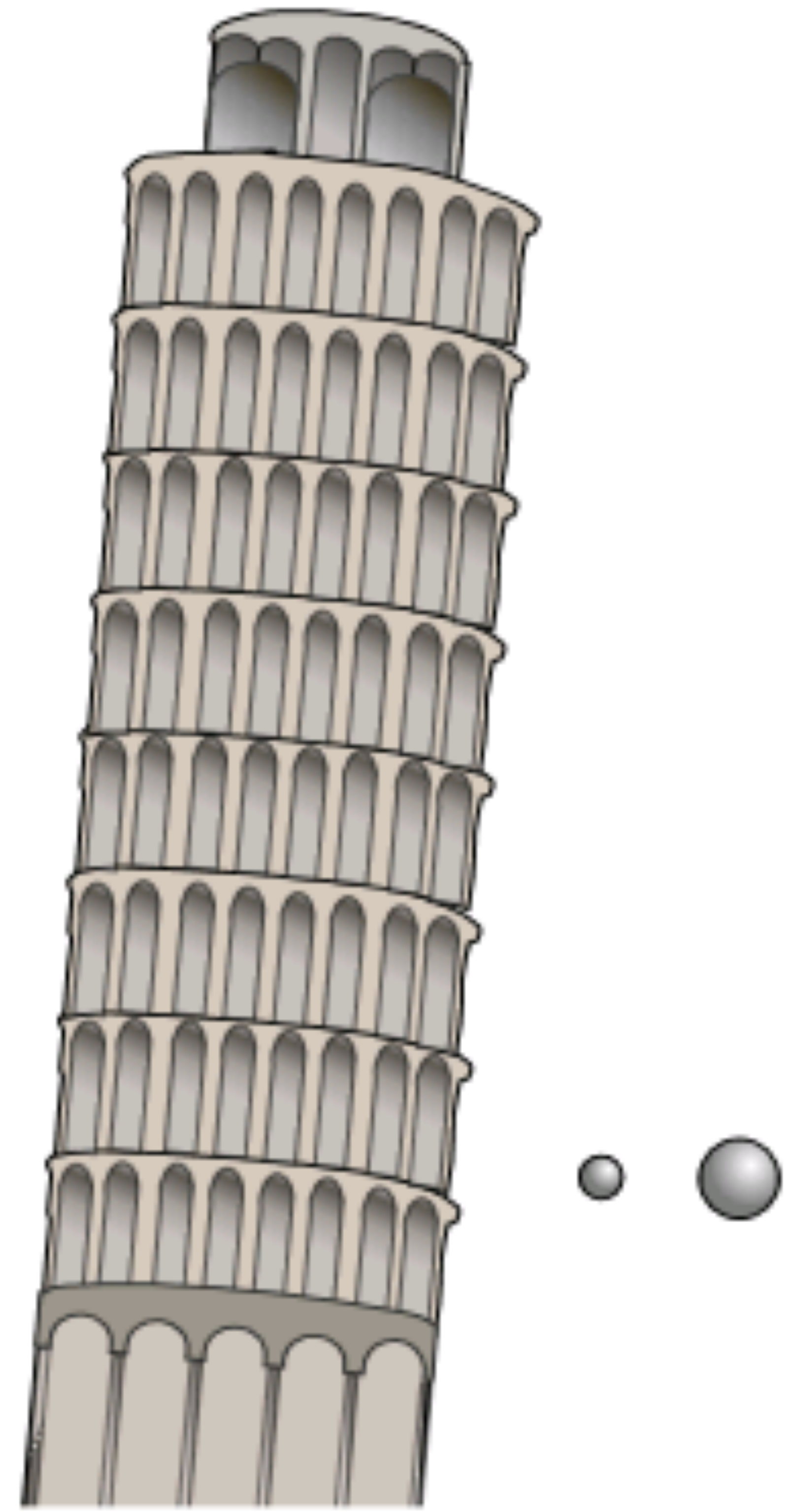
$$g = \frac{GM}{R^2}$$

- This equation shows that ***g*** **is a constant for all objects located at the same** ***R*****.**

Gravitational Acceleration at Earth's Surface



Old idea



Galileo

Galileo’s Experiment on the Moon

In 1971, Apollo 15 astronaut David Scott on the Moon recreated Galileo's famous experiment with feather and hammer.

The Hubble Space Telescope is in an orbit at an altitude of 600 km, the Earth's radius is 6500 km. What is the gravitational acceleration in the orbit compared to the "g=9.8 m/s^2" on the surface of the Earth?

Answer: g is only 1.2x smaller.

Astronauts feel **weightless** (sometimes called zero-g) not because there is no gravity, but because they are constantly falling. **This falling frame is a non-inertial reference frame,** in which an **artificial force** called "centripetal force" cancels gravity.

Use g to Measure the Mass of the Earth

$$g = \frac{GM}{R^2} \Rightarrow GM = gR^2$$

- The surface gravitational acceleration, *g*, can be measured using a pendulum's period and its length:

$$P = 2\pi\sqrt{\frac{L}{g}}$$

- Or, *g* can be measured using the free fall time *t* of a ball over a height of *h:*

$$h = \frac{1}{2}gt^2 \rightarrow g = \frac{2h}{t^2}$$

- The radius of the Earth is known since **Eratosthenes**
- Note that we can measure ***GM***, but neither ***G*** nor ***M*** separately. Astronomical mass measurements rely on laboratory measurements of ***G***.

What happened on October 4, 1957?

exactly 66 years ago

"All the News That's Fit to Print"

The New York Times.

LATE CITY EDITION

NEW YORK, SATURDAY, OCTOBER 5, 1957.

SOVIET FIRES EARTH SATELLITE INTO SPACE; IT IS CIRCLING THE GLOBE AT 18,000 M. P. H.; SPHERE TRACKED IN 4 CROSSINGS OVER U. S.

HOFFA IS ELECTED TEAMSTERS' HEAD; WARNS OF BATTLE

COURSE RECORDED

Navy Picks Up Radio Signals—4 Report Sighting Device

Device Is 8 Times Heavier Than One Planned by U.S.

560 MILES HIGH

Visible With Simple Binoculars, Moscow Statement Says

FAUBUS COMPARES HIS STAND TO LEE'S

ARGENTINA TAKES EMERGENCY STEPS

SATELLITE SIGNAL BROADCAST HERE

The world's first artificial satellite

- size of a beach ball (22.8 inches diameter)
- weighed 183.9 lbs
- orbital period: 98 min
- perigee: 230 km+6500 km,
- apogee: 940 km+6500 km.

Consequences:

- boosted US space effort, as they fear of inter-continental ballistic missiles carrying nuclear weapons

The first cosmonaut:

- November 3, 1957: Sputnik 2 launched with first on-board passenger (**Laika, a dog!**)
- Sputnik 2 was ~1000 lbs, orbited for 200 days!

January 31, 1958: Launch of Explorer 1

After *Sputnik 1*, US effort increases under the order of **President Eisenhower.** Only 84 days after *Sputnik 1*, the US launched *Explorer 1,* carrying scientific instruments built by **Prof. James Van Allen's team.**

Explorer 1 was placed in an orbit with a perigee of 224 miles and an apogee of 1,575 miles (from Earth surface), having a period of 115 minutes. Its total weight is only 31 lbs, including 18 lbs of instrument

Iowa Instrumentation built to detect cosmic rays

- Highly-energetic charged particles (90% protons, 9% Helium ions, & 1% electrons).
- Once in orbit, Explorer 1 detected fewer cosmic rays than predicted
- Van Allen's hypothesis that there were "belts" trapping the Cosmic rays in them, which led to the discovery of the **Van Allen radiation belt**.

Jet Propulsion Laboratory Director Dr. James Pickering, Dr. James van Allen of the State University of Iowa, and Army Ballistic Missile Agency Technical Director Dr. Wernher von Braun triumphantly display a model of the Explorer I, America's first satellite, shortly after the satellite's launch on January 31, 1958.

Part II: N-body Problem

center of mass and Virial Theorem

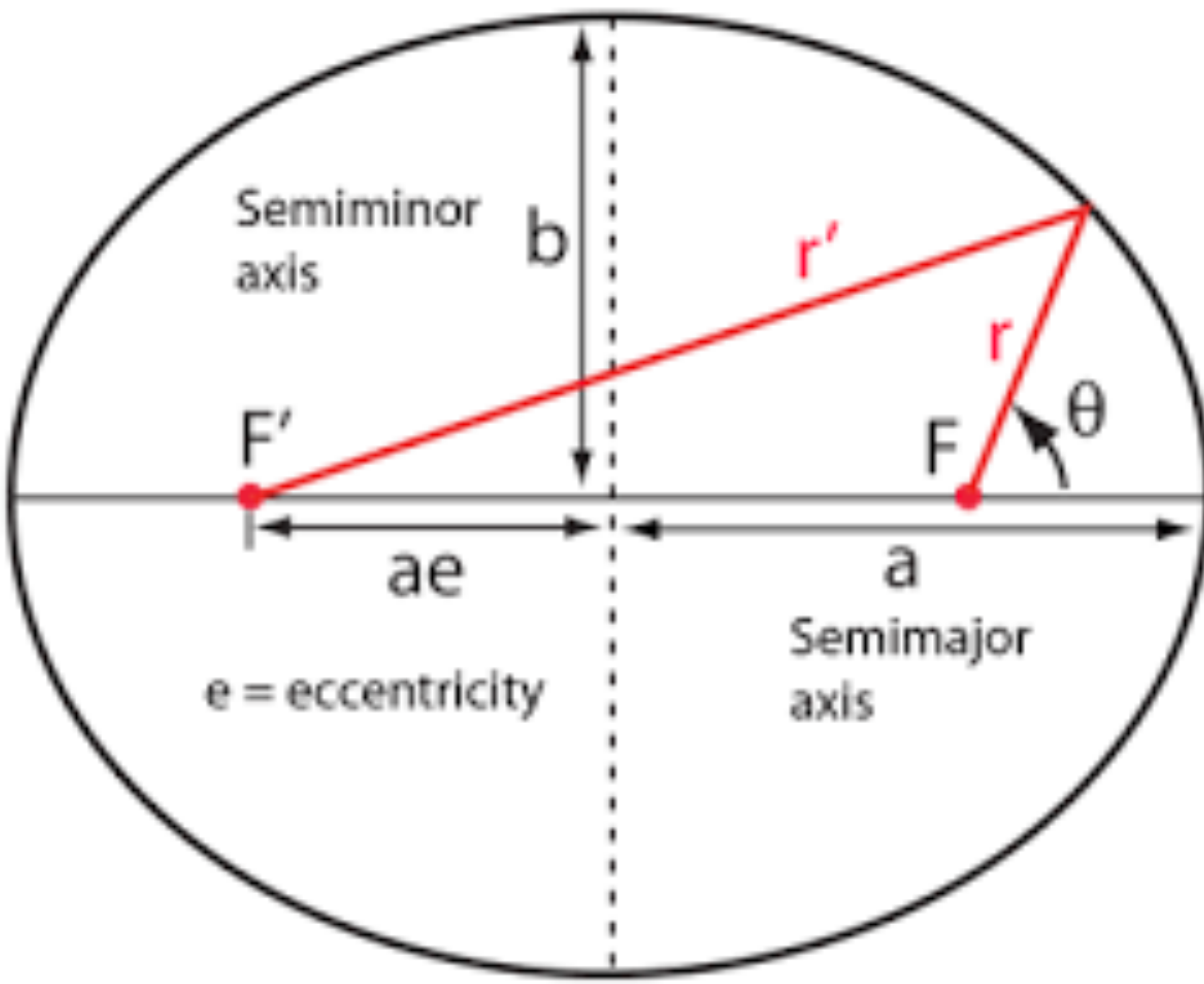
Center of Mass:

the inertial reference frame for two-body problem

One-body problem:

The Sun treated as an **immovable** object at the focus of the elliptical orbit

This is a great approximation when the central object dominates in mass



Mathematical Form of Kepler's 1st Law

In the *polar coordinate system* centered on the focus F, the trace of an elliptical orbit is:

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \text{ where } e = \sqrt{1-\frac{b^2}{a^2}}$$

Aphelion distance:

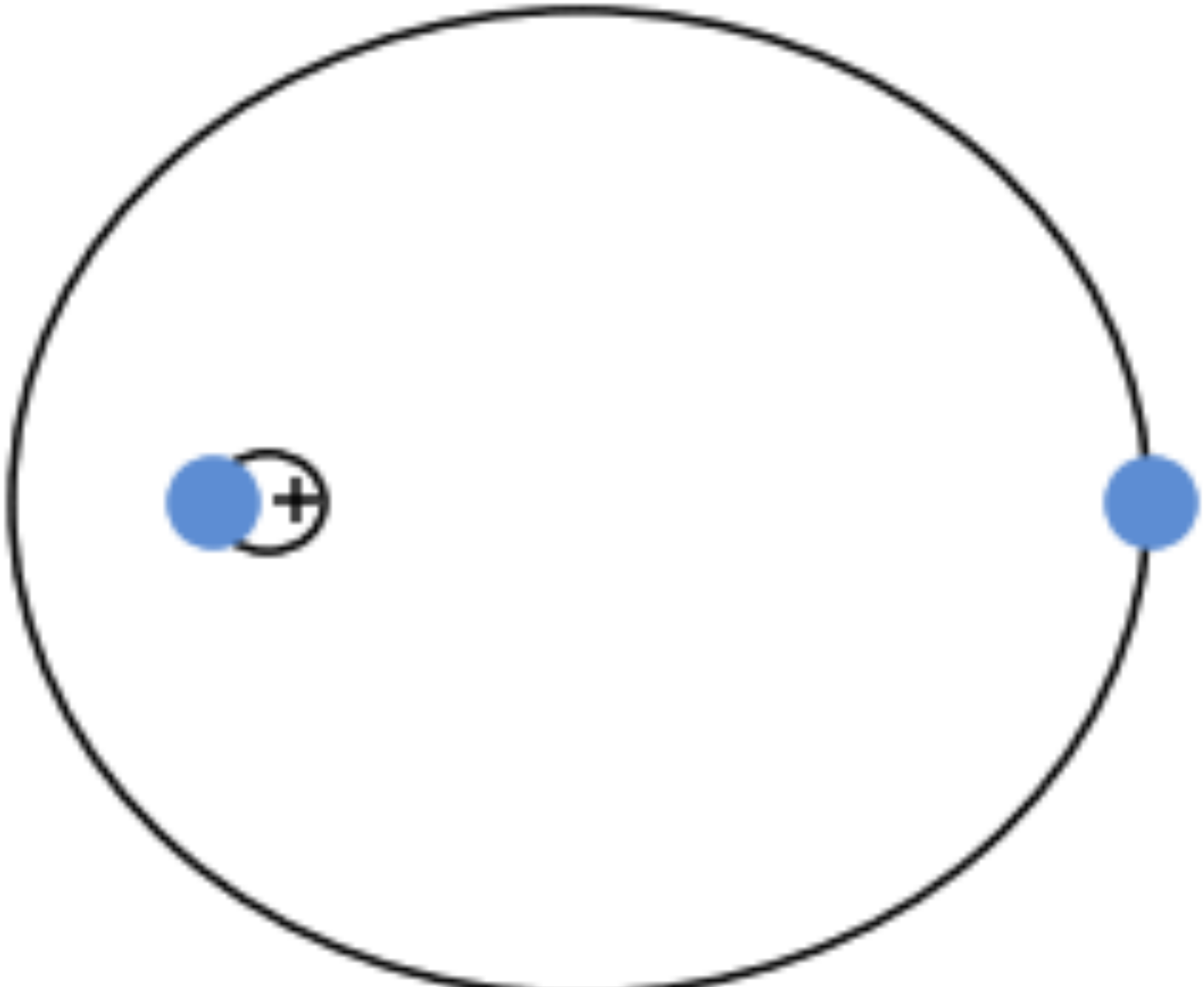
$$r_{\mathrm{ap}} = a(1+e)$$

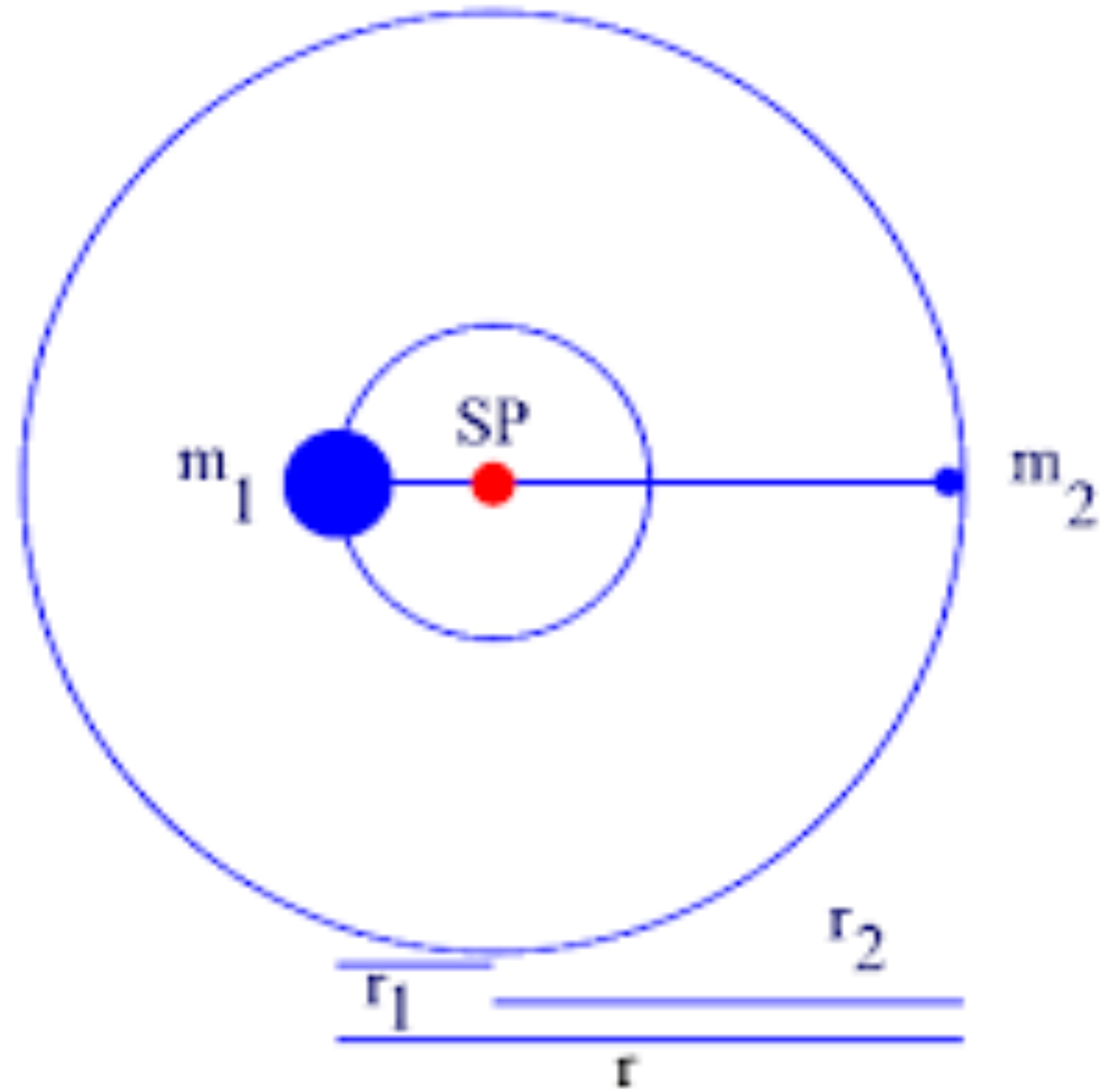
Perihelion distance:

$$r_{\mathrm{peri}} = a(1-e)$$

Two-body problem:

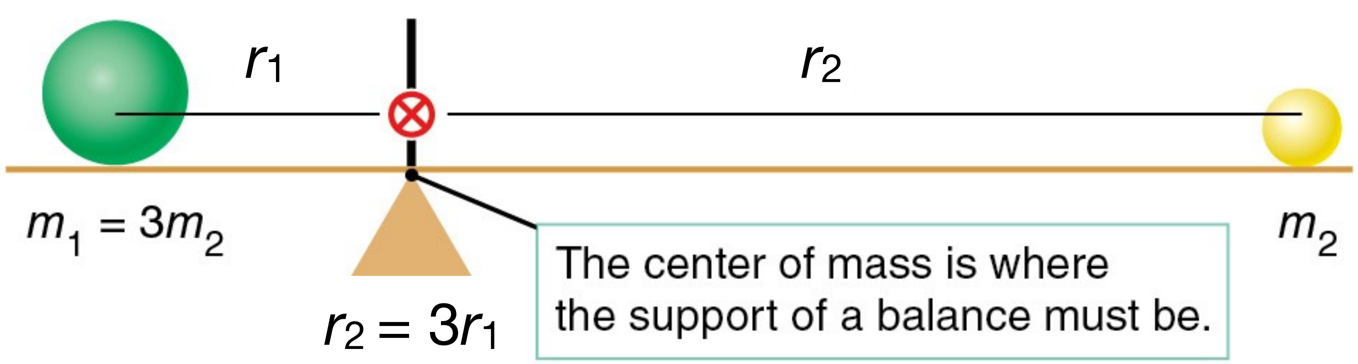
Because gravity is mutual, both objects involved in a gravitational system (a two-body system) must orbit around a common center of mass.

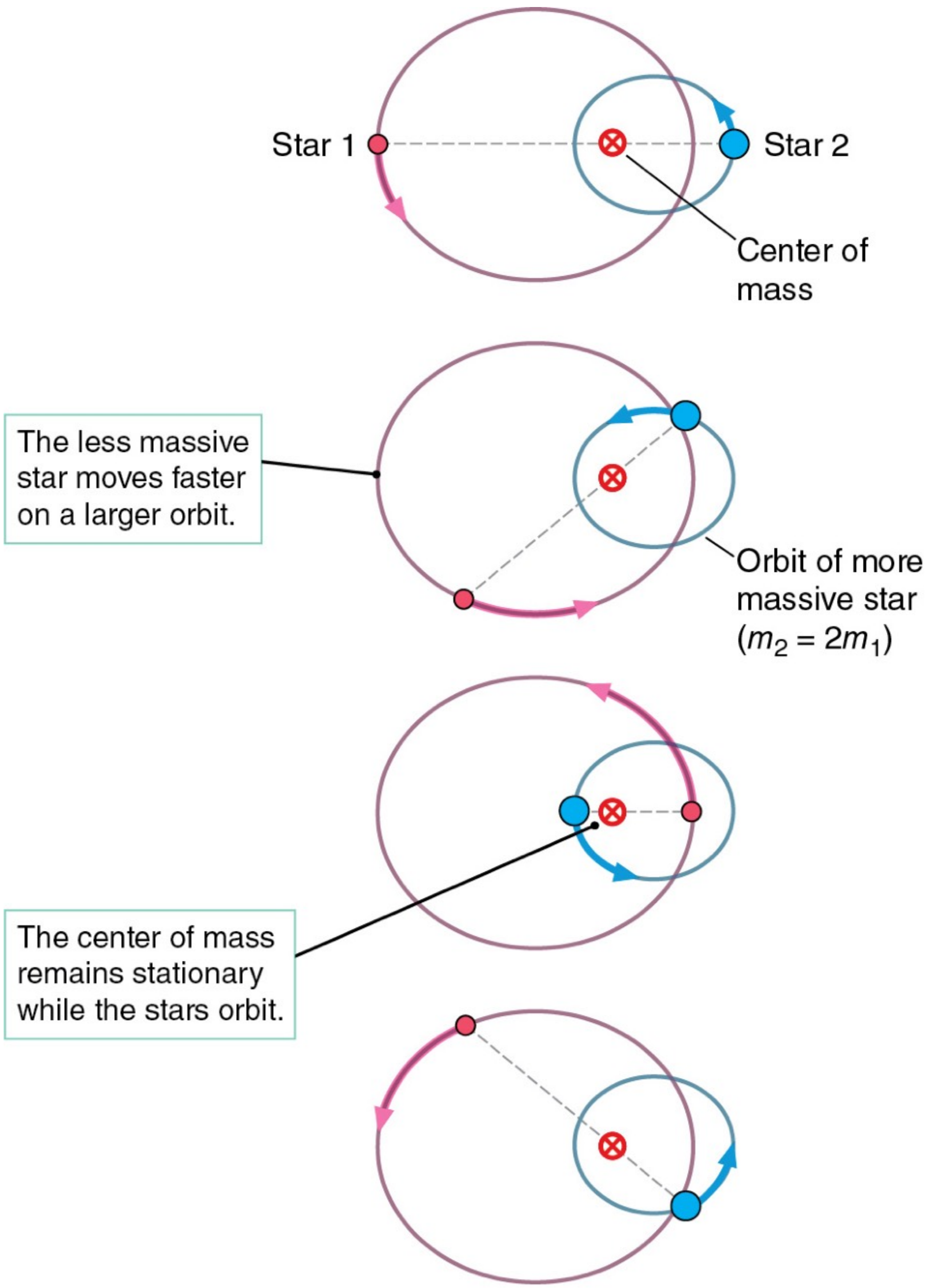




Center of Mass - Two-Body Problems

- Many stars are **binary stars** orbiting a common **center of mass**.
- A less massive star moves faster on a larger orbit.





Center of mass "seesaw" equation:

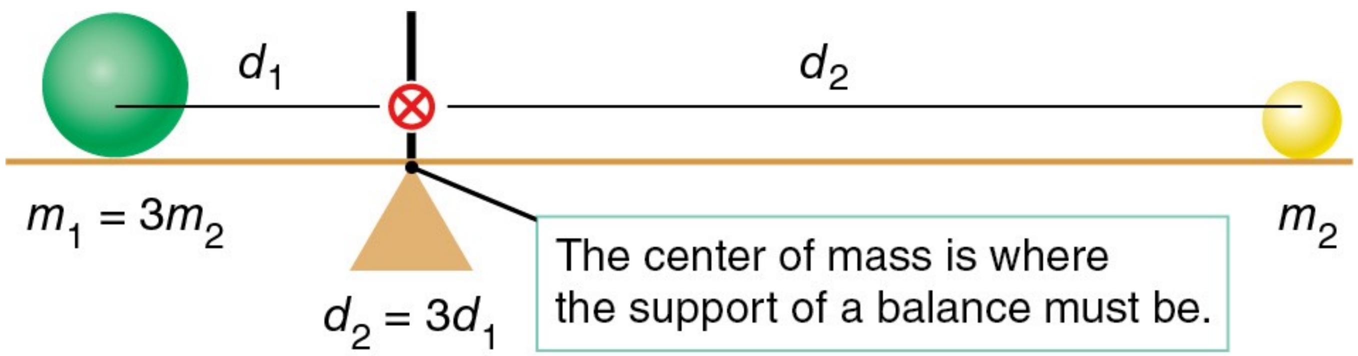
$$m_1 r_1 = m_2 r_2$$

Apply the Seesaw Equation on the Earth-Luna system:

$M_E/M_L \sim 80 \sim (R_E/R_L)^3$, E-L Distance = D = 384,400 km

What's the distance between the CoM and Earth's Center?

Is the CoM inside or outside of the Earth, given R_E = 6400 km?



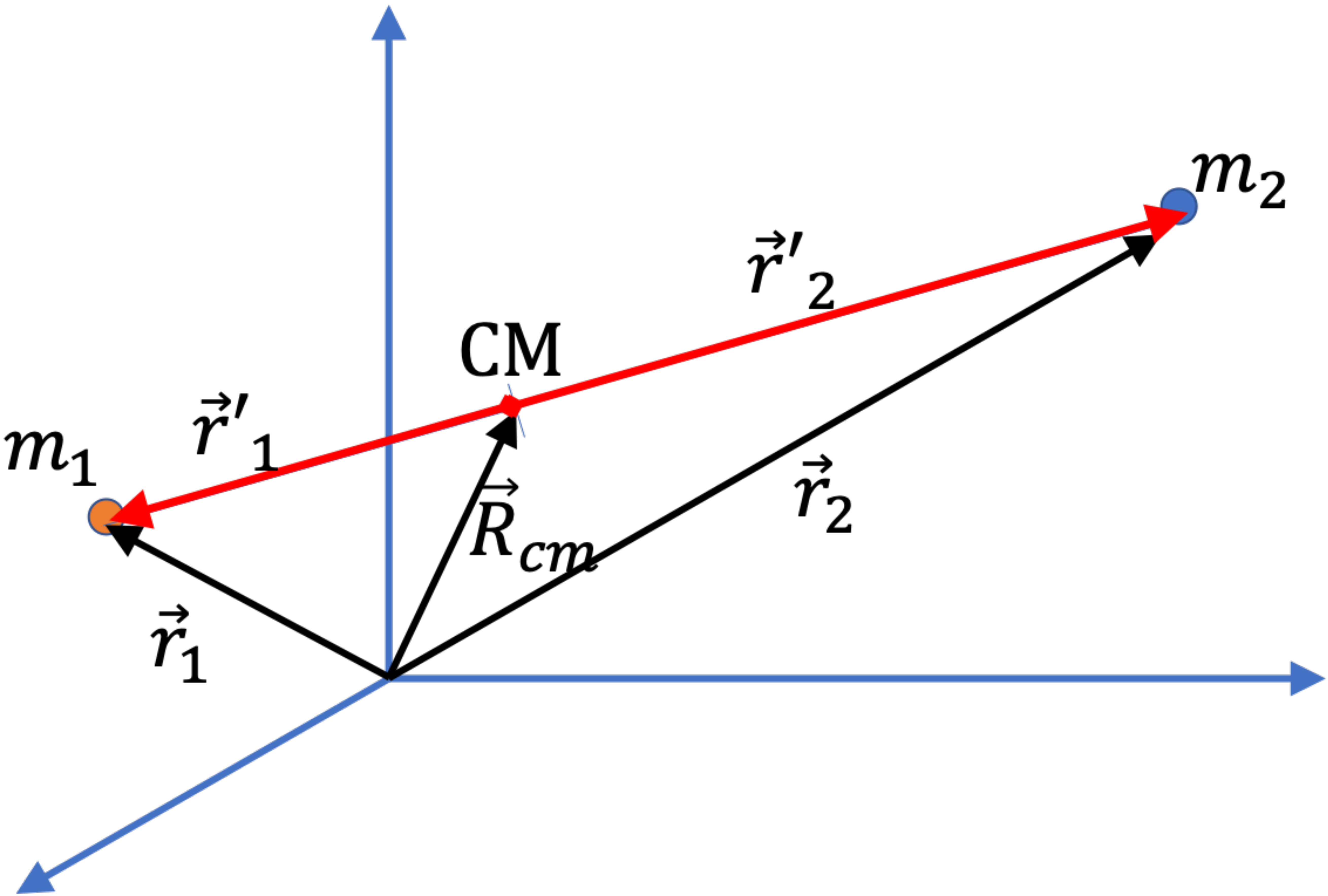
Center of mass
"seesaw" equation:
$$m_1 r_1 = m_2 r_2$$

r1 + r2 = D
m1 r1 = m2 r2
=> r1 (1 + m1/m2) = D
=> r1 = D/(1+m1/m2) = 384400/(1+80) = 4746 km
The CoM is 1700 km below surface.

Reduced Mass and Total Mass:

Mathematically reducing two-body problem to one-body problem by replacing masses

Two-body Problem - General Reference Frame



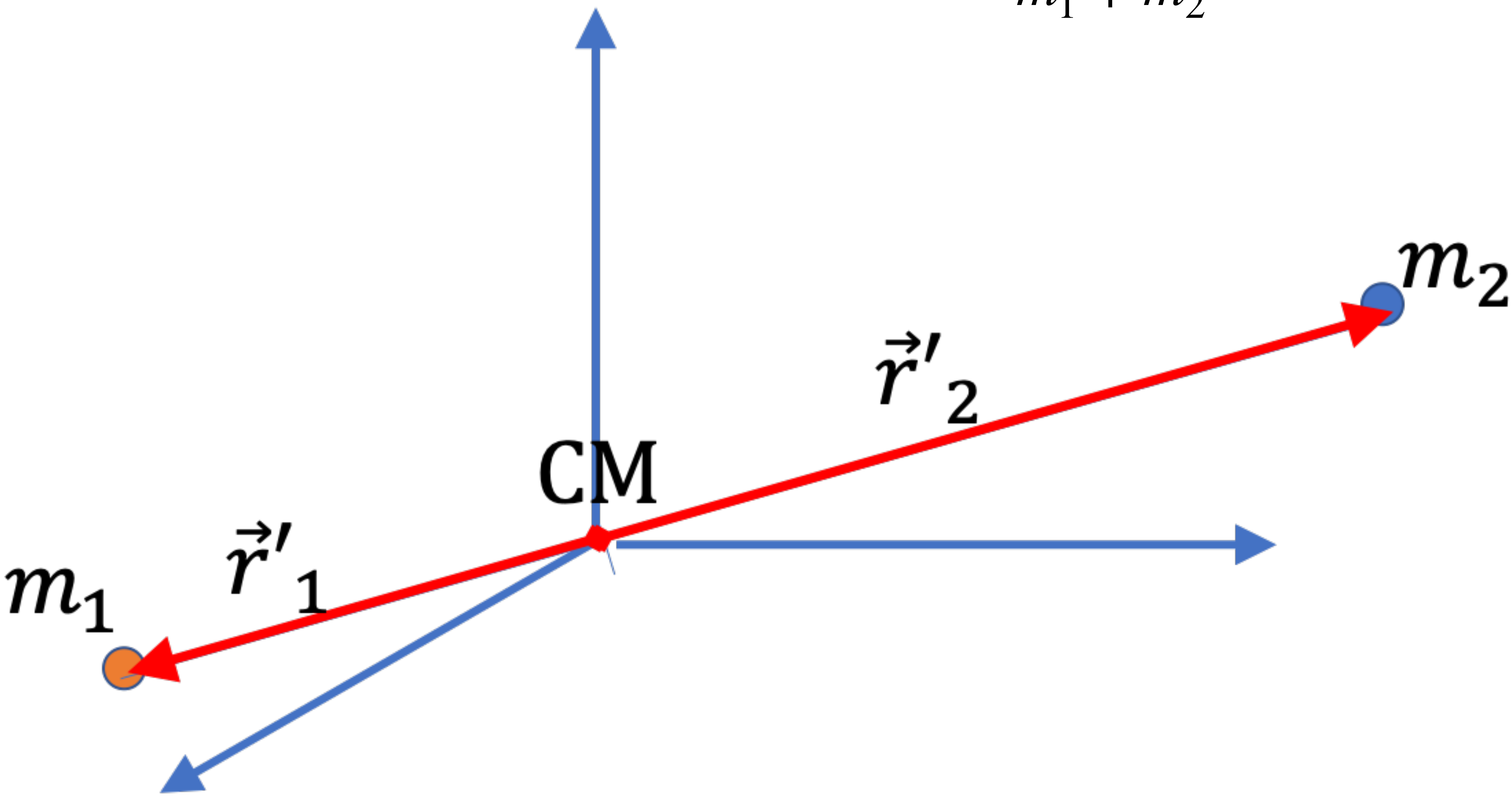
Two-body Problem - The Center-of-Mass Reference Frame

$$m_1\overrightarrow{r_1} + m_2\overrightarrow{r_2} = 0$$

$$\overrightarrow{r_2} - \overrightarrow{r_1} = \vec{r}$$

$$\vec{r}_1 = -\frac{m_2}{m_1 + m_2}\vec{r}$$

$$\vec{r}_2 = \frac{m_1}{m_1 + m_2}\vec{r}$$



Two-Body Problem reduced to One-Body Problem

define reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\vec{r}_1 = -\frac{m_2}{m_1 + m_2}\vec{r} = -\frac{\mu}{m_1}\vec{r} \qquad \vec{v}_1 = -\frac{\mu}{m_1}\vec{v}$$

$$\vec{r}_2 = \frac{m_1}{m_1 + m_2}\vec{r} = \frac{\mu}{m_2}\vec{r} \qquad \vec{v}_2 = \frac{\mu}{m_2}\vec{v}$$

Then write down the total kinetic and gravitational potential energy

$$E = \frac{1}{2}m_1|\vec{v}_1|^2 + \frac{1}{2}m_2|\vec{v}_2|^2 - G\frac{m_1 m_2}{|\vec{r}_2 - \vec{r}_1|}$$

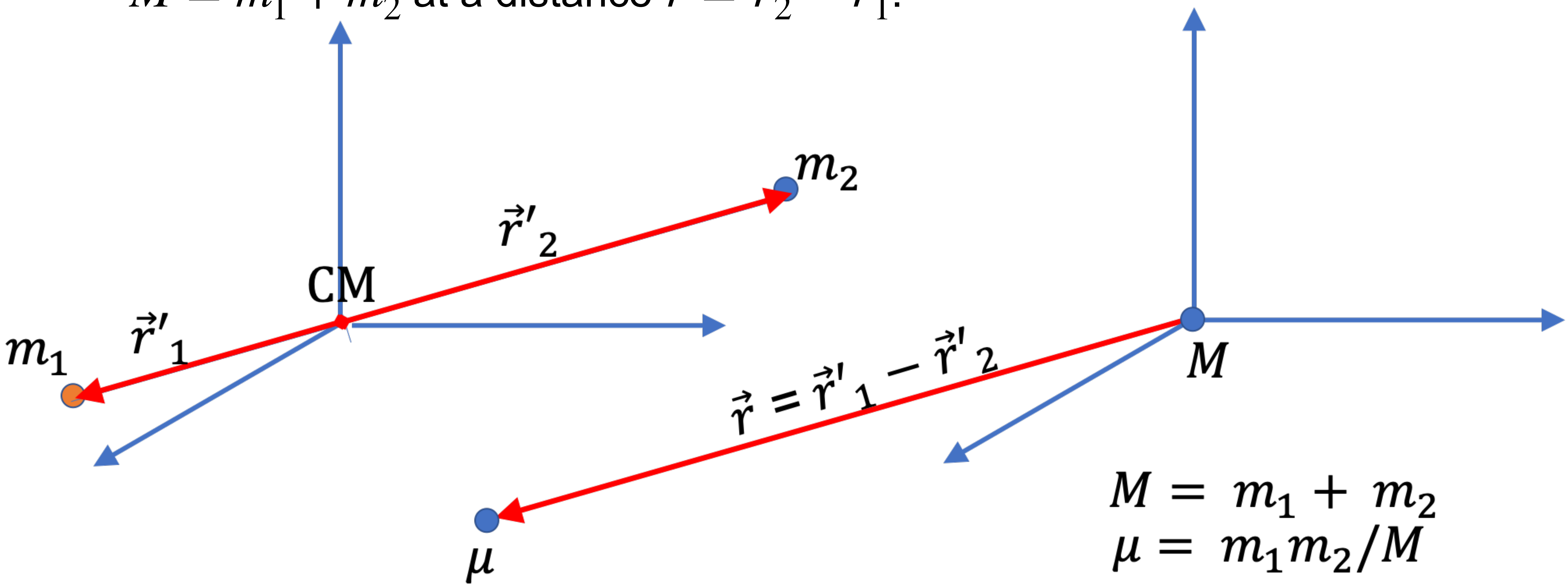
$$= \frac{1}{2}m_1\left(\frac{\mu}{m_1}\right)^2 v^2 + \frac{1}{2}m_2\left(\frac{\mu}{m_2}\right)^2 v^2 - G\frac{(m_1 + m_2)\cdot m_1 m_2/(m_1 + m_2)}{r}$$

$$= \frac{1}{2}\mu\left(\frac{\mu}{m_1} + \frac{\mu}{m_2}\right)v^2 - G\frac{M\mu}{r} \Rightarrow E = \frac{1}{2}\mu v^2 - G\frac{M\mu}{r}$$

- The two-body problem is equivalent to a one-body problem with the reduced mass $\mu = m_1 m_2/(m_1 + m_2)$ moving about a fixed total mass $M = m_1 + m_2$ at a distance $\vec{r} = \vec{r}_2 - \vec{r}_1$.

Kepler's 3rd Law for Two-body Systems

- The two-body problem is **equivalent** to a one-body problem with the reduced mass $\mu = m_1 m_2/(m_1 + m_2)$ moving about a fixed total mass $M = m_1 + m_2$ at a distance $\vec{r} = \vec{r}_2 - \vec{r}_1$.



$$M = m_1 + m_2$$
$$\mu = m_1 m_2 / M$$

K3 One-body problem:

$$\frac{m}{1\ M_{\rm sun}} = \left(\frac{a}{1\ \rm AU}\right)^3 \left(\frac{P}{1\ \rm year}\right)^{-2}$$

K3 Two-body problem:

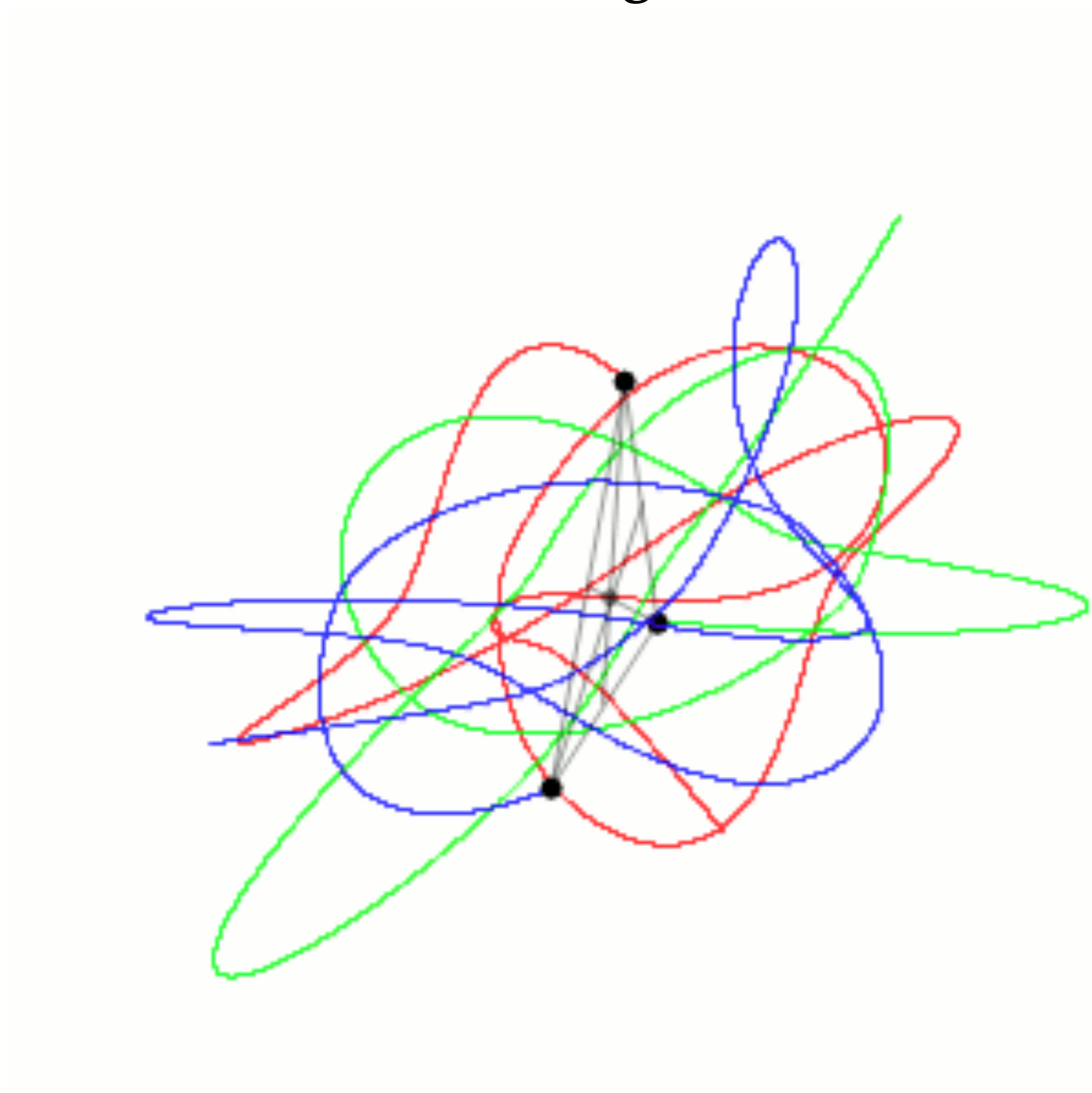
$$\frac{m_1 + m_2}{1\ M_{\rm sun}} = \left(\frac{a_1 + a_2}{1\ \rm AU}\right)^3 \left(\frac{P}{1\ \rm year}\right)^{-2}$$

N-body Problem:

complex self-gravitating systems
follow a simple law -
the virial theorem

Three-body problem up to N-body problem:

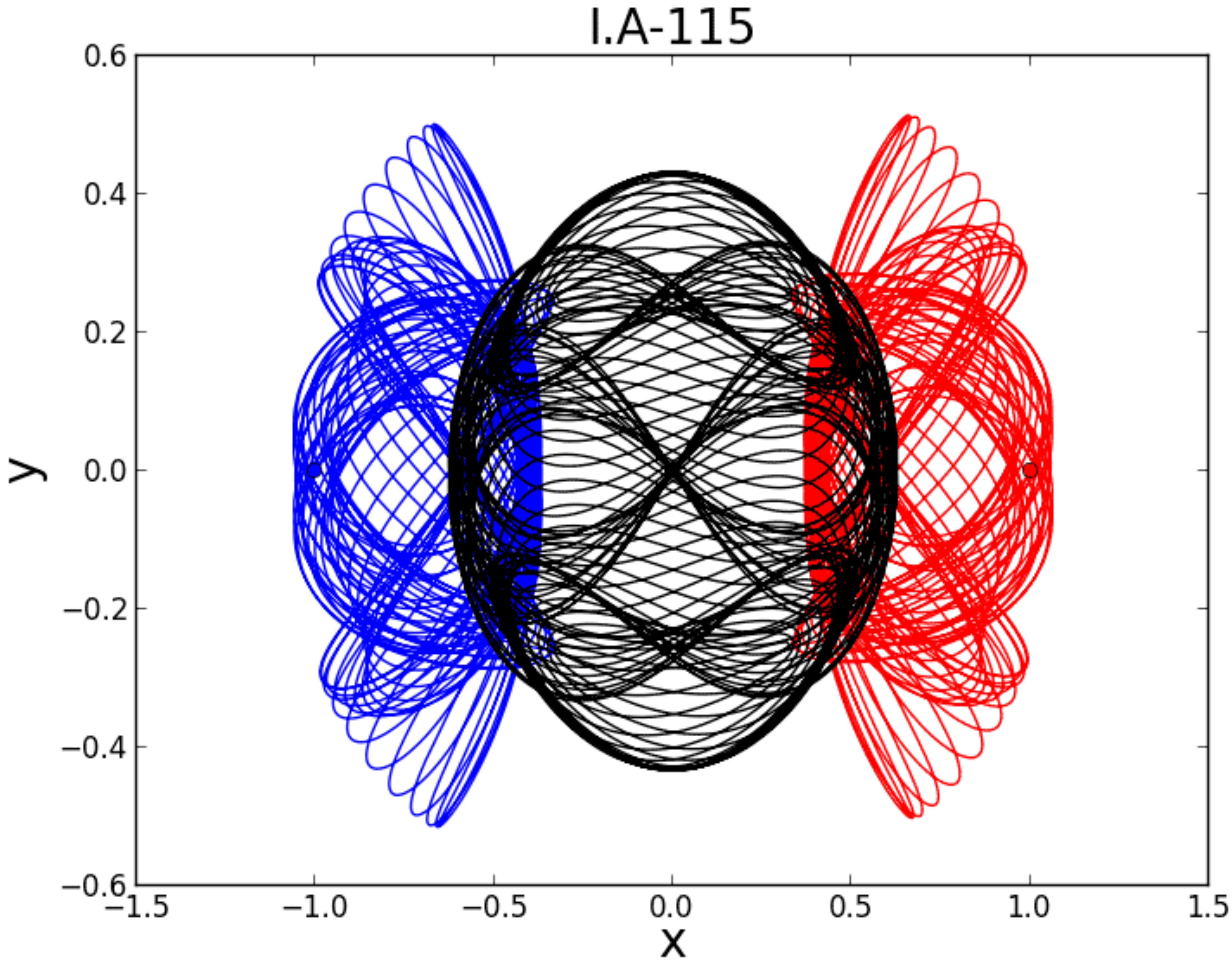
Chaotic and Irregular Orbits



Three identical bodies with zero initial velocities

Three-body problem up to N-body problem:

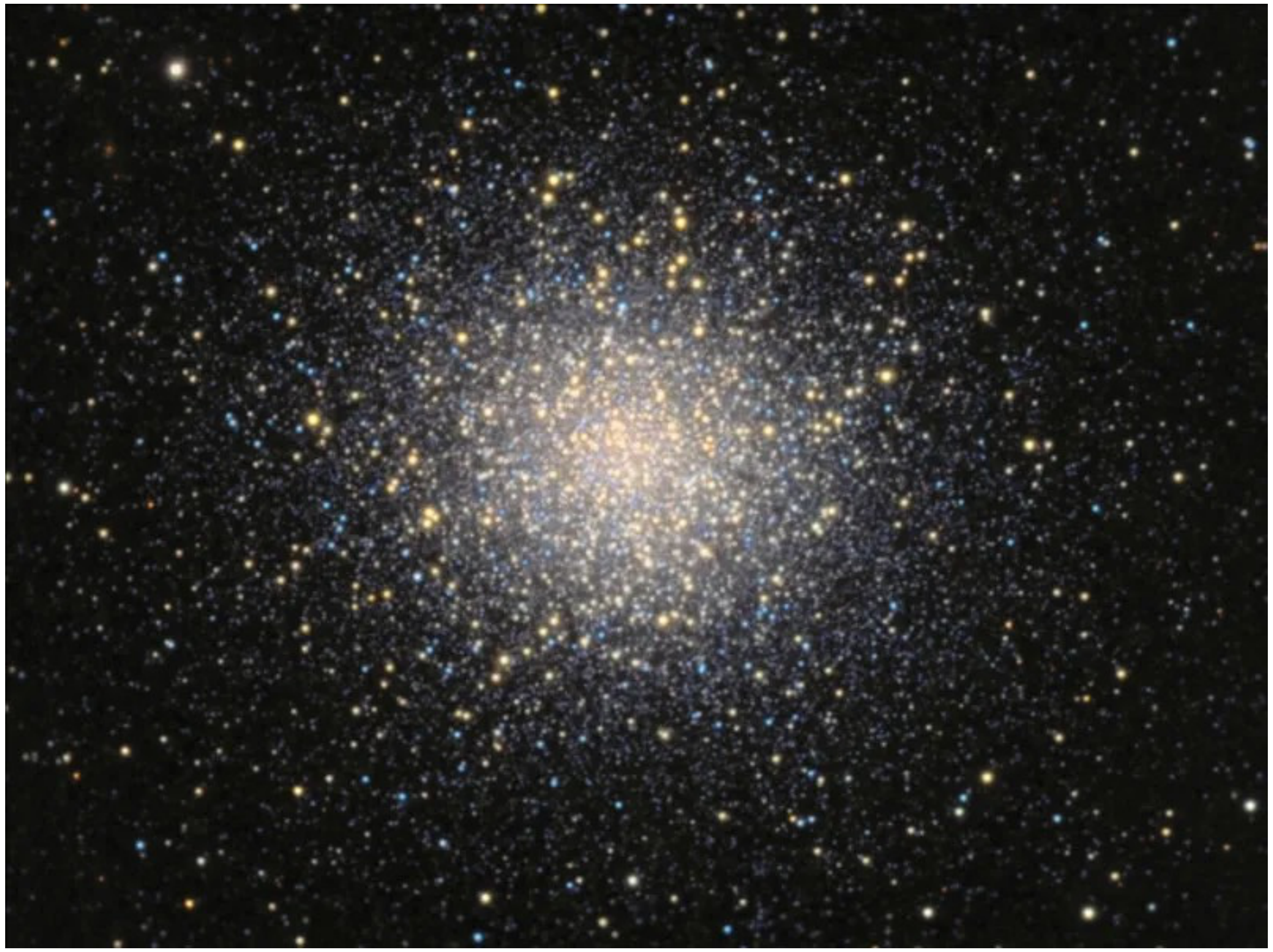
Chaotic and Irregular Orbits

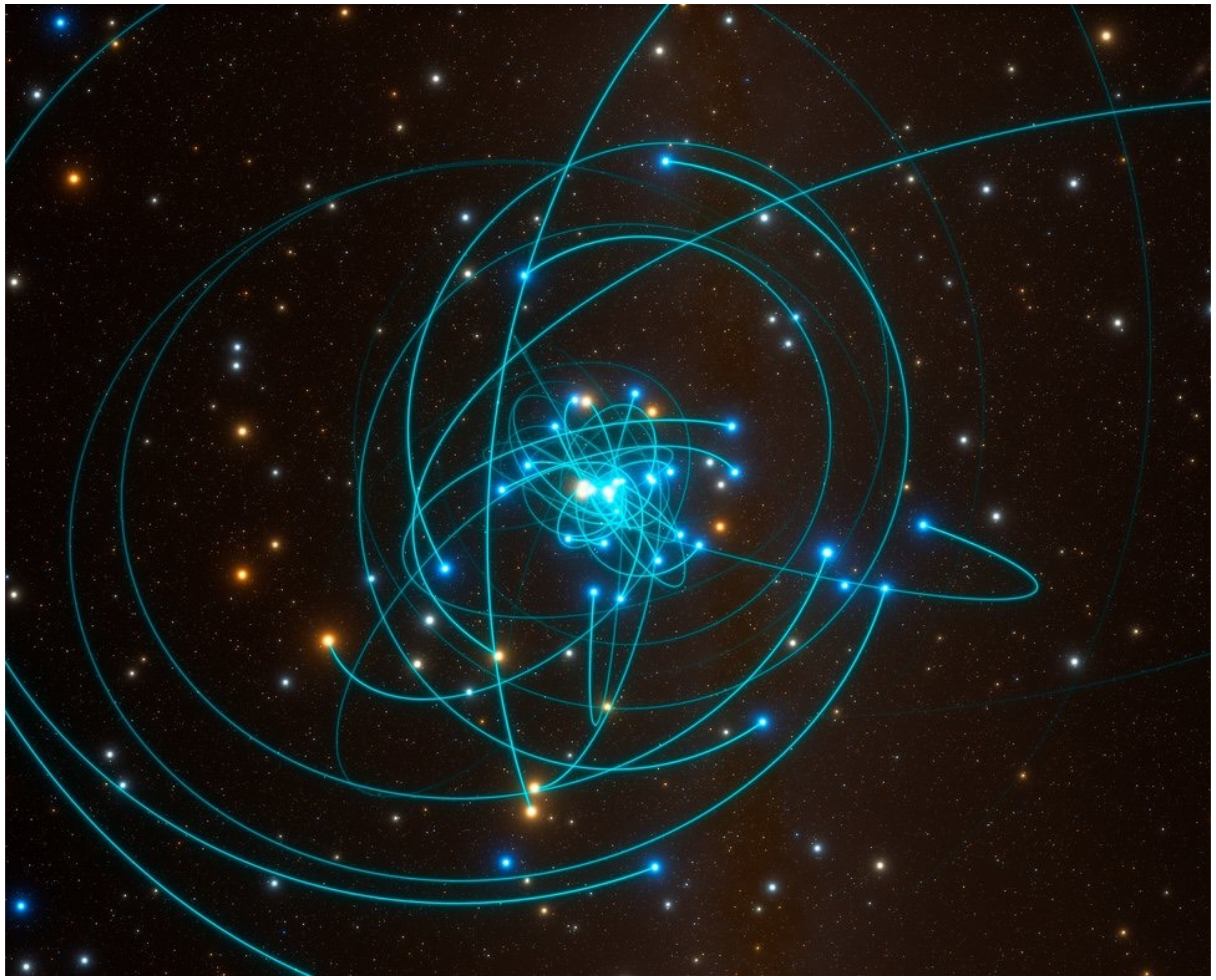


A rare case of periodic orbits of equal-mass co-planer three-body system

Think about a self-gravitating system in dynamical equilibrium like a globular cluster

- The **distribution of stars** determine its **mass distribution**
- The **mass distribution** determines its **gravitational potential**
- The **gravitational potential** determines the **orbits of the stars**
- The **orbits of stars** determine the **distribution of stars**, which closes the loop

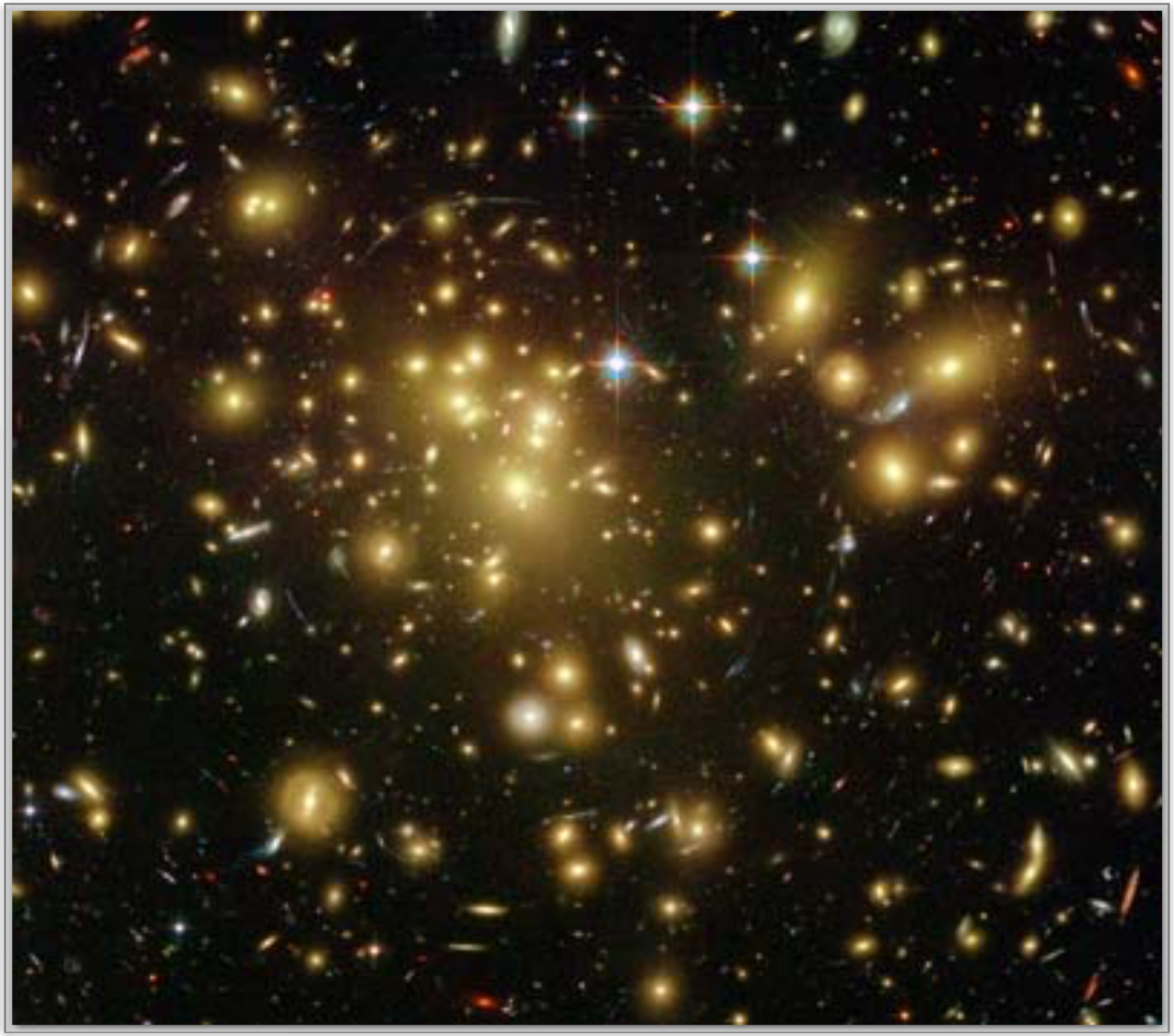




All self-gravitating systems (N-body) obey the Virial Theorem

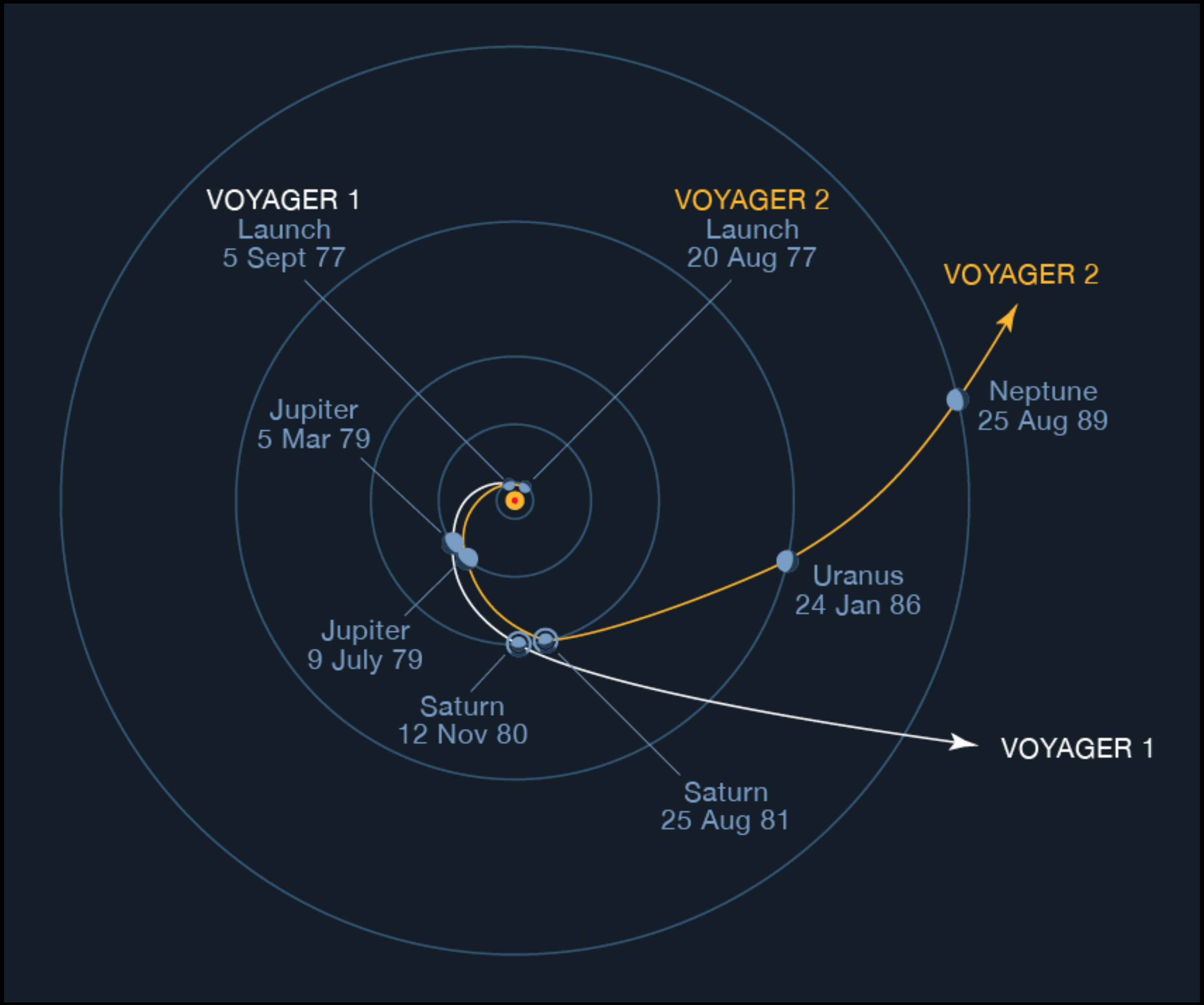
$$2\bar{K} + \bar{U} = 0 \Rightarrow \bar{v}^2 = \frac{G\tilde{M}}{\bar{R}} \;\&\; G\tilde{M} = \bar{v}^2\bar{R}$$

This applies to all self-gravitating systems:
planetary systems, molecular clouds, stars, star clusters, galaxies, galaxy clusters



Part III: Orbital Dynamics

vis-viva Eq., special velocities, transfer orbit, rocket Eq.



Vis Viva Equation
(Conservation of Energy)

Derivation

Kinetic energy: the Energy of Motion

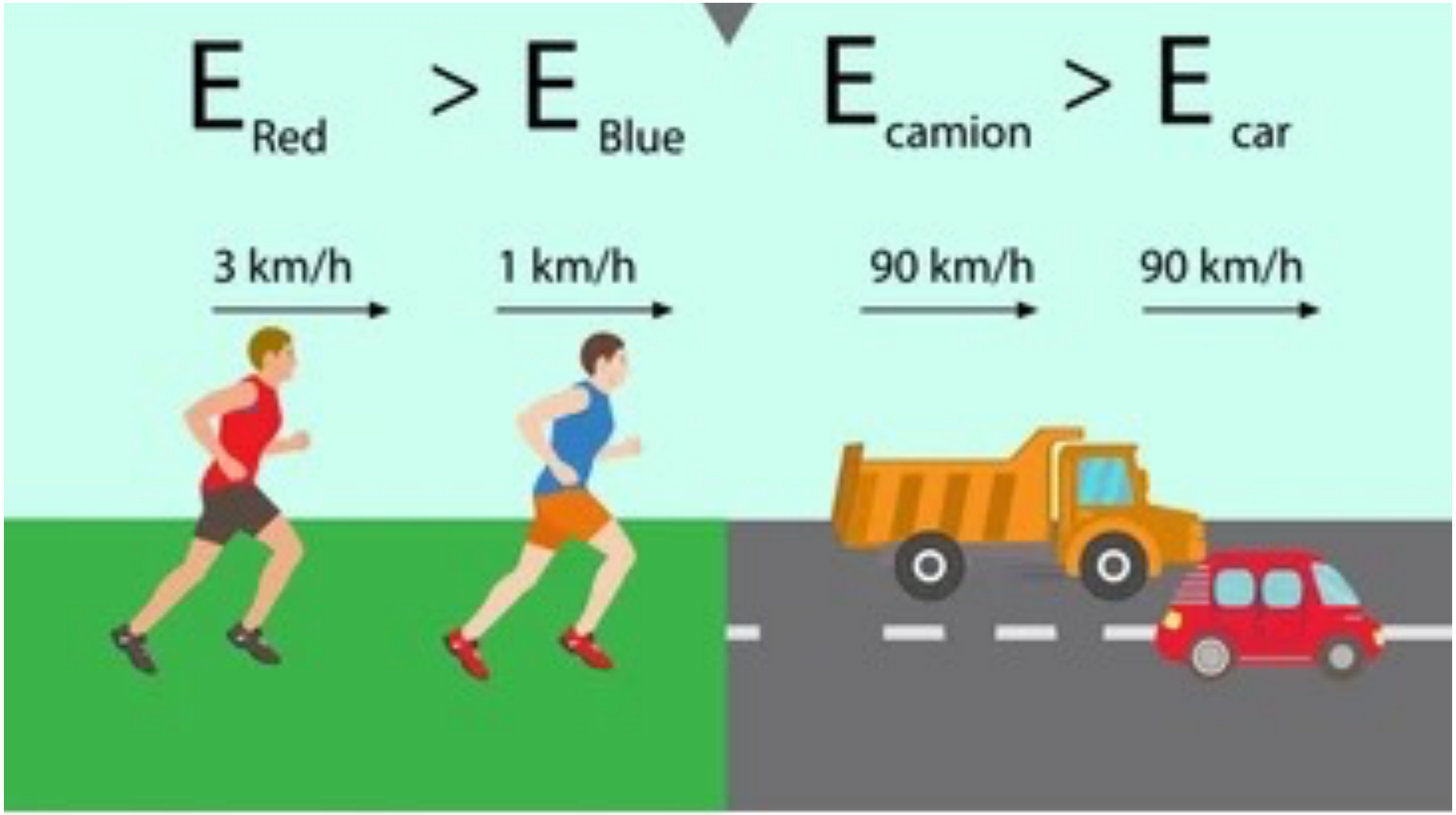
Otzi, **the Iceman**, was killed by the kinetic energy of an arrow in **3230 BC**

In 2001, X-rays and a CT scan revealed that Ötzi had an arrowhead lodged in his left shoulder when he died and a matching small tear on his coat.

Kinetic energy

Kinetic energy of mass *m* moving a speed *V*

$$E_{ke} = \frac{1}{2} m V^2$$



Kinetic energy: the Energy of Motion

Work measures **energy transfer** that occurs when an object is moved over a **distance** by an **external force.**

In the simplest scenario, the only external force is constant and the object accelerates from zero velocity along a straight line:

$$W = F\,s$$

because **work is energy transfer**, we have:

$$E_k(t) - E_k(t=0) = W = F\,s$$

we can choose t = 0 as the moment when the object has zero velocity, (implying zero kinetic energy: $E_k(t=0) = 0$), so that:

$$E_k(t) = W = F\,s$$

And for constant external force, we have constant acceleration:

$$F = m\,a$$

$$s = a\,t^2/2$$

Plug these in, we have

$$E_k(t) = F\,s = ma\;a\,t^2/2 = mV^2/2$$

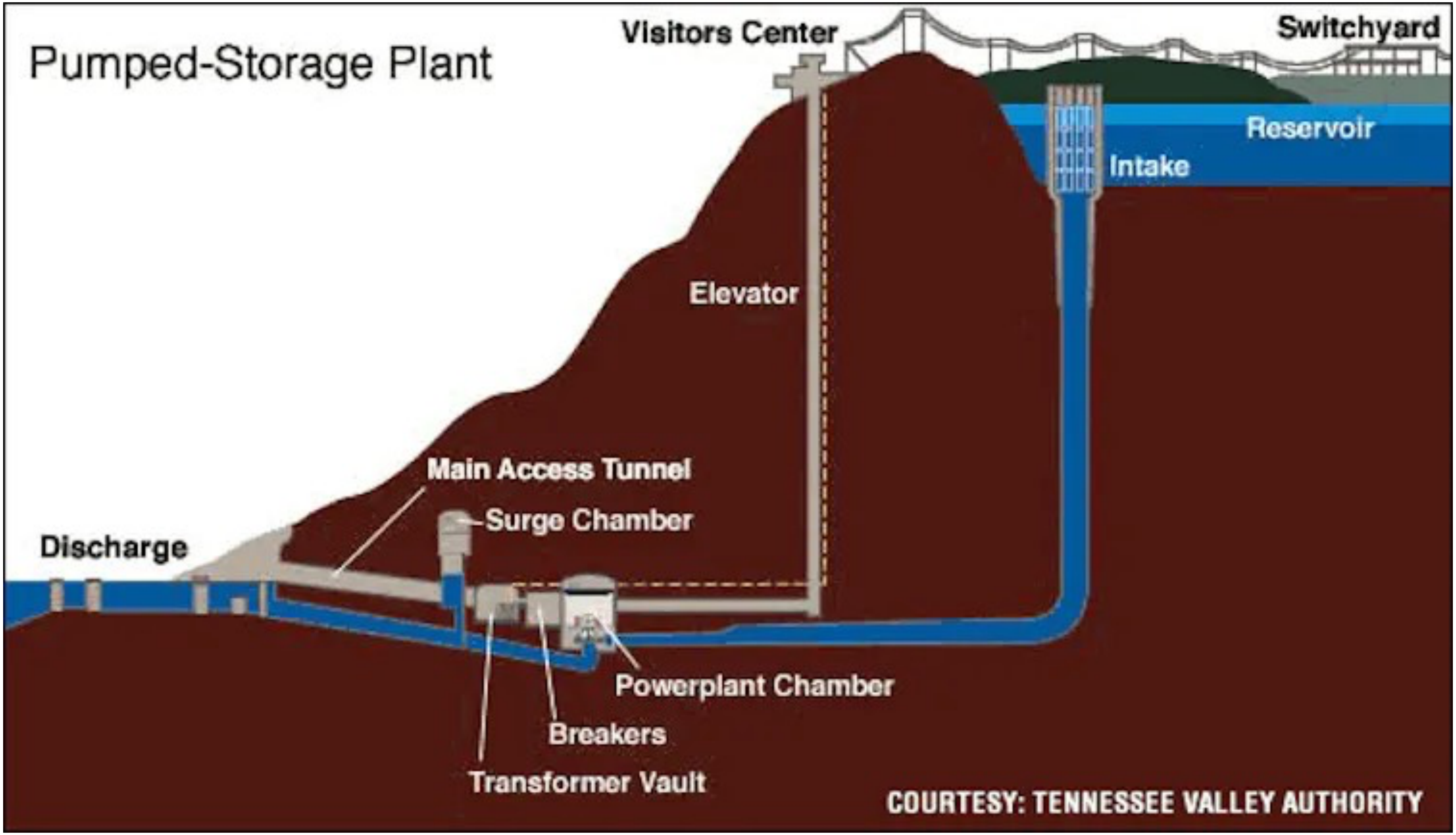
Gravitational Potential Energy

Gravitational Potential Energy

Potential Energy Battery: store electricity in an elevated reservoir

if we define zero potential energy for zero height, we have

$$E_g(h) = mgh$$



Gravitational potential energy (derivation)

Work, in physics, measures **energy transfer** that occurs when an object is moved over a **distance** by an **external force.**

Suppose now the only external force is a constant gravity and the object accelerates from zero velocity along a straight vertical line:

$$W = F\,s$$

because work is energy transfer, we have:

$$E_g(t) - E_g(t=0) = E_g(h=0) - E_g(h) = W = F\,s = -mgh$$

The **negative sign** in the last term is due to the fact that the distance traveled (s) is measured in the opposite direction of height (h)

if we define zero potential energy for zero height: $E_g(h=0)=0$, we have

$$E_g(h) = - - mgh = mgh$$

Gravitational potential energy (derivation)

Assuming constant gravity over the distance traveled, we had derived:

$$E_g(h) = mgh$$

But we know that gravity varies with distance, so this approximation does not work very well when the distance travelled is large compared to the size of the central object

Newton's Universal Law of Gravitation: $F = G\frac{m_1 m_2}{r^2}$

To deal with a variable external force, we need to do an integral:

$$E_g(r) = -\int_{\text{inf}}^{r} -\frac{GMm}{r^2}dr = -\frac{GMm}{r}$$

Gravitational energy and kinetic energy

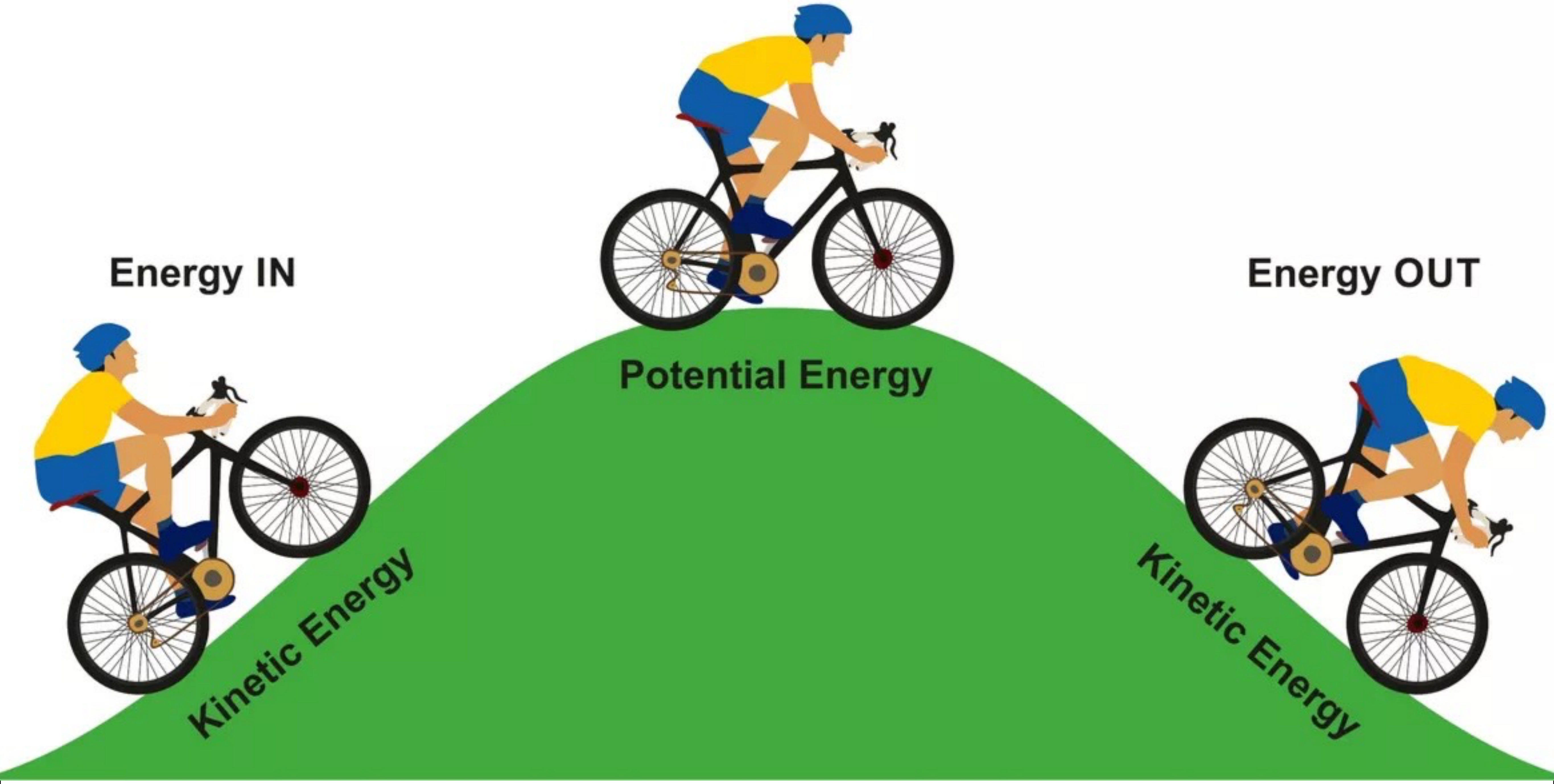
Gravitational potential energy of mass *m* in *M*'s gravity

$$E_g = \frac{-GMm}{R}$$

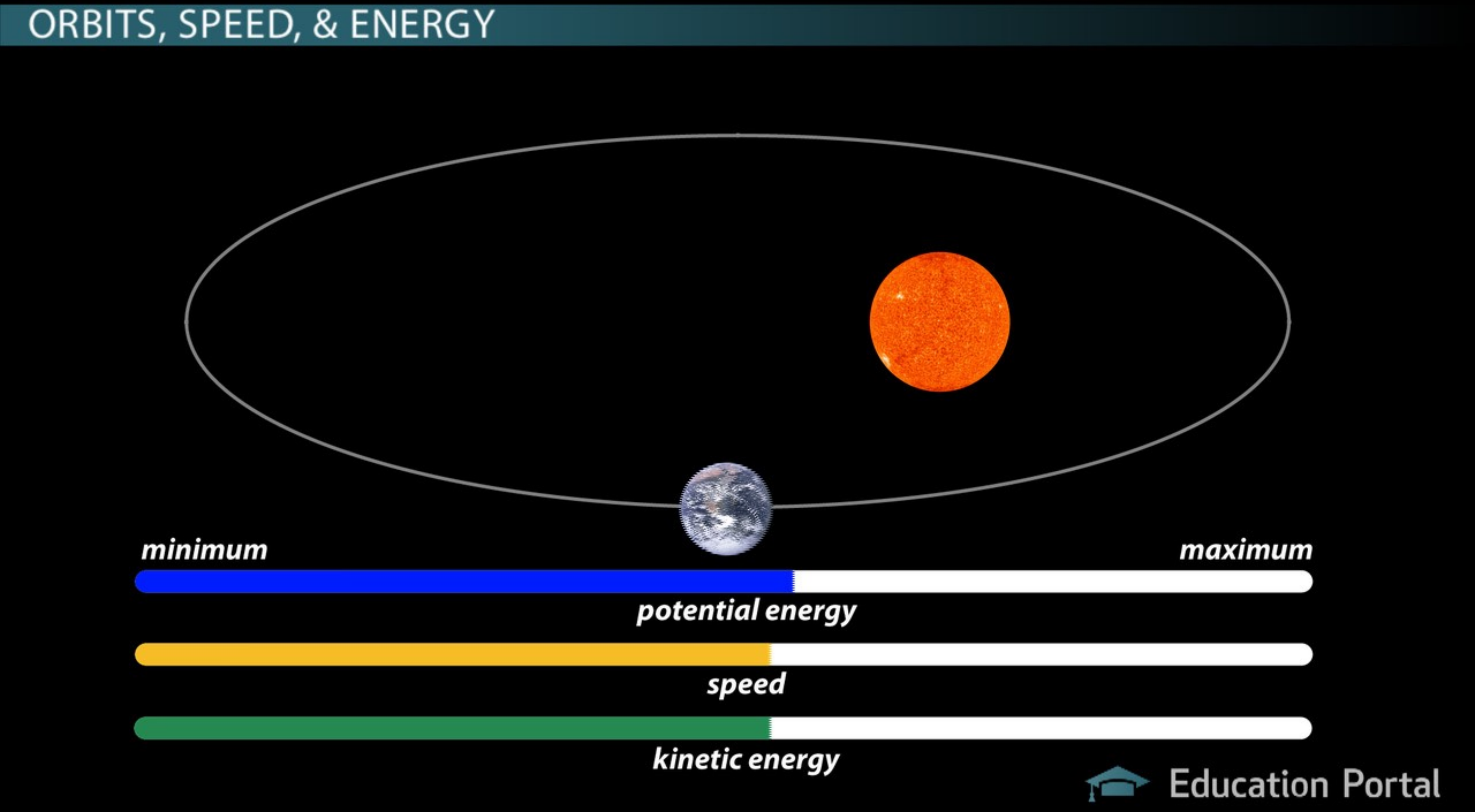
Kinetic energy of mass *m* moving a speed *V*

$$E_{ke} = \frac{1}{2} m V^2$$

Conservation of Energy (demo)



a planet orbiting around the Sun has both
kinetic energy and gravitational potential energy



From Virial Theorem to vis-viva Equation (energy conservation)

Virial Theorem: $2\bar{K} + \bar{U} = 0 \Rightarrow \bar{K} + \bar{U} = \bar{U}/2$

Energy Conservation: $\bar{K} + \bar{U} = \bar{U}/2 = \text{constant}$

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a}$$

the above can be rearranged to yield the **vis-viva Equation**:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

Applications of the vis-viva equation:

circular velocity

escape velocity

perihelion & aphelion velocities

Rocket Engines

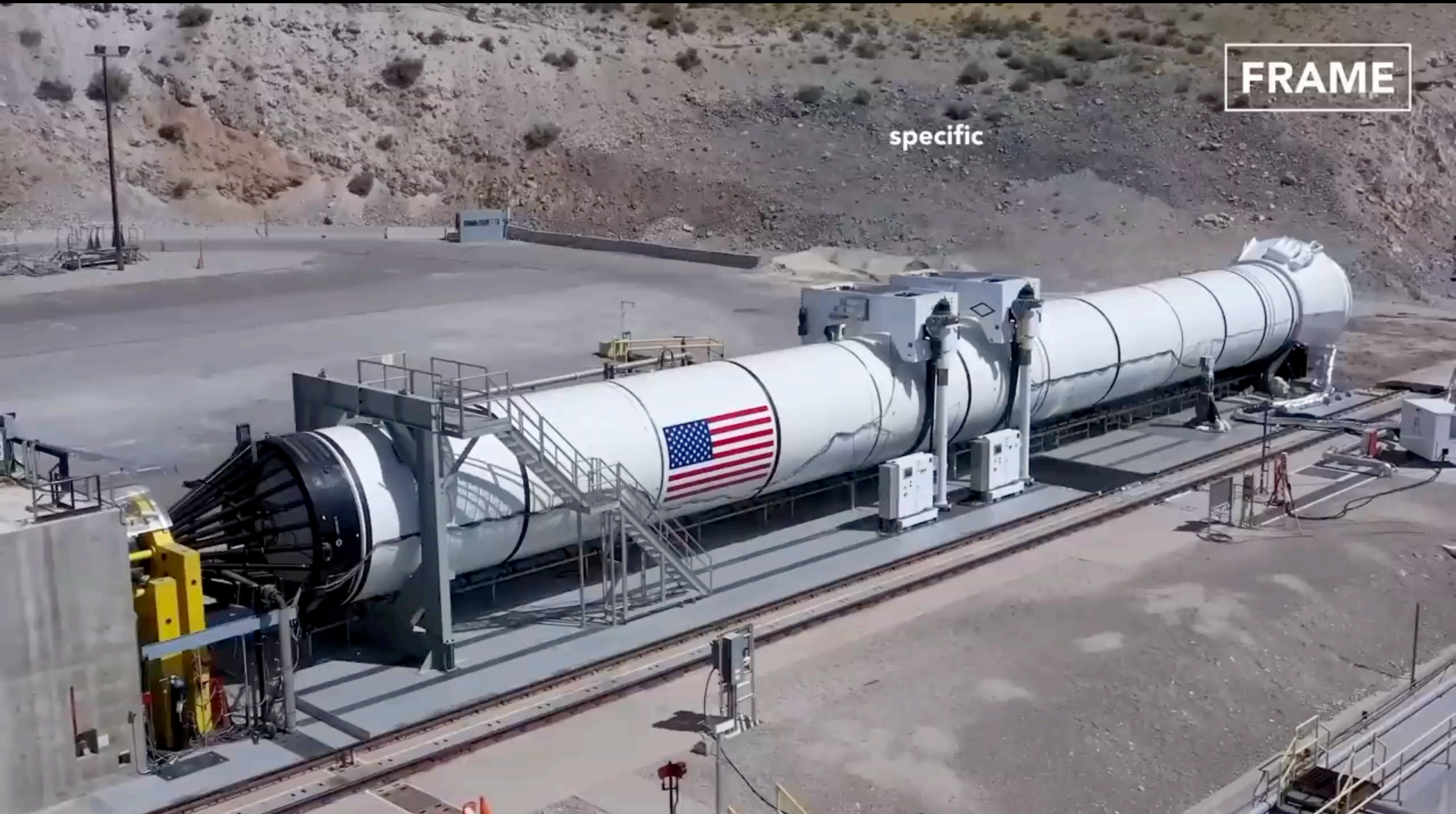
videos and demos

Rockets: Our Vehicles to the Deep Space

How rocket engines are made?

Mars in 4K, made with Curiosity data

How rocket engines are tested?



Vis Viva Equation
(Conservation of Energy)

Applications

From Virial Theorem to vis-viva Equation (energy conservation)

Virial Theorem: $2\bar{K} + \bar{U} = 0 \Rightarrow \bar{K} + \bar{U} = \bar{U}/2$

Energy Conservation: $\bar{K} + \bar{U} = \bar{U}/2 = \text{constant}$

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Applications of the vis-viva equation:

circular velocity

escape velocity

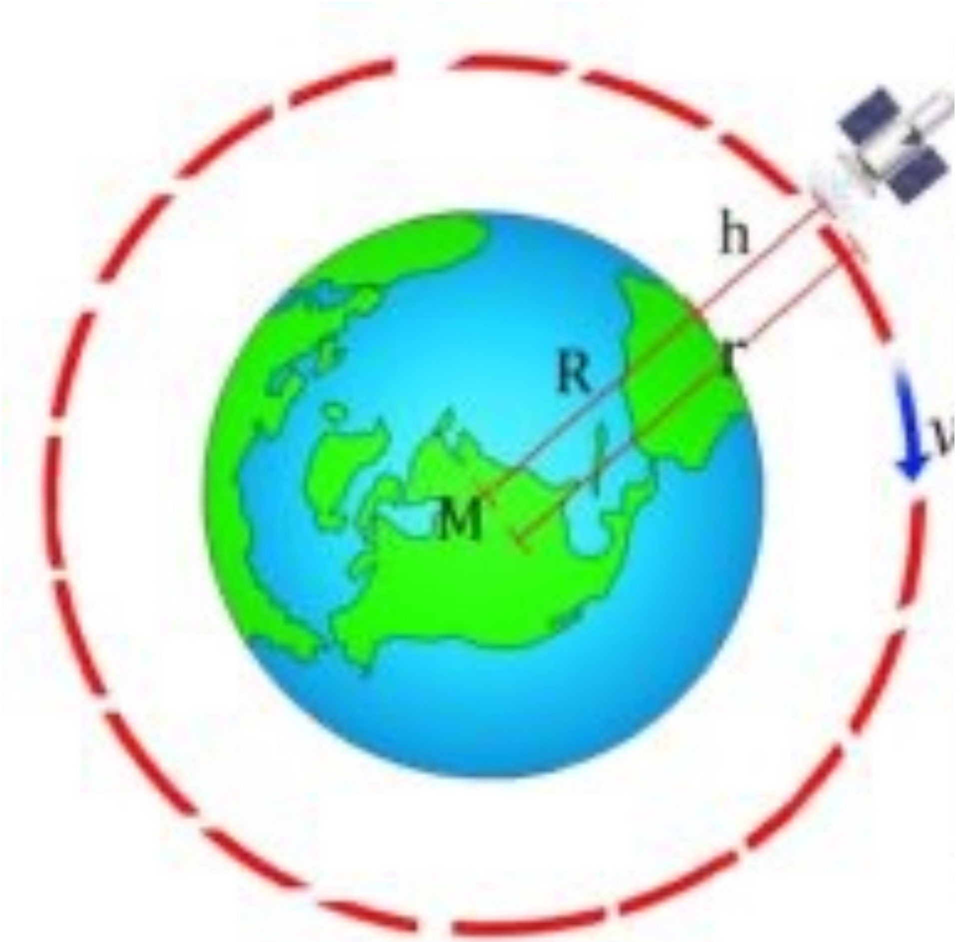
perihelion & aphelion velocities

Vis Viva Application 1: Circular Velocity

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

for circular orbit, *a = r, we get the circular velocity:*

$$v_{\rm circ} = \sqrt{GM/r}$$



Practice: Calculate circular velocity at the Earth's surface

- In order to orbit around a planet, an object must achieve a velocity the circular velocity at the planet's surface:

$$v_{\text{circ}} = \sqrt{\frac{GM}{r}}$$

- What is the circular velocity at Earth's surface?

$$G = 6.67 \times 10^{-11}\text{m}^3/\text{kg}/\text{s}^2$$

$$M_E = 6.0 \times 10^{24}\text{kg}$$

$$R_E = 6370\text{km}$$

- This is also known as the **1st cosmic velocity**

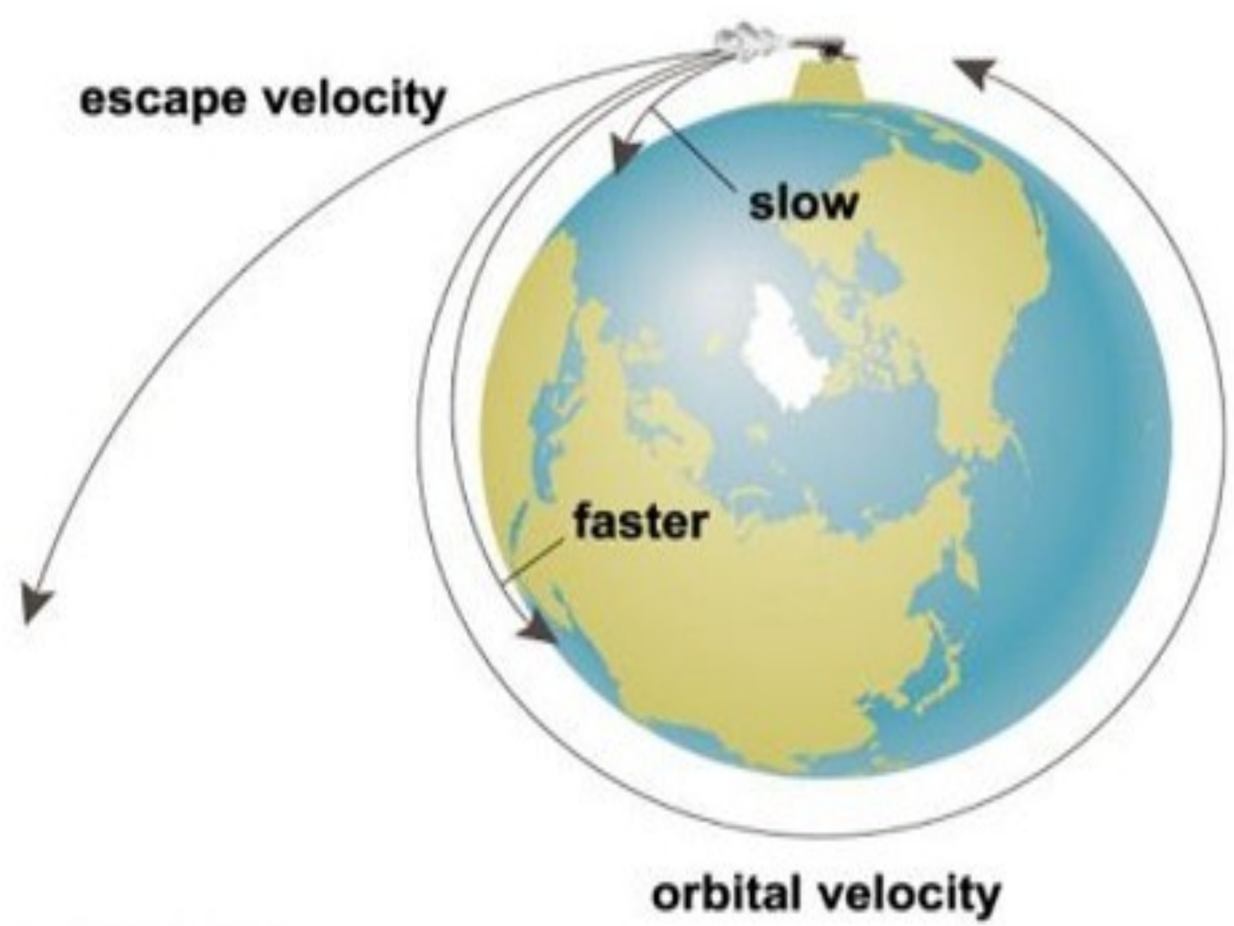
(v_1 = 7.9 km/s)

Vis Viva Application 2: Escape Velocity from Planetary Surface

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

for an escape orbit, *a = infinity, we get:*

$$v_{\text{esc}} = \sqrt{2GM/r} = \sqrt{2}v_{\text{circ}}$$



Practice: Calculate escape velocity from the Earth's surface

- In order to escape from a planet's gravity, an object must achieve a velocity greater than the planet's escape velocity at its surface:

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{r}} = \sqrt{2}\,v_{\mathrm{circ}}$$

- What is the escape velocity from Earth's surface?

$$G = 6.67 \times 10^{-11}\mathrm{m}^3/\mathrm{kg}/\mathrm{s}^2$$

$$M_E = 6.0 \times 10^{24}\mathrm{kg}$$

$$R_E = 6370\mathrm{km}$$

- This is also known as the **2nd cosmic velocity**

(v_2 = 11.2 km/s)

Practice: Calculate escape velocity from the Sun at Earth's Orbit

- In order to escape from the Sun's gravity, an object must achieve a velocity greater than the Sun's escape velocity from its current heliocentric distance:

$$v_{\rm esc} = \sqrt{\frac{2GM}{r}} = \sqrt{2}v_{\rm circ}$$

- What is the escape velocity from the Sun?

$$G = 6.67 \times 10^{-11}\ {\rm m^3/kg/s^2}$$

$$M_\odot = 2 \times 10^{30}\ {\rm kg}$$

$$r = 1.5 \times 10^{8}\ {\rm km} = 1\ {\rm AU}$$

- This is NOT the **3rd cosmic velocity (*$v_3 \neq$ 42 km/s*)**, which is defined as the velocity needed to launch an object to escape the Solar system from Earth's surface.
- **How would you calculate *v_3*?**

BTW, the Google Calculator is a great tool for astronomers!

Google

sqrt(2 * G * solar mass/1 AU)

Formula | Images | Pdf | Meaning | Videos | Shopping | News | Books | Maps

About 34,200,000 results (1.12 seconds)

sqrt((2 * G * solar mass) / (1 AU)) =

42 129.2243 m / s

Feedback

Application 3: Perihelion and Aphelion Velocities - Related to Transfer Orbit

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

At perihelion: $r_{peri} = a(1-e)$, so:

$$v_{\mathrm{peri}} = \sqrt{\frac{GM(1+e)}{a(1-e)}}$$

At aphelion: $r_{ap} = a(1+e)$, so:

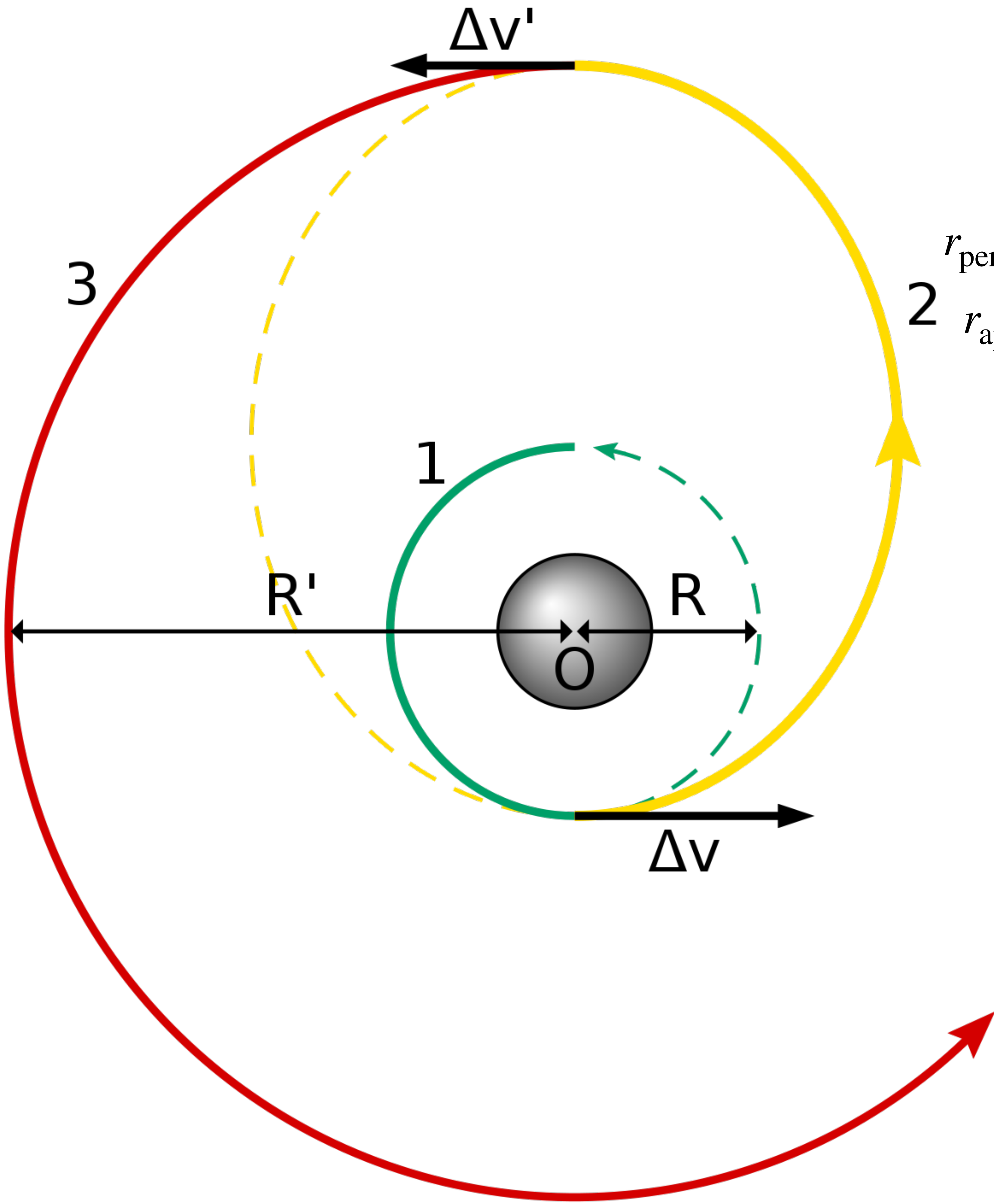
$$v_{\mathrm{ap}} = \sqrt{\frac{GM(1-e)}{a(1+e)}}$$

Are these equations consistent with Kepler's 2nd law?

Hohmann Transfer Orbit

the most fuel efficient transfer,
a major application of vis viva Eq.

MAVEN's Trajectory to Mars is a Hohmann Transfer Orbit



A Hohmann transfer orbit has a perihelion on the inner orbit (e.g., Earth), and an aphelion on the outer orbit (e.g., Mars):

$$r_{\text{peri,transfer}} = R = a_{\text{transfer}}(1 - e_{\text{transfer}})$$

$$r_{\text{ap,transfer}} = R' = a_{\text{transfer}}(1 + e_{\text{transfer}})$$

Using the relationship between peri/aphelion distances and semimajor axis and eccentricity, we can solve for a & e:

$$a_{\text{transfer}} = \frac{1}{2}(R + R')$$

$$e_{\text{transfer}} = 1 - \frac{r_{\text{peri}}}{a} = 1 - \frac{2R}{R + R'}$$

Because the semimajor axis of the transfer orbit is the average of R and R’, the energy of the transfer orbit is greater than that of R but less than that of R’

Application 3: Perihelion and Aphelion Velocities - Related to Transfer Orbit

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

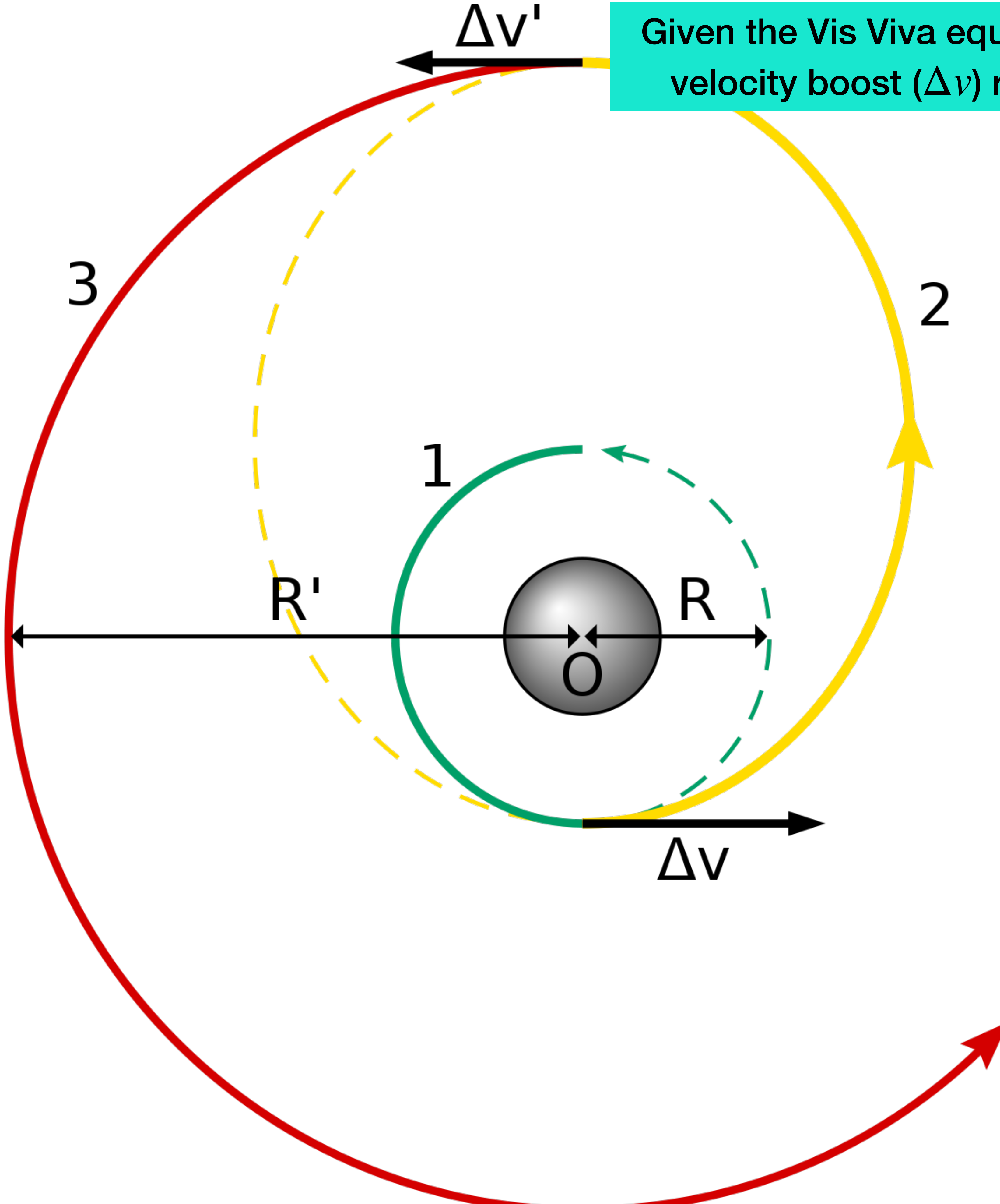
At perihelion: $r_{peri} = a(1-e)$, so:

$$v_{\mathrm{peri}} = \sqrt{\frac{GM(1+e)}{a(1-e)}}$$

At aphelion: $r_{ap} = a(1+e)$, so:

$$v_{\mathrm{ap}} = \sqrt{\frac{GM(1-e)}{a(1+e)}}$$

Given the Vis Viva equation, derive the expression for the velocity boost (Δv) required to enter the transfer orbit



First, we write down the circular velocity on orbit 1:

$$v_{\text{circ}} = \sqrt{\frac{GM}{R}}$$

Then, we use Vis Viva Eq to write down the velocity at perihelion on the transfer orbit 2:

$$v_{\text{peri}} = \sqrt{GM\left(\frac{2}{r_{\text{peri}}} - \frac{1}{a_{\text{transfer}}}\right)}$$

$$= \sqrt{GM\left(\frac{2}{R} - \frac{2}{R+R'}\right)}$$

The required velocity boost is simply the difference between the two:

$$\Delta v = v_{\text{peri}} - v_{\text{circ}}$$

Example: Δv to Geostationary transfer orbit

Circular velocity at R (v_{circ}): 7.7 km/s

Semimajor axis of low orbit (R): 6,700 km

geostationary orbit (R'): 42,200 km

First, we write down the circular velocity on orbit 1:

$$v_{\rm circ} = \sqrt{\frac{GM}{R}}$$

Then, we use Vis Viva Eq to write down the velocity at perihelion on the transfer orbit 2:

$$v_{\rm peri} = \sqrt{GM\left(\frac{2}{r_{\rm peri}} - \frac{1}{a_{\rm transfer}}\right)}$$

$$= \sqrt{GM\left(\frac{2}{R} - \frac{2}{R+R'}\right)}$$

The required velocity boost is simply the difference between the two:

$$\Delta v = v_{\rm peri} - v_{\rm circ}$$

The Three Stages to Arrive at Mars

Stage 1: Δv to enter Mars transfer orbit

Circular velocity of Earth (v_{circ}): 30 km/s

Semimajor axis of Earth (R): 1 AU

Semimajor axis of Mars (R'): 1.5 AU

First, we write down the circular velocity on orbit 1:

$$v_{\rm circ} = \sqrt{\frac{GM}{R}}$$

Then, we use Vis Viva Eq to write down the velocity at perihelion on the transfer orbit 2:

$$v_{\rm peri} = \sqrt{GM\left(\frac{2}{r_{\rm peri}} - \frac{1}{a_{\rm transfer}}\right)}$$

$$= \sqrt{GM\left(\frac{2}{R} - \frac{2}{R + R'}\right)}$$

The required velocity boost is simply the difference between the two:

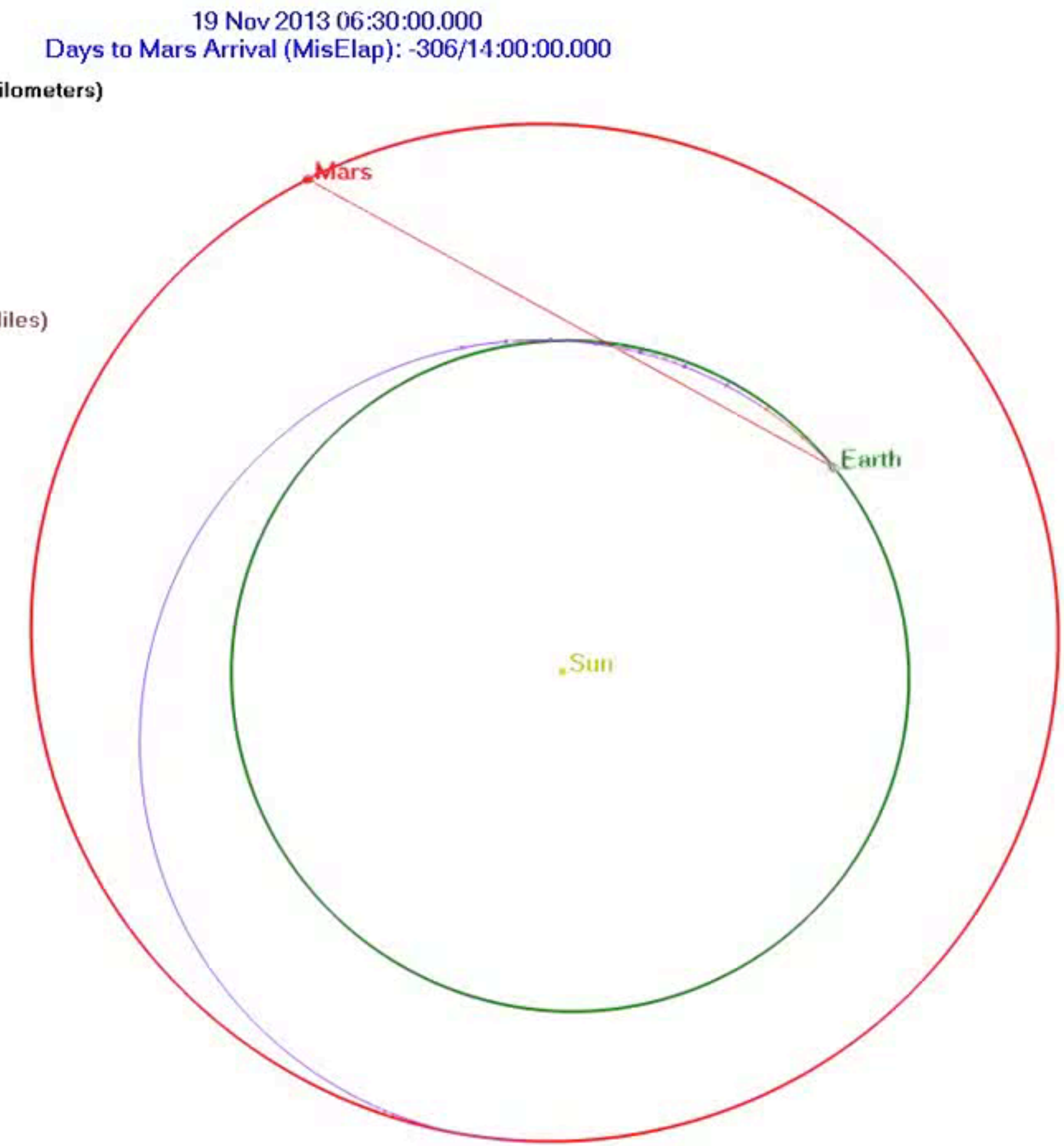
$$\Delta v = v_{\rm peri} - v_{\rm circ}$$

Answer: 2.8 km/s

How can this be less than the escape velocity from Earth?
(v_2 = 11.2 km/s)

Stage 2: Timing Spacecraft's Rendezvous with the Outer Planet:

when to initiate the transfer orbit depends on the time it takes to reach Mars

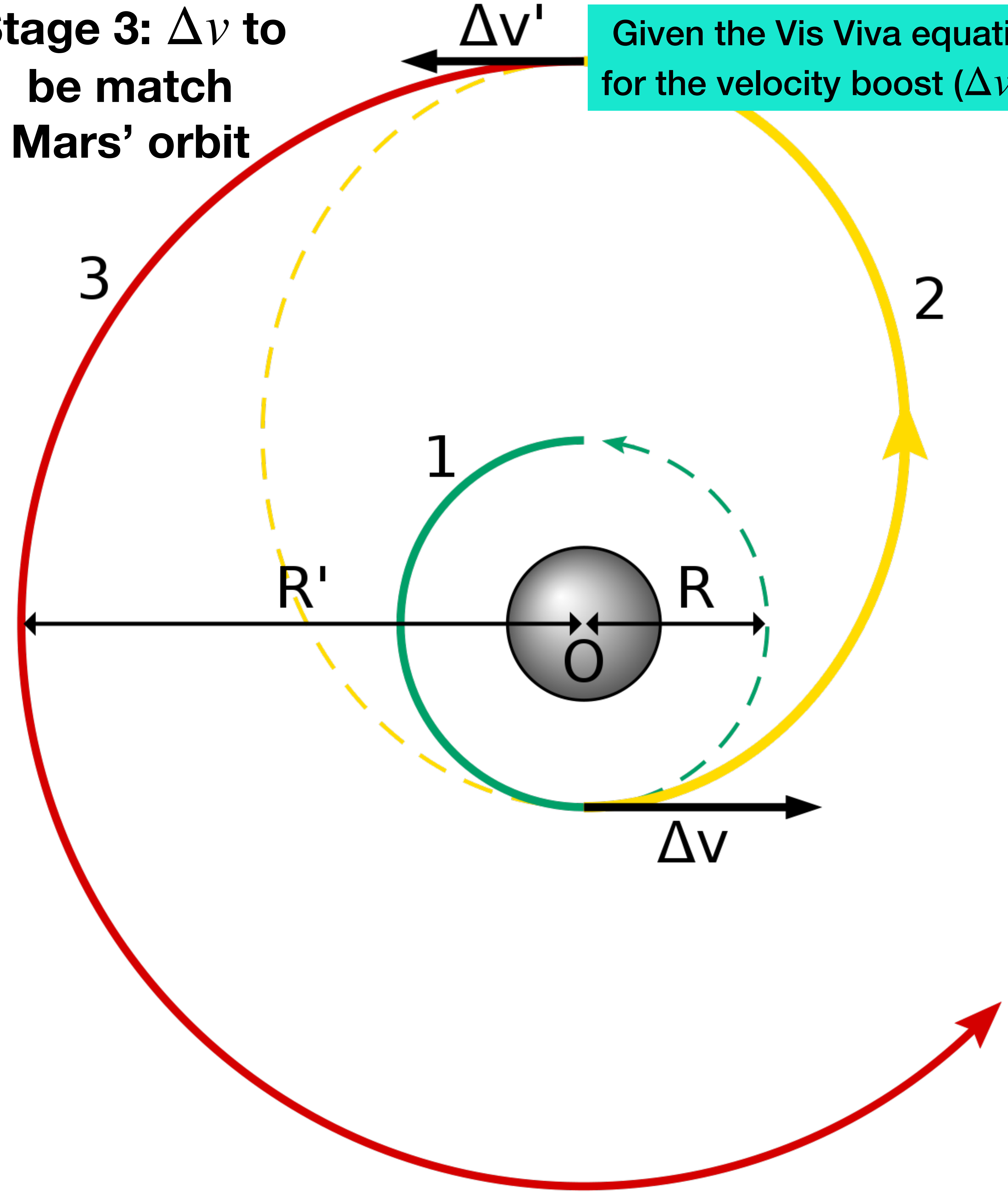


MAVEN Range and Velocity (units of Kilometers)	
Earth_Range (km):	192293
Velocity_wrt_Earth (km/sec):	4.045
Mars_Range (km):	264525423
Velocity_wrt_Mars (km/sec):	33.724
Sun_Range (km):	147718010
Velocity_wrt_Sun (km/sec):	33.019

MAVEN Range and Velocity (units of Miles)	
Earth_Range (mi):	119485
Velocity_wrt_Earth (mi/sec):	2.513
Mars_Range (mi):	164368478
Velocity_wrt_Mars (mi/sec):	20.955
Sun_Range (mi):	91707716
Velocity_wrt_Sun (mi/sec):	20.517

How would you calculate the time it takes for the spacecraft to reach Mars?

Stage 3: Δv to be match Mars' orbit



Given the Vis Viva equation, can you derive the expression for the velocity boost ($\Delta v'$) required to enter the outer orbit?

First, we write down the circular velocity on orbit 2:

$$v_{\text{circ}} = \sqrt{\frac{GM}{R'}}$$

Then, we use Vis Viva Eq to write down the velocity at aphelion on the transfer orbit 2:

$$v_{\text{ap}} = \sqrt{GM\left(\frac{2}{r_{\text{ap}}} - \frac{1}{a_{\text{transfer}}}\right)}$$

$$= \sqrt{GM\left(\frac{2}{R'} - \frac{2}{R+R'}\right)}$$

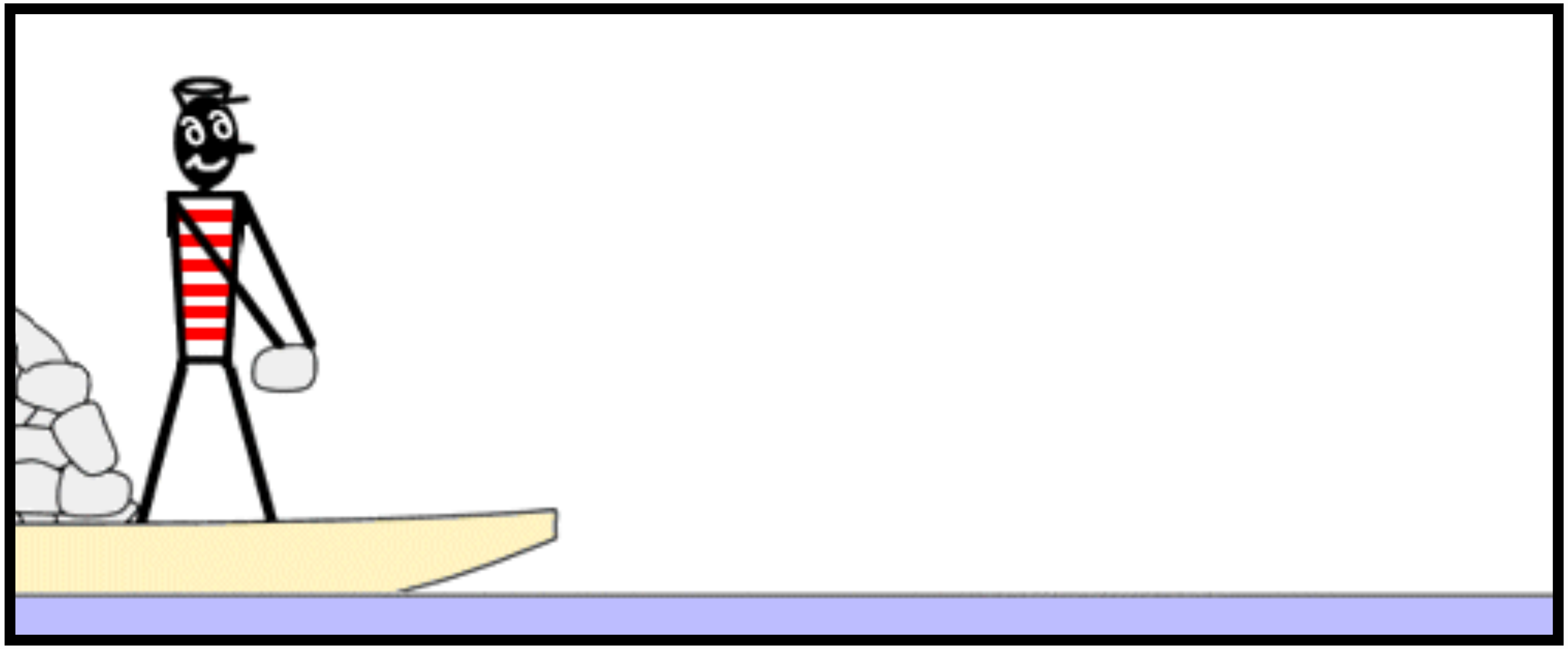
The required velocity boost is simply the difference between the two:

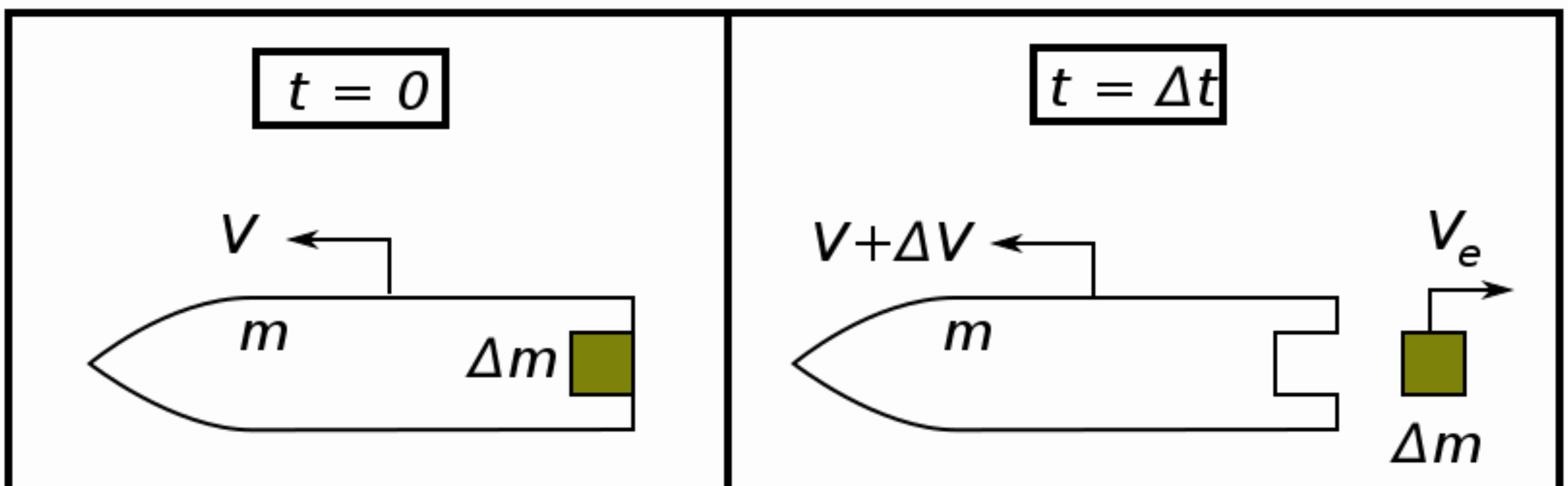
$$\Delta v = v_{\text{circ}} - v_{\text{ap}}$$

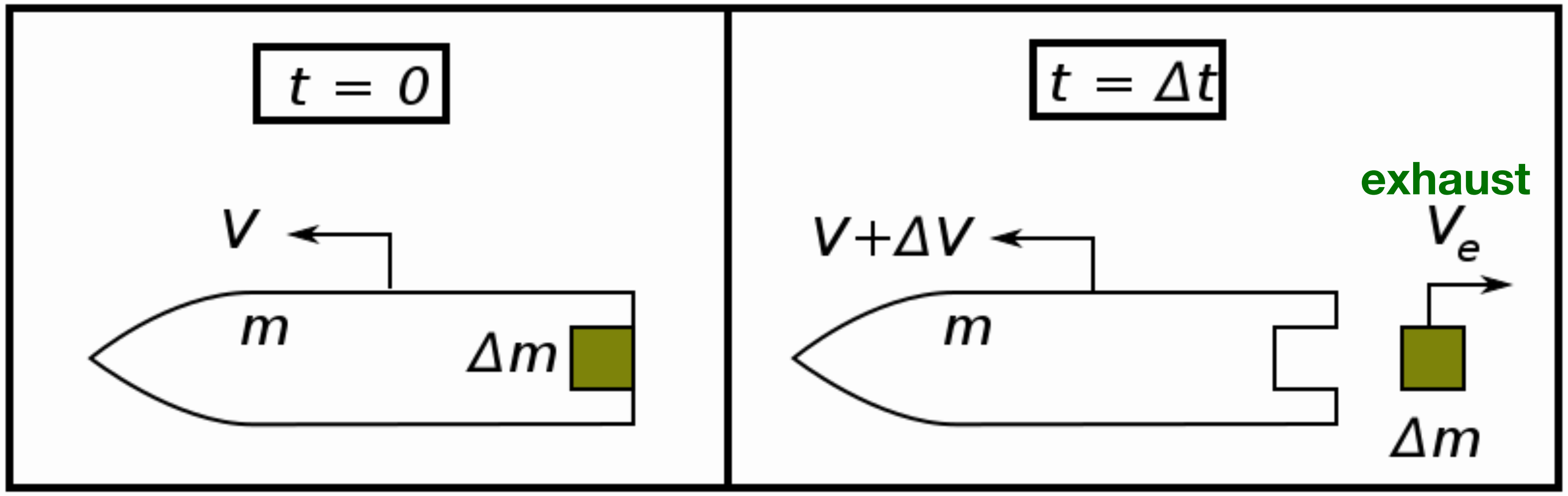
The Rocket Equation

the economics of reaching the desired Δv
derivation & application

Derivation of the Rocket Equation







In the **inertial** reference frame that is **at rest**, we can write down the total momentum at the two different times:

$$P_0 = (m + \Delta m)V$$

$$P_{\Delta t} = m(V + \Delta V) + \Delta m V_e$$

The velocity of the exhaust in the rest-frame (***V_e***) is related to the velocity of the exhaust in the rocket frame (***v_e***):

$$V_e = V - v_e$$

Without an external force, ***the momentum is conserved***:

$$P_0 = P_{\Delta t} \rightarrow (m + \Delta m)V = m(V + \Delta V) + \Delta m(V - v_e)$$

$$\rightarrow m\Delta V = \Delta m v_e \rightarrow \Delta V = v_e \frac{\Delta m}{m}$$

Based on the conservation of momentum, we have reached:

$$P_0 = P_{\Delta t} \rightarrow (m + \Delta m)V = m(V + \Delta V) + \Delta m(V - v_e)$$

$$\rightarrow m\Delta V = \Delta m v_e \rightarrow \Delta V = v_e \frac{\Delta m}{m}$$

We also realize that Δm is a **decrease** in rocket mass *m*:

$$\Delta m = -dm$$

while ΔV is an **increase** in rocket velocity *V*:

$$\Delta V = dV$$

We can now convert the top equation to a **differential equation**:

$$dV = -v_e \frac{dm}{m}$$

and then integrate both side from the beginning of the rocket burn (V_i, m_i) to the end of the burn (V_f, m_f):

$$\int_{V_i}^{V_f} dV = -v_e \int_{m_i}^{m_f} \frac{dm}{m} \rightarrow V_f - V_i = v_e \ln \frac{m_i}{m_f}$$

Tsiolkovsky rocket equation

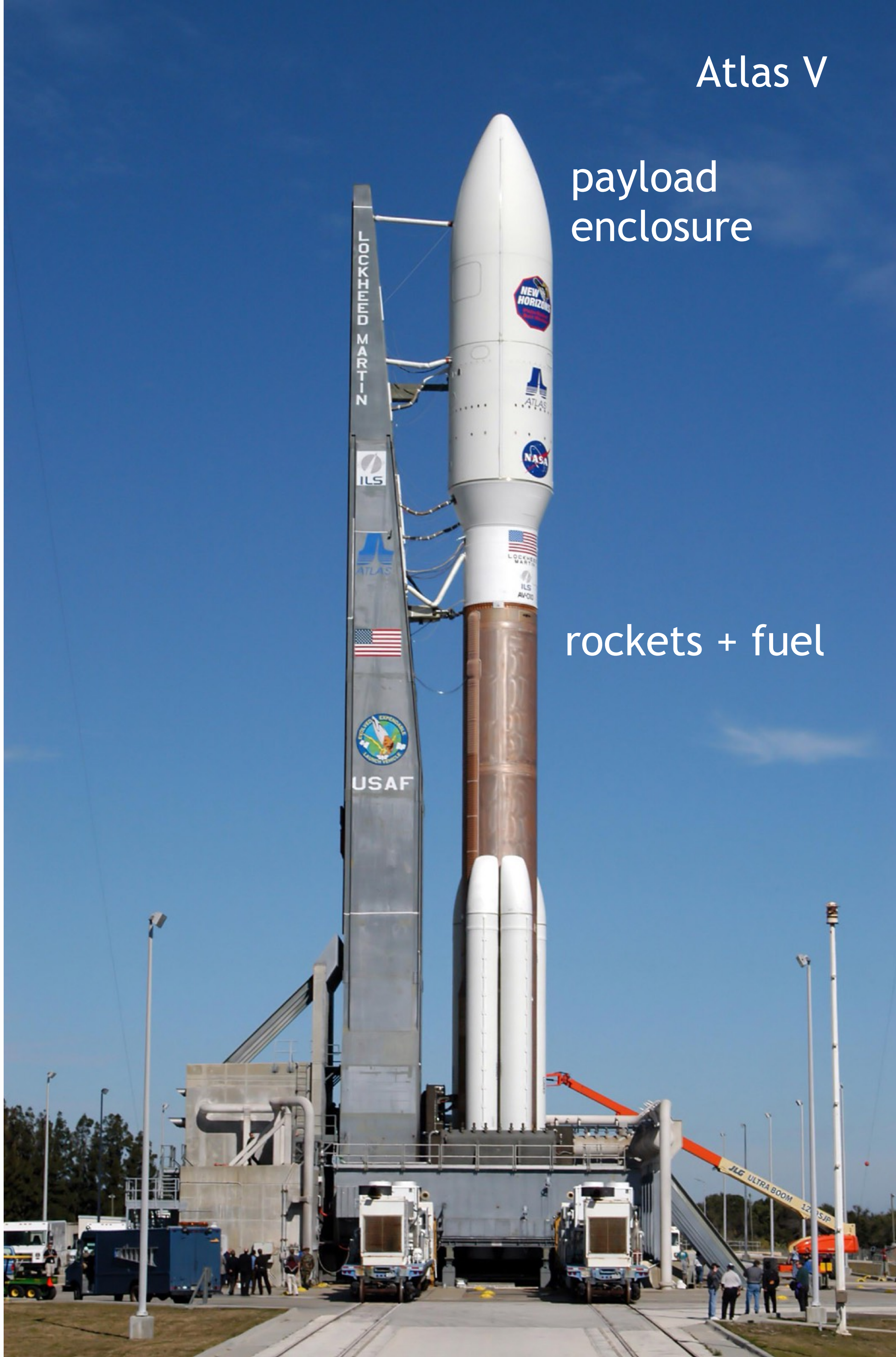
$$\Delta V = V_f - V_i = v_e \ln \frac{m_i}{m_f}$$

where in the above equation:

v_e is the velocity of the exhaust *relative to rocket*

m_i is the total mass of the spacecraft before the burn
m_f is the total mass of the spacecraft after the rocket burn

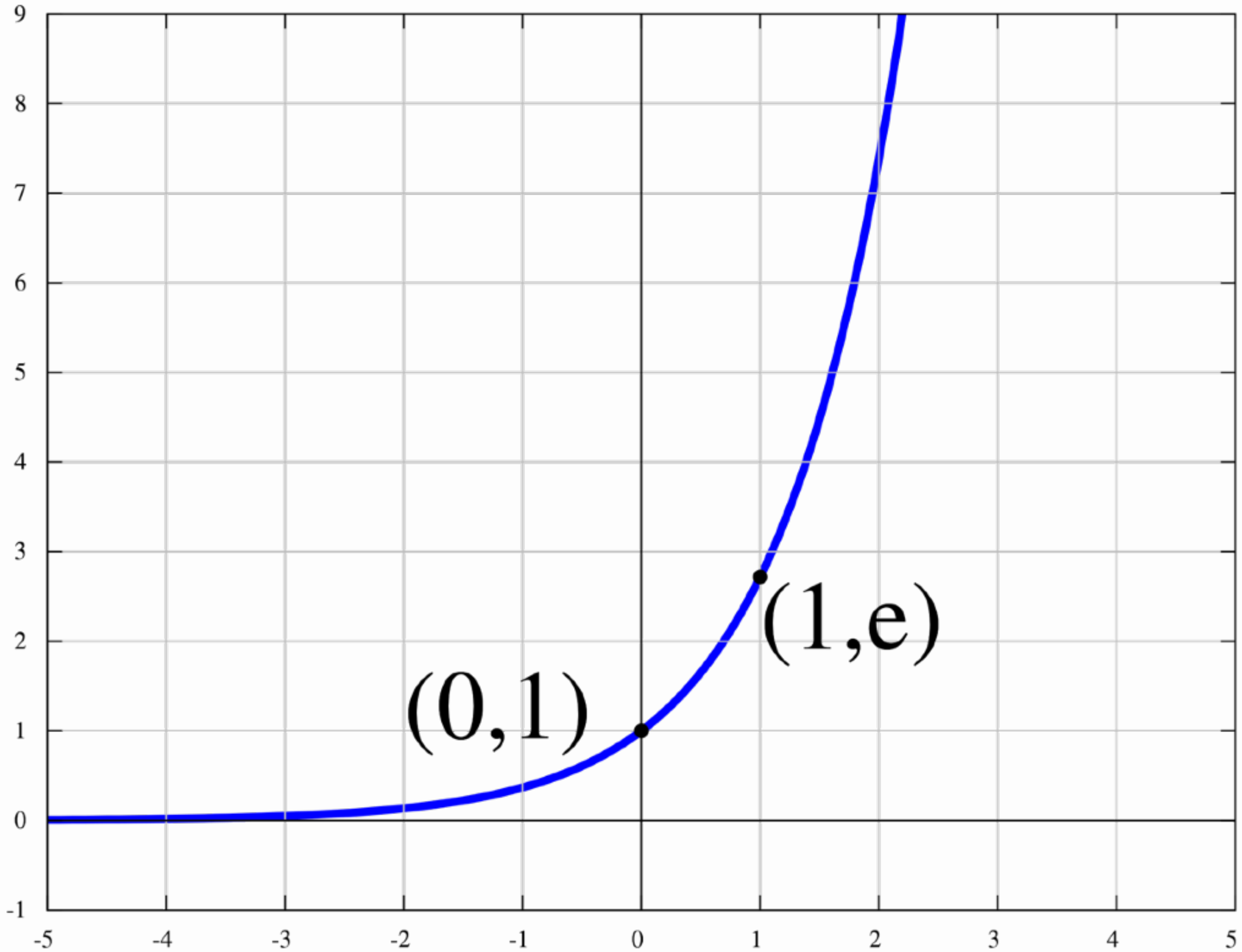
V_i is the velocity before the burn
V_f is the velocity after the burn

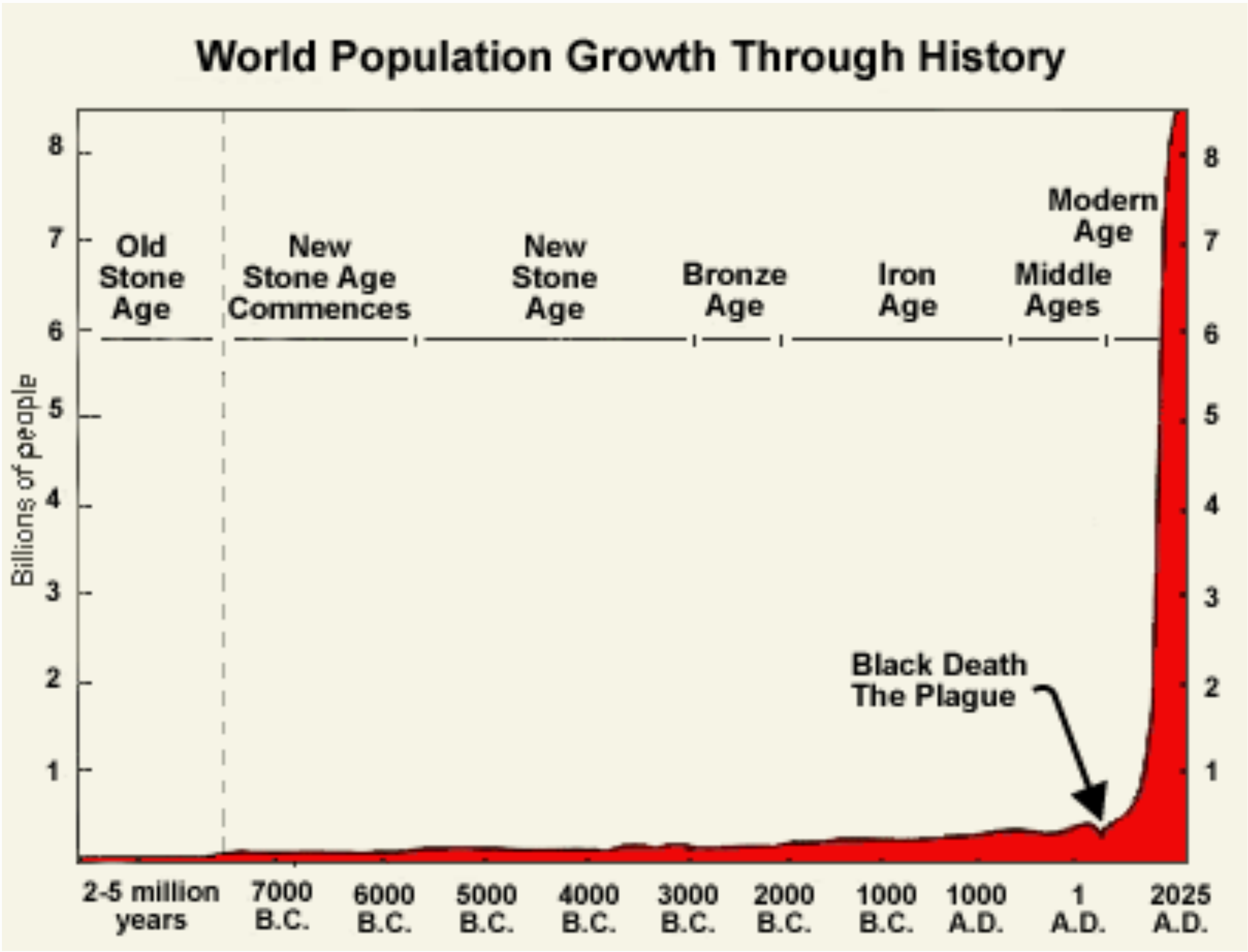


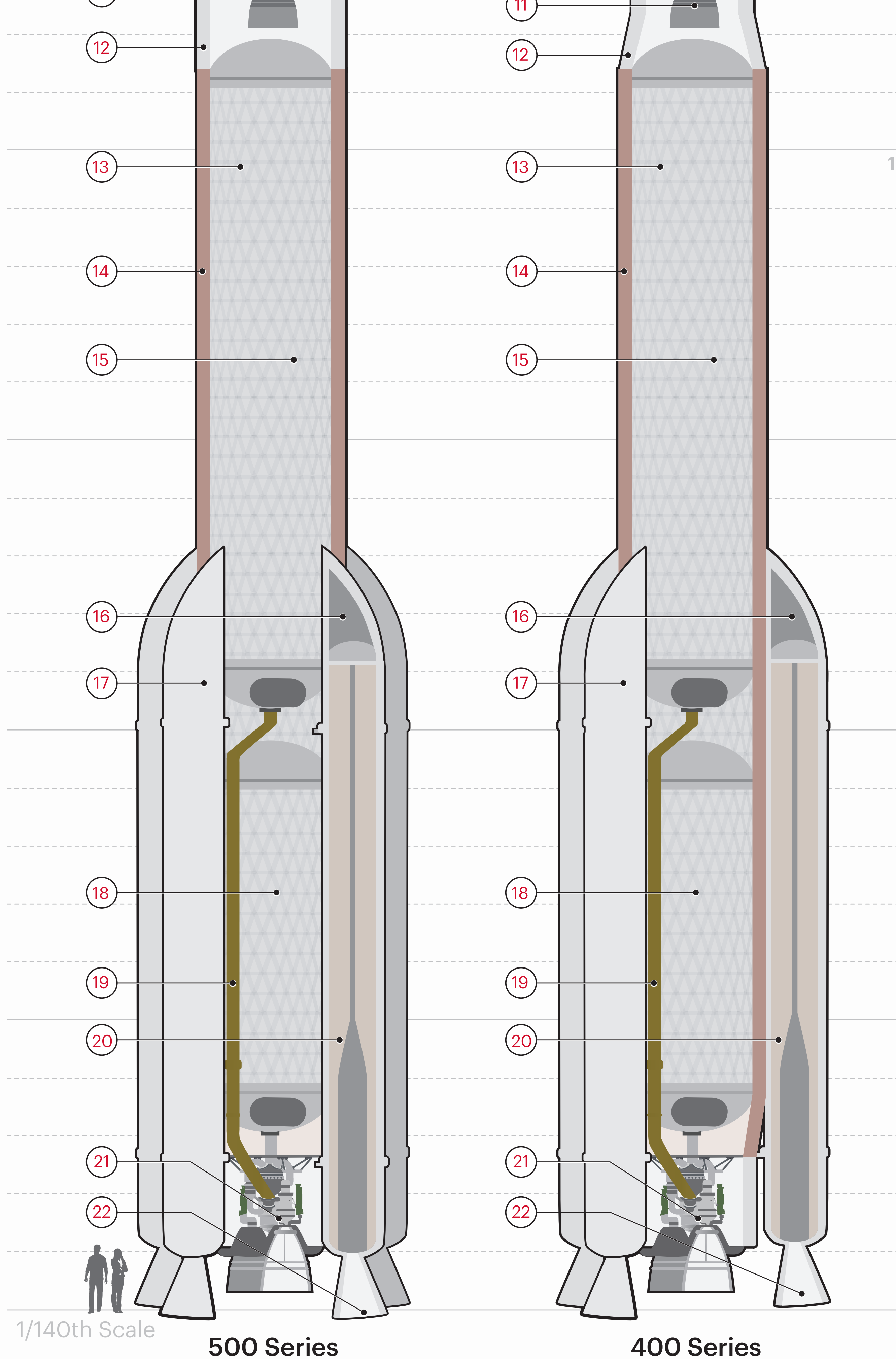
The Rocket Equation shows that the **mass ratio** is an *exponential* function of **velocity ratio**, so higher ΔV requires even higher initial to final mass ratio, until the ratio becomes prohibitively high

$$\Delta V = V_f - V_i = v_e \ln \frac{m_i}{m_f} \quad \Rightarrow \quad \frac{m_i}{m_f} = \exp \frac{\Delta V}{v_e}$$

(Euler's number *e = 2.71828, log e = 0.434*)



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1/140th Scale
500 Series
400 Series

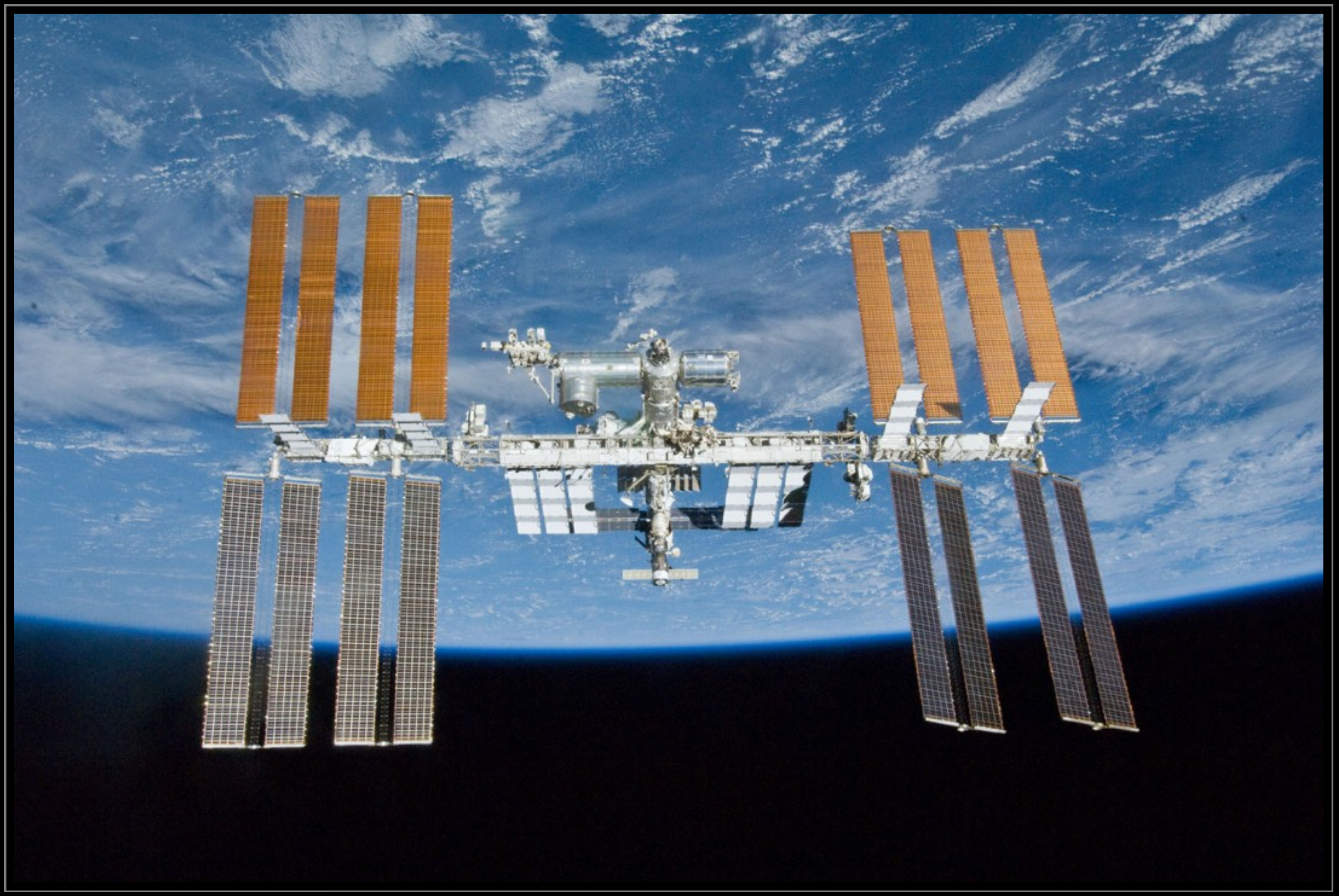

LOCKHEED MARTIN
ILS
USAF
NASA

Controllable parameter: exhaust velocities (v_e)

$$\Delta V = v_e \ln \frac{m_i}{m_f}$$

Typical performances of common propellants

Propellant mix		Effective exhaust velocity (m/s)
liquid oxygen/ liquid hydrogen		4462
liquid oxygen/ kerosene (RP-1)		3510
nitrogen tetroxide/ hydrazine		3369
n.b. All performances at a nozzle expansion ratio of 40		



Estimate how much fuel a rocket needs to carry to supply **100 liter of water** (100 kg) to the international space station (v_1 = 7.9 km/s)? Assume an exhaust velocity of 3 km/s.

$$\Delta V = V_f - V_i = v_e \ln \frac{m_i}{m_f} \quad \Rightarrow \quad \frac{m_i}{m_f} = \exp \frac{\Delta V}{v_e}$$

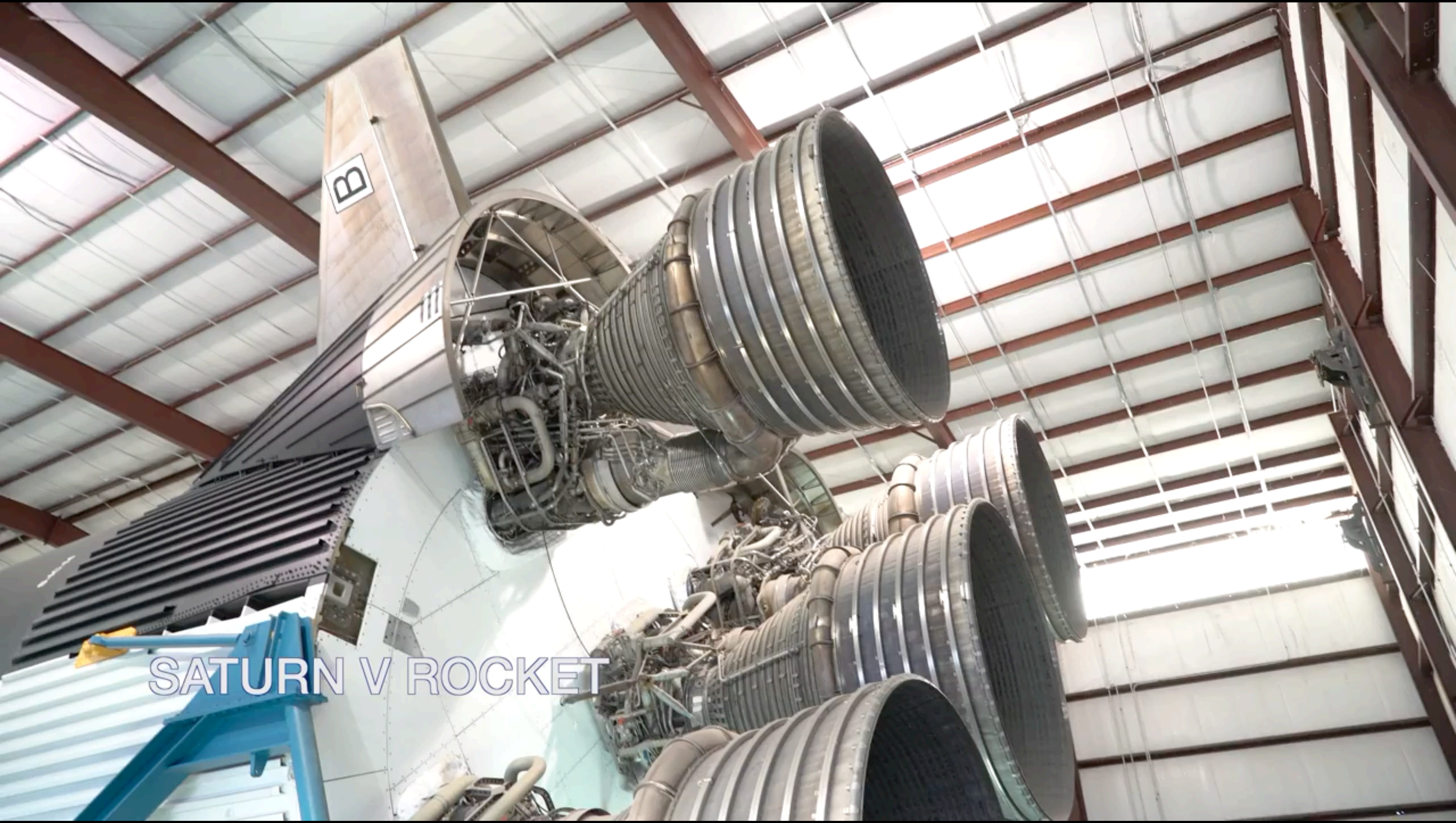
for ΔV = 8 km/s, v_e = 3 km/s, & $m_f = 10^2$ kg, we have

m_i/m_f = exp(8/3) = 14.4 and $m_i = 1.4\text{x}10^3$ kg

What about delivering payload to the moon?

For v_2 = 11.2 km/s (escape velocity from Earth) => $m_i/m_f = \exp(11.2/3) = 42$

Saturn V at Johnson Space Center, Houston, Texas



The rocket equation exposed a fundamental problem for <u>interstellar</u> travel with conventional rockets

$$\Delta V = V_f - V_i = v_e \ln \frac{m_i}{m_f} \quad \Rightarrow \quad \frac{m_i}{m_f} = \exp \frac{\Delta V}{v_e}$$

For example:

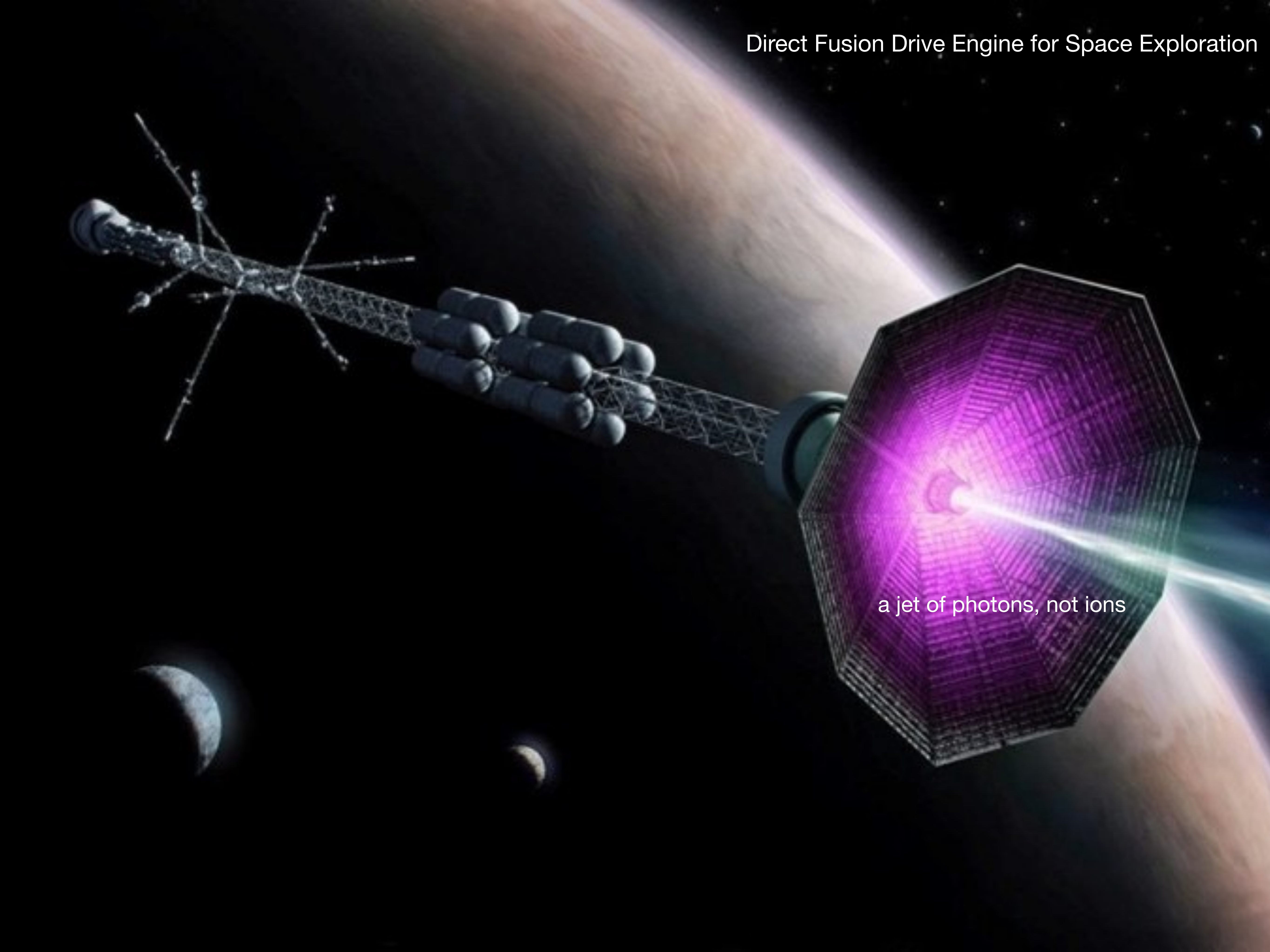
$V_f = 0.1\ c = 30{,}000$ km/s,

$v_e = 3$ km/s

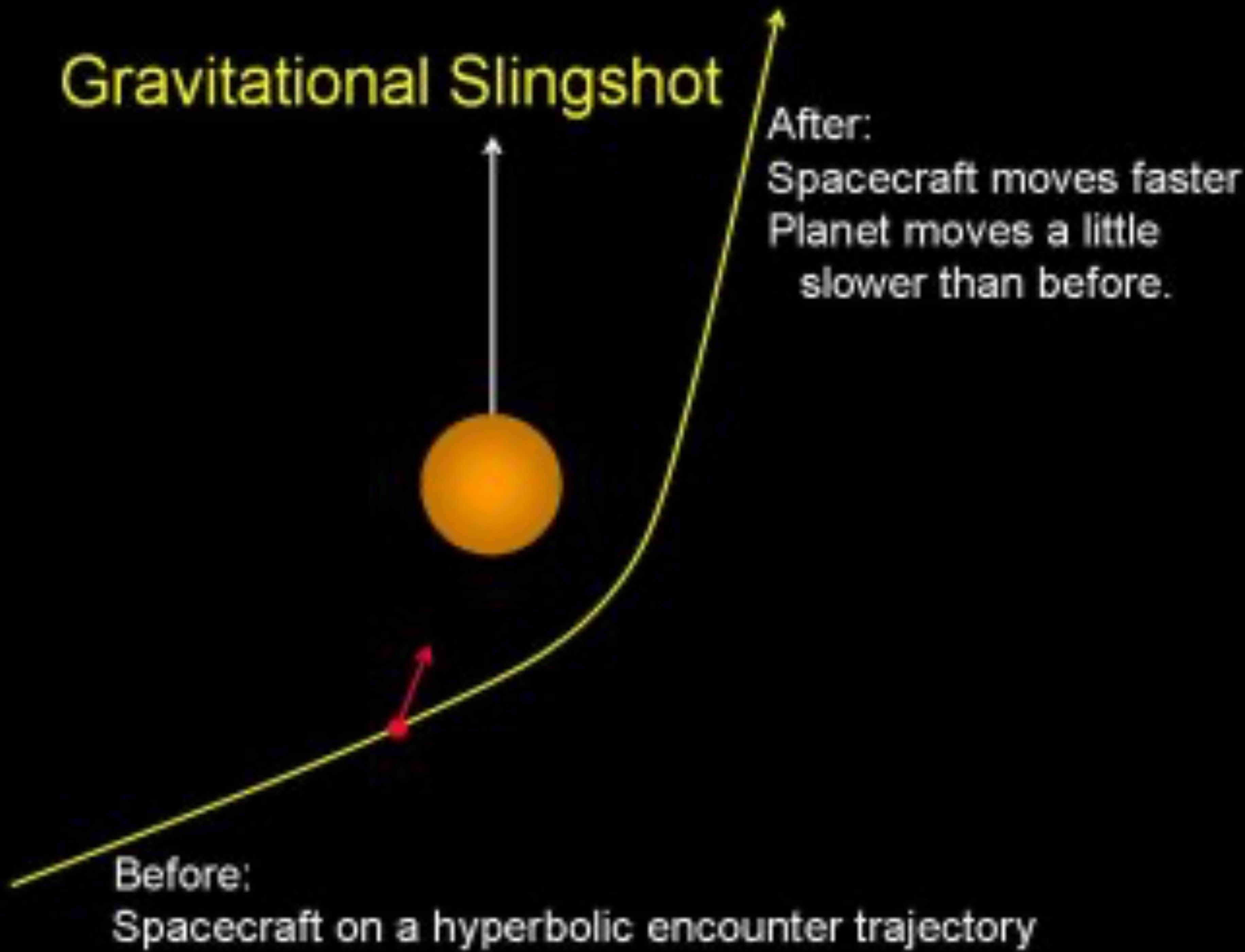
would require

$m_i / m_f = e^{10000} = 10^{\log(e)*10000} = \mathbf{10^{4338}}$

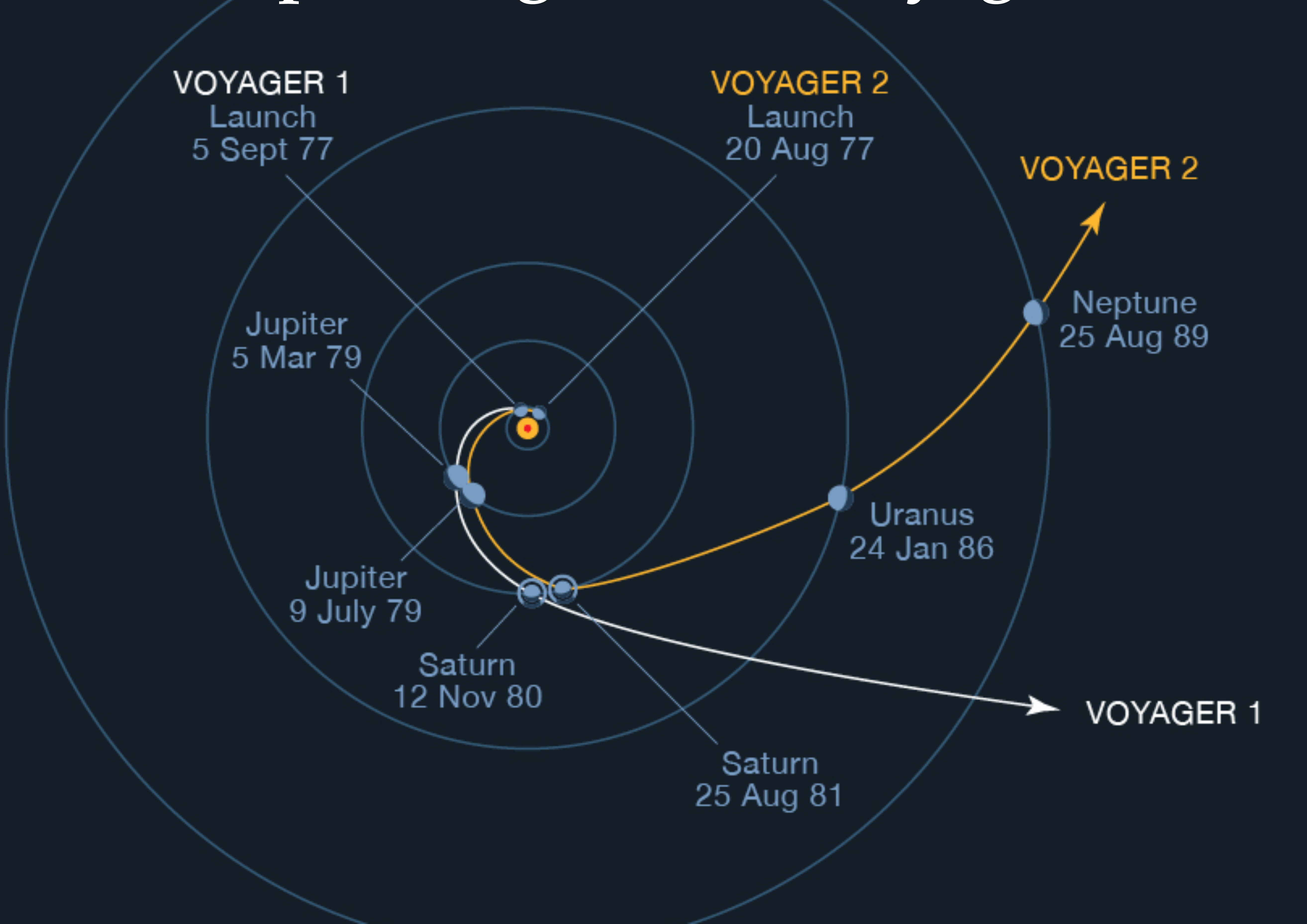
This means it is impossible for conventional rockets to travel to the stars in a human lifetime


Direct Fusion Drive Engine for Space Exploration
a jet of photons, not ions

When rocket alone becomes insufficient, we use the planets' gravity to pull



The Multiple slingshots of Voyagers 1 & 2



Chap 4: Equations of Orbital Mechanics

Newton's law of gravitation & surface gravity

$$F_G = \frac{GMm}{d^2} \Rightarrow g = \frac{GM}{d^2}$$

vis-viva Equation: deals with all velocities, incl. circular velocity & escape velocity

$$v^2 = GM\left(\frac{2}{d} - \frac{1}{a}\right) \Rightarrow$$

For objects orbiting around the Sun, we have a simpler version:

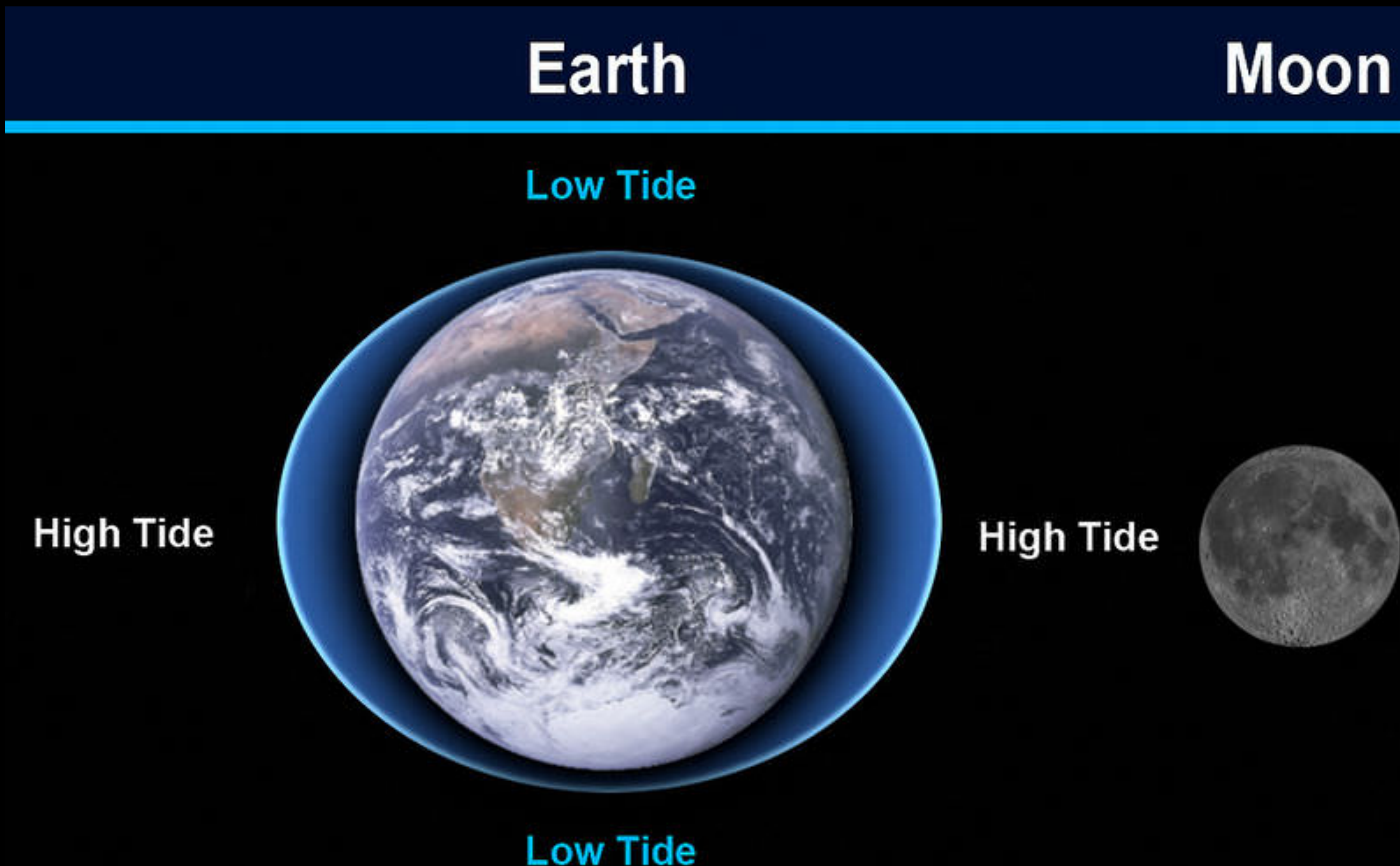
$$v = \sqrt{\frac{GM_\odot}{1\ \mathrm{AU}}}\sqrt{\frac{2\ \mathrm{AU}}{d} - \frac{1\ \mathrm{AU}}{a}}$$

$$= 30\ \mathrm{km/s}\sqrt{\frac{2\ \mathrm{AU}}{d} - \frac{1\ \mathrm{AU}}{a}}$$

Rocket Equation: $\Delta v = v_e \ln \frac{m_i}{m_f} \quad \Leftarrow\Rightarrow \quad \frac{m_i}{m_f} = \exp \frac{\Delta v}{v_e}$

Part IV: Tidal Forces

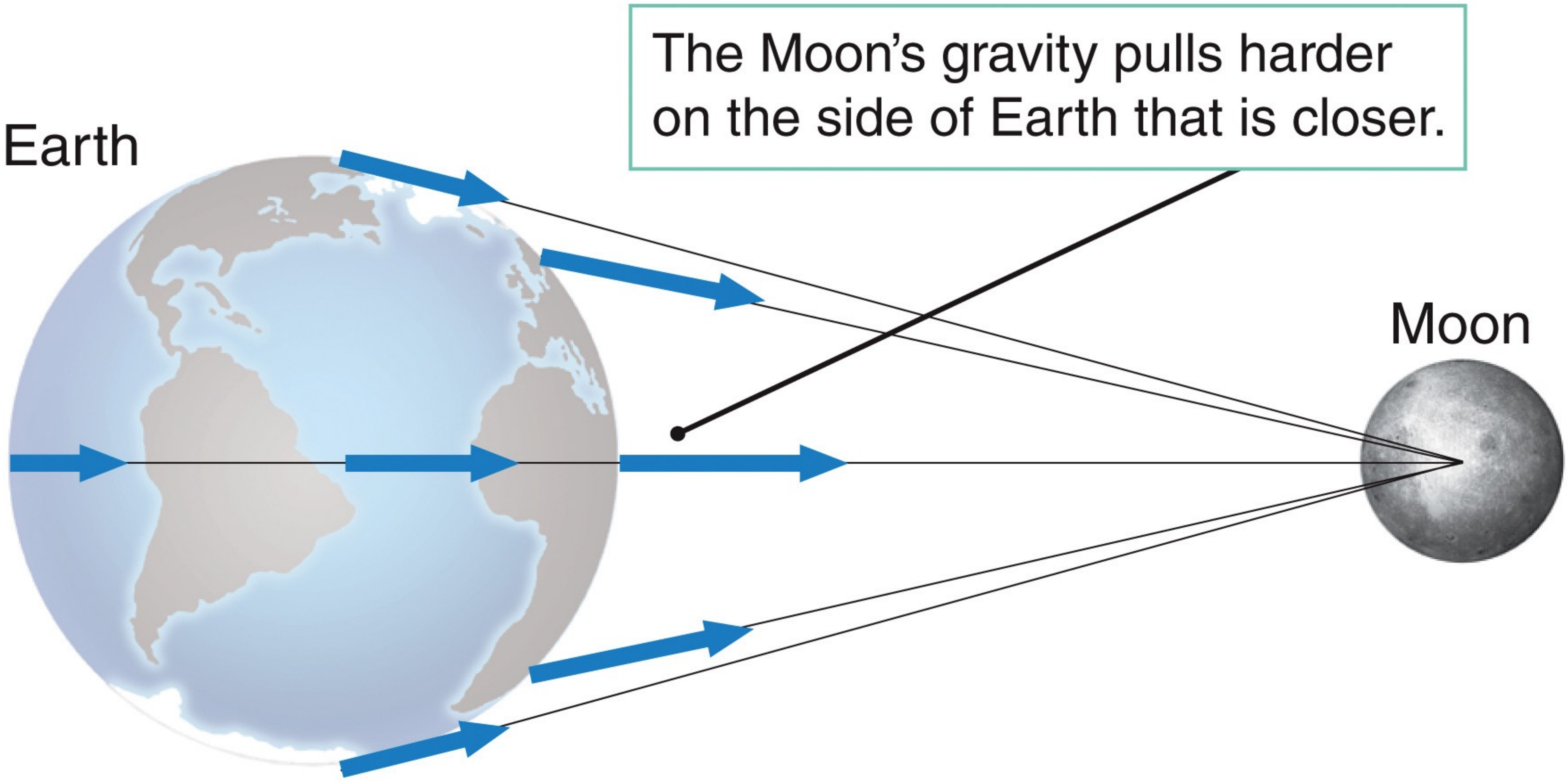
Ocean Tides, F_{tidal} Equation, & Roche Limit



Ocean Tides:

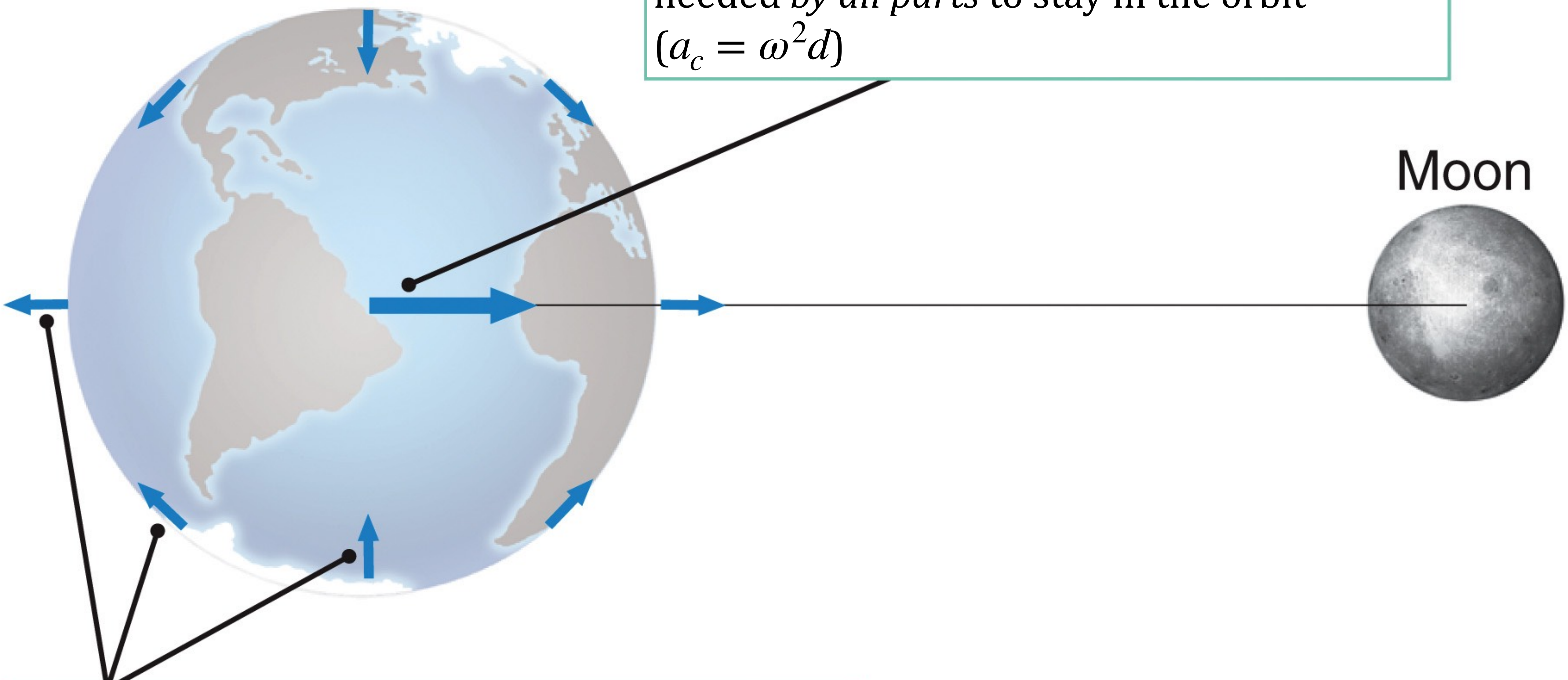
Daily Cycles and Monthly Cycles

Tidal Acceleration



Tidal Acceleration

The *average* gravitational acceleration (GM_{moon}/d^2) provides the acceleration needed *by all parts* to stay in the orbit ($a_c = \omega^2 d$)

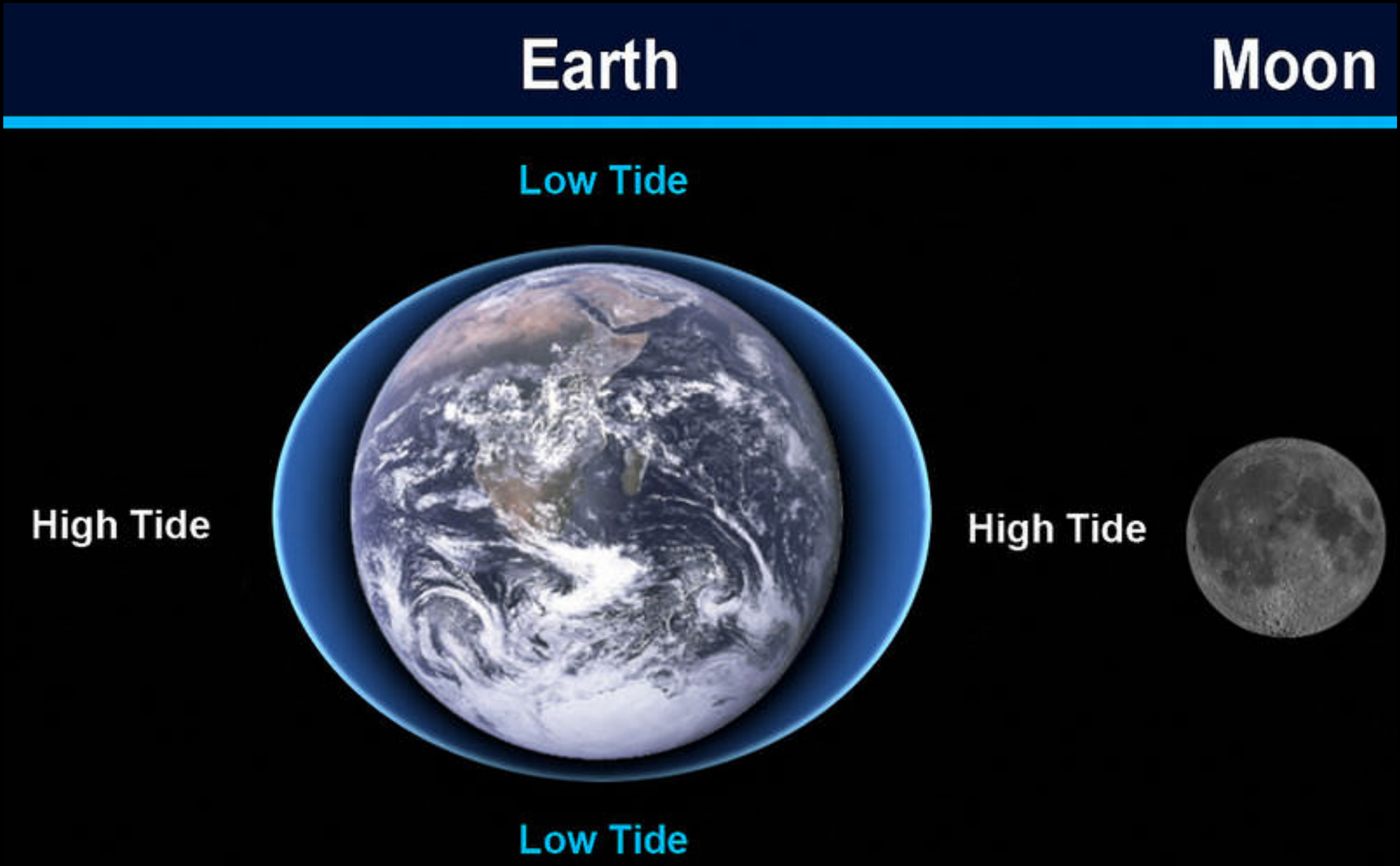


The difference between the *actual* gravitational acceleration and the *mean* gravitational acceleration provides an extra acceleration at each point, called *tidal acceleration*.

Daily Cycles of Low and High Tides

Six Hour Time Lapse of the Ocean Low to High Tide, Nova Scotia, Canada

The Tidal Bulge in the Ocean due to the Moon

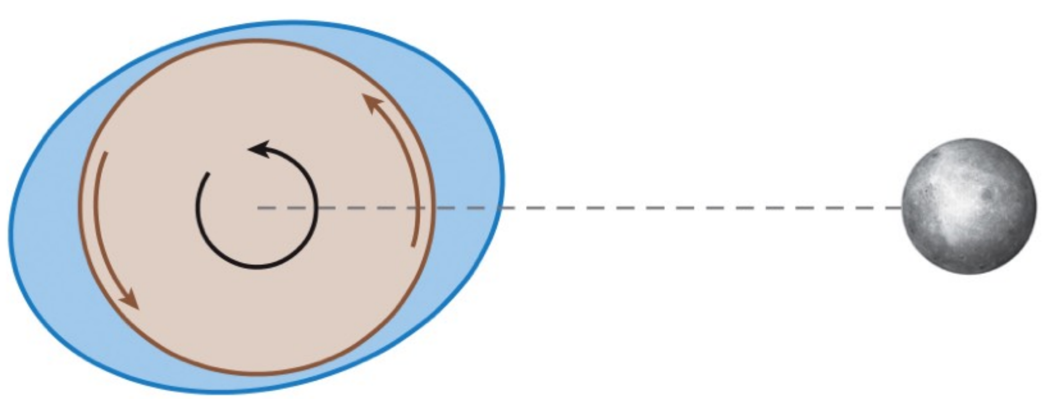


Earth's Rotation:
the daily cycles of High and Low tides

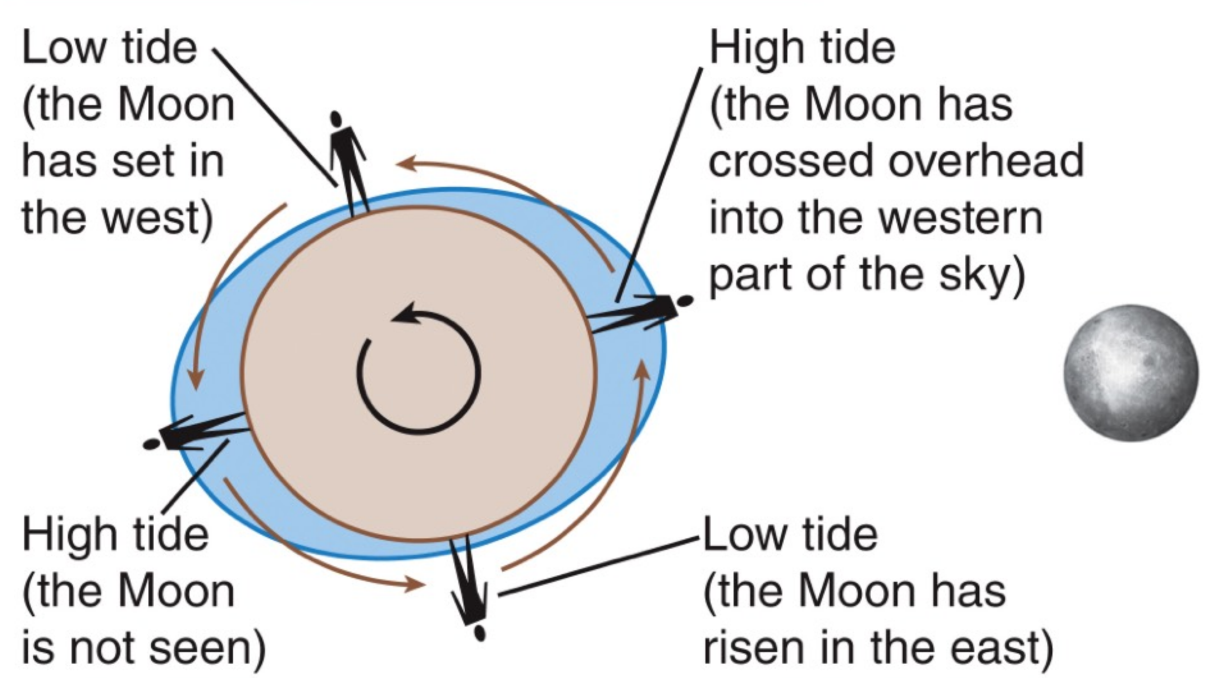
- Earth rotates under the tidal bulge (*shaped like a football*).
- We get two high and two low tides each day, **6 ¼ hours apart.**

Why 6 1/4 hours?

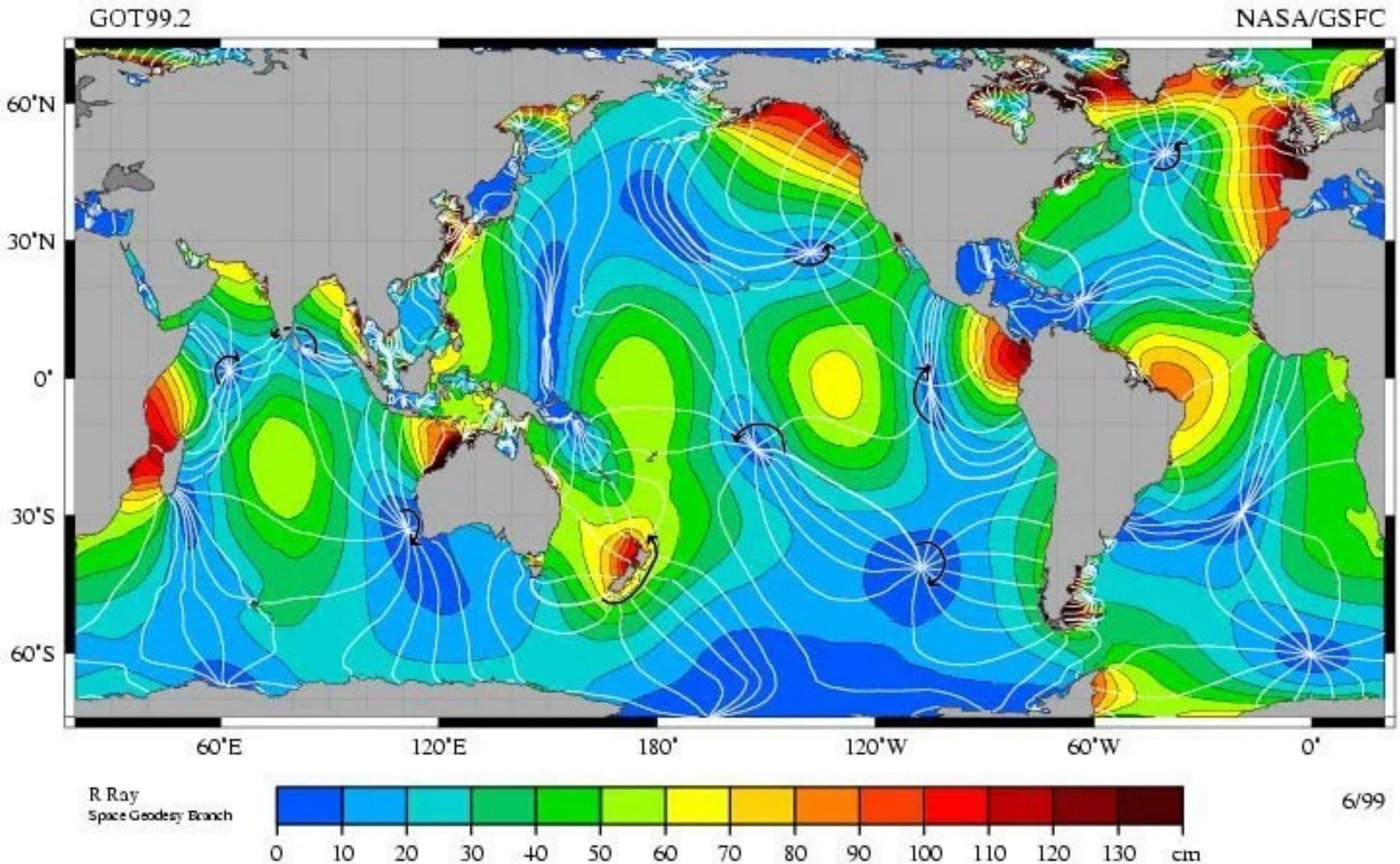
Because of friction, Earth's rotation drags its tidal bulge around, out of perfect alignment with the Moon.



Ocean tides rise and fall as the rotation of Earth carries us through the ocean's tidal bulges.

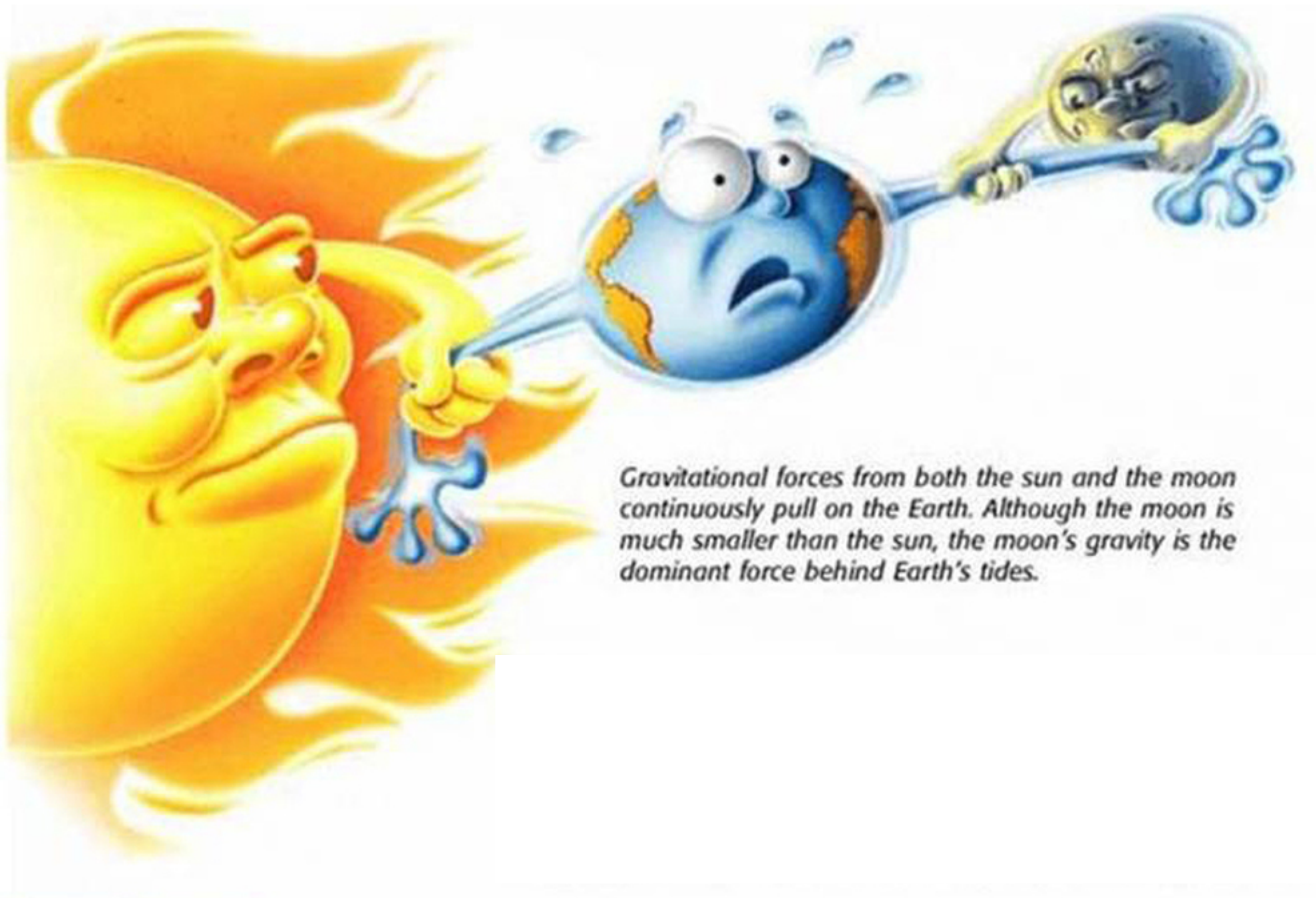


The complex tidal ranges of the Earth’s ocean



But why tides have variable strengths?

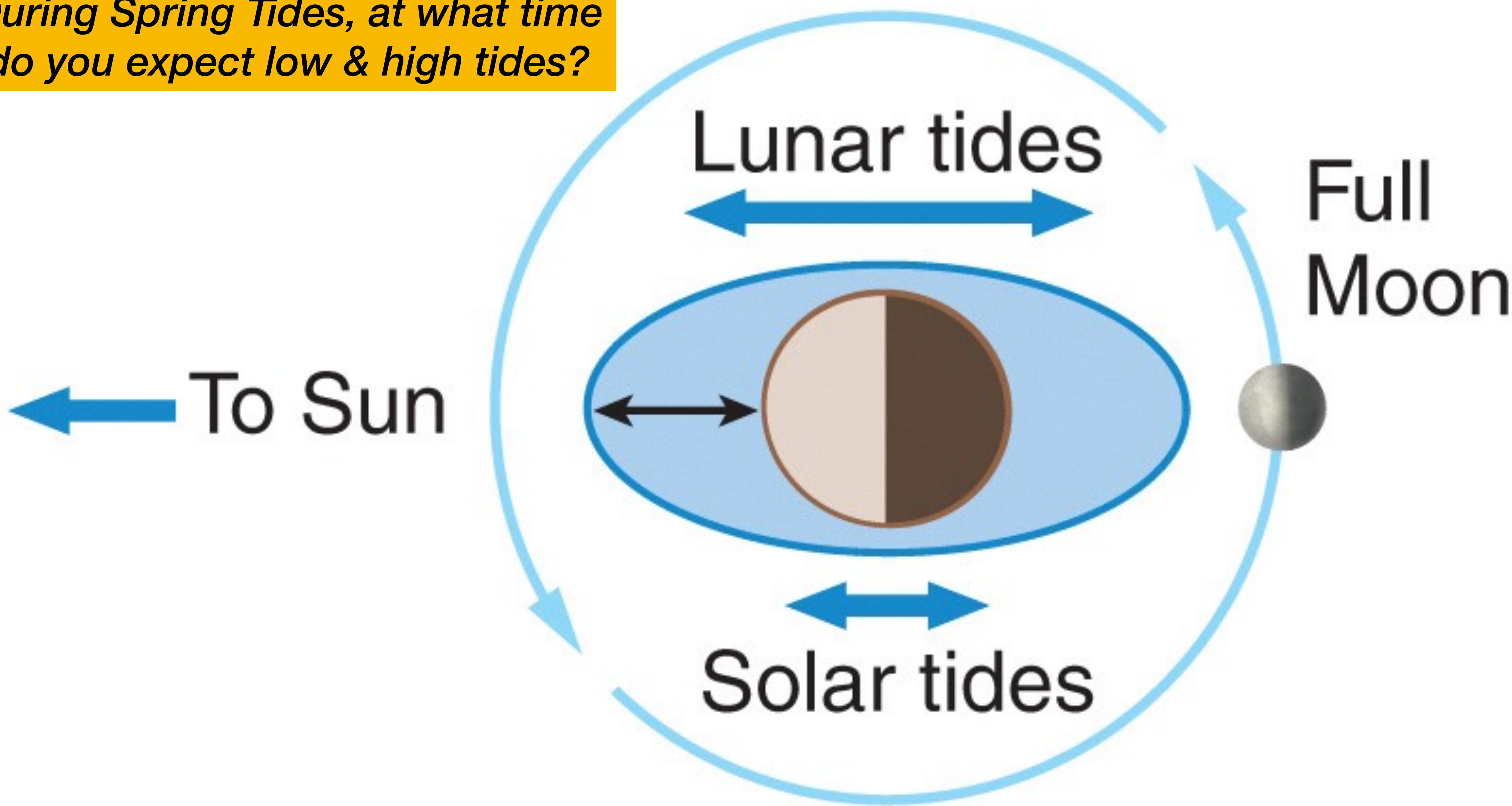
The Earth is always under the influence of both the Lunar and the Solar Tides



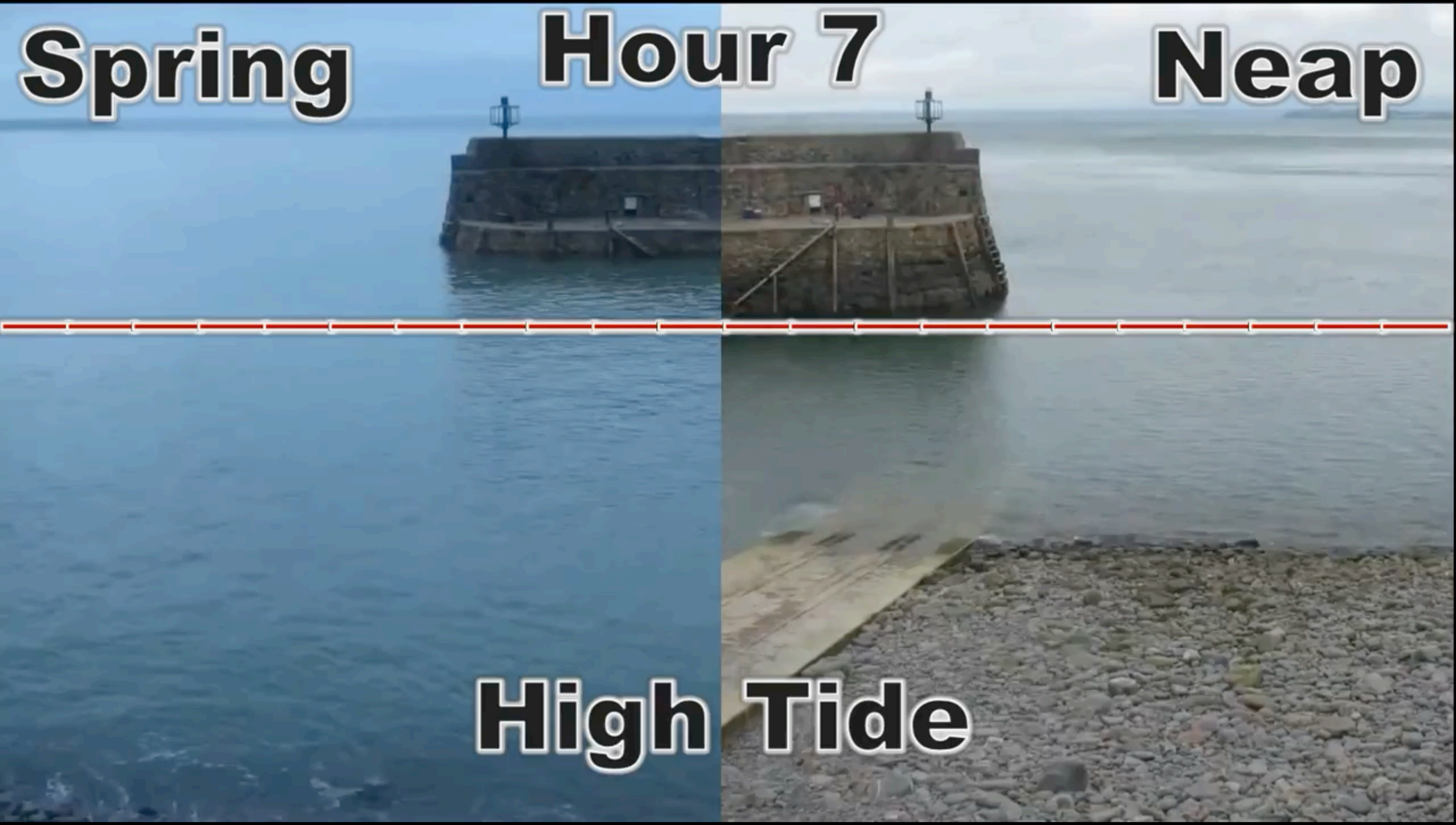
Lunar + Solar Tides:
the monthly cycle of Spring & Neap tides

Spring tides: full or new moon

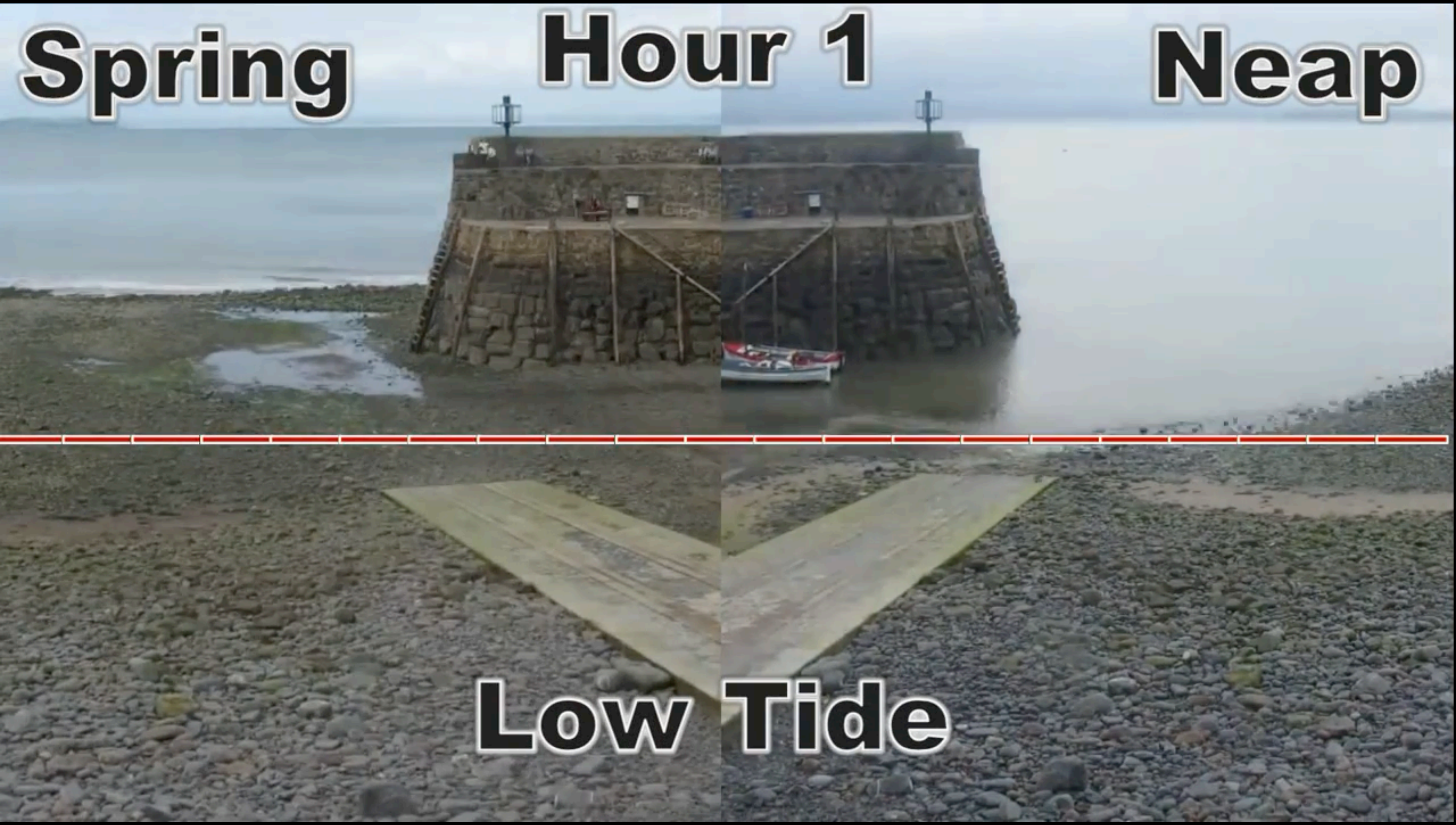
During Spring Tides, at what time do you expect low & high tides?



Spring tides have the highest high tides



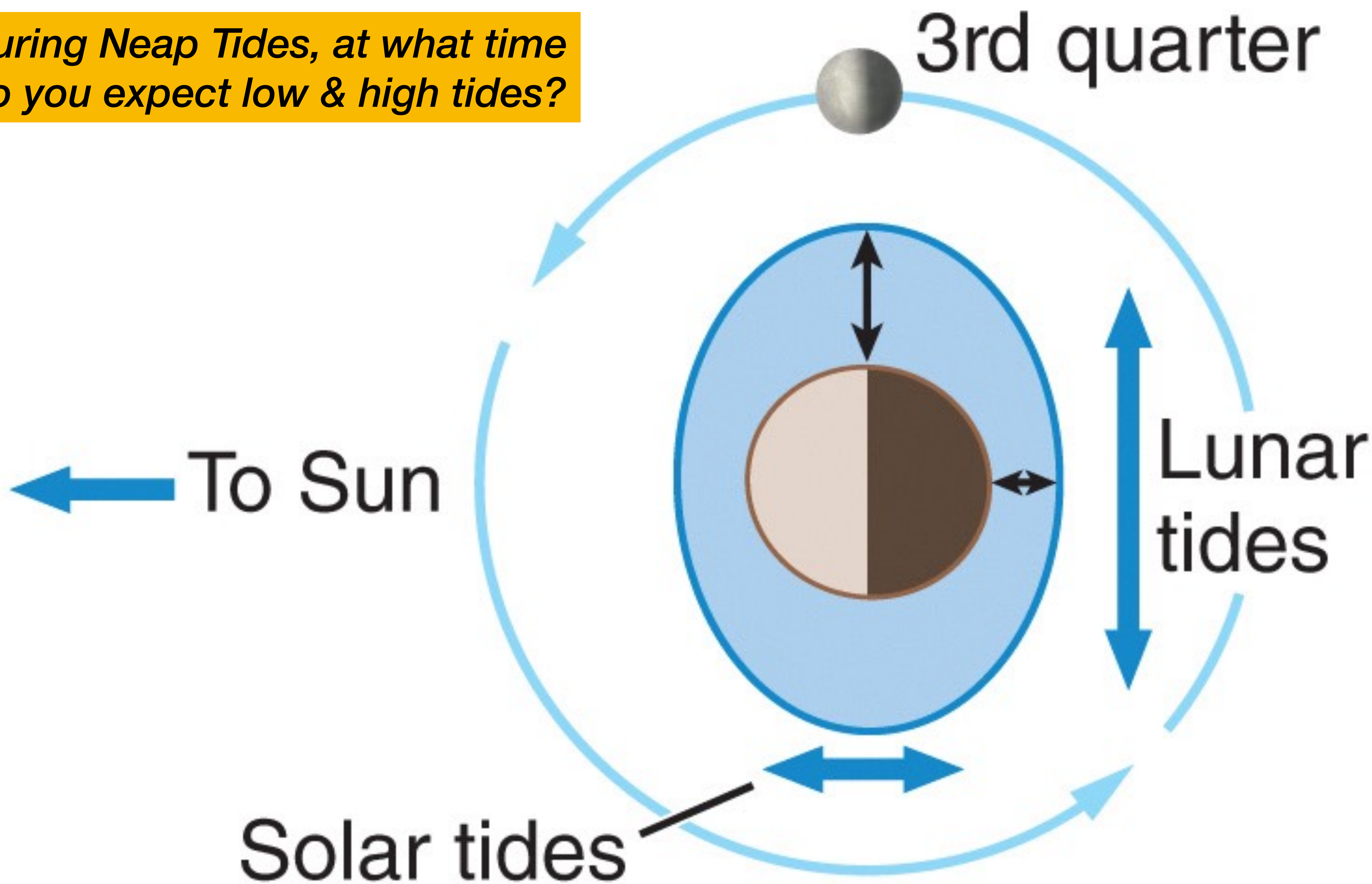
Spring tides have the lowest low tides



Lunar + Solar Tides: the monthly cycle of Spring & Neap tides

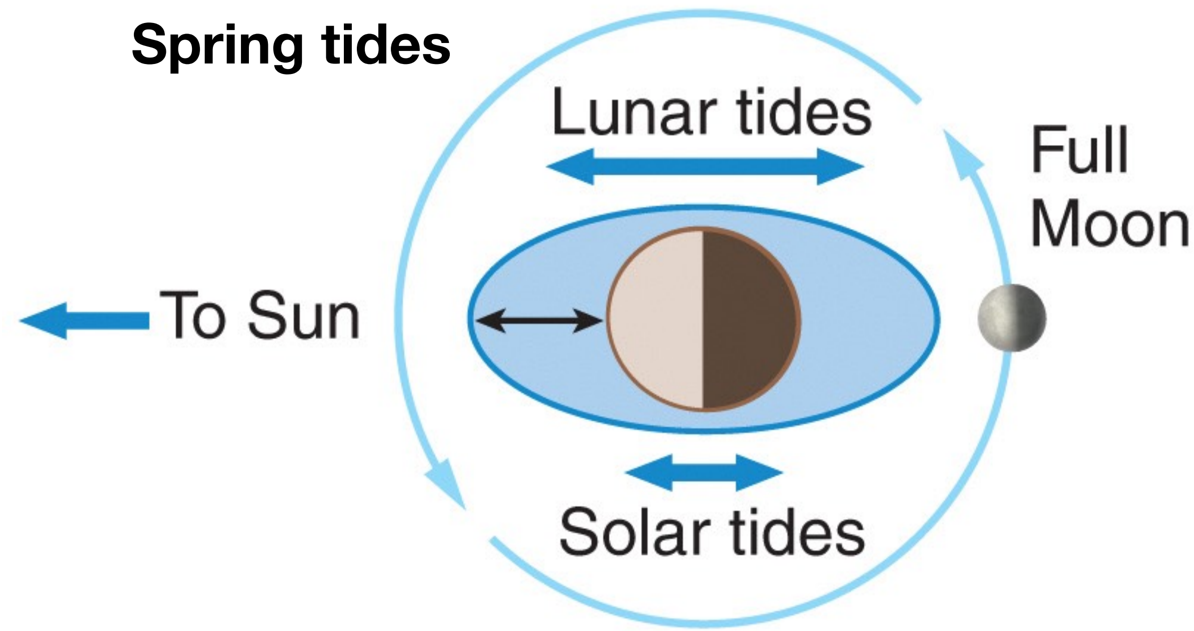
Neap tides: quarter moons

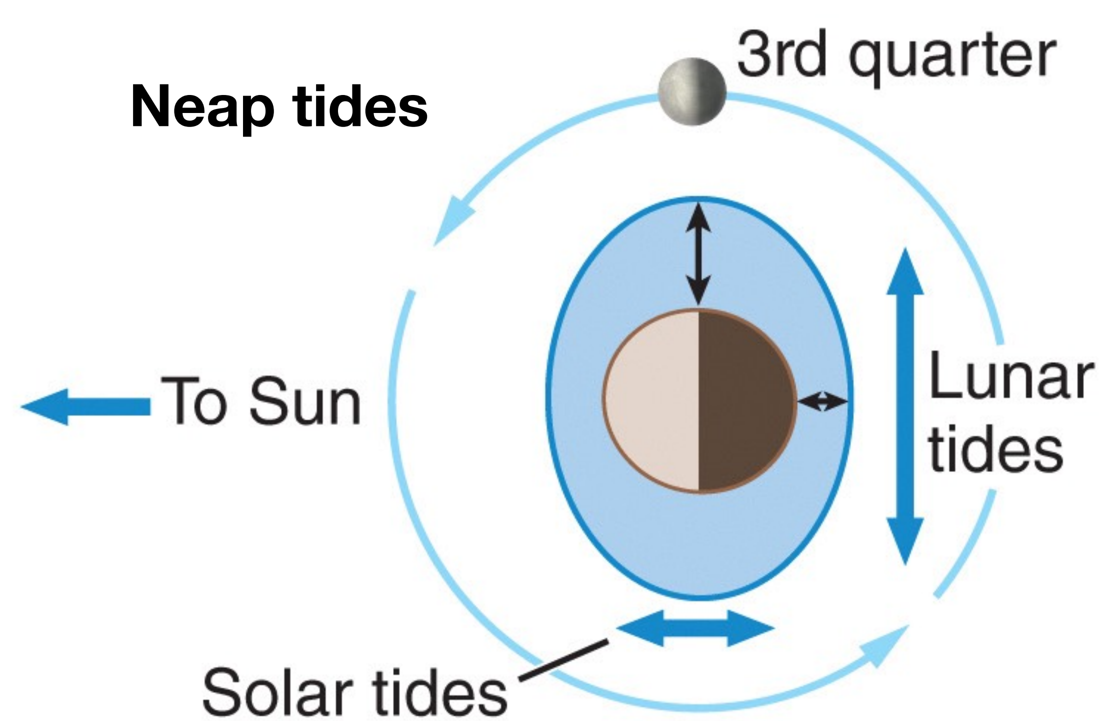
During Neap Tides, at what time do you expect low & high tides?



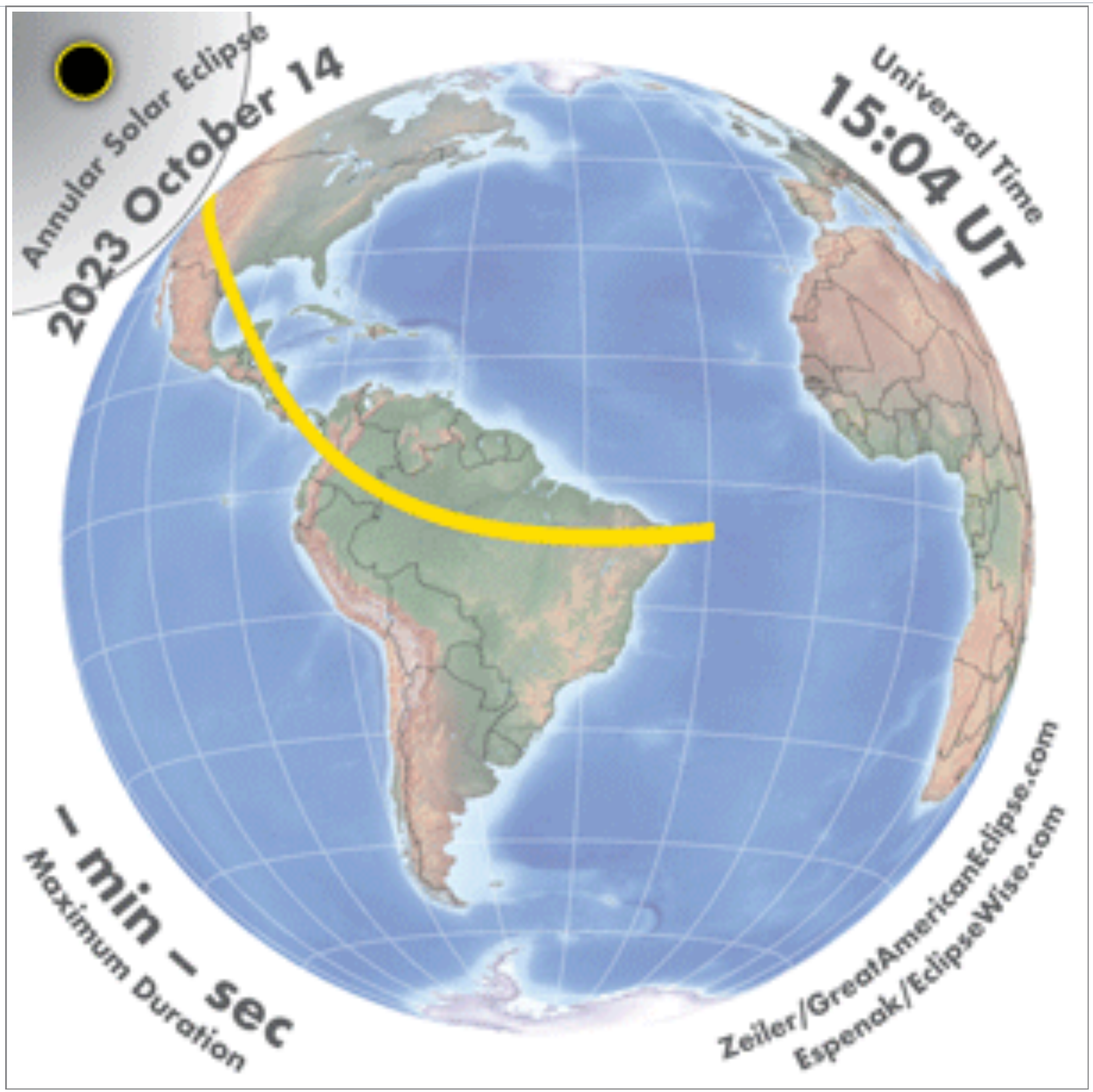
The orbital motion of the Moon causes the monthly cycle of Spring & Neap tides

- **Two Spring (strong) tides** occur when the Sun and Moon are aligned (**new or full phases**) and are more extreme than normal.
- **Two Neap (weak) tides** occur when the Moon, Earth, and Sun are at right angles (**quarter phases**). The Sun and Moon pull in perpendicular directions and partially cancel each other. These high tides are lower than normal.



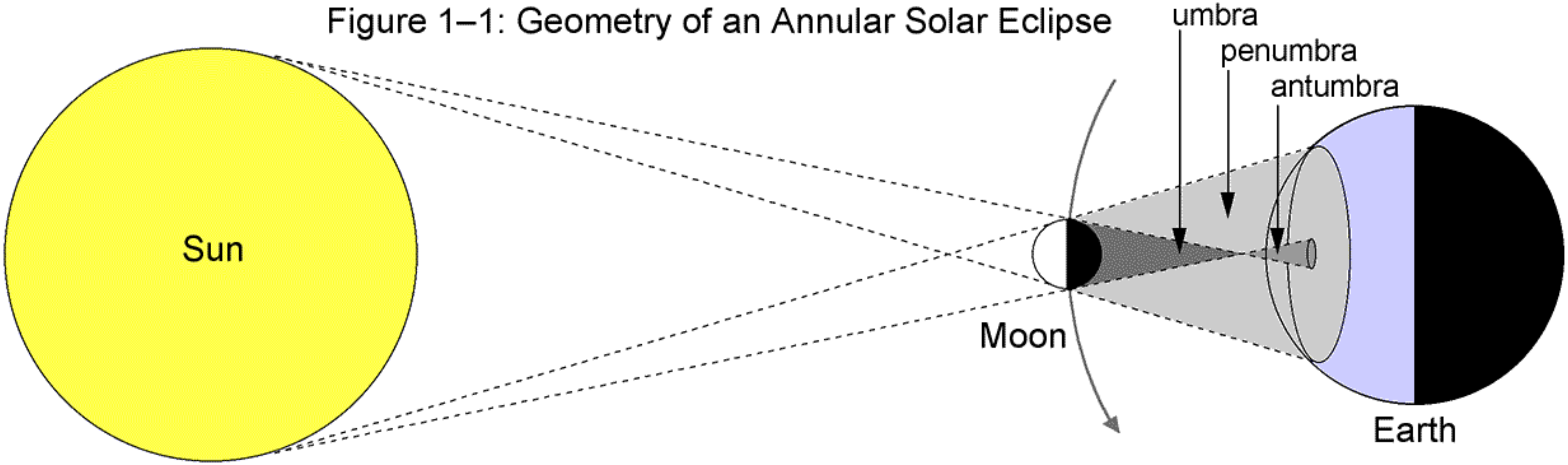


Path of the Moon's Shadow on October 14, 2023



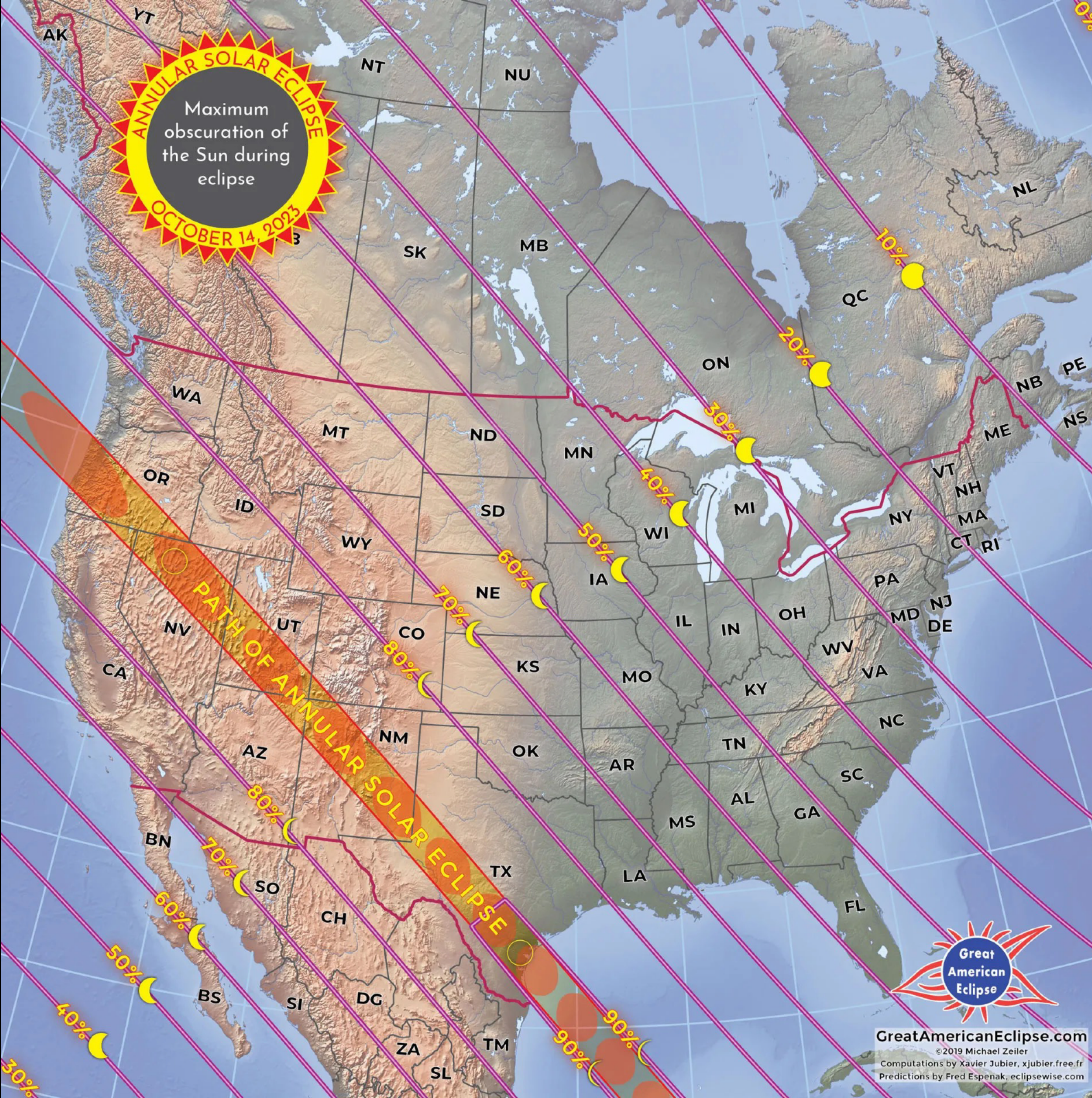
Solar Eclipse: Umbra, Penumbra, & Antumbra

Figure 1–1: Geometry of an Annular Solar Eclipse



Courtesy of "Thousand Year Canon of Solar Eclipses: 1501 – 2500", Fred Espenak, AstroPixels Publishing, 2015.

2023 Annular Solar Eclipse Path and Limit Lines



Solar Eclipse Oct 14, 2023

In Des Moines, the partial eclipse will begin at 10:27 a.m. and end at 1:17 p.m.

The peak of the eclipse here — 54% of maximum — will occur at 11:49 a.m.

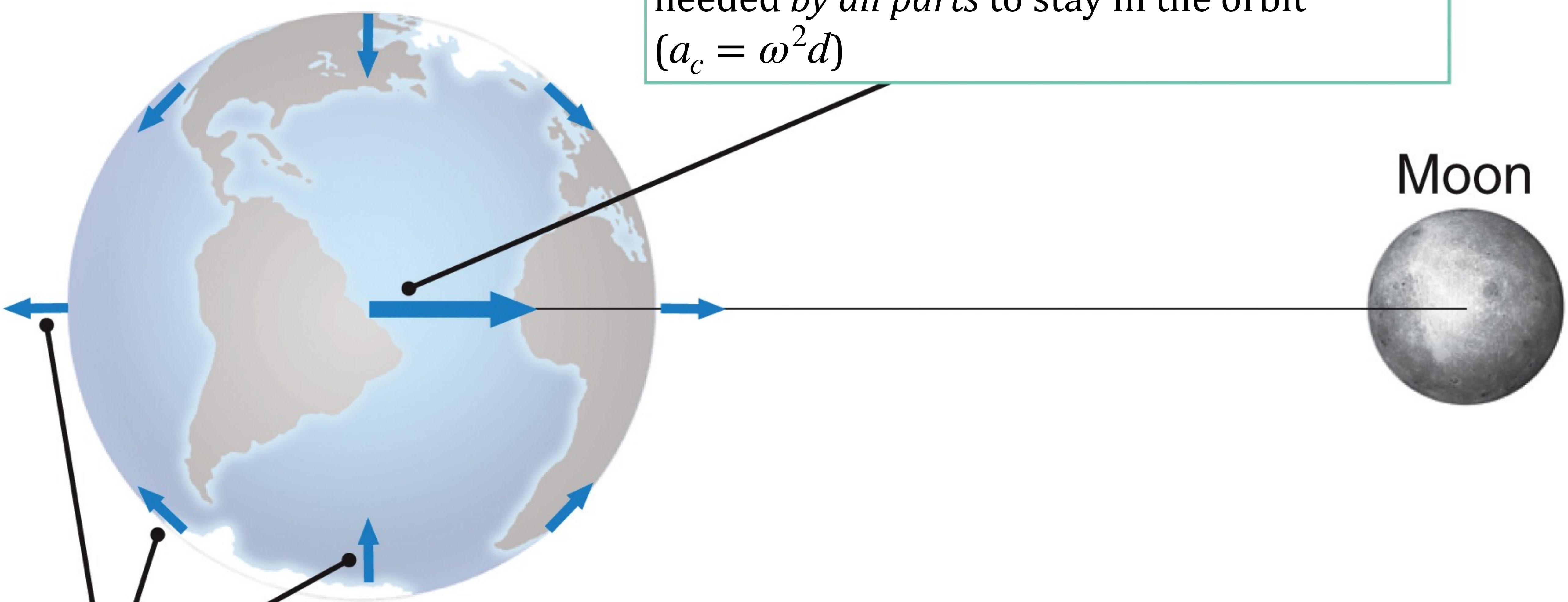
NASA youtube live stream:

https://www.youtube.com/watch?v=LIY79zjud-Q

Tidal Acceleration Equation

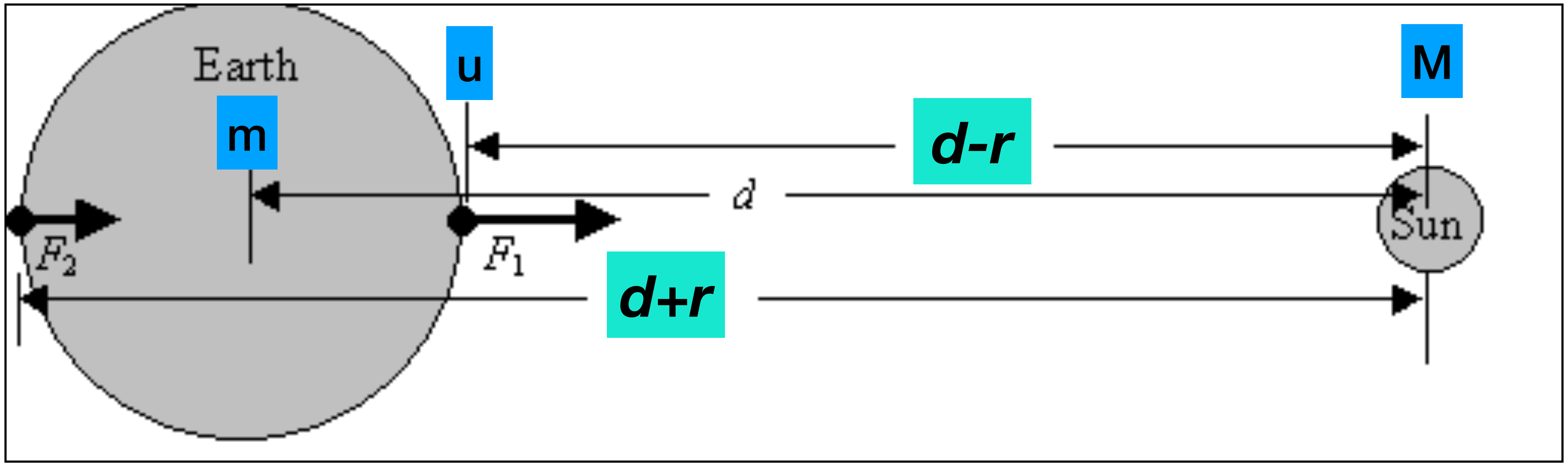
Tidal Acceleration

The *average* gravitational acceleration (GM_{moon}/d^2) provides the acceleration needed *by all parts* to stay in the orbit ($a_c = \omega^2 d$)



The difference between the *actual* gravitational acceleration and the *mean* gravitational acceleration provides an extra acceleration at each point, called *tidal acceleration*.

Gravity on the nearer side for a test mass u



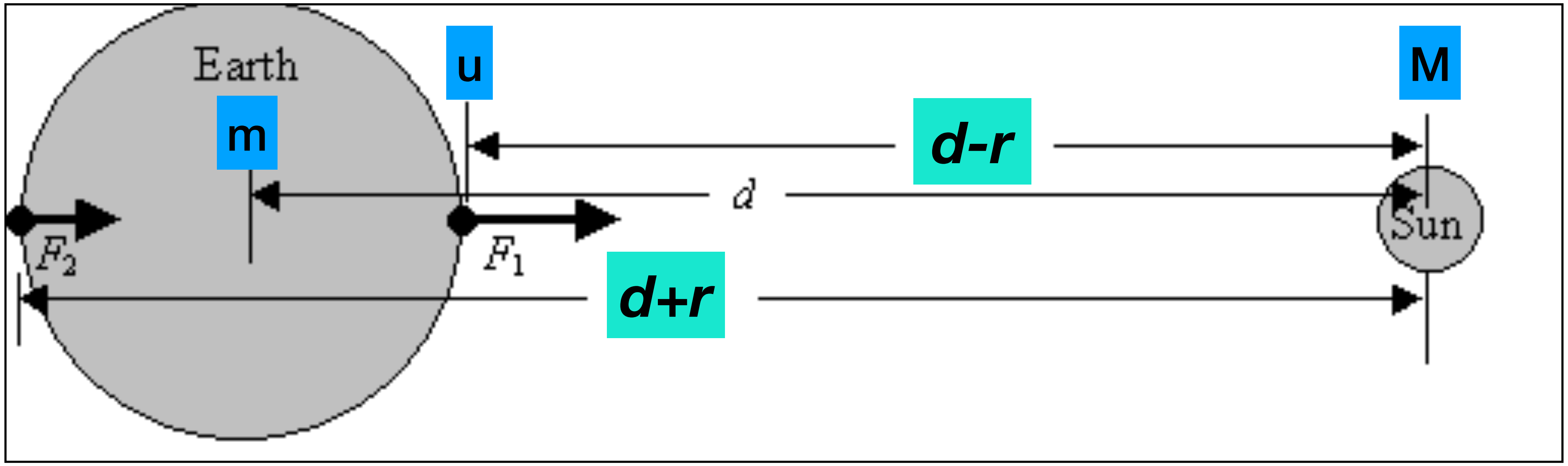
$$F_1 = \frac{GMu}{(d-r)^2} = \frac{GMu}{d^2}(1-\frac{r}{d})^{-2}$$

Approximation based on Taylor expansion for ε <<1

$$(1+\varepsilon)^x \approx 1 + x\varepsilon$$

$$F_1 = \frac{GMu}{d^2}(1-\frac{r}{d})^{-2} \approx \frac{GMu}{d^2}(1+2\frac{r}{d})$$

Tidal Force as a Differential Force



$$F_1 = \frac{GMu}{d^2}(1 - \frac{r}{d})^{-2} \approx \frac{GMu}{d^2}(1 + 2\frac{r}{d}) \qquad \bar{F} = \frac{GMu}{d^2}$$

$$\Rightarrow F_{\text{tidal}} = F_1 - \bar{F} \approx \frac{GMu}{d^2}\frac{2r}{d} = \frac{2GMu}{d^3} \cdot r$$

Tidal Force = Gradient of Gravity x Object Size

(applicable when size << distance)

Tidal Acceleration Comparison: Moon vs. Sun

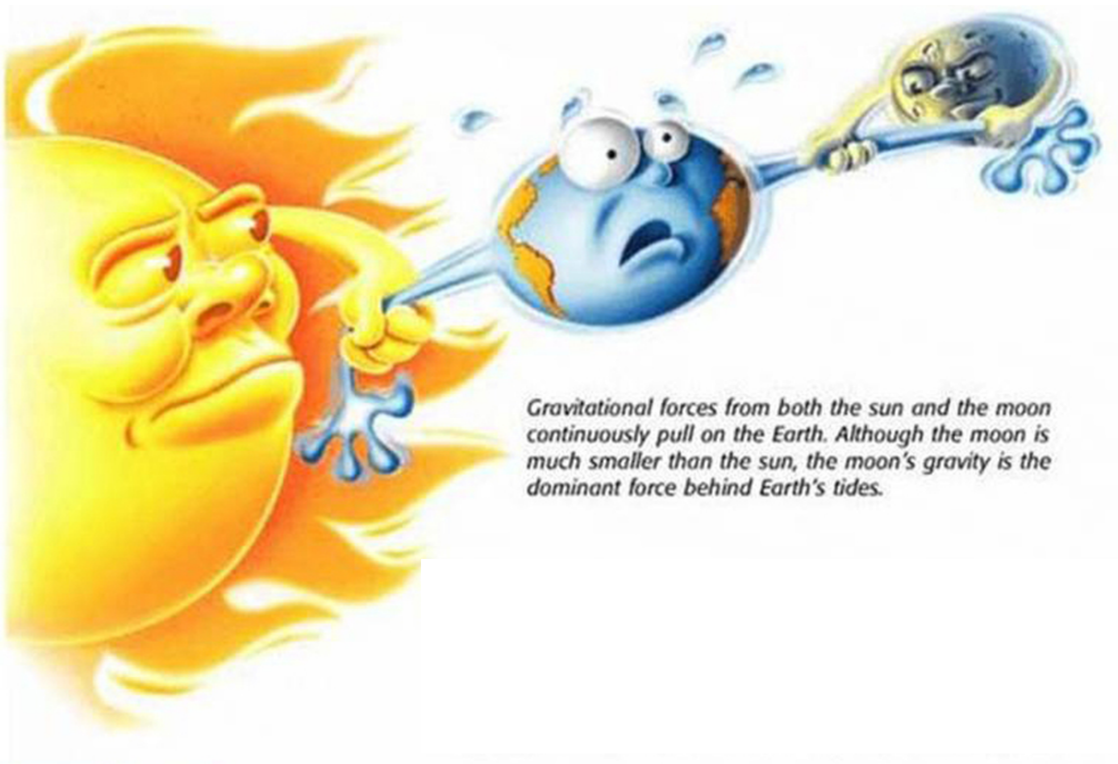
On Earth, we experience the tidal forces from both the Moon and the Sun. They are called **lunar tides** and **solar tides**, respectively.

Which tide is stronger? By how many times?

distance ratio: $d_{Sun}/d_{Moon} = 400$

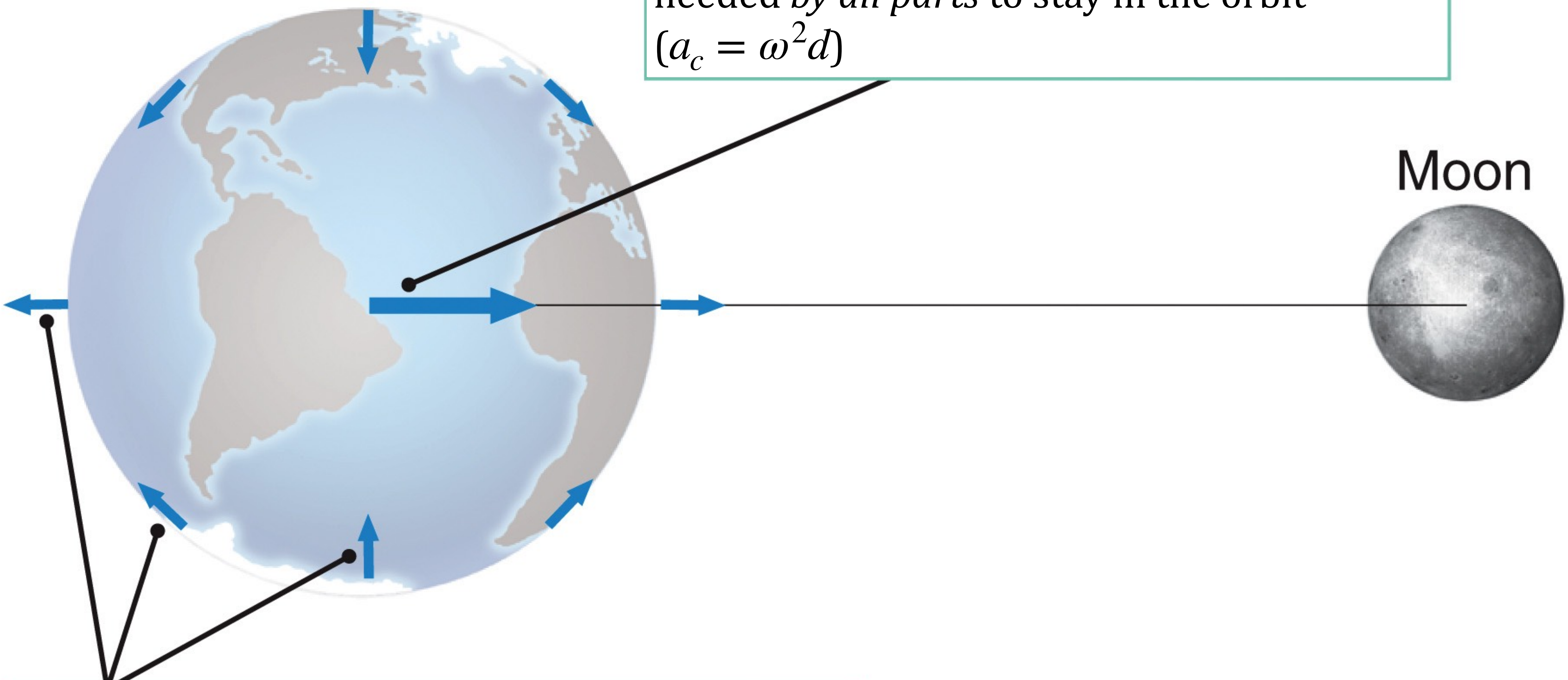
mass ratio: $M_{Sun}/M_{Moon} = 2.4\times10^{7} = 24$ million

$$a_{\text{tidal}} = \frac{F_{\text{tidal}}}{u} = \frac{2GM}{d^3} \cdot r$$



Tidal accelerations are everywhere on our planet, why the planet don't fall apart? What is balancing the tidal force?

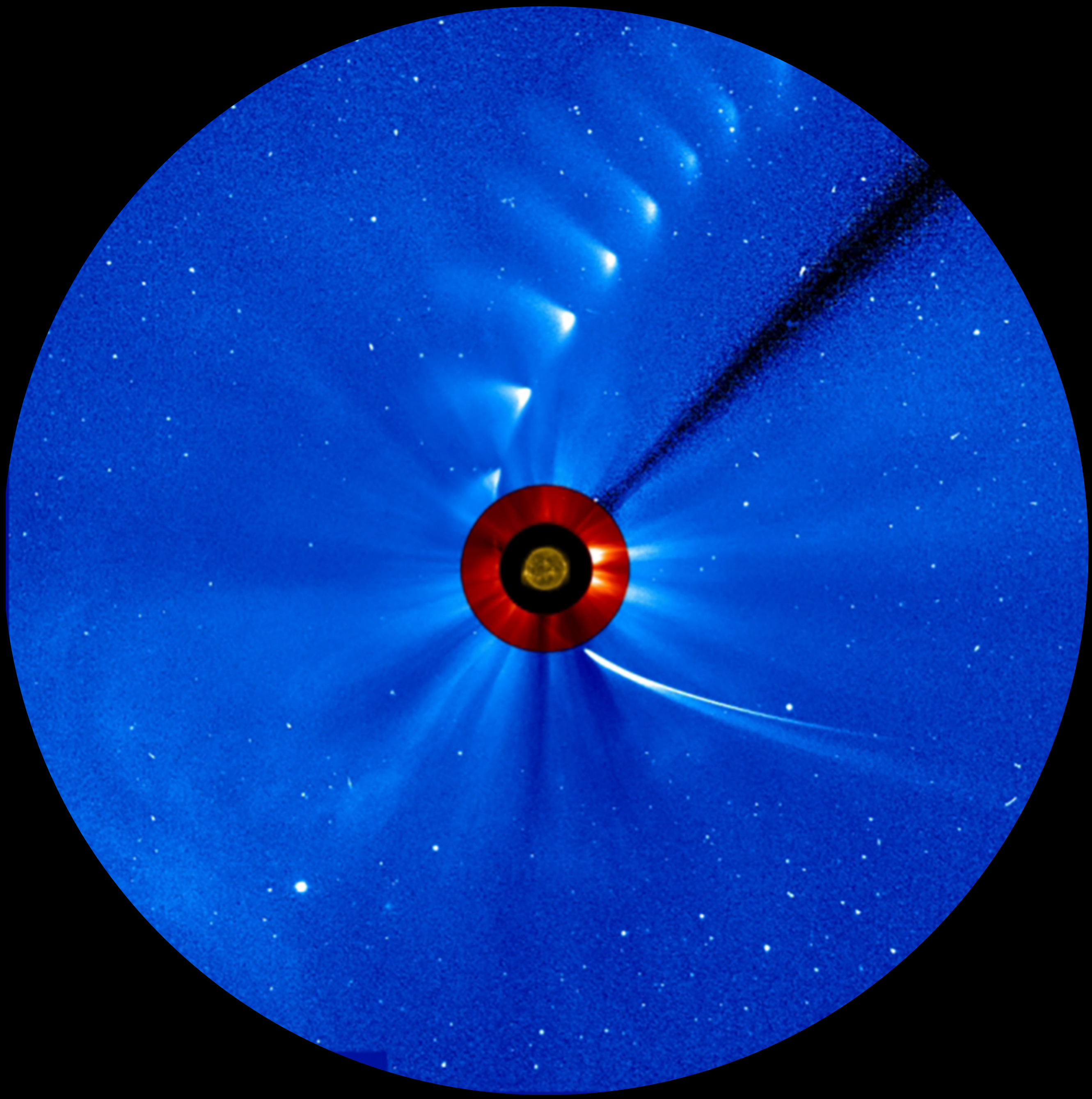
The *average* gravitational acceleration (GM_{moon}/d^2) provides the acceleration needed *by all parts* to stay in the orbit ($a_c = \omega^2 d$)

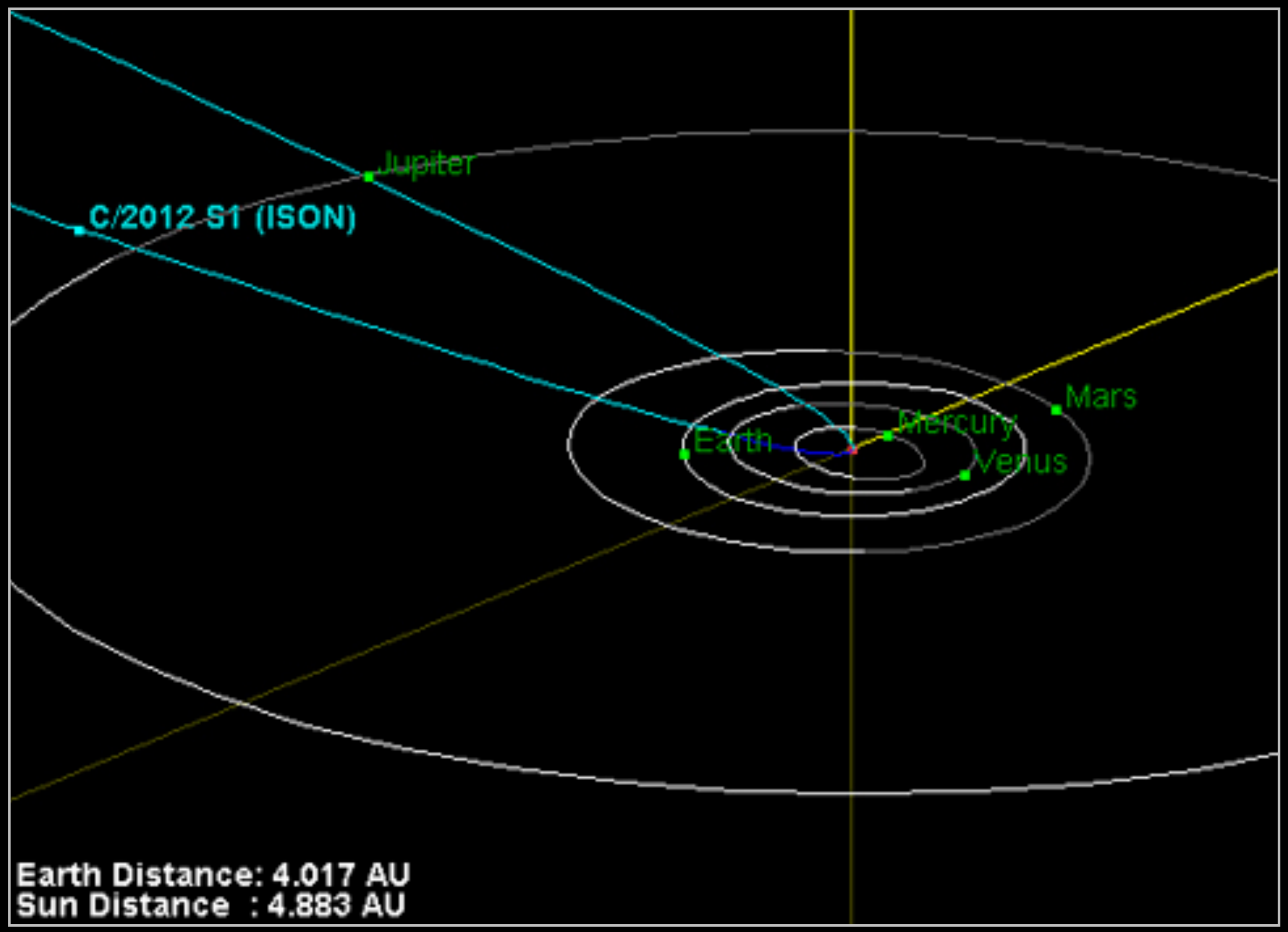


The difference between the *actual* gravitational acceleration and the *mean* gravitational acceleration provides an extra acceleration at each point, called *tidal acceleration*.

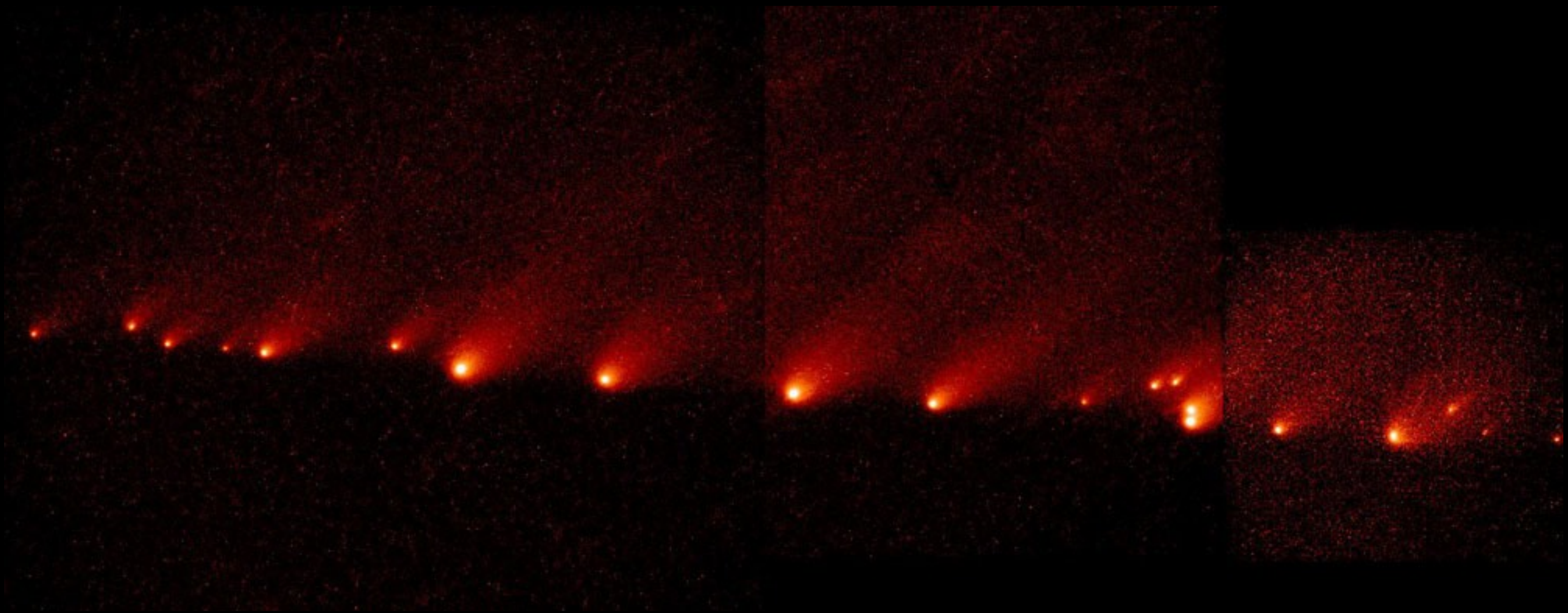
The most dramatic tidal effect: Tidal Disruption

Comets are loosely bound objects, they can be disrupted by tidal force when they get dangerously close to our star or large planets





Shoemaker-Levy 9's tidal disruption by Jupiter in 1992
the encounter was within 1.62 Jupiter radii

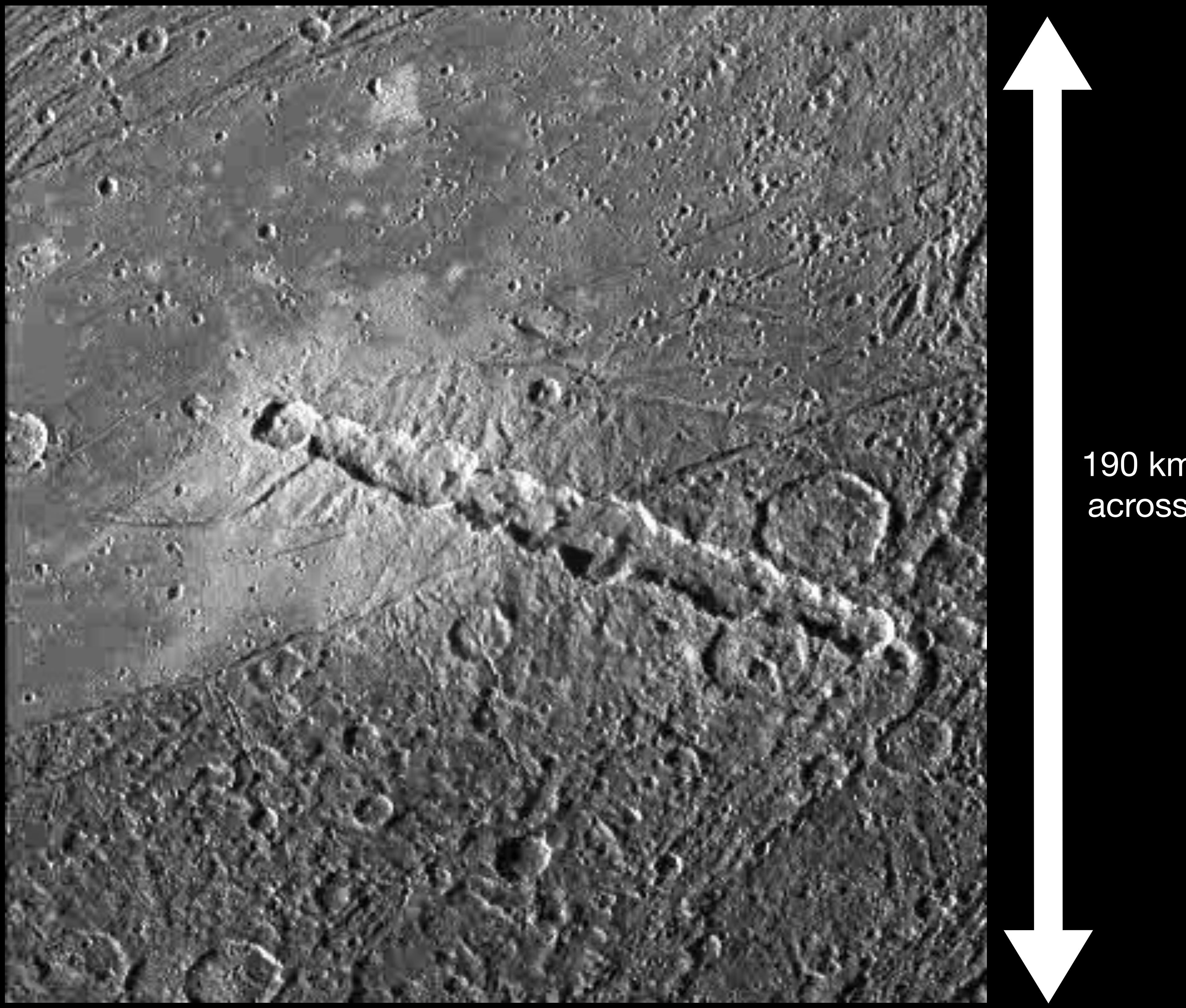


The individual nuclei of Comet Shoemaker-Levy 9, as imaged by the Hubble Space Telescope on May 17, 1994.

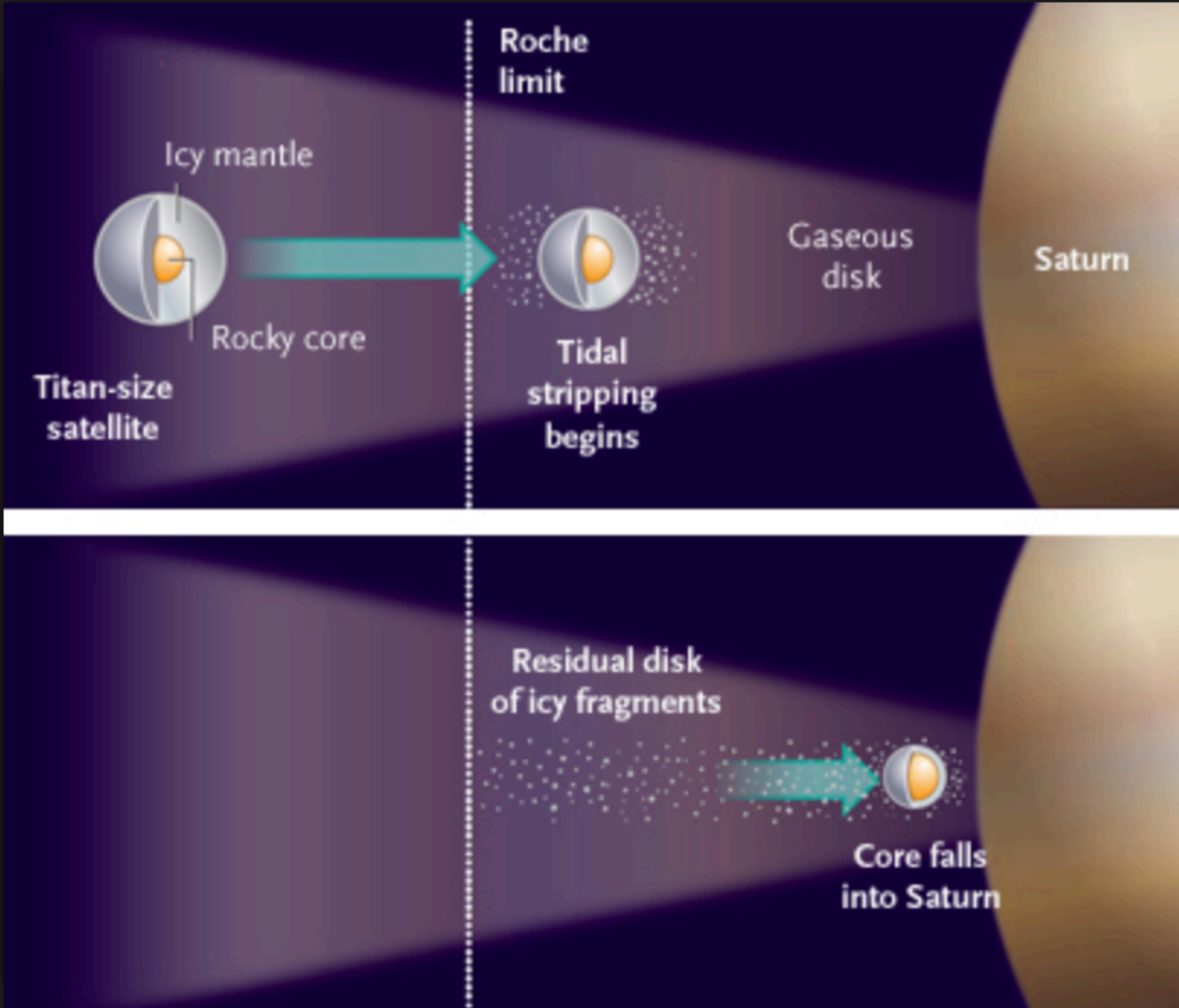

Shoemaker-Levy 9's crash into Jupiter in 1994
Impact Scars on Jupiter


Shoemaker-Levy 9’s crash into Jupiter in 1994
Impact Scar “G” on Jupiter

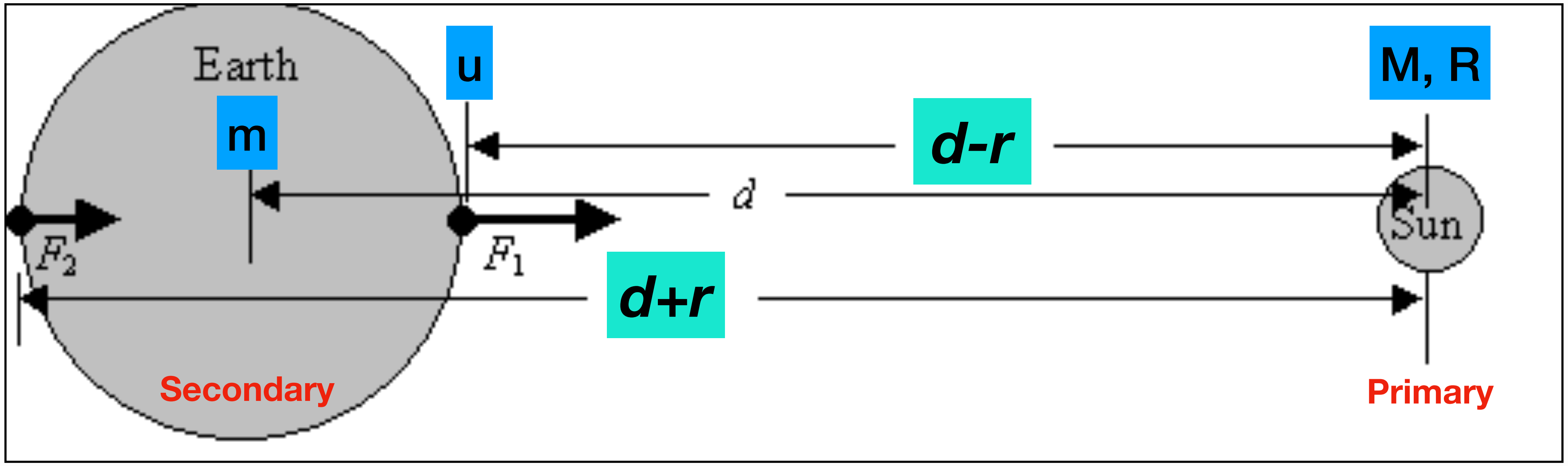
A Chain of Craters on Ganymede indicates a similar tidal disruption



How close is too close?
The *Roche Limit*


Roche limit
Icy mantle
Rocky core
Titan-size satellite
Tidal stripping begins
Gaseous disk
Saturn
Residual disk of icy fragments
Core falls into Saturn

Tidal disruption occurs when tidal force exceeds self-gravity



Tidal Force on a test mass u = Gradient of Gravity from Primary x Radius of Secondary

$$F_{\text{tidal}} = \frac{2GMu}{d^3} \cdot r$$

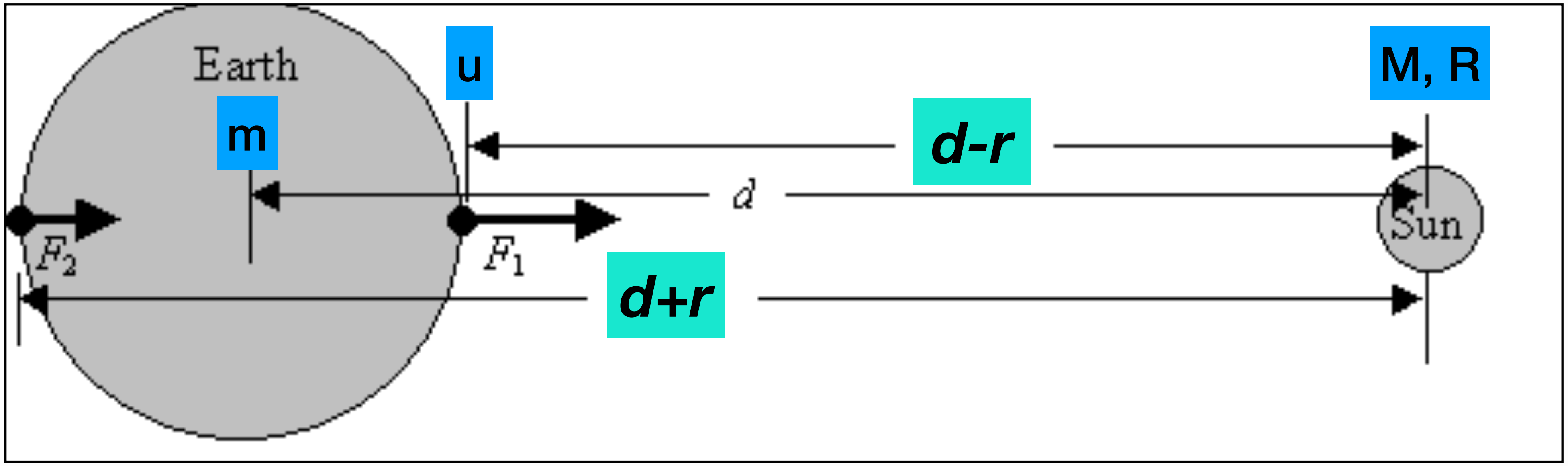
At what distance does the tidal force equals self-gravity?

$$F_{\text{sg}} = \frac{Gmu}{r^2}$$

to answer that, we need to solve for d given $F_{tidal} = F_{sg}$:

$$F_{\text{tidal}} = F_{\text{sg}} \Rightarrow d = d_{\text{Roche}} = r\left(\frac{2M}{m}\right)^{\frac{1}{3}}$$

The Roche Limit is the distance we just solved



$$d_{\text{Roche}} = r\left(\frac{2M}{m}\right)^{\frac{1}{3}} = R\left(\frac{2\rho_M}{\rho_m}\right)^{\frac{1}{3}} = 1.26R\left(\frac{\rho_M}{\rho_m}\right)^{\frac{1}{3}}$$

where in the last step, we used the following identities to replace the masses with densities:

$$M = \rho_M \cdot \frac{4}{3}\pi R^3$$ *primary object*

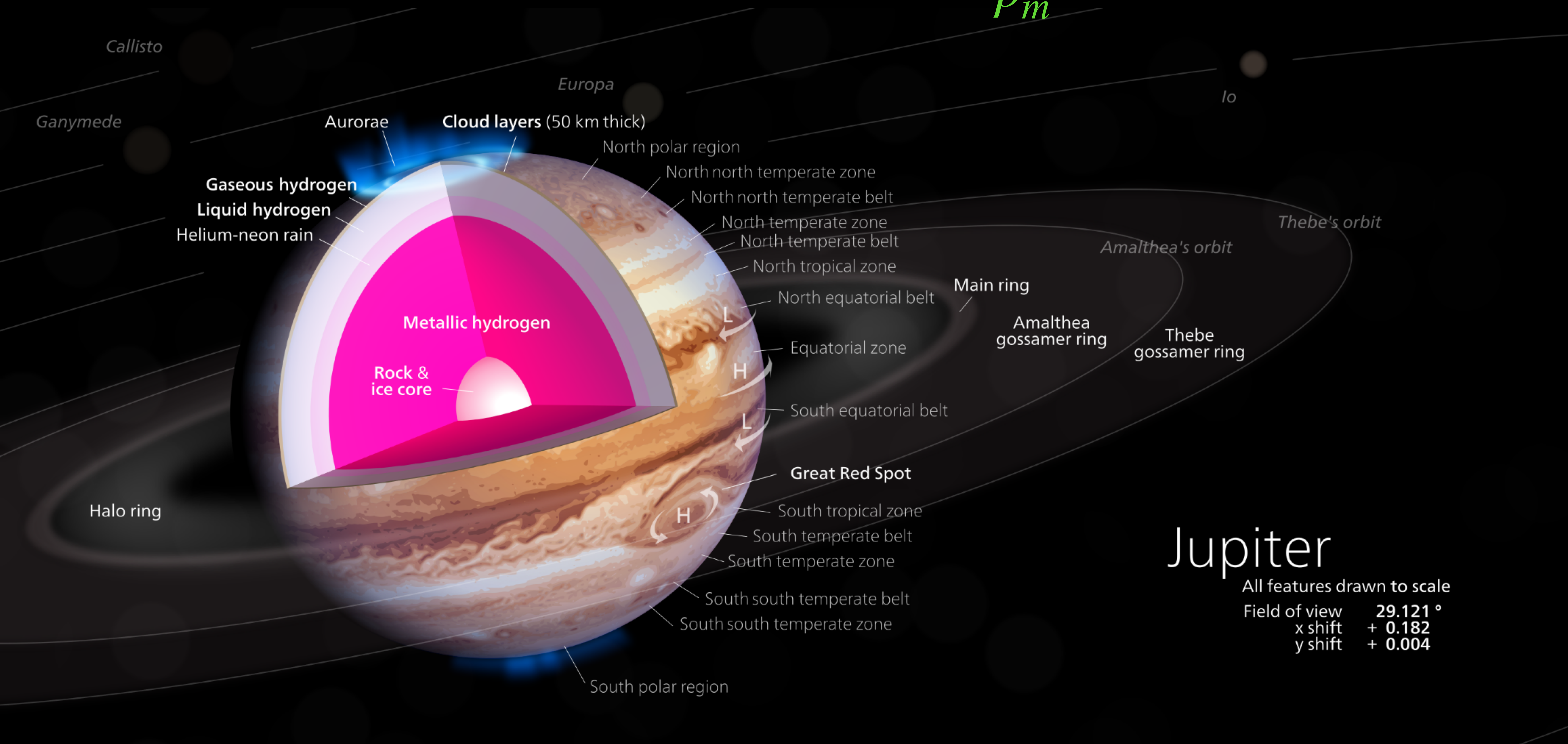
$$m = \rho_m \cdot \frac{4}{3}\pi r^3$$ *secondary object*

*The Roche limit is determined by the **densities** of **both** the **primary** and the **secondary** objects and the **radius of the primary***

Practice: Calculate Roche Limit of Jupiter-Comet System

Mean Density of Jupiter: 1.3 g/cm^3
Mean Density of Comet: 0.6 g/cm^3
Calculate the rigid-body Roche limit in Jupiter radii

$$d_{\text{Roche,Rigid}} = 1.26 \cdot R \cdot \left(\frac{\rho_M}{\rho_m}\right)^{\frac{1}{3}}$$



Practice: Calculate Roche Limit of Saturn-Pan System

Mean Density of Saturn: 0.687 g/cm^3

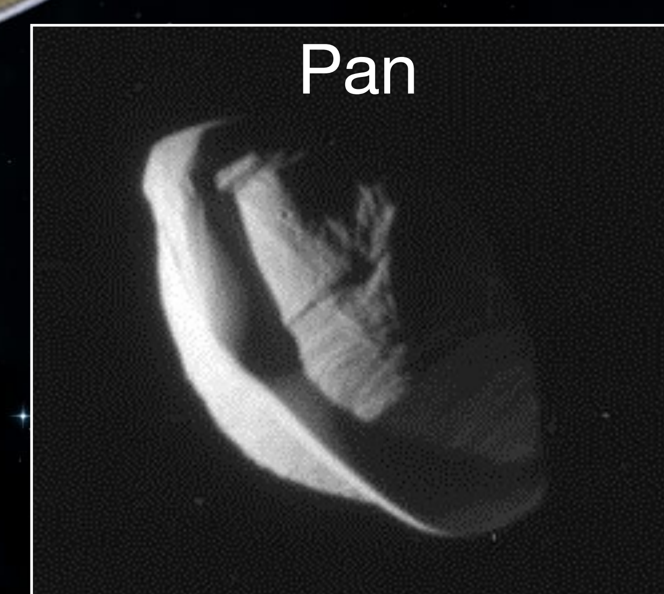
Saturn Radius: 58,232 km

Pan is the innermost satellite, its density is 0.42 g/cm3

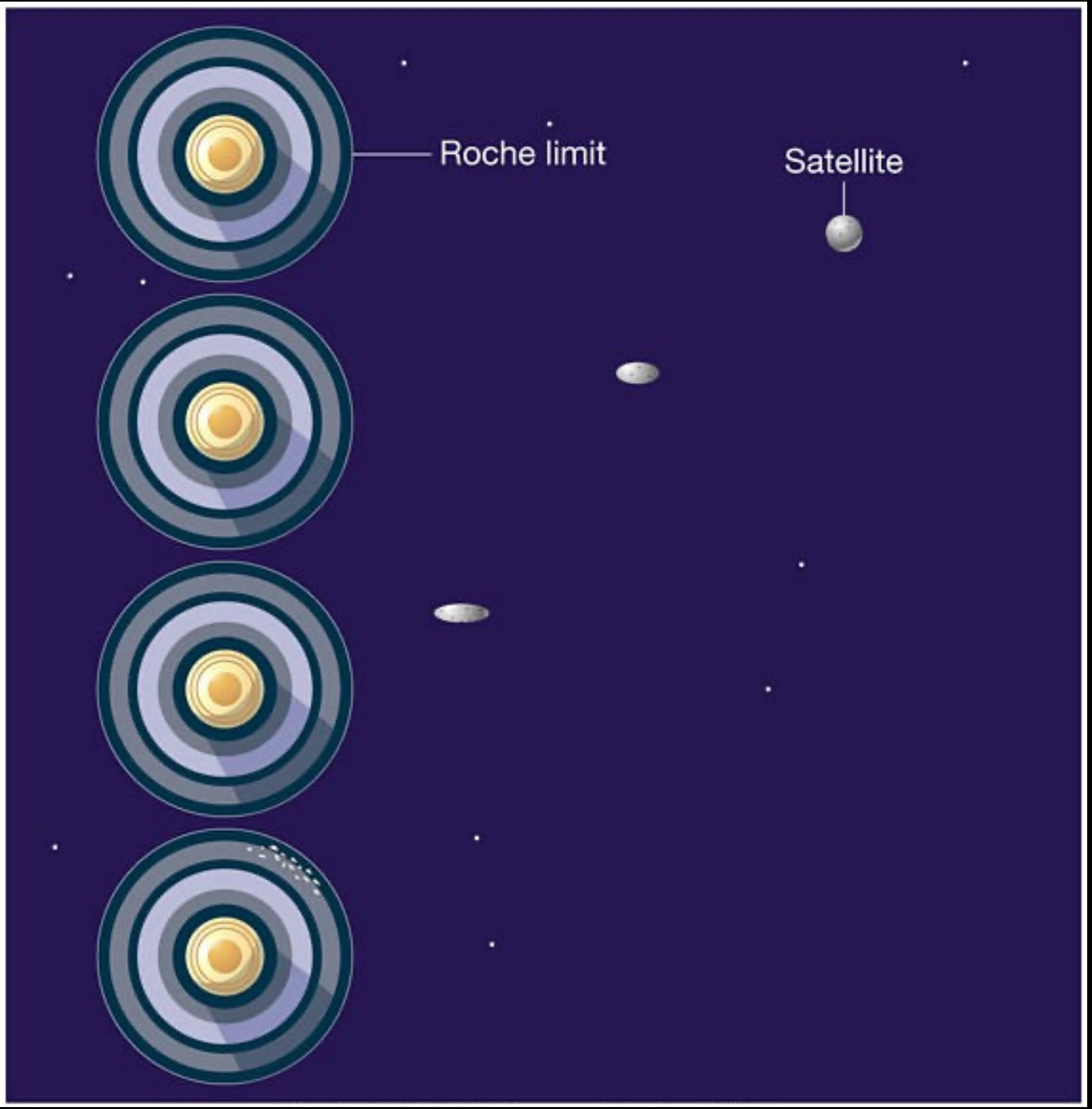
Calculate the Roche limit in km

compare it with the distance of Pan to Saturn: 133,584 km

$$d_{\text{Roche}} = R\left(\frac{2\rho_M}{\rho_m}\right)^{\frac{1}{3}} = 1.26 \cdot R \cdot \left(\frac{\rho_M}{\rho_m}\right)^{\frac{1}{3}}$$



The tidal deformation of a satellite falling towards Saturn making the rigid-body assumption inaccurate



Tidal acceleration

$$a_{\text{tidal}} = \frac{2GM}{d^3} \cdot r$$

The Roche Limit for Rigid Body (not accounting for deformation)

$$d_{\text{Roche,Rigid}} = 1.26 \cdot R \cdot \left(\frac{\rho_M}{\rho_m}\right)^{\frac{1}{3}}$$

The Roche Limit for Fluid Body (accounting for deformation)

$$d_{\text{Roche,Fluid}} = 2.44 \cdot R \cdot \left(\frac{\rho_M}{\rho_m}\right)^{\frac{1}{3}}$$

Tidal Locking of the Moon

Tidal acceleration is asymmetric

$$a_{\text{tidal}} = \frac{F_{\text{tidal}}}{u} = \frac{2GM}{d^3} \cdot r$$

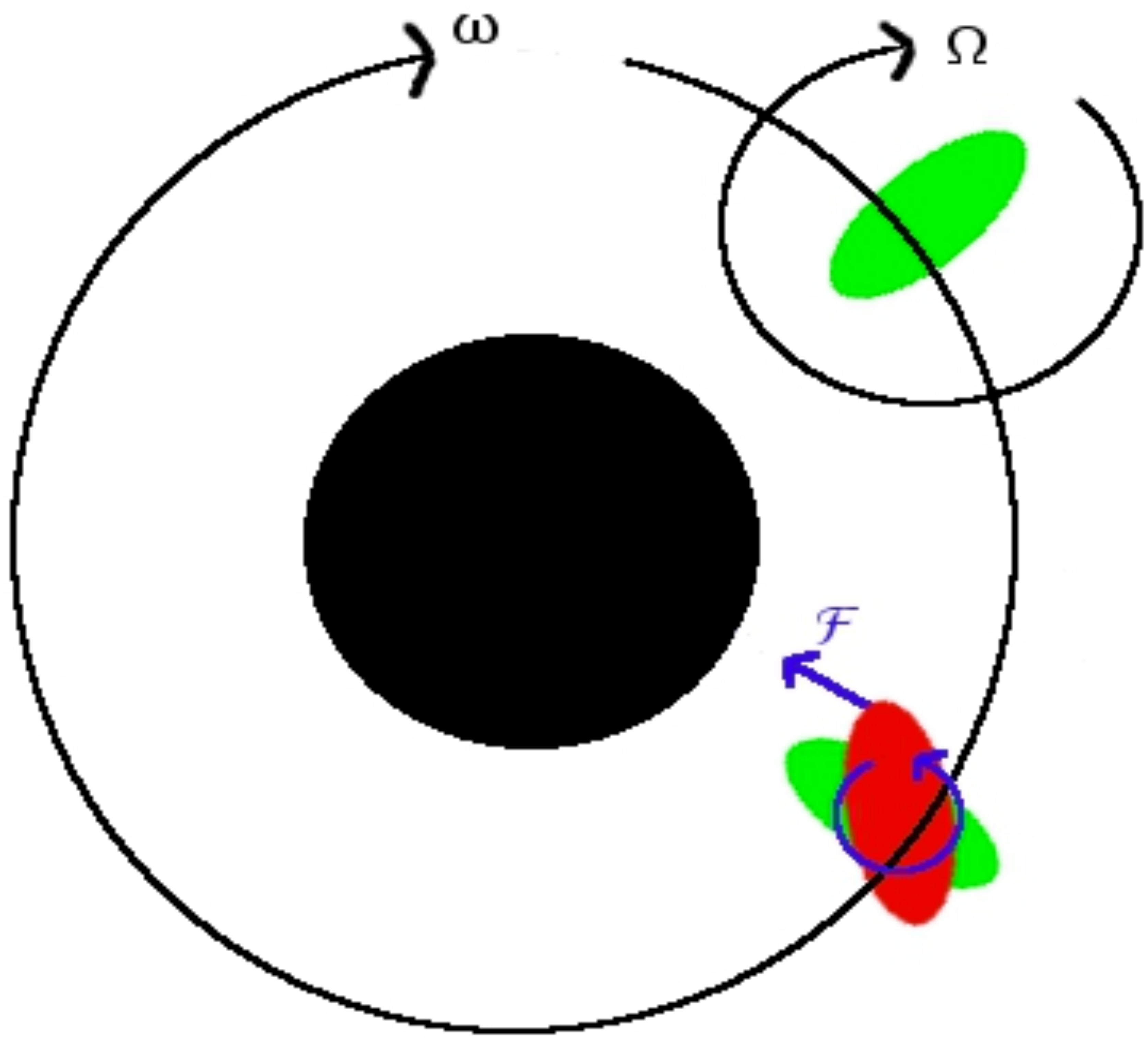
The tidal force on the Moon from the Earth does NOT equal to the tidal force on the Earth from the Moon, even though the distance d is the same in both cases, their $M \times r$ are different.

In other words, there is no Newton's 3rd law in tidal forces!

Given a mass ratio of 81 and a size ratio of 4, calculate the ratio of the tidal force from Earth to Moon and that from Moon to Earth

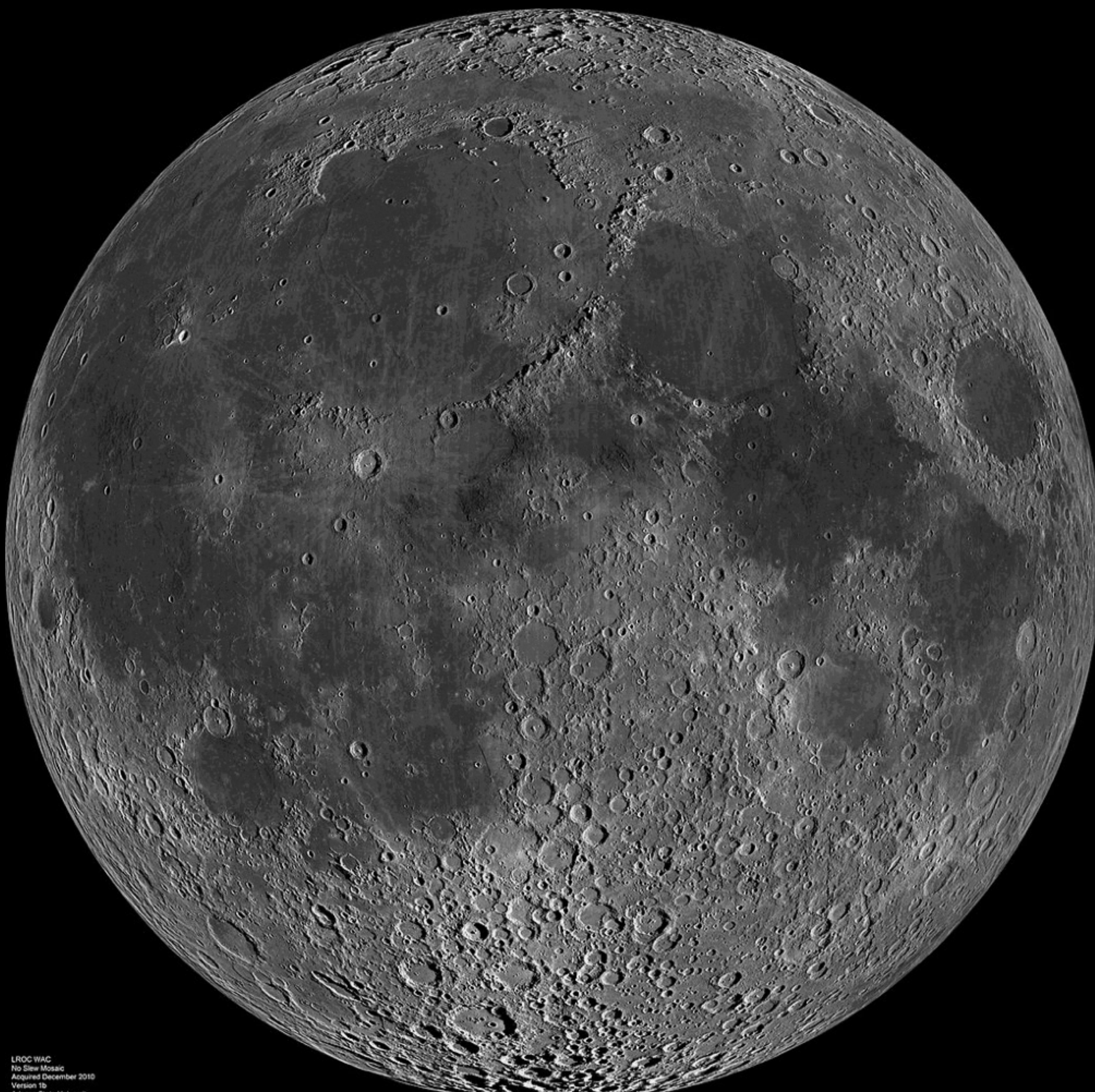
$$\frac{a_t(\text{Earth} \rightarrow \text{Moon})}{a_t(\text{Moon} \rightarrow \text{Earth})} = \frac{M_{\text{Earth}} r_{\text{Moon}}}{M_{\text{Moon}} r_{\text{Earth}}} = 81 \times 0.25 = 20$$

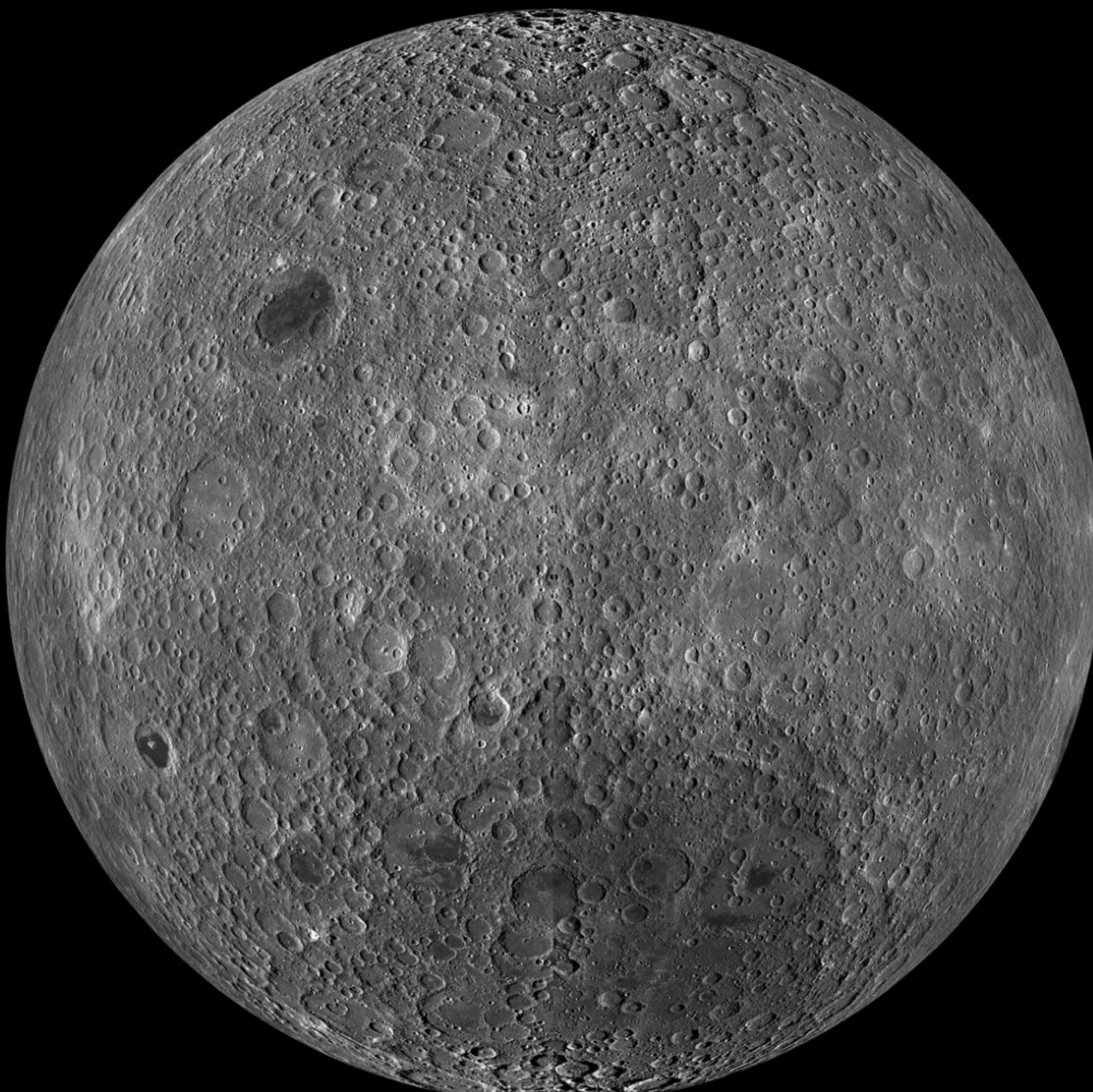
Tidal Locking of the Moon



From Earth, we can only see the near side of the Moon because of tidal locking

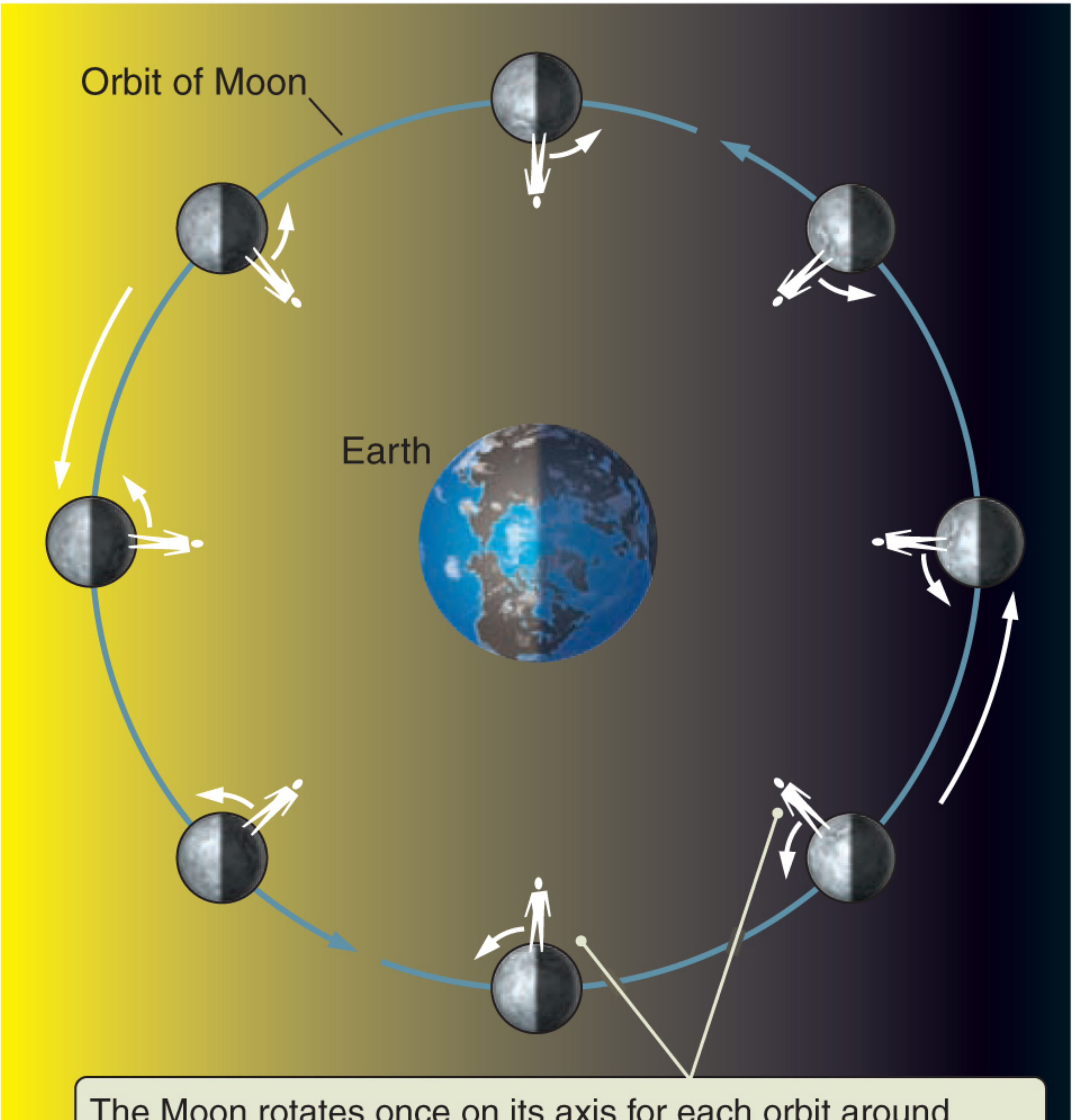
Near-Side & Far-Side of the Moon





LROC WAC
No Slew Mosaic
Acquired December 2010
Version 1b
Arizona State University

Synchronous Rotation of the Moon due to Tidal Locking



The Moon rotates once on its axis for each orbit around Earth, and so keeps the same face toward Earth at all times.

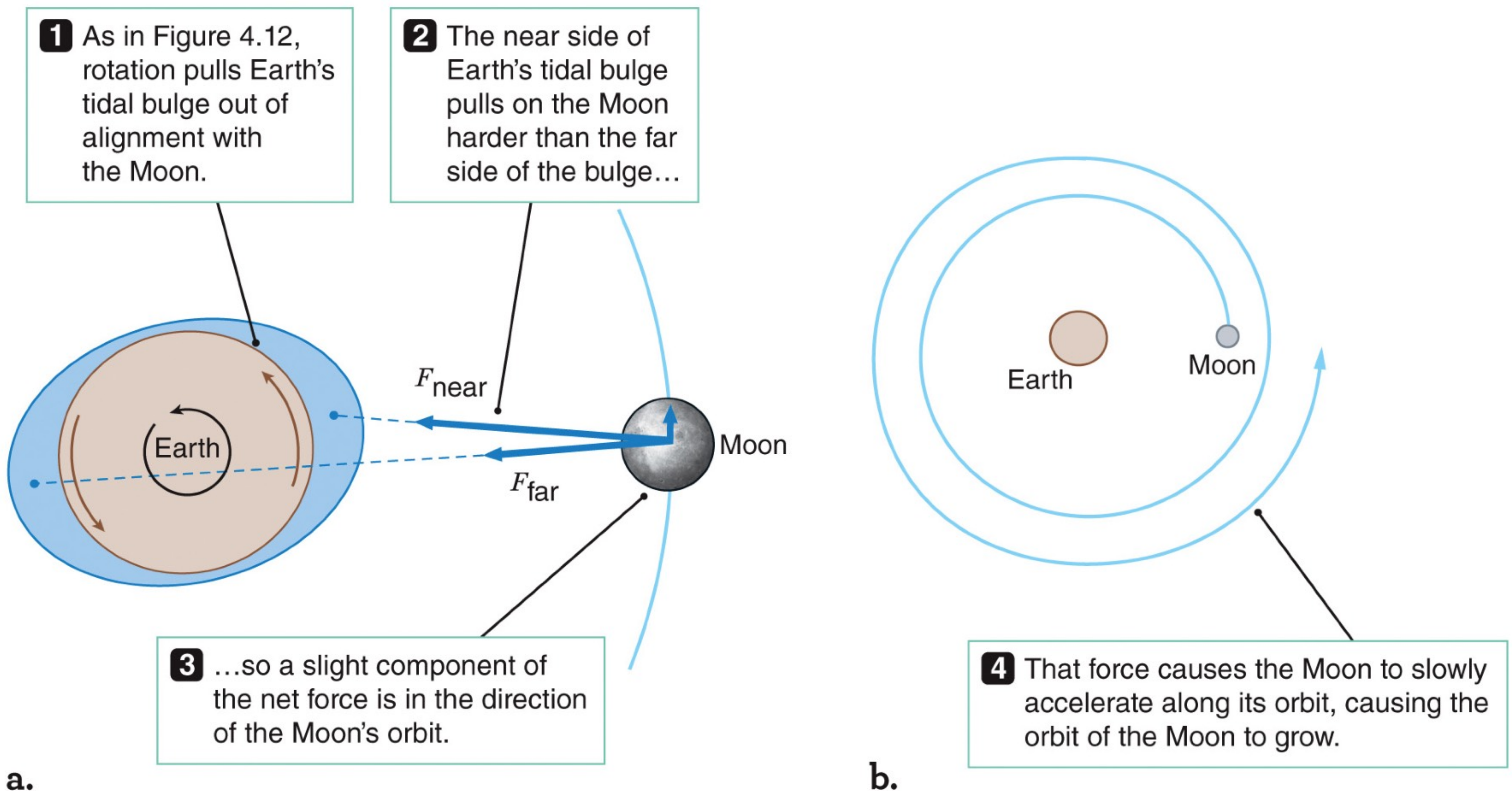
- We only see one face of the Moon.
- *Synchronous rotation*.
- Completes one full rotation in one full orbit around Earth.
- This is caused by tidal locking

Tides affect the solid part of Earth, too.

- § A gravitational pull can stretch and deform a solid body.
- § Results in friction, which generates heat.
- § Friction also opposes the rotation of Earth, causing Earth to very gradually **slow its rotation**.
- § Days lengthen by about 0.0015 seconds every century.

The Moon Is Getting Farther Away

- Because of tides, Earth is not a perfect sphere.
- Earth's leading edge creates an acceleration on the Moon in its orbit, resulting in a bigger orbit, 3.83 cm/year.
- The Moon's orbital period increases by 0.014 seconds per century.

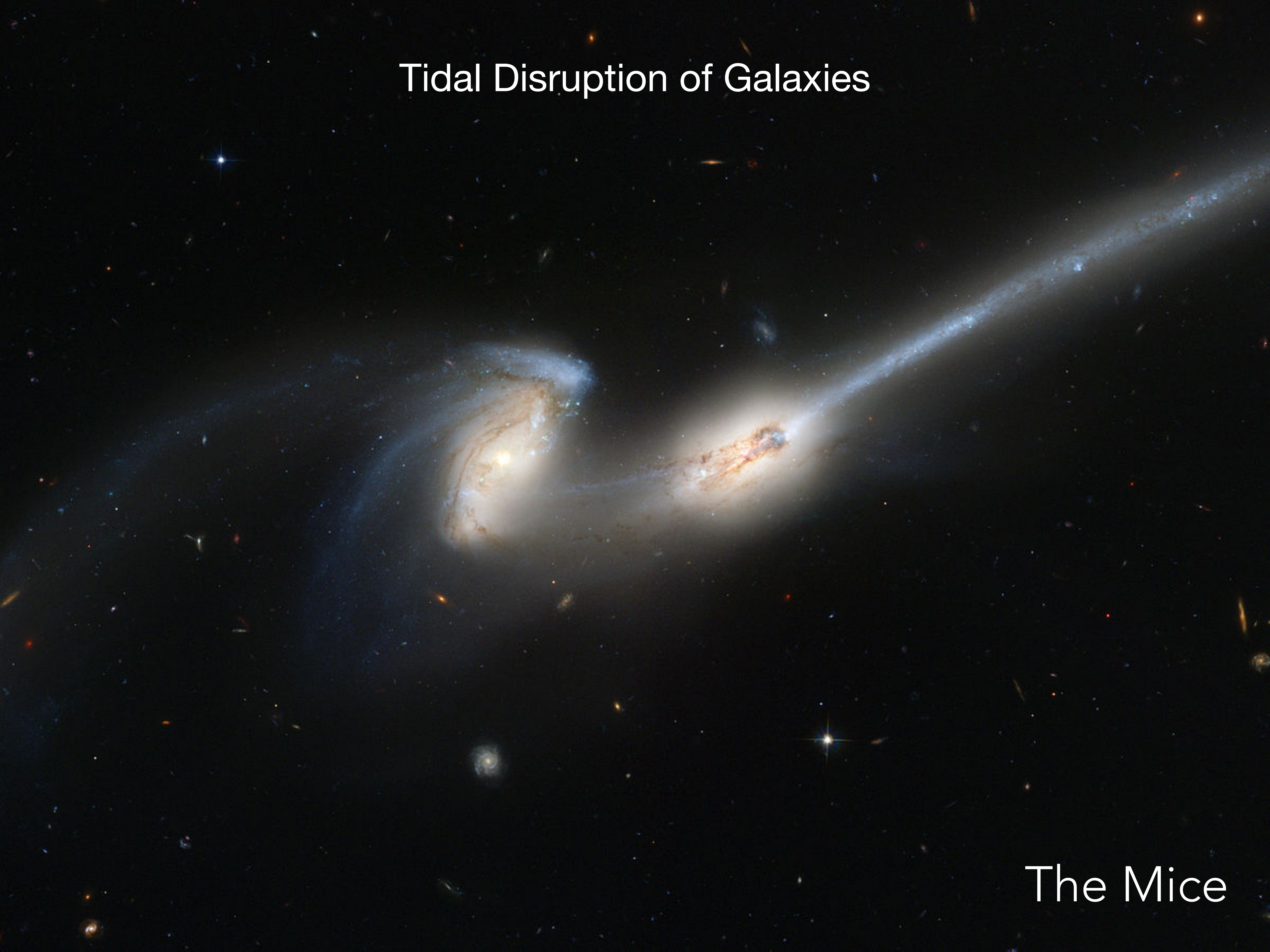


Tidal Disruption of Galaxies

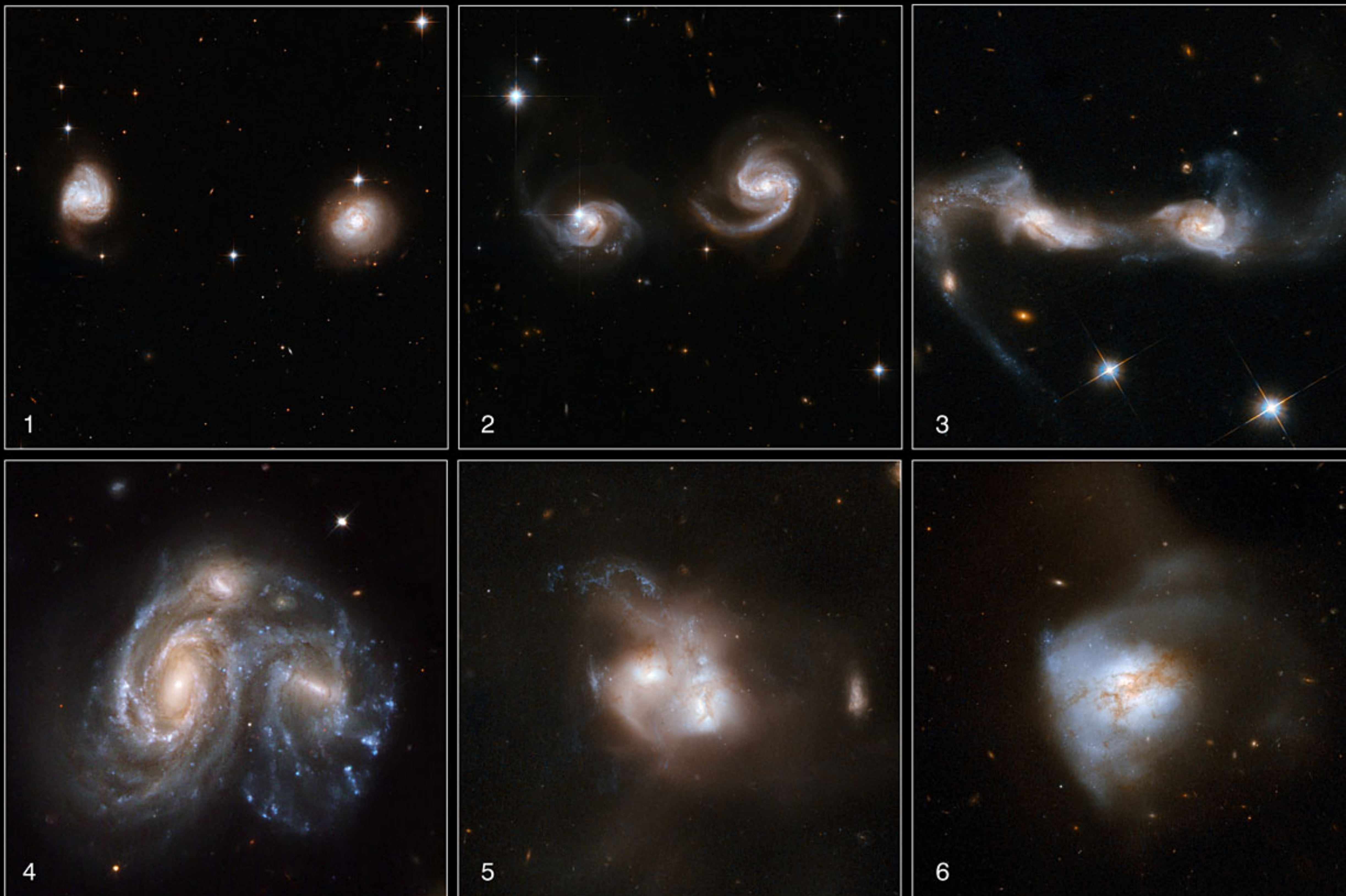
Galaxies are also loosely bound systems, so tidal disruptions are common

Stephan's Quintet (JWST image)

Tidal Disruption of Galaxies



Nearby Merging Galaxies



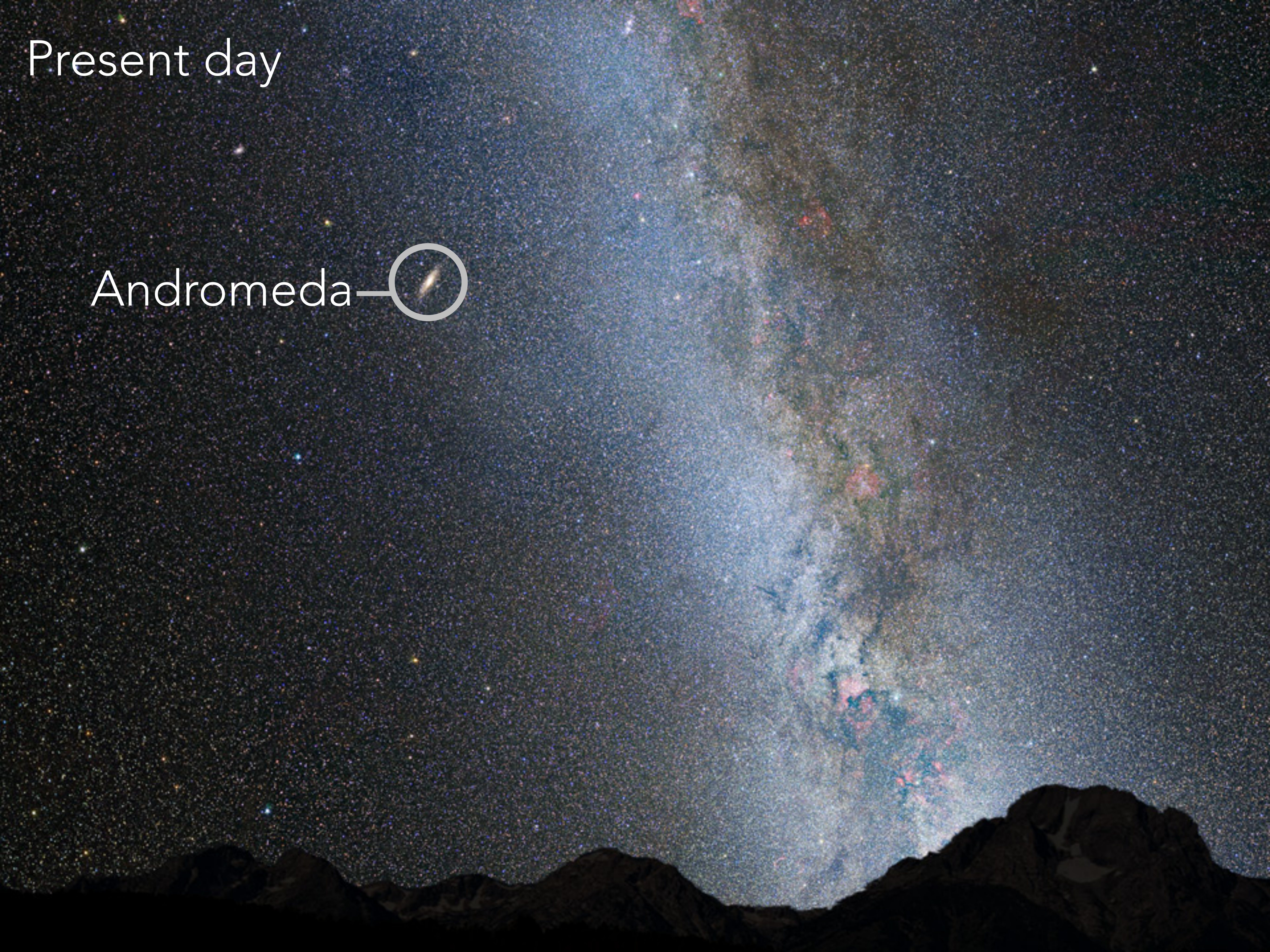
state-of-the-art numerical simulation today

Tidal Disruption of the Milky Way & the Andromeda Galaxy

The Milky Way and its neighbour Andromeda
are destined to merge within the next 5 billion years

...or so.

This simulation shows what might happen
to the gas (shades of blue) and newly formed stars (red)
when the two galaxies come together.


Present day
Andromeda


in 3.7 billion years


in 3.8 billion years


in 3.9 billion years