Chapter 5: Light & Spectroscopy
Chapter 5: The Nature of Light

• Light is a massless particle called photon
  • Photoelectric effect

• Light is an electromagnetic wave
  • Reflection, refraction, diffraction, and interference
  • Speed of light, wavelength, and frequency: \( c = \lambda \nu \)
  • Light ranges from radio waves to gamma rays: \( E = h\nu \)

• How light carries information?
  • source direction: light propagates through a straight line
  • luminosity/distance: distance square law \( F = L/4\pi d^2 \)
  • spectrum of light: distribution of energy as a function of wavelength
Chapter 5: Spectroscopy

• Blackbody radiation - Temperature \( T \)
  • Surface flux vs. \( T \): Stefan-Boltzmann law
  • Peak Radiation vs. \( T \): Wien’s displacement law
  • Application: Equilibrium temperature of planets
• Kirchhoff’s laws: the formation of three types of spectra
• What information do we get out of spectroscopy?
  • composition: specific atomic and molecular transitions
  • kinematics: Doppler shift & line broadening
  • cosmological distances: Hubble’s law
Light as a massless particle

photons
Einstein’s 1922 Nobel Price was awarded “for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect”
PHOTOELECTRIC EFFECT: SILICON SOLAR PANELS

Inside a photovoltaic cell

- Energy from light
- Transparent negative terminal
- Positive terminal
- Glass
- N-type layer (semiconductor)
- Junction
- P-type layer (semiconductor)
- Freed electrons
- Holes filled by freed electrons
- Electron flow (current)

Source: U.S. Energy Information Administration
PHOTOELECTRIC EFFECT: SILICON CAMERAS
Light as an EM wave

diffraction & interference
Interference: Double Slit

two narrow slits

projection screen
The challenge of directly imaging exoplanets
To see the planets, we need to block the star
Diffraction: Poisson Spot

- Screen with shadow of circular object.
- Object which casts a circular shadow
- Point light source
Use non-circular patterns on a Starshade to reduce undesired diffraction pattern (Poisson spot)
Use a Starshade to Study Exoplanets

Studying Other Worlds with the Help of a Starshade
PHOTOLITHOGRAPHY: MICROCHIP PRODUCTION

NOTE THE THINNER ASSIST FEATURES AROUND MAIN FEATURES

A photomask to be projected onto the Silicate wafer

Detailed patterns on the mask
Robert Noyce - “The Mayor of Silicon Valley”
Born in Burlington, Iowa; graduated from Grinnell College, Physics Major
Light as an EM wave

speed of light
If light is a wave, how fast does it propagate?

• **Galileo: 1607**
  - measured travel time between two lanterns spaced 2 km apart (couldn’t detect any time delay)

• **Roemer: 1676**
  - timed eclipses of Jupiter’s moons – noticed different timings because of changes in the Earth-Jupiter distance

• **Fizeau and Foucault: 1849-1862**
  - used stationary and rotating mirrors to deflect light enough to measure speed (~300,000 km/s)
Galileo’s round-trip time experiment (1607)

Galileo unsuccessfully attempted to measure the speed of light by asking an assistant on a distant hilltop to open a lantern the moment Galileo opened his lantern.

For hilltops separated by 10 km, round-trip time delay for light is only 66 microsec!
Galileo’s round-trip time experiment on the Moon (1969)

Lunar Ranging Retro Reflector from the Apollo 11 mission
Roemer’s Timing Experiment of Jovian satellites (1676)

- **Jupiter**
  - $d = 139,822$ km
  - $\rho = 1,326$ kg/m$^3$
  - $P_{\text{rot}} = 9.9$ hr

- **Io**
  - $d = 3,643$ km
  - $\rho = 3,528$ kg/m$^3$
  - $P = 1.8$ days

- **Europa**
  - $d = 3,121$ km
  - $\rho = 3,013$ kg/m$^3$
  - $P = 3.6$ days

- **Ganymede**
  - $d = 5,262$ km
  - $\rho = 1,936$ kg/m$^3$
  - $P = 7.2$ days

- **Callisto**
  - $d = 4,820$ km
  - $\rho = 1,834$ kg/m$^3$
  - $P = 16.7$ days
Roemer’s Timing Experiment of Jovian satellites (1676)

In 1676, Danish astronomer Olaus Røemer discovered that the exact time of eclipses of Jupiter’s moons varied based on how near or far Jupiter was to Earth.

This occurs because it takes varying amounts of time for light to travel the varying distance between Earth and Jupiter.

The change of eclipsing period as Earth moves away / towards Jupiter is a Doppler effect.
- In 1676, measured by Rømer by timing the eclipses of Jovian moons (He measured 225,000 km/s).
- Current best measurement at 299,792 km/s.
Speed of Light Demo

15 cm & 20 m optical fibers
Oscilloscope
Pulsed laser source
Speed of Light in Vacuum vs. in Medium

Light travels at a constant speed in vacuum

\[ c = 299,792.458 \text{ km/s} \]

Light travels slower in air, water, glass, etc.

→ refractive index: \( n = \frac{c}{v} \) (e.g., \( n = \frac{1.33}{1.44} \) for water/optical fiber)
Speed of Light & Wavelength in Medium

• Speed of light decreases when light enters from vacuum to a medium (air, water, glass, etc.)
• Because the frequency of light doesn’t change, this leads to a decrease in wavelength in medium
• Ground-based astronomical observations are carried out in air, so the wavelength measured in air need to be corrected by the refractive index to compare it with the wavelength in vacuum

\[ c_{\text{medium}} = \frac{c}{n_{\text{medium}}} \]
\[ \lambda_{\text{medium}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{medium}}} \]
Global internet backbone map: undersea fiber optics cables
Starlink Internet: low latency (20ms) with 12,000 Low Earth-orbit Satellites
Light as an EM wave

wavelength & frequency
Today, we understand that light has characteristics of both particles and waves. Light behaves according to the same equations that govern electric and magnetic fields that move at 300,000 km/s so light is also called *electromagnetic radiation*.

Electromagnetic radiation consists of oscillating electric and magnetic fields. The distance between two successive wave crests is called the **wavelength** and is designated by the letter $\lambda$. 
Wavelength, frequency, and speed

- **Wavelength** ($\lambda$): length between crests (unit: nm, um, ...)
- **Frequency** ($f$): number of waves that pass by per unit time (unit: Hz, MHz, ...)

Wave Speed = Wavelength x Frequency
Wavelength, frequency, and speed

- Wavelength and frequency are inversely proportional.
  - A long wavelength means low frequency.
  - A short wavelength means high frequency.
- The speed of light, \( c \), is constant.

\[
\text{Wavelength} = \frac{\text{Speed}}{\text{Frequency}} \quad \text{or} \quad \lambda = \frac{c}{f}
\]
The Wide Range in $\lambda$ and $f$: from radio to Gamma rays

- Visible: a small range of wavelengths that humans can see
- Red visible light = longest wavelength ($\lambda \sim 750$ nm)
- Violet visible light = shortest wavelength ($\lambda \sim 380$ nm)
- Gamma rays, X-rays, UV, visible, IR, microwave, radio
Practice: Wavelength-Frequency Conversion

\[ \text{Wavelength} = \frac{\text{Speed}}{\text{Frequency}} \quad \text{or} \quad \lambda = \frac{c}{f} \]

- \( c = 3 \times 10^8 \, \text{m/s} \)

- The 2.7 Kelvin Cosmic Microwave Background radiation peaks at a wavelength of 1 mm, what’s the frequency of the wave?

- The Very Large Array works in the frequency range between 1 GHz and 10 GHz. What’s the range in wavelength?
Example: Find the wavelength of the light wave coming from a radio station broadcasting on 770 AM (770 kHz):

\[
\text{Wavelength} = \frac{\text{Speed}}{\text{Frequency}} \quad \text{or} \quad \lambda = \frac{c}{f}
\]

Knowing the speed of light and one other variable, either the wavelength or frequency of the light in question, you can find the remaining quantity.

\[
\lambda = \frac{c}{f} = \left( \frac{3 \times 10^8 \text{ m/s}}{7.7 \times 10^5/\text{s}} \right) = 390 \text{ m}
\]
light carries energy
Einstein’s 1922 Nobel Price was awarded “for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect”
PHOTOELECTRIC EFFECT

(a) Light

\[ E = h\nu \]

Electron

(b) Slope = h

Metal

\[ E_m \]

\[ -q\Phi \]
Energy of Photons: Photoelectric Effect

• Result #1: if you increase intensity of light beam you get more of the same electrons

• Result #2: if you increase frequency of light beam you get higher energy electrons

→ Photon’s energy depends on its frequency

\[ E = hf = \frac{hc}{\lambda} \]

\( h \) is Planck’s constant (6.6e-34 Joule/Hz), 
\( f \) is frequency, \( E \) the energy; 
\( c \) is speed of light and \( \lambda \) is wavelength
Energy - Wavelength Conversion

- Energy is often given in units of electron-volt (eV), which is the amount of kinetic energy gained by a single electron accelerating through an electric potential difference of one volt.
- Wavelength is often given in units of micron (um).
- $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$, $h = 6.626 \times 10^{-34} \text{ J/Hz}$, $c = 3 \times 10^8 \text{ m/s}$, given $E = \frac{hc}{\lambda}$, calculate the wavelength (in micron) of photons with energies of 1 eV.

$$\lambda = 1.24 \mu\text{m} \left( \frac{E}{1 \text{ eV}} \right)$$
Energy - Wavelength Conversion

• Chandra X-ray observatory can detect photons with energies between 0.5 and 8 keV, what’s the wavelength range in nm?

\[ \lambda = 1.24 \ \mu m \left( \frac{E}{1 \text{ eV}} \right) \]
Inverse Square Law of Flux

- **Luminosity** is the total amount of energy per unit time (i.e., power) emitted by the source (unit: Watt = Joule/s)
- **Flux** is the amount of arriving energy per unit time per unit area (unit: Watt/m²) at a distance \( d \) from source
- **Flux** decreases as the distance from the source increases, obeying an inverse square law:

\[
F = \frac{L}{4\pi d^2}
\]
Luminosity does NOT depend on distance

- **Luminosity** is the total power *(energy per unit time)* emitted by the source (SI unit: Watt = Joule/s; cgs unit: erg/s)
- Equations below assume the source emit light *isotropically*

\[
L = F(d) \times 4\pi d^2 = F(\text{surface}) \times 4\pi R^2
\]
Practice: solar constant calculation

• The Sun has a **luminosity** of 3.86e26 Watts
• The Earth is at the mean **distance** of 1.5e11 m (1 AU)
• What is the flux at the distance of the Earth for an area **perpendicular** to the Sun direction?

\[ F = \frac{L}{4\pi d^2} \]
Flux & Received Power

- **Flux** is the amount of arriving energy per unit time per unit area (flux = power per unit area; unit: Watt/m²)
- What’s the total amount of received power in a given area?

\[ P = F \times \text{Projected Area} \]

Projected Area = Surface Area * sin(altitude)

The same amount of sunlight strikes the ground at a shallower angle and so is spread out over a larger area.
Practice: Total Arrived Power from the Sun

- The flux of the Sun at the Earth’s distance is 1.4 kW/m²
- The Earth has a radius of 6400 km
- What is the total solar power intercepted by the Earth?

\[ P = F \times \text{Projected Area} \]

Answer: 1.8e17 W = 180 Billion MegaWatts
Palo Verde Generation Station (largest nuclear power plant in US):
3.8 MegaWatt, located in Arizona
The Three Gorges Dam Power Station: 22,500 MegaWatts
Blackbody Emission

Temperature & Spectral Shape
Temperature and Emitted Light

- **Temperature** is a measure of internal energy—and the average kinetic energy (speed) of atoms and molecules.
- **Kelvin scale**: water freezes/boils at 273 K/373 K
- **Absolute zero**: thermal motion stops

Gas contains atoms and molecules moving about in all directions.

Temperature is related to the average speed of the gas particles.

Doubling the gas temperature in a fixed box increases the average particle speed by 1.4x.
Different dense objects at the same temperature emit similar spectra of light.

- Molten rock: 1,200 °C
- Molten steel: 1,200 °C
The hotter the object, the bluer its spectrum becomes.
To quantify the color of objects more precisely, we do spectroscopy. A Spectrum is a plot of Flux Density (flux per unit wavelength range) vs. Wavelength.
The Spectrum of the Sun from UV to IR

Sunlight spectrum in space as a function of wavelength

- **Peak**
- **Exponential tail**
- **Power-law tail**
The Planck Curve: Intensity (Brightness) vs. Temperature

\[ I_\lambda = B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \]

This gives power emitted per unit projected surface area per unit solid angle per unit wavelength.

Intensity is also called Brightness or Spectral Irradiance.
The Planck Function: Two Different Forms

\[ I_\lambda = B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \]

this gives **power** emitted per unit projected surface area per unit solid angle per unit wavelength

\[ I_\nu = B(\nu, T) \equiv \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \]

this gives **power** emitted per unit projected surface area per unit solid angle per unit frequency
The Planck Curve

\[ I_\lambda = B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{kT}} - 1} \]

- What happens when \( T \) increases?
  - Peak shifts to shorter wavelength
  - Total area under the Planck curve increases
  - Flux density increases at all wavelengths
Using an infrared camera to measure temperature

Why can’t we use an optical camera for this?
The Blackbody Spectrum of the Cosmic Microwave Background

Cosmic background radiation has a Planck spectrum.

Planck spectrum, $T = 2.73$ K

COBE measurements of the CMB

The uncertainties in the measurements are much less than the thickness of the line.
Blackbody Emission

Temperature & Peak Wavelength
Making an object hotter makes it more luminous...
...and shifts the peak of its Planck spectrum to shorter wavelengths.

- $T = 6000 \, \text{K}$, $\lambda_{\text{peak}} = 480 \, \text{nm}$
- $T = 5000 \, \text{K}$, $\lambda_{\text{peak}} = 580 \, \text{nm}$
- $T = 4000 \, \text{K}$, $\lambda_{\text{peak}} = 730 \, \text{nm}$
- $T = 3000 \, \text{K}$, $\lambda_{\text{peak}} = 970 \, \text{nm}$
- $T = 2000 \, \text{K}$, $\lambda_{\text{peak}} = 1450 \, \text{nm}$
Wien’s Displacement Law

- The peak wavelength of a blackbody is inversely proportional to its temperature.
  \[
  \lambda_{\text{peak}} = \frac{2.9 \text{ mm K}}{T}
  \]
- Peak wavelength \( \lambda_{\text{peak}} \) is the wavelength of light of a blackbody that is emitted the most.
- Here the wavelength is in nanometers and the temperature is in kelvins.
- “Hotter means bluer.”
Wien’s Displacement Law - Derivation

• Start with Planck function, do derivative over lambda (this is the gradient of the curve), find where the gradient equals zero

\[ u_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}. \]

Differentiating \( u(\lambda, T) \) with respect to \( \lambda \) and setting the derivative equal to zero gives:

\[ \frac{\partial u}{\partial \lambda} = 2hc^2 \left( \frac{hc}{kT\lambda^7} \frac{e^{hc/\lambda kT}}{(e^{hc/\lambda kT} - 1)^2} - \frac{1}{\lambda^6} \frac{5}{e^{hc/\lambda kT} - 1} \right) = 0, \]

which can be simplified to give:

\[ \frac{hc}{\lambda kT} \frac{e^{hc/\lambda kT}}{e^{hc/\lambda kT} - 1} - 5 = 0. \]

By defining:

\[ x \equiv \frac{hc}{\lambda kT}, \]

the equation becomes one in the single variable \( x \):

\[ \frac{xe^x}{e^x - 1} - 5 = 0. \]

\[ \lambda_{\text{peak}} = \frac{hc}{xkT} = (2.897 \text{ mm} \cdot \text{K}) / T. \]
Practice: Wien’s Displacement Law

• The Sun has a mean surface temperature of 5800 K, calculate the wavelength at which the emission peaks. Give your answer in nanometer (nm).

\[ \lambda_{\text{peak}} = \frac{2.9 \text{ mm K}}{T} \]

Answer: 500 nm
The Spectrum of the Sun outside of the Earth’s atmosphere

\[ \lambda = 525 \text{ nm} \]
Practice: Wien’s Displacement Law

• The Earth has a mean surface temperature of 288 K, calculate the wavelength at which the emission peaks. Give your answer in micron (μm).

\[ \lambda_{\text{peak}} = \frac{2.9 \text{ mm K}}{T} \]

Answer: 10,000 nm = 10 micron
Blackbody Emission
Temperature & Surface Flux
Two Planck Curves

Note large change in integrated radiation (area under curve) with small change in peak wavelength
Surface Flux – Stefan-Boltzmann Law

- **Flux** is the total amount of energy emitted per square meter every second (the luminosity per area).

\[
F = \sigma_{SB} T^4
\]

\[
T(K) = T(C) + 273.15
\]

\[
\sigma_{SB} = \frac{2\pi^5 k^4}{15c^2 h^3} = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}
\]

where \( T \) is the temperature in K, \( F \) is the flux, and \( \sigma \) (sigma) is called the **Stefan-Boltzmann constant**.

- Hotter objects emit *much* more energy (per square meter per second) than cool objects: *surface flux \( \sim T^4 \)*
Stefan-Boltzmann Law - Derivation

• Start with intensity given by the Planck function, first integrate over 2PI solid angle, then integrate over frequency from zero to infinity, and

**Intensity**: power emitted per \( \text{unit projected surface area per unit solid angle per unit frequency} \)

\[
I_\nu = B(\nu, T) \equiv \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}
\]

**Surface Flux**: power emitted per \( \text{unit surface area} \)

\[
F = \int_{\nu=0}^{\infty} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} B(\nu, T) \cos \theta \sin \theta \, d\theta \, d\phi \, d\nu
\]

To evaluate this integral, do a substitution,

\[
u = \frac{h\nu}{kT}
\]

\[
du = \frac{h}{kT} \, d\nu
\]

\[
F = \frac{2\pi h}{c^2} \left( \frac{kT}{h} \right)^4 \int_0^\infty \frac{u^3}{e^u - 1} \, du.
\]
With the Stefan-Boltzmann law, find Earth’s surface flux using its average temperature of +15 C.

\[ F = \sigma_{SB} T^4 \]

\[ T(K) = T(C) + 273.15 \]

\[ \sigma_{SB} = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \]

Answer: 390 W/m²

About a quarter of the Solar constant (1366 W/m²)

Is this a coincidence?
Application of Blackbody Radiation

Equilibrium Temperatures of Planets
heating & cooling - CPU temperature
what happens to the CPU when the cooling fan stops working?

Its temperature increases until it stabilizes again (reaching a new equilibrium).
Calculating a Planet’s Equilibrium Temperature

**Heating**: The solar power received by the planet depends on its heliocentric distance, modulated by its albedo (*a*; which is a measure of its reflectivity).

\[
\begin{align*}
\left( \frac{\text{Energy absorbed by the planet each second}}{\text{area of the planet}} \right) &= \left( \frac{\text{Absorbing}}{\text{Solar Flux}} \right) \times \left( \frac{\text{Fraction of sunlight absorbed}}{} \right) \\
&= \pi R_{\text{planet}}^2 \times \frac{L_{\text{Sun}}}{4\pi d^2} \times (1 - a)
\end{align*}
\]
Calculating a Planet’s Equilibrium Temperature

- **Cooling**: The power radiated away from the planet depends on its **surface area** and its **surface temperature**.
- Here we had assumed *uniform* surface temperature, what if this wasn’t the case?

\[
\left( \frac{\text{Energy radiated}}{\text{by planet each second}} \right) = \left( \frac{\text{Surface area of planet}}{\text{per square meter}} \right) \times \left( \frac{\text{Energy radiated}}{\text{per second}} \right)
\]

\[= 4\pi R^2 \times \sigma T^4\]
Calculating a Planet’s Equilibrium Temperature

The **equilibrium temperature** of a planet is the temperature at which the cooling rate balances the heating rate:

\[
\left( \begin{array}{c}
\text{Energy radiated} \\
\text{by the planet each second}
\end{array} \right) = \left( \begin{array}{c}
\text{Energy absorbed} \\
\text{by the planet each second}
\end{array} \right)
\]

\[
4\pi R_{\text{planet}}^2 \sigma T^4 = \pi R_{\text{planet}}^2 \frac{L_{\text{Sun}}}{4\pi d^2} (1 - a)
\]

- Solving for \( T \):

\[
T = \left( \frac{L_{\text{Sun}} (1 - a)}{16 \sigma \pi d^2} \right)^{1/4}
\]
Planetary Temperatures: Predictions vs. actual values

- The balance between solar heating and radiative cooling predicts the equilibrium surface temperature of planets without atmospheres.
- The presence of atmosphere increases planet's surface temperature via greenhouse effect
- Does this imply that heating ≠ cooling for planets with atmosphere?
The Earth’s Thermal Emission Seen from Space
Equilibrium Temperature with Atmosphere

- Greenhouse gases are nearly transparent to incoming solar radiation (mostly optical light), but they strongly absorb the planet’s reemitted thermal emission (mid-IR light).
- They reduce the ability of the planet to cool, by a factor of $1/(1 + \tau)$, where $\tau$ is the mean opacity of the atmosphere in mid-IR.

\[
\begin{pmatrix}
\text{Energy radiated by the planet each second} \\
\text{each second}
\end{pmatrix}
= \begin{pmatrix}
\text{Energy absorbed by the planet each second} \\
\text{each second}
\end{pmatrix}
\]

\[
\frac{4\pi R_{\text{planet}}^2 \sigma T^4}{1 + \tau} = \pi R_{\text{planet}}^2 \frac{L_{\odot}}{4\pi d^2} (1 - a)
\]

- Solving for $T$:

\[
T^4 = \frac{L_{\odot}(1 - a)}{16\pi \sigma d^2} (1 + \tau)
\]
Practice: Greenhouse effect

- For fast-rotating planets without greenhouse gas:
  \[ T^4 = \frac{L_\odot (1 - a)}{16 \pi \sigma_{SB} d^2} \]

- For fast-rotating planets with greenhouse gas:
  \[ T^4 = \frac{L_\odot (1 - a)}{16 \pi \sigma_{SB} d^2} (1 + \tau) \]

  where \( \tau \) is the mean opacity of the atmosphere in mid-IR

- Suppose you calculated the Earth’s equilibrium temperature of 260 K (-13 C) using the first equation, but the actual mean temperature is 288 K (+15 C). What is the mean opacity of the Earth’s atmosphere?

- When the Earth is 2 deg C hotter than today (290 K), what will be the mean opacity of the atmosphere? How much percent was the increase compared to today’s opacity?
Blackbody Emission recap
The Planck Curve

\[ I_\lambda = B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \]

- What happens when \( T \) increases?
  - Peak shifts to shorter wavelength
  - Total area under the Planck curve increases
  - Flux density increases at all wavelengths
Using an infrared camera to measure temperature

Why can’t we use an optical camera for this?
Equations of Blackbody Emission

Planck’s Function:

\[ I_\lambda = B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5} \left( \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \right) \]

Wien’s Displacement Law:

\[ \lambda_{\text{peak}} = \frac{2.9 \, \text{mm} \, \text{K}}{T} \]

Stefan-Boltzmann Law

\[ F = \sigma_{\text{SB}} T^4 \]

\[ T(K) = T(C) + 273.15 \]

\[ \sigma_{\text{SB}} = \frac{2\pi^5k^4}{15c^2h^3} = 5.67 \times 10^{-8} \, \text{W} \, \text{m}^{-2} \, \text{K}^{-4} \]

Equilibrium Temperature:

\[ (T_{\text{eq}}^{\text{atm}})^4 = \frac{L_\odot(1 - a)}{16\pi\sigma_{\text{SB}}d^2}(1 + \tau) \]
Equilibrium Temperature of Fast-Rotating Planets

- For fast-rotating planets with greenhouse gas:
  \[(T_{eq}^{\text{atm}})^4 = \frac{L_\odot (1 - a)}{16\pi\sigma_{SB}d^2}(1 + \tau)\]

- The surface temperature of the planet increases as (a) the distance to the Sun decreases, and/or (b) the albedo of the planet decreases, and/or (c) the greenhouse gas increases.
Practice: Greenhouse effect

- For fast-rotating planets without greenhouse gas:
  \[
  (T_{eq})^4 = \frac{L_\odot (1 - a)}{16\pi\sigma_{SB}d^2}
  \]

- For fast-rotating planets with greenhouse gas:
  \[
  (T_{eq}^{\text{atm}})^4 = \frac{L_\odot (1 - a)}{16\pi\sigma_{SB}d^2} (1 + \tau)
  \]

  where \(\tau\) is the mean opacity of the atmosphere in mid-IR.

- Suppose you calculated the Earth’s equilibrium temperature of 260 K (-13 C) using the first equation, but the actual mean temperature is 288 K (+15 C). What is the mean mid-IR opacity of the Earth’s atmosphere?

- When the Earth is 2 deg C hotter than today (290 K), what will be the mean mid-IR opacity of the atmosphere? How much percent was the increase compared to today’s opacity?
OK, Earth is heating up because of the increased mid-IR opacity, how can we cool it down?
Radiator Fins of Planet *Trantor* in *Foundation & Empire* (Asimov 1952)
The Snow Line of the Solar System

$$T_{eq} = \left( \frac{L_\odot(1 - a)}{16\pi\sigma_{SB}d^2} \right)^{\frac{1}{4}} = 255 \ K \ d_{AU}^{-0.5} \left( \frac{1 - a}{1 - 0.3} \right)^{\frac{1}{4}}$$

At 2.5 AU, $T_{eq} = 170$ K for albedo = 0.1

Inner Rocky Planets & Asteroids (too hot to have water ice on surface)

Outer Gas Planets, Icy Comets and Asteroids (water ice in atmospheres & on surface)
Thermodynamic properties of water

Sublimation of ice
Circumstellar Habitable Zone: the distance range that could support liquid water under sufficient atmospheric pressure.
Three Types of Spectra

Kirchhoff’s Laws
Kirchhoff’s Three Laws of Spectroscopy

**Law 1** A hot opaque body, such as a perfect blackbody, or a hot, dense gas produces a *continuous spectrum* -- a complete rainbow of colors with without any specific spectral lines.
Law 3  A cloud of cool gas in front of a source of a continuous spectrum produces an absorption line spectrum - a series of dark spectral lines among the colors of the continuous spectrum.
Kirchhoff’s Laws: Continuous and Absorption Spectra
Absorption Lines in Quasar Spectra Reveal the Intergalactic Medium
Law 2  The same gas cloud produces an **emission line spectrum** - a series of bright spectral lines against a dark background - when viewed sideways
Example Emission Line Spectra

- **Unknown gas**
- **Helium**
- **Oxygen**
- **Neon**
- **Argon**
- **Xenon**
Summary: Kirchhoff’s Laws
Each chemical element produces its own unique set of spectral lines.
Absorption lines in the Solar spectrum indicate the presence of ionized iron (Fe II)
Emission Lines in the Spectra of Planetary Nebulae

[Diagram showing emission lines such as [OIII], Hα, [NII], and [ArIII].]
speed-wavelength-frequency relation
\[ c = \lambda f \]

Energy of a photon
\[ E = hf \]

speed of light in medium
\[ c_{\text{medium}} = \frac{c}{n_{\text{medium}}} \]

Inverse distance square law of received flux
\[ L = F(d) \times 4\pi d^2 \]

Total received power from radiative flux
\[ P = F \times \text{Projected Area} \]
Planck’s Function: \[ I_\lambda = B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{kT}} - 1} \]

Wien’s Displacement Law: \[ \lambda_{\text{peak}} = \frac{2.9 \text{ mm K}}{T} \]

\[ F = \sigma_{\text{SB}} T^4 \]
\[ T(K) = T(C) + 273.15 \]
\[ \sigma_{\text{SB}} = \frac{2\pi^5k^4}{15c^2h^3} = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \]

Equilibrium Temperature: \[ \left( T_{\text{eq}}^{\text{atm}} \right)^4 = \frac{L_\odot(1-a)}{16\pi\sigma_{\text{SB}}d^2}(1 + \tau) \]