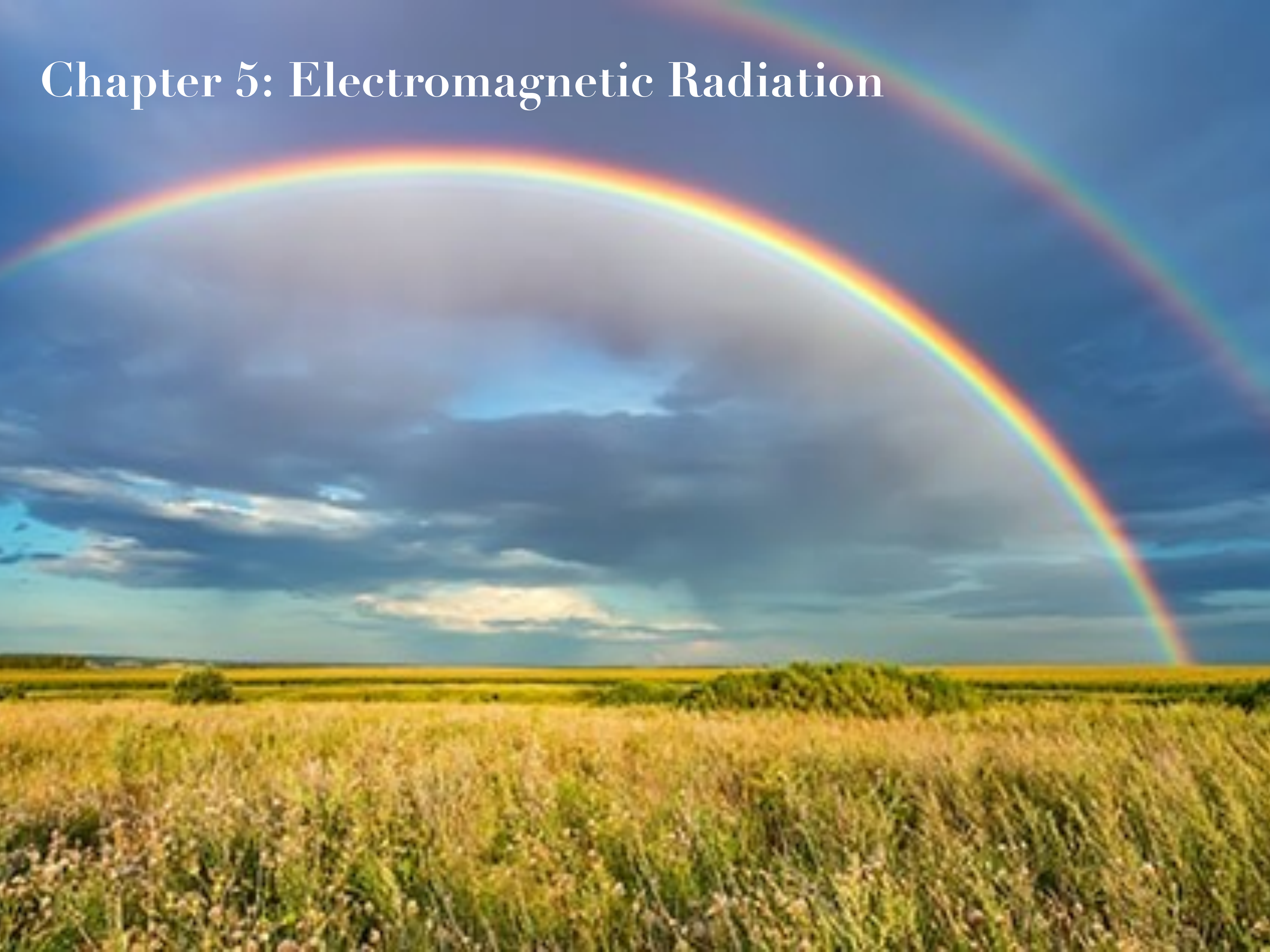
Chapter 5: Electromagnetic Radiation



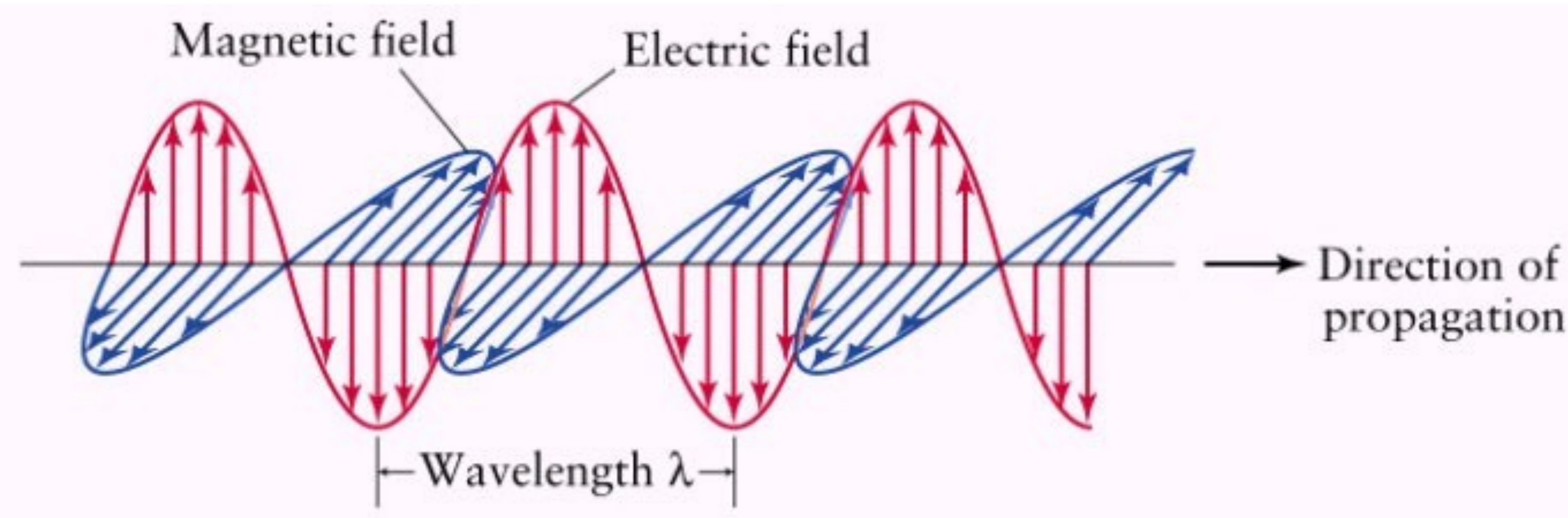
Part I: The Nature of Light

- Light is an electromagnetic wave
  - diffraction and interference
  - **Speed of light, wavelength, and frequency**: $c = \lambda\nu$
- Light is a massless particle (photon)
  - Photoelectric effect, reflection, and refraction: $\lambda_{\text{medium}} = \lambda_{\text{vacuum}}/n_{\text{medium}}$
  - **Energy of Light**: $E = h\nu$, **Momentum of Light**: $E^2 = (pc)^2 + (m_0c^2)^2$
- How light carries information?
  - source direction: light propagates through a straight line
  - luminosity / distance: **distance square law** $F = L/4\pi d^2$
  - **spectrum** of light: distribution of energy as a function of wavelength

Light as an EM wave

wavelength & frequency

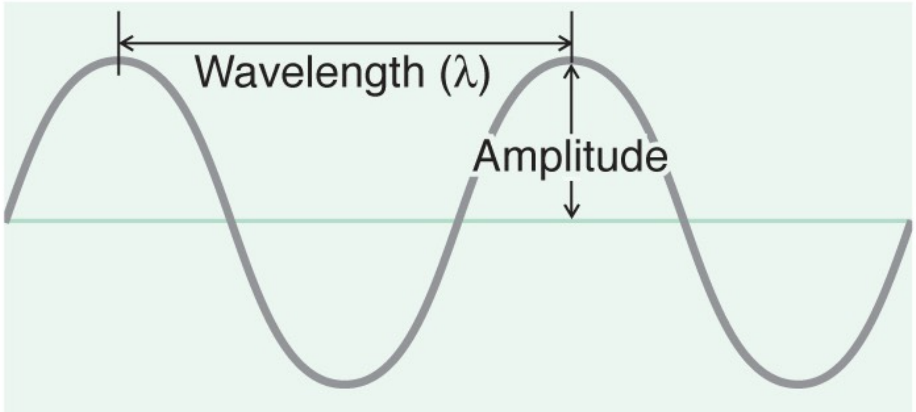
Light consists of self-propagating and oscillating electric and magnetic fields, so light is also called ***electromagnetic radiation***.



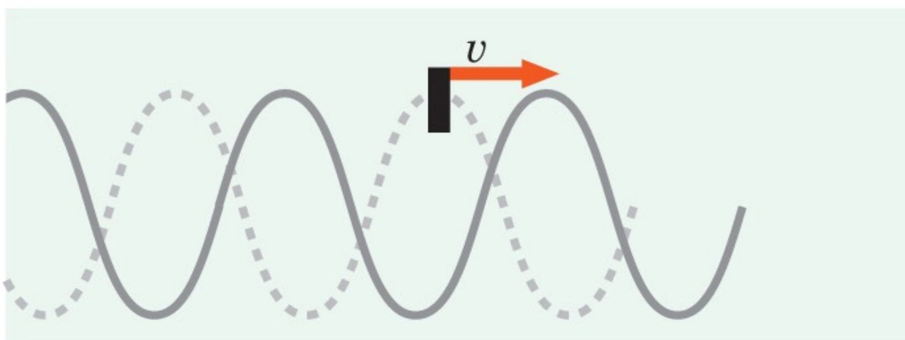
The distance between two successive crests is called ***wavelength*** *(λ).*
The frequency of the oscillation measured at a fixed point is the ***frequency*** *(ν) of the EM radiation*

Light does not need a medium to propagate.

Wavelength, frequency, and speed



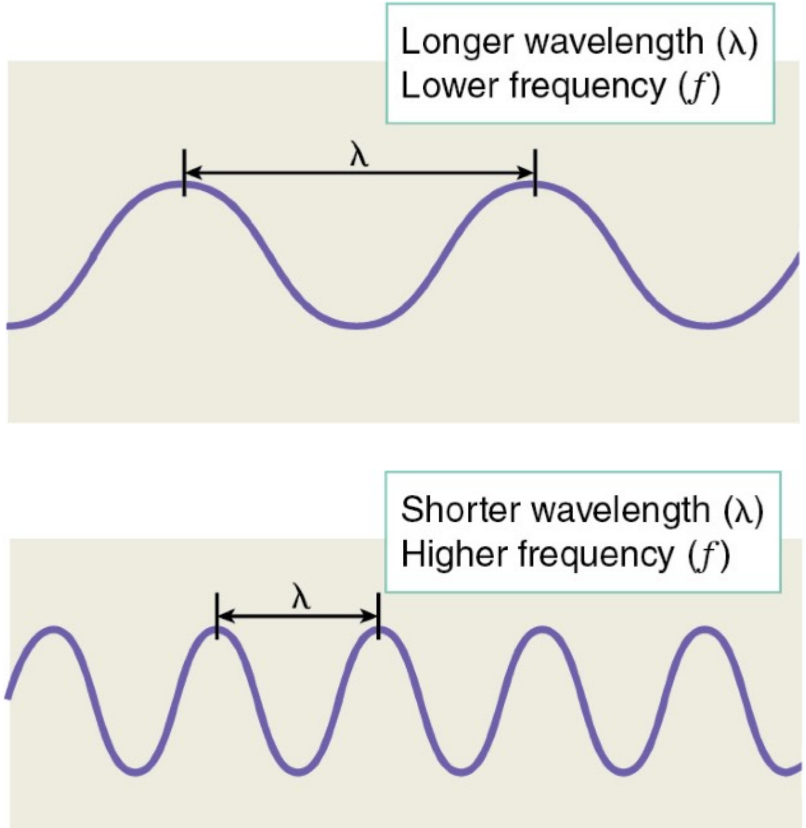
- *Wavelength* (λ): length between crests (unit: nm, um, …)



- *Frequency* (ν): number of waves that pass by per unit time (unit: Hz, MHz, …)

Wave Speed = Wavelength x Frequency

Wavelength, frequency, and speed

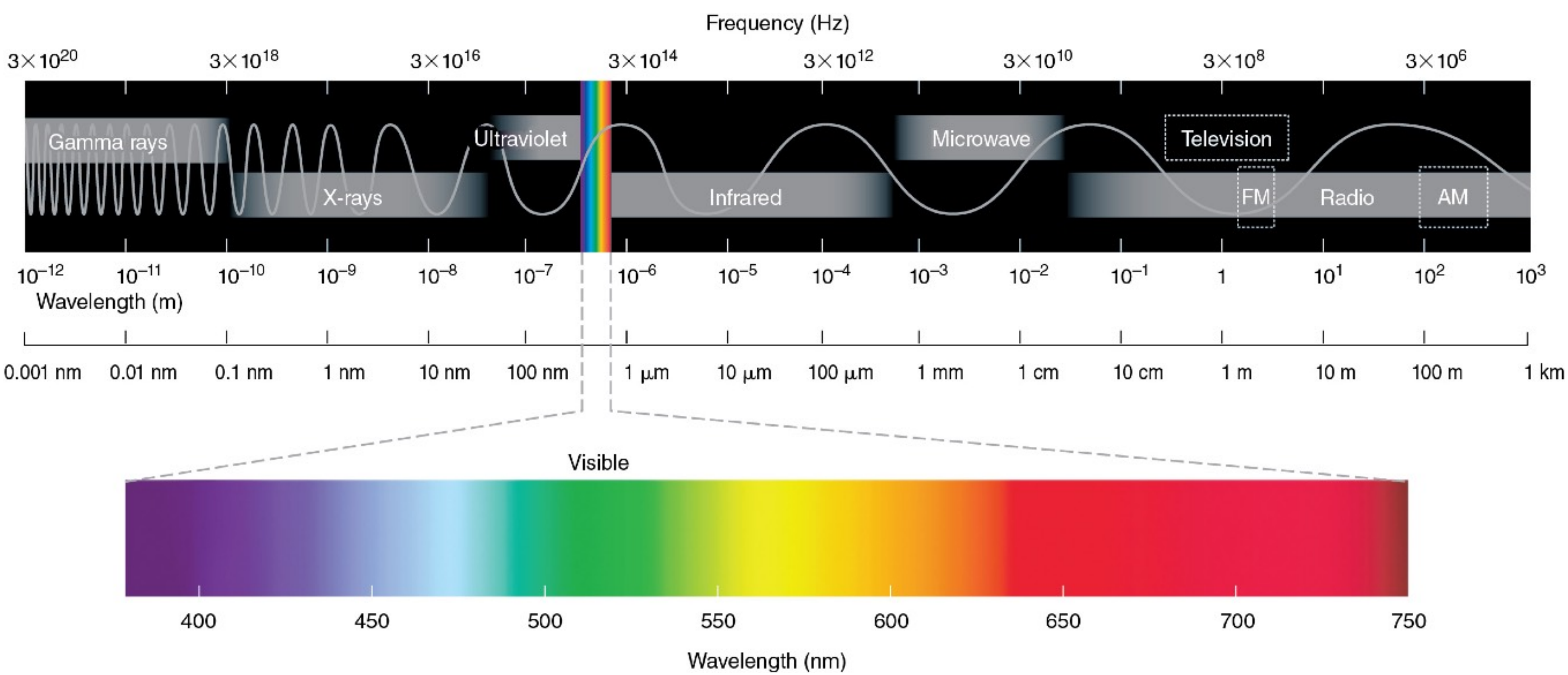


- Wavelength and frequency are inversely proportional.
  - A long wavelength means low frequency.
  - A short wavelength means high frequency.
- That is because the speed of light, *c*, is constant, and there is a simple relation among the three properties:
  - $c = \lambda\nu$

$$\text{Wavelength} = \frac{\text{Speed}}{\text{Frequency}}$$

The Wide Range in λ and ν: from radio to Gamma rays

- Visible: a small range of wavelengths that humans can see
- Red visible light = longest wavelength (λ ~ 750 nm)
- Violet visible light = shortest wavelength (λ ~ 380 nm)
- Gamma rays, X-rays, UV, visible, IR, microwave, radio



Practice: Wavelength-Frequency Conversion

$$\text{Wavelength} = \frac{\text{Speed}}{\text{Frequency}}$$

- c = 3e8 m/s
- The 2.7 Kelvin Cosmic Microwave Background radiation peaks at a wavelength of 1 mm, what's the frequency of the wave?
- The Very Large Array works in the frequency range between 1 GHz and 10 GHz. What's the range in wavelength?

Working It Out 5.1: Working with EM Radiation

- Example: Find the wavelength of the light wave coming from a radio station broadcasting on 770 AM (770 kHz):

$$\text{Wavelength} = \frac{\text{Speed}}{\text{Frequency}}$$

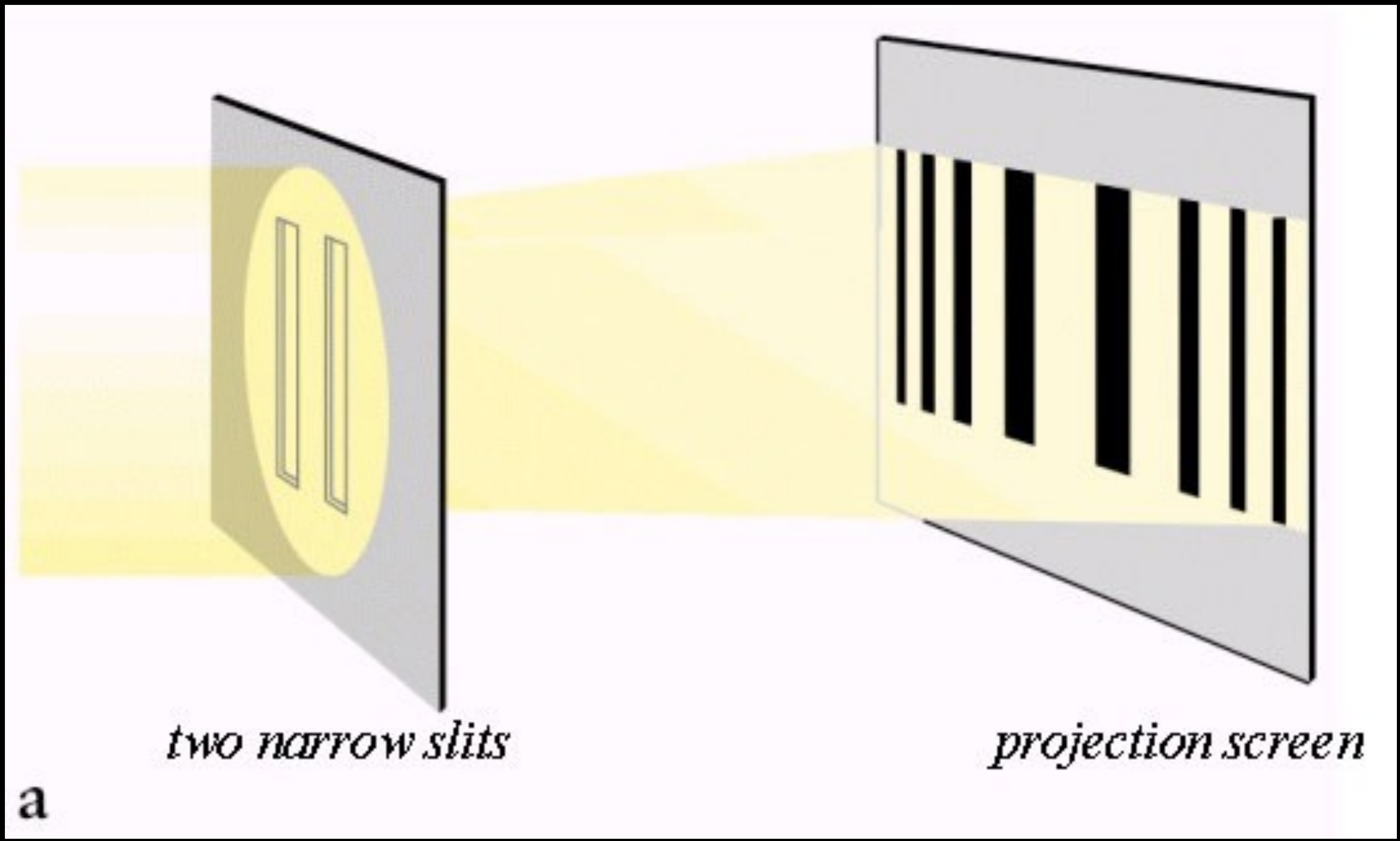
- Knowing the speed of light and one other variable, either the wavelength or frequency of the light in question, you can find the remaining quantity.

$$\left(\frac{3 \times 10^8 \text{ m/s}}{7.7 \times 10^5/\text{s}}\right) = 390 \text{ m}$$

Light as an EM wave

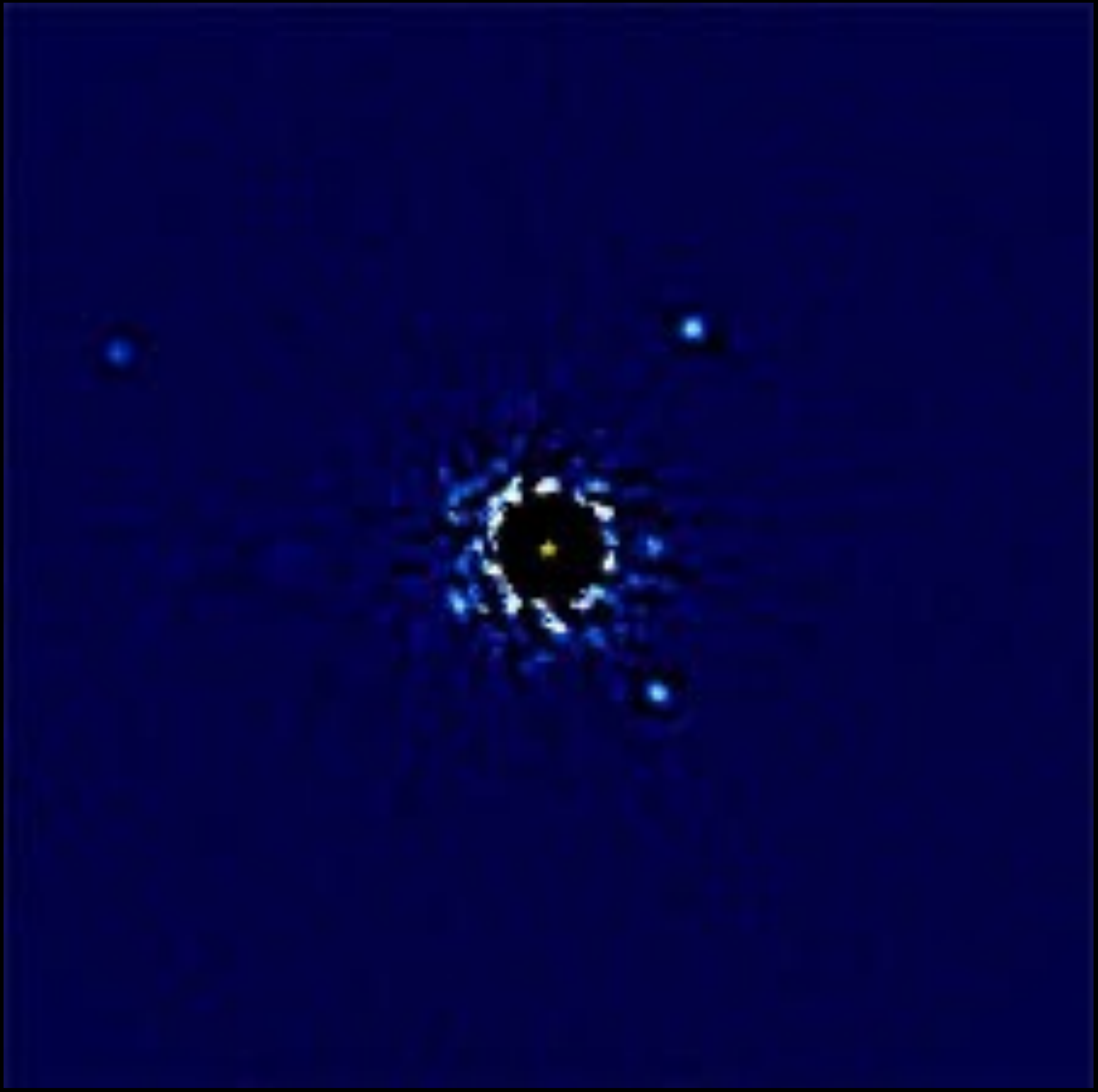
diffraction & interference

Interference: Double Slit

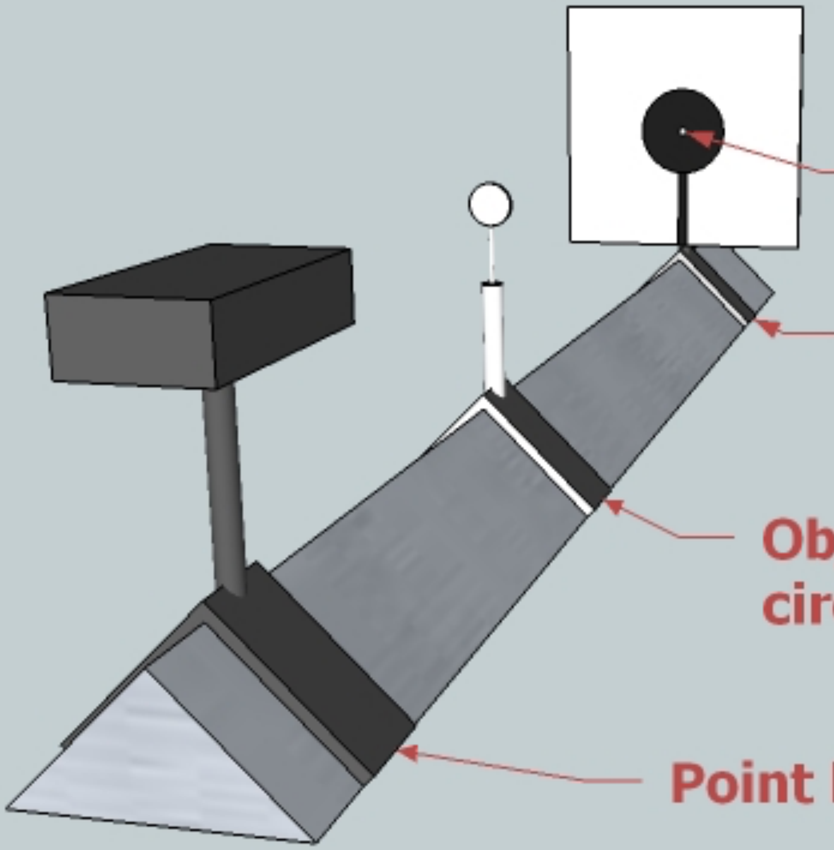


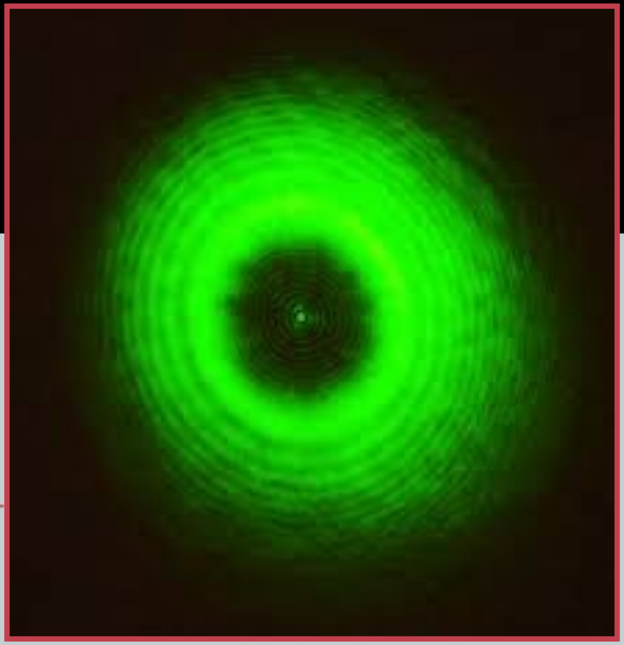
The challenge of directly imaging exoplanets

To see the planets, we need to block the star

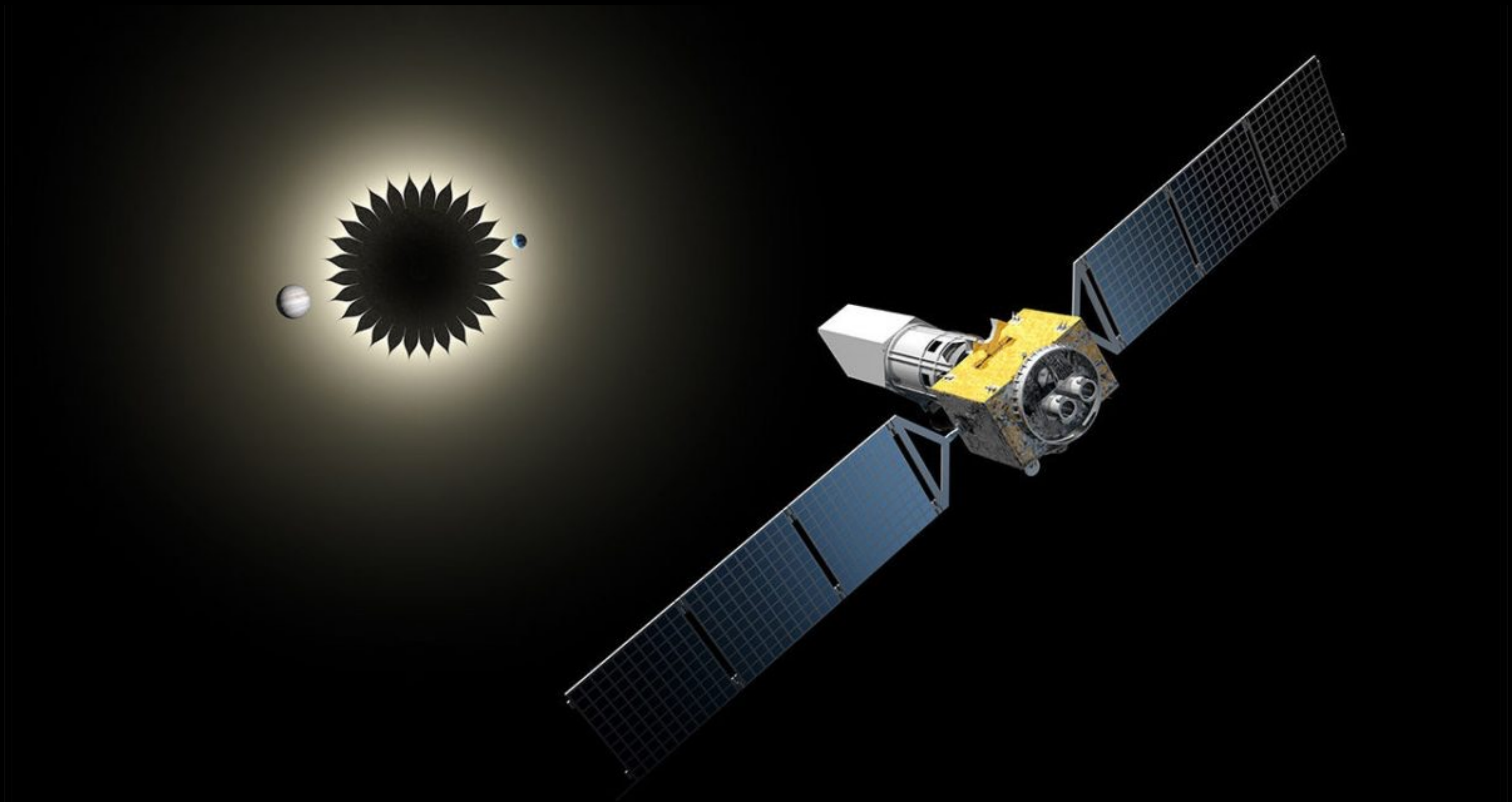


Diffraction: Poisson Spot

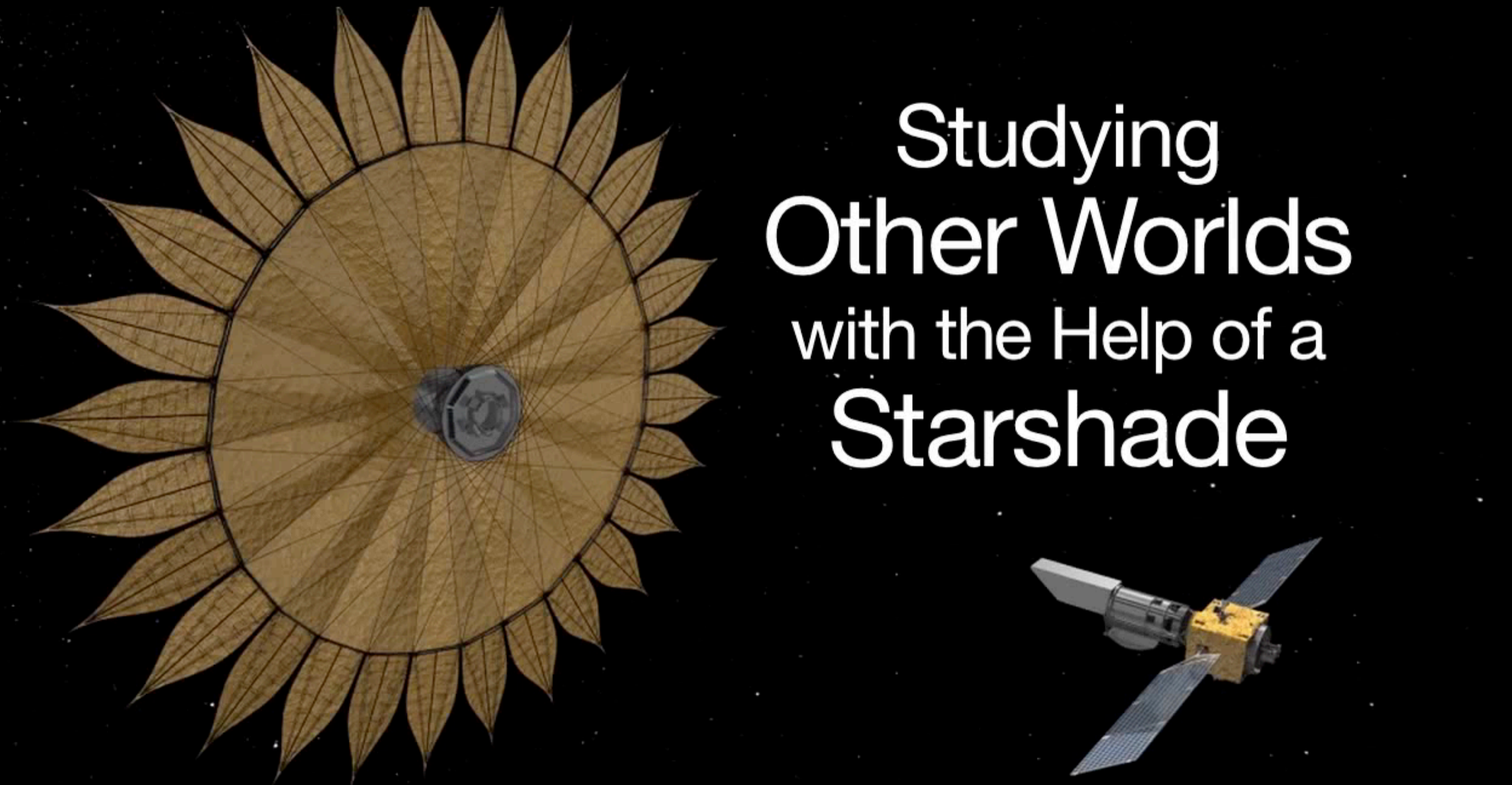




Use non-circular patterns on a Starshade to reduce undesired diffraction pattern (Poisson spot)



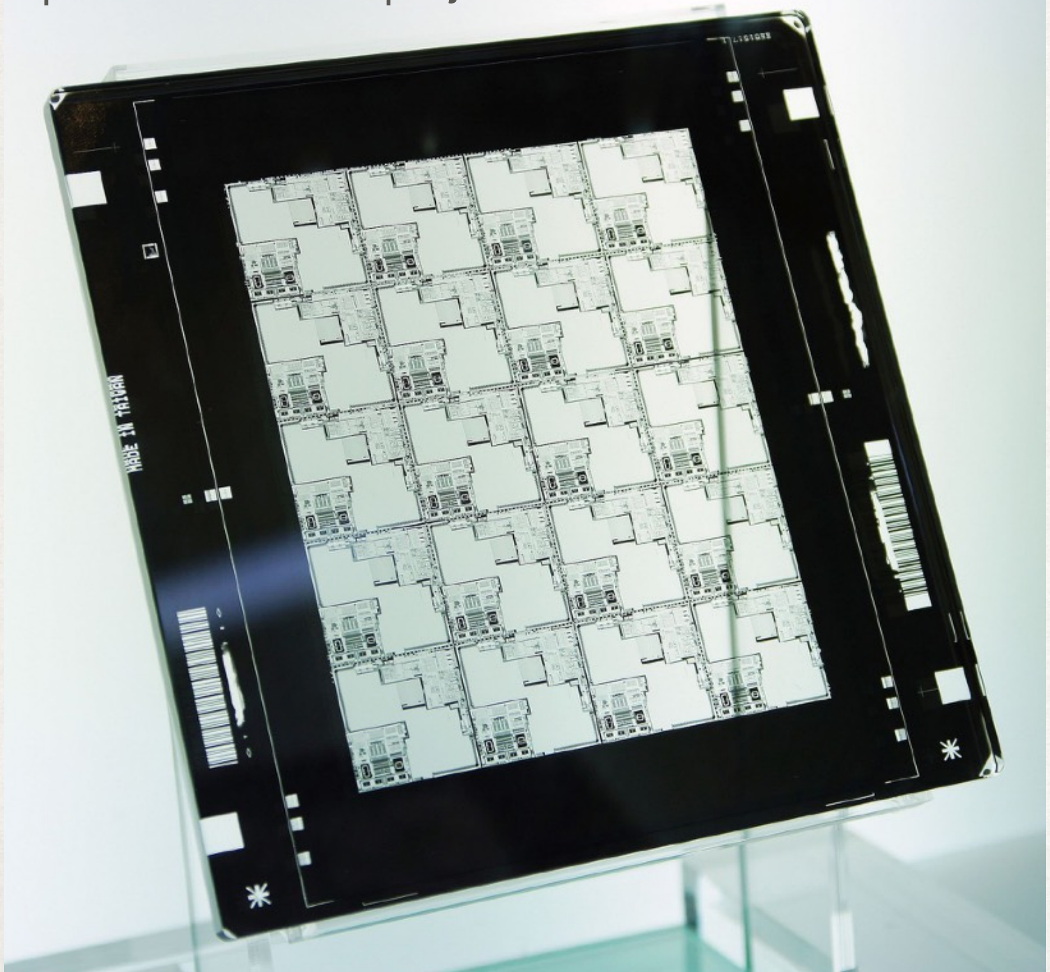
Use a Starshade to Study Exoplanets



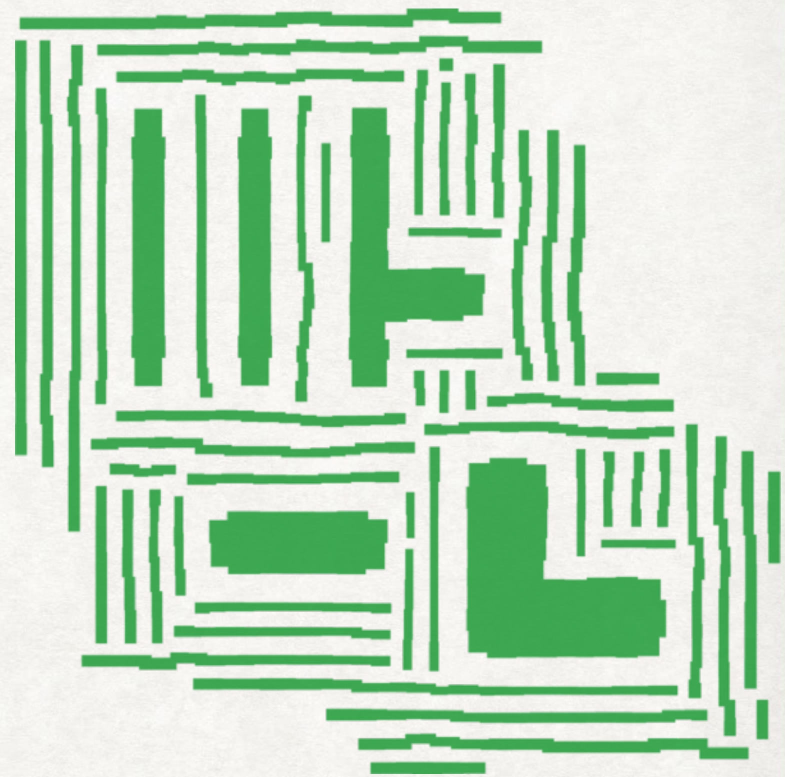
PHOTOLITHOGRAPHY: MICROCHIP PRODUCTION

NOTE THE THINNER ASSIST FEATURES AROUND MAIN FEATURES

A photomask to be projected onto the Silicate wafer

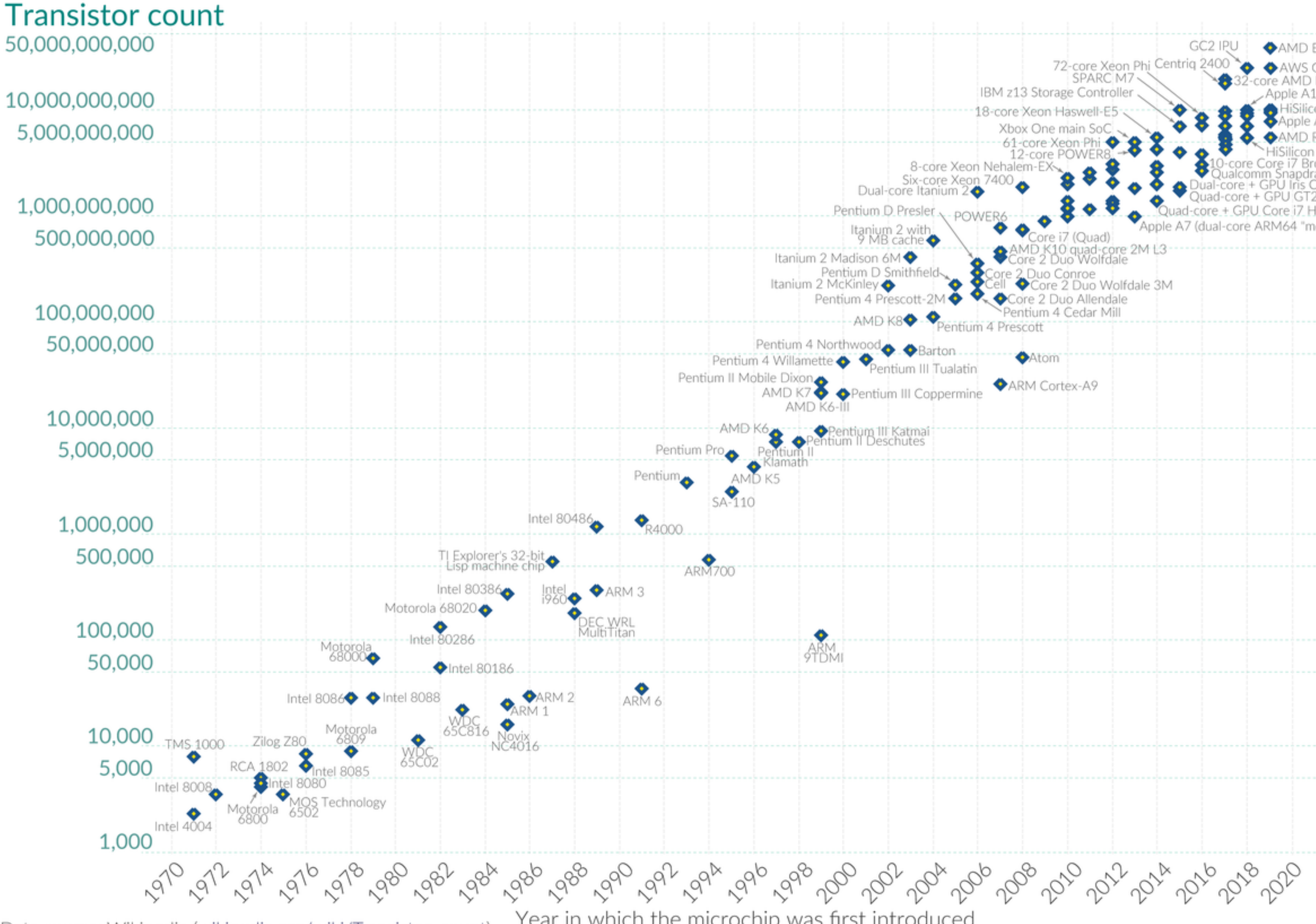


Detailed patterns on the mask



Moore's Law: The number of transistors on microchips doubles every two years

Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years. This advancement is important for other aspects of technological progress in computing – such as processing speed or the price of computers.



Data source: Wikipedia (wikipedia.org/wiki/Transistor_count)

PHOTOLITHOGRAPHY: CHIP PRODUCTION

The ability to project a clear image of a small feature onto the wafer is limited by the wavelength of the light that is used, and the ability of the reduction lens system to capture enough diffraction orders from the illuminated mask. Current state-of-the-art photolithography tools use deep ultraviolet (DUV) light from excimer lasers with wavelengths of 248 and 193 nm (the dominant lithography technology today is thus also called "excimer laser lithography"), which allow minimum feature sizes down to 50 nm. Excimer laser lithography has thus played a critical role in the continued advance of the Moore's Law for the last 20 years (see below[19]).

Robert Noyce (1927-1990) - “The Mayor of Silicon Valley”

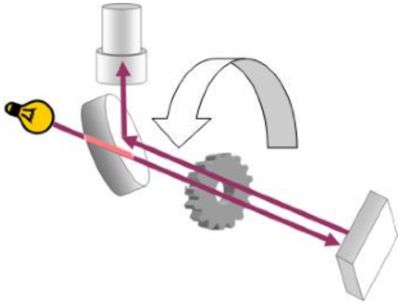
Born in Burlington, Iowa; graduated from Grinnell College, Physics Major

Light as an EM wave

speed of light

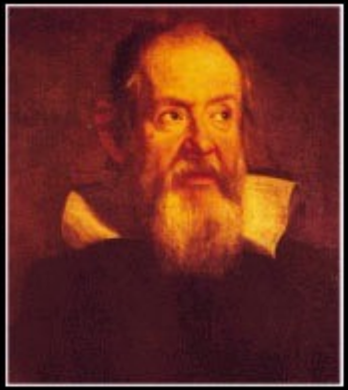
If light is a wave, how fast does it propagate?

- Galileo: 1607
  - measured travel time between two lanterns spaced 2 km apart (couldn't detect any time delay)
- **Roemer: 1676**
  - timed eclipses of Jupiter's moons – noticed their changing periods because of changes in the Earth-Jupiter distance
- Fizeau and Foucault: 1849-1862
  - used stationary and rotating mirrors to deflect light enough to measure speed (~300,000 km/s)



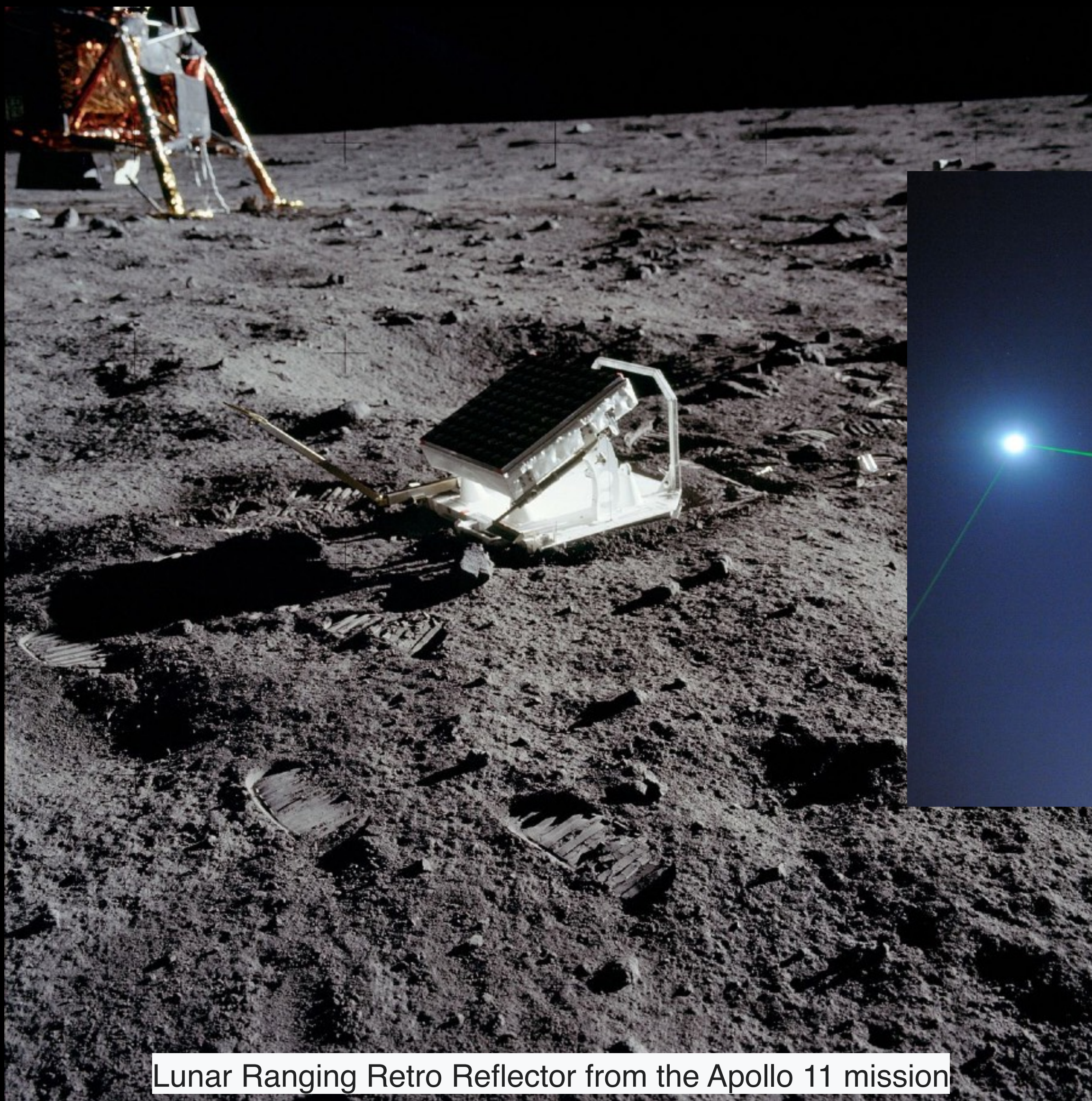
Galileo's round-trip time experiment (1607)

Galileo unsuccessfully attempted to measure the speed of light by asking an assistant on a distant hilltop to open a lantern the moment Galileo opened his lantern.



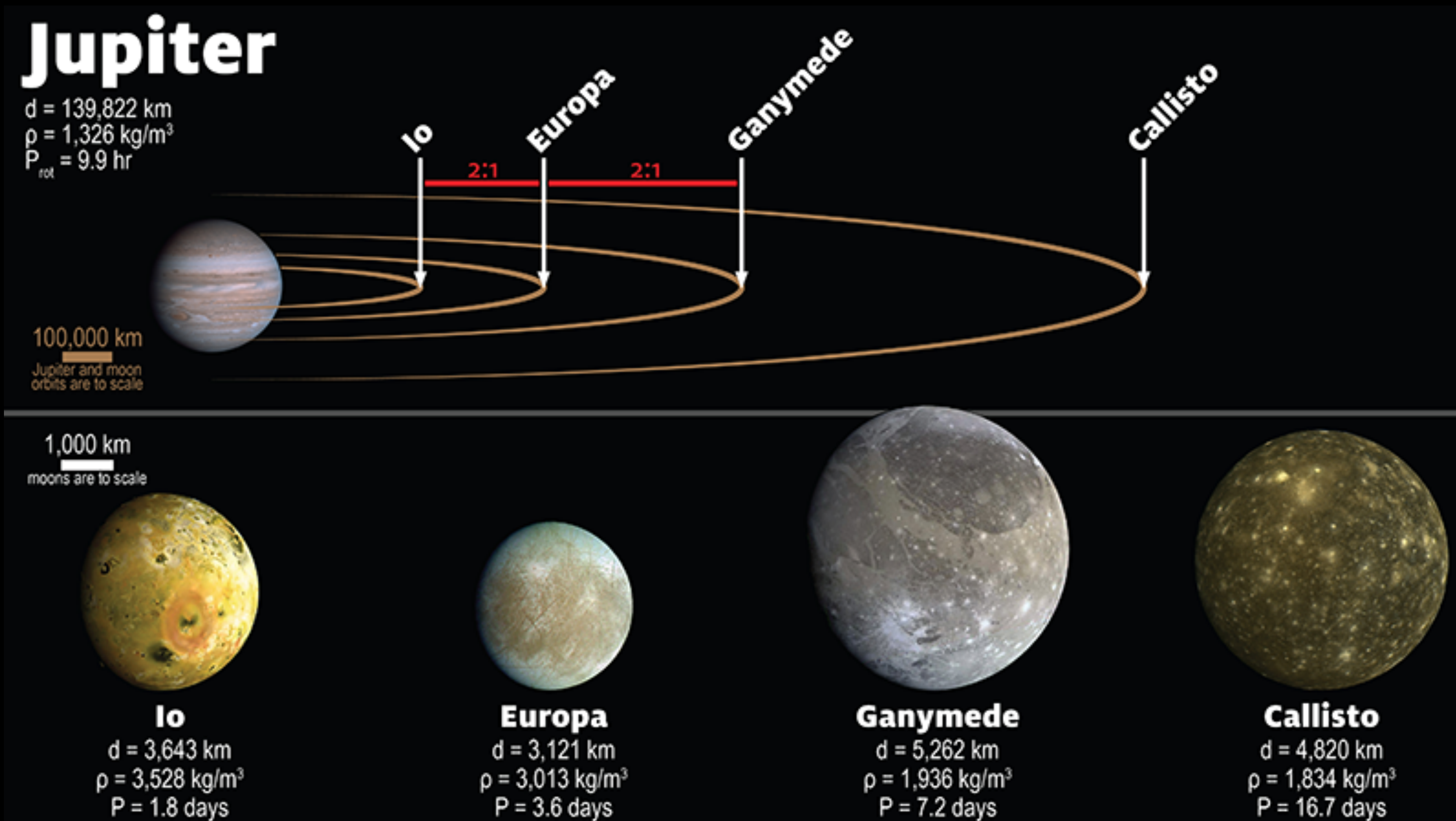
For hilltops separated by 10 km, round-trip time delay for light is only 66 microsec!

Galileo’s round-trip time experiment on the Moon (1969)

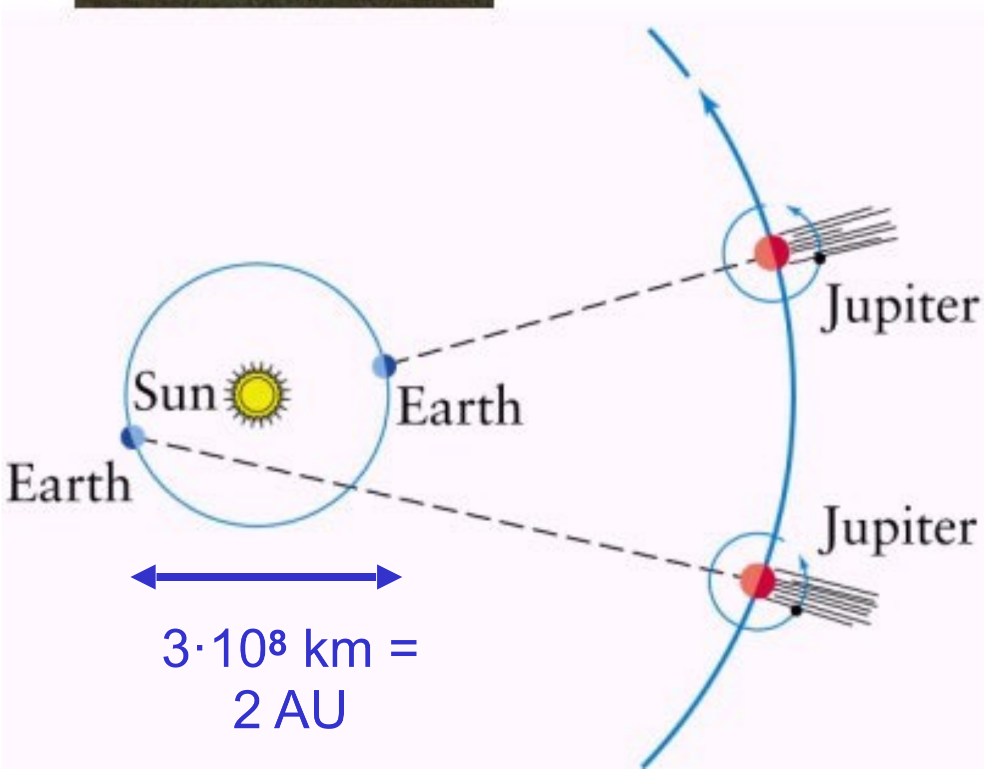


Lunar Ranging Retro Reflector from the Apollo 11 mission

Roemer's Timing Experiment of Jovian satellites (1676)



Roemer's Timing Experiment of Jovian satellites (1676)

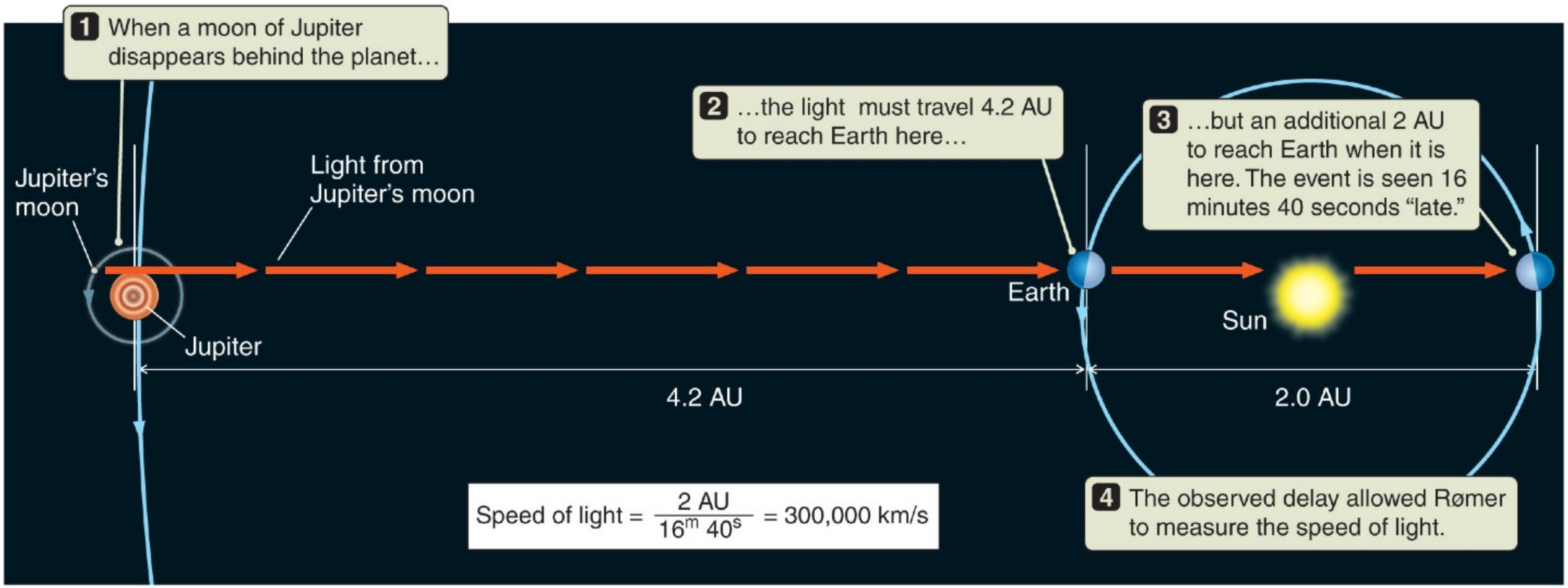


In 1676, Danish astronomer Olaus Røemer discovered that the exact time of eclipses of Jupiter's moons varied based on how near or far Jupiter was to Earth.

This occurs because it takes varying amounts of time for light to travel the varying distance between Earth and Jupiter.

*The change of eclipsing period as Earth moves away / towards Jupiter is a **Doppler effect***

- In 1676, measured by Rømer by timing the eclipses of Jovian moons (He measured 225,000 km/s).
- Current best measurement at 299,792 km/s.

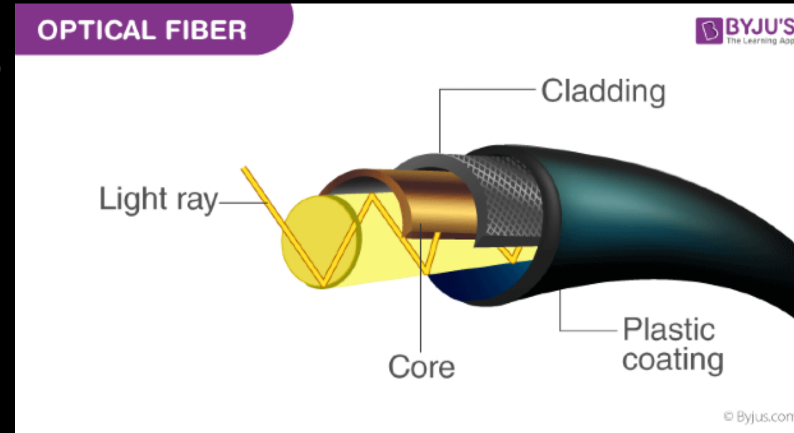


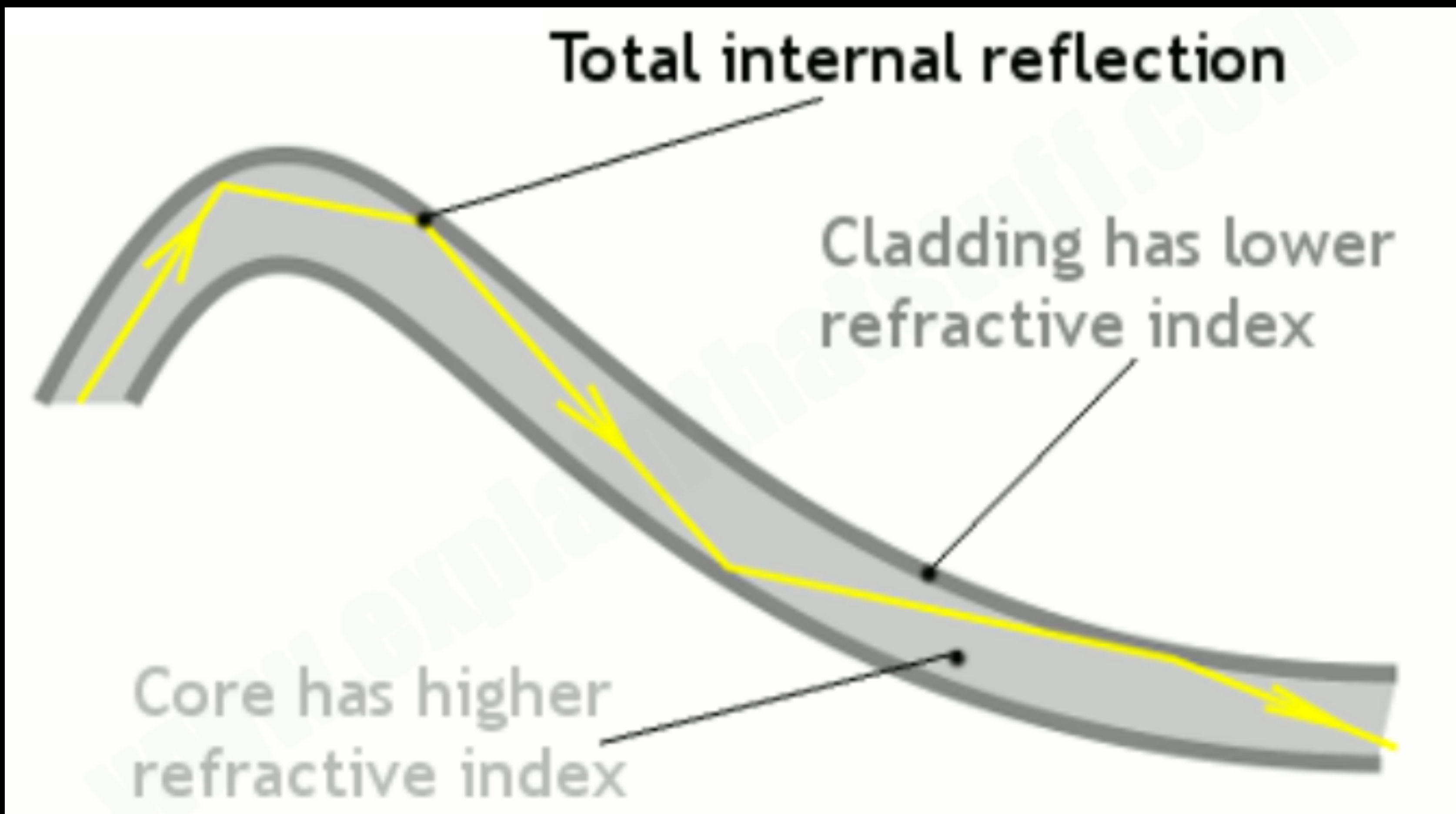
Speed of Light Demo

15 cm & 20 m optical fibers

Oscilloscope

Pulsed laser source





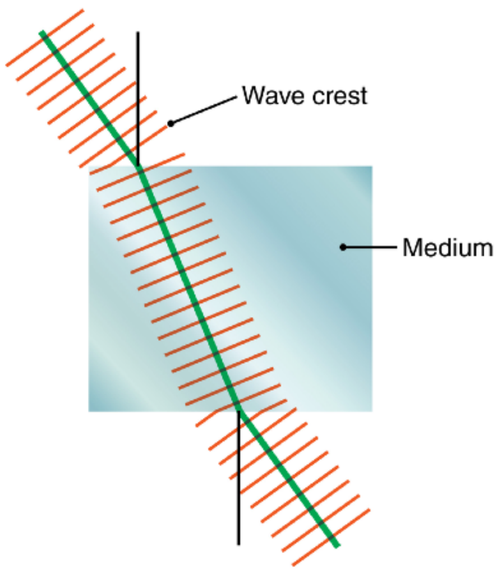
Speed of Light in Vacuum vs. in Medium

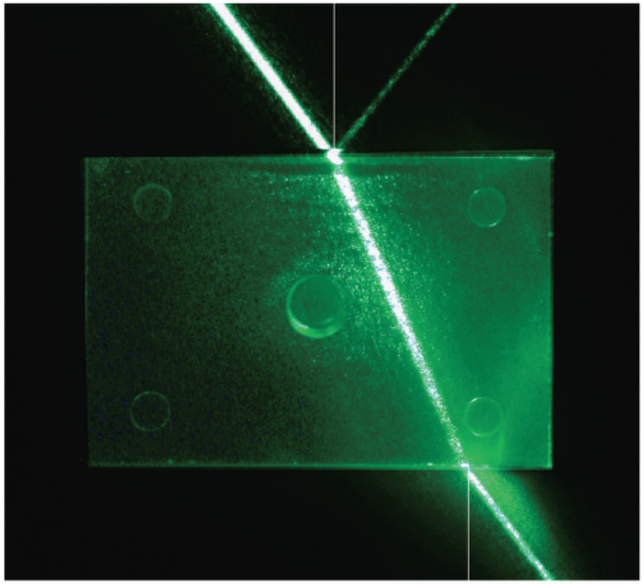
Light travels at a constant speed in vacuum

c = 299,792.458 km/s

Light travels slower in a medium (e.g., air, water, glass)

refractive index: n = c/v (e.g., n = 1.33/1.44 for water/optical fiber)





Speed of Light & Wavelength in Medium

- Speed of light decreases when light enters from vacuum to a medium (air, water, glass, etc.)
- Because the frequency of light doesn't change, this leads to a decrease in wavelength in medium
- Ground-based astronomical observations are carried out in air, so the wavelength measured in air need to be corrected by the refractive index to compare it with the wavelength in vacuum

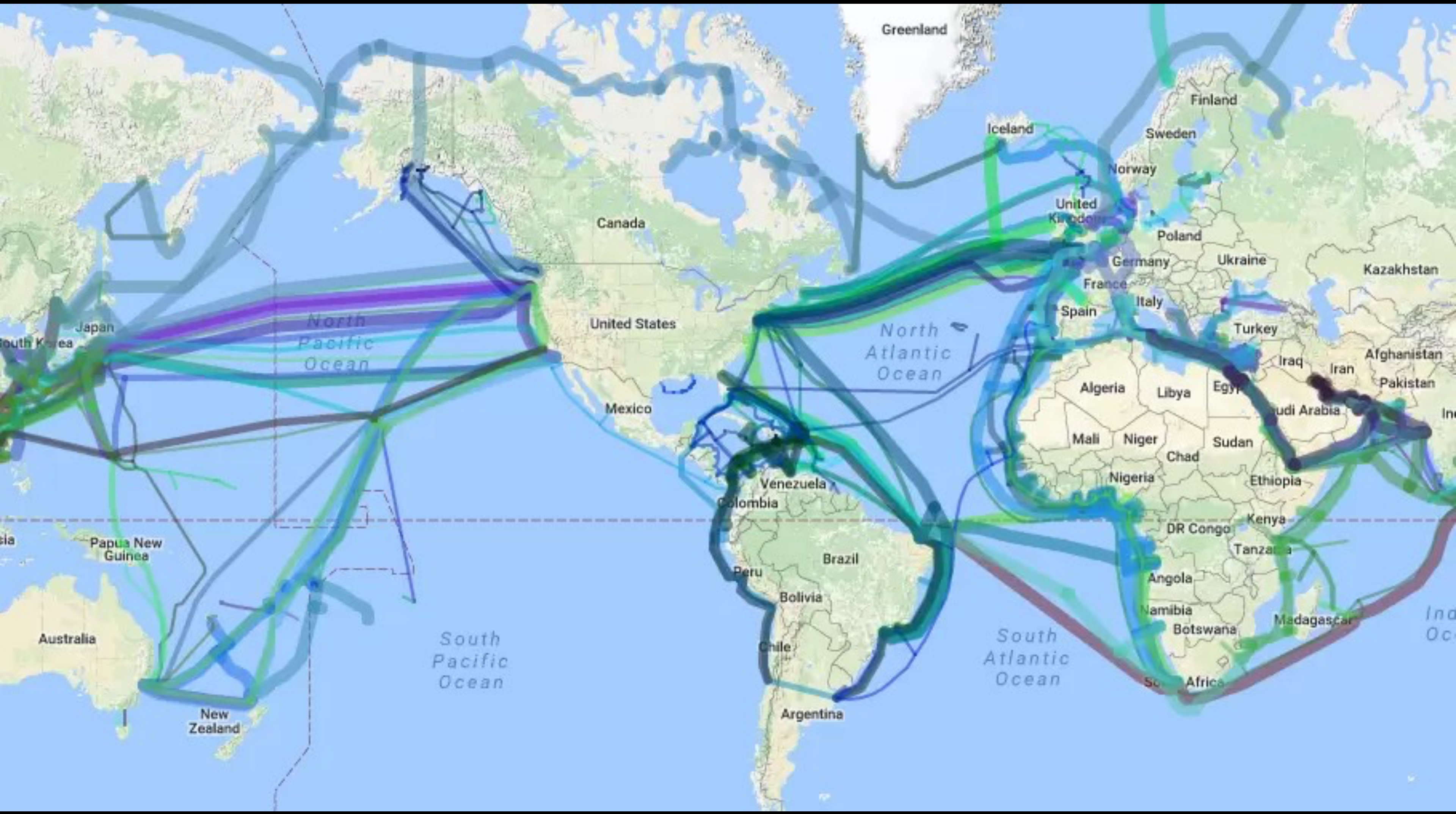
speed of light in medium

$$c_{\mathrm{medium}} = c/n_{\mathrm{medium}}$$

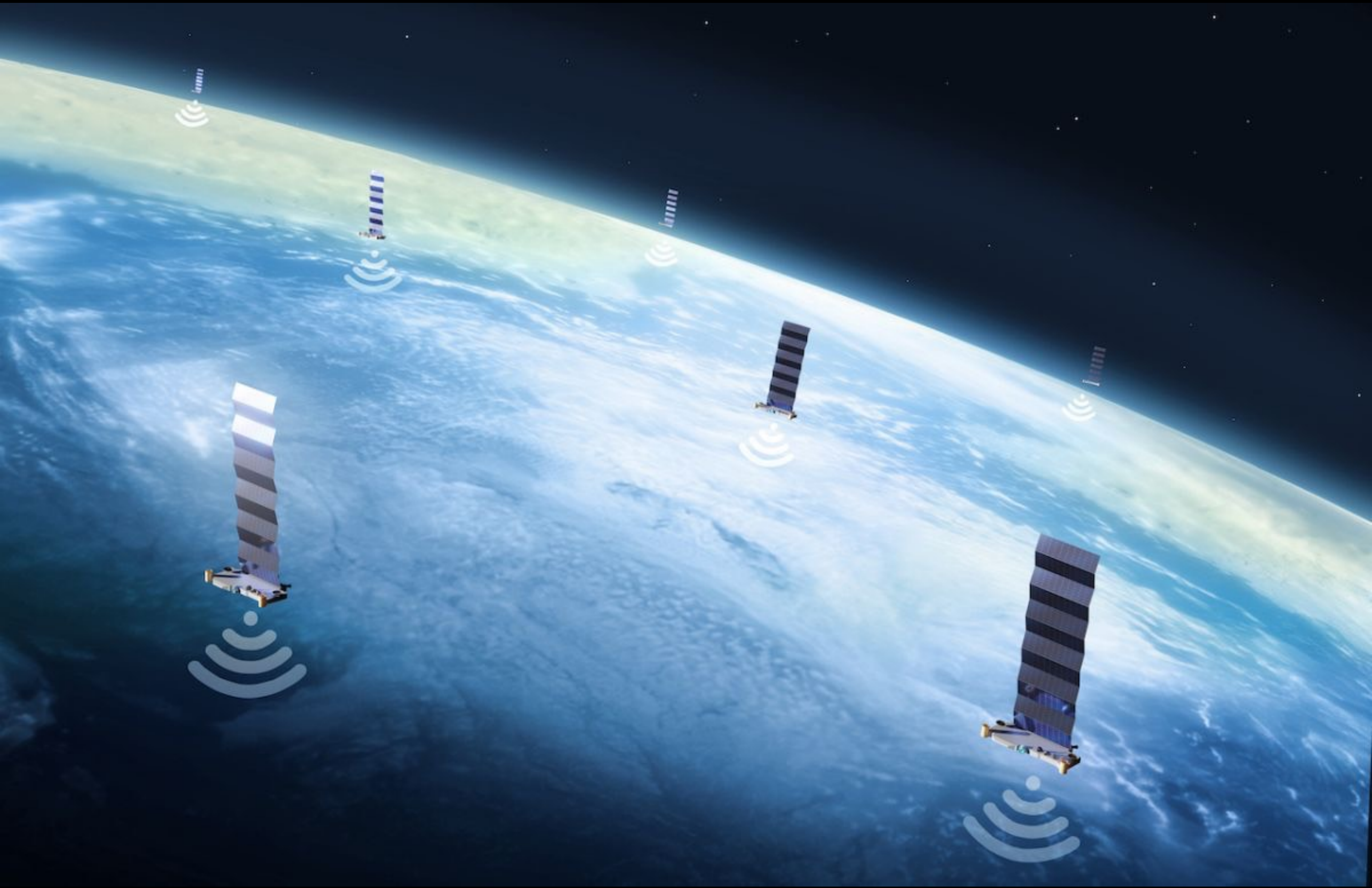
wavelength of light in medium

$$\lambda_{\mathrm{medium}} = \lambda_{\mathrm{vacuum}}/n_{\mathrm{medium}}$$

Global internet backbone map: undersea fiber optics cables



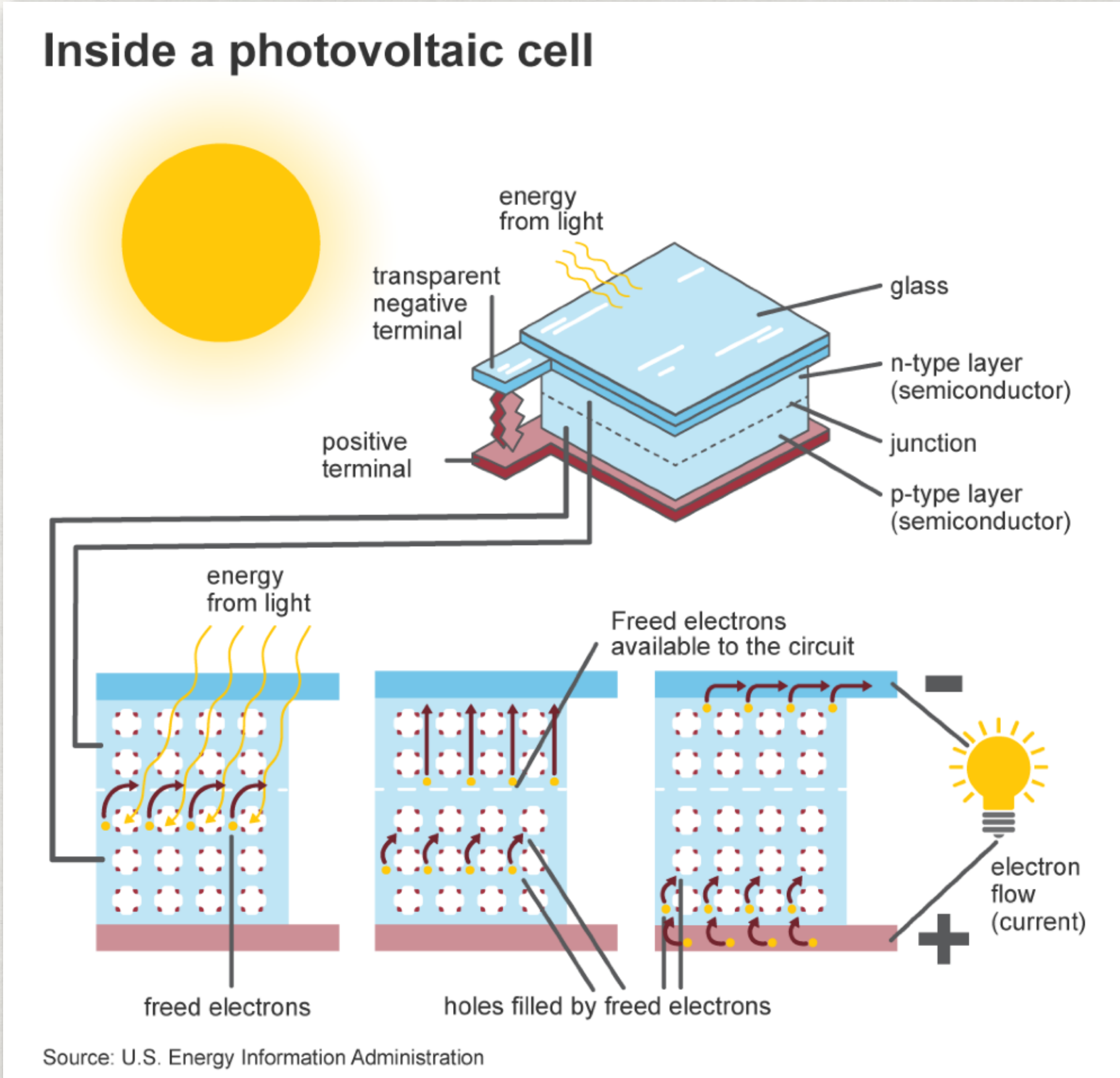
Starlink Internet: low latency (20ms) with 12,000 Low Earth-orbit Satellites



Light as a massless particle

photons

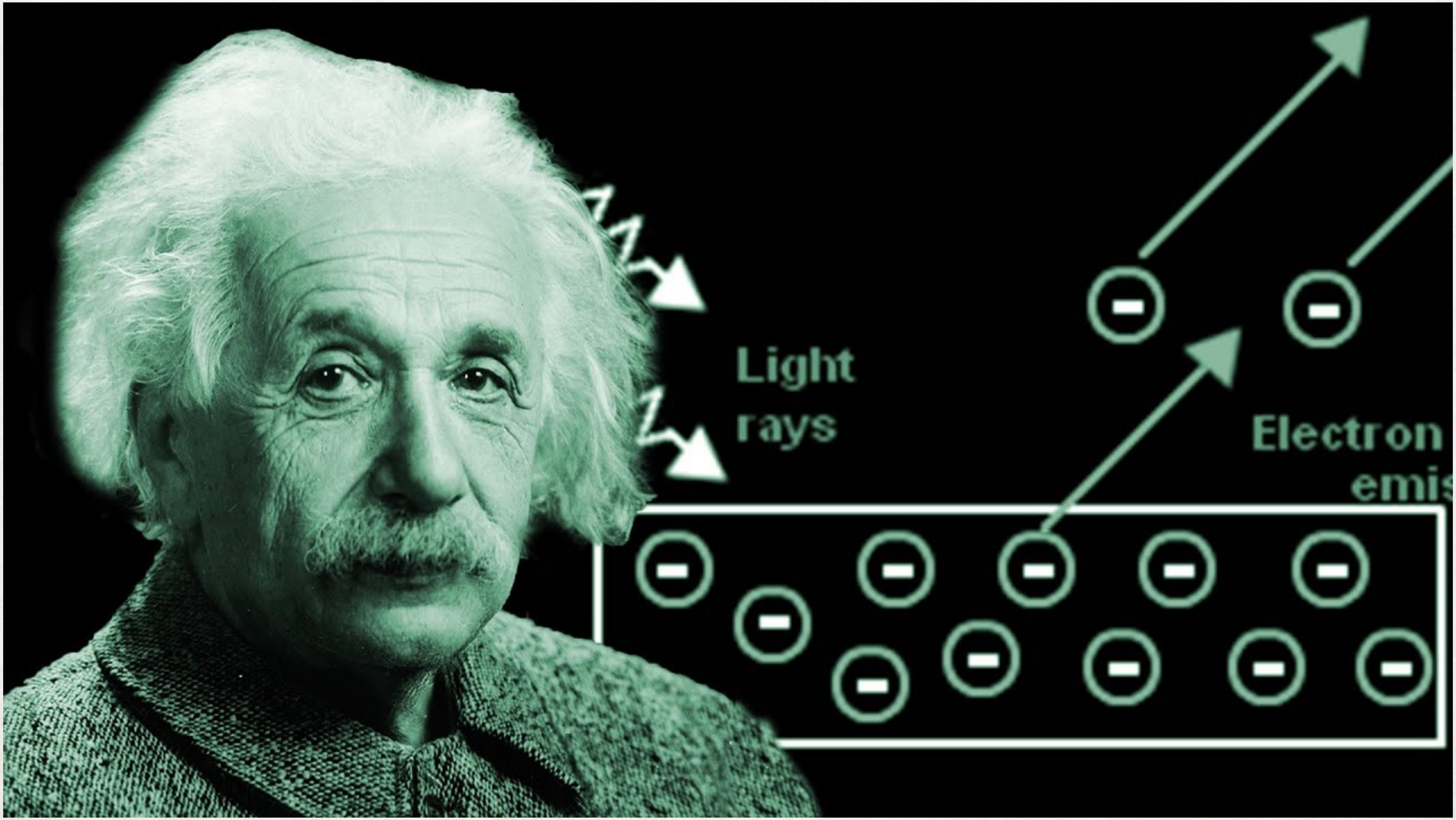
PHOTOELECTRIC EFFECT: SILICON SOLAR PANELS



PHOTOELECTRIC EFFECT: SILICON CAMERAS

PHOTOELECTRIC EFFECT: METAL PLATE EXPERIMENTS

Einstein's 1922 Nobel Price was awarded "for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect"



Energy of Photons: Photoelectric Effect

- Result #1: if you increase **intensity** of light beam you get **more of the same electrons**

- Result #2: if you increase **frequency** of light beam you get **higher energy electrons**

Photon's energy depends on its frequency

$$E = h\nu = hc/\lambda$$

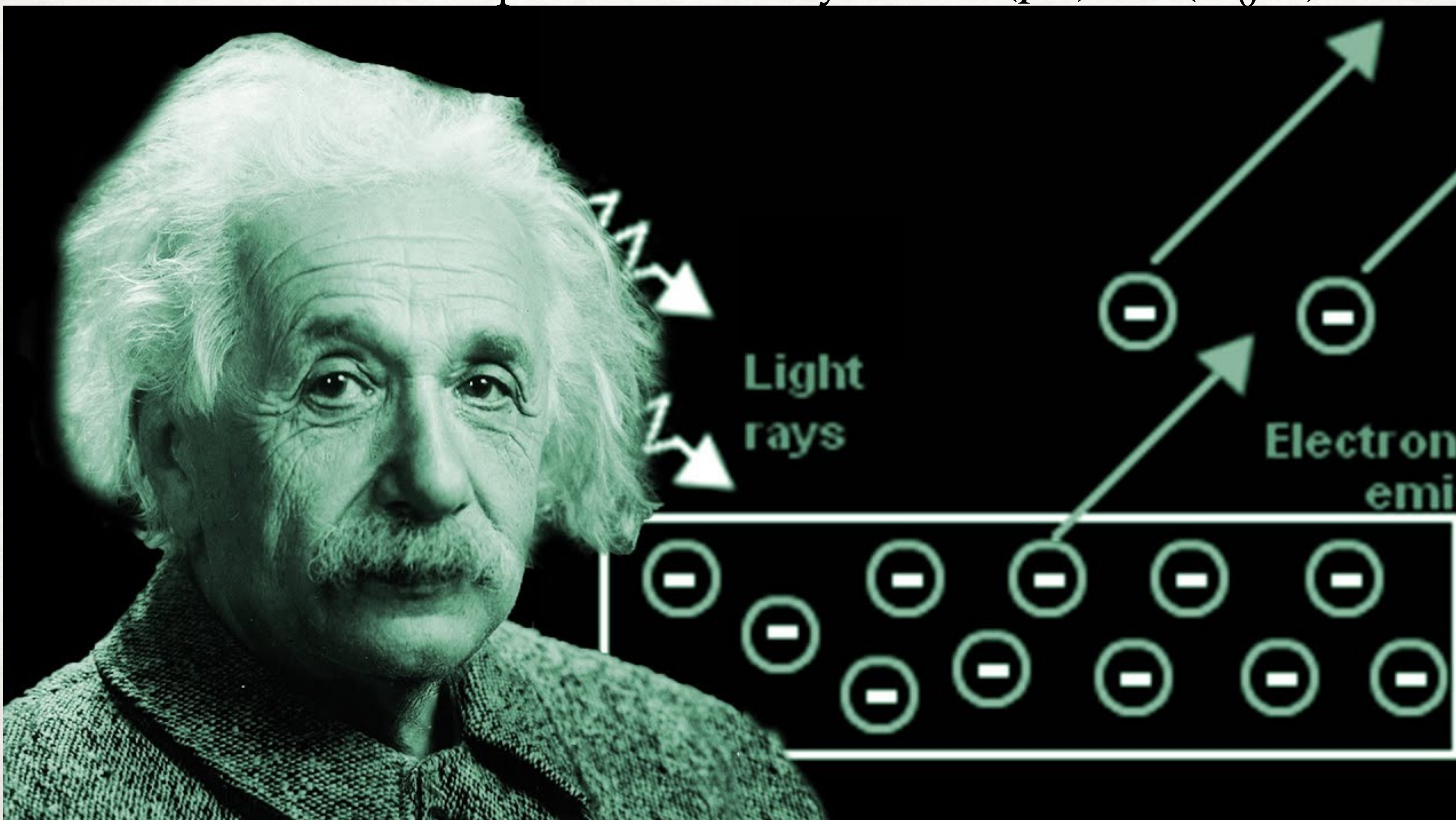
h is Planck's constant (6.6e-34 Joule/Hz),
ν is frequency, E the energy;
c is speed of light and λ is wavelength

PHOTOELECTRIC EFFECT: ENERGY & MOMENTUM

Energy of Light: $E = h\nu$

Momentum of Light: $p = h\nu/c = h/\lambda$

derived from special relativity: $E^2 = (pc)^2 + (m_0c^2)^2$



Energy (eV) to Wavelength (μm) Conversion

- Energy is often given in units of electron-volt (eV), which is the amount of kinetic energy gained by a single electron accelerating through an electric potential difference of one volt

- Wavelength is often given in units of micron (um)

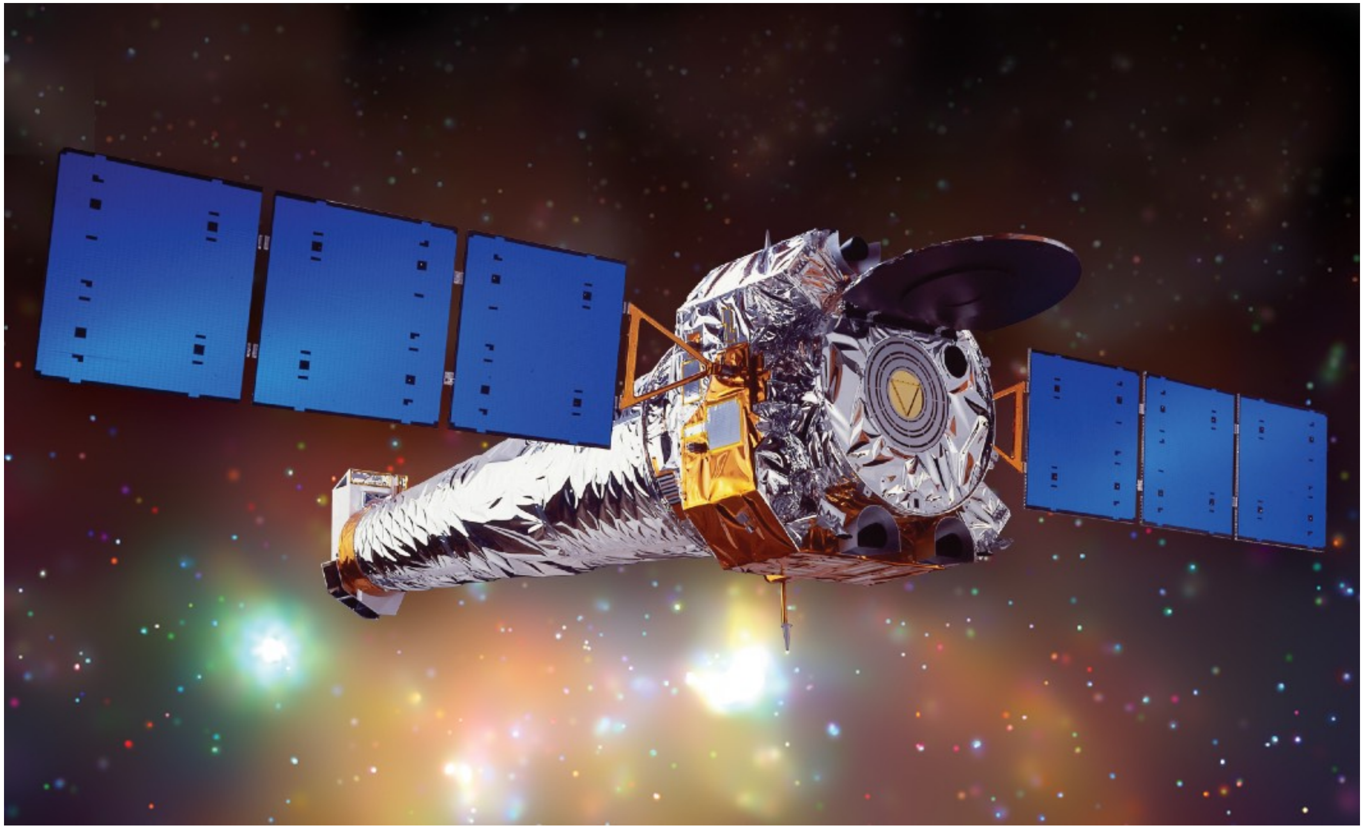
- 1 eV = 1.602e-19 J, h = 6.626e-34 J/Hz, c = 3e8 m/s, given $E = hc/\lambda$, calculate the wavelength (in micron) of photons with energies of 1 eV.

$$\lambda = 1.24\ \mu\text{m} \left(\frac{E}{1\ \text{eV}}\right)$$

Energy (eV) to Wavelength (μm) Conversion

- Chandra X-ray observatory can detect photons with energies between 0.5 and 8 keV, what's the wavelength range in nm?

$$\lambda = 1.24\ \mu\mathrm{m} \left(\frac{E}{1\ \mathrm{eV}} \right)$$



Basic Properties of Light

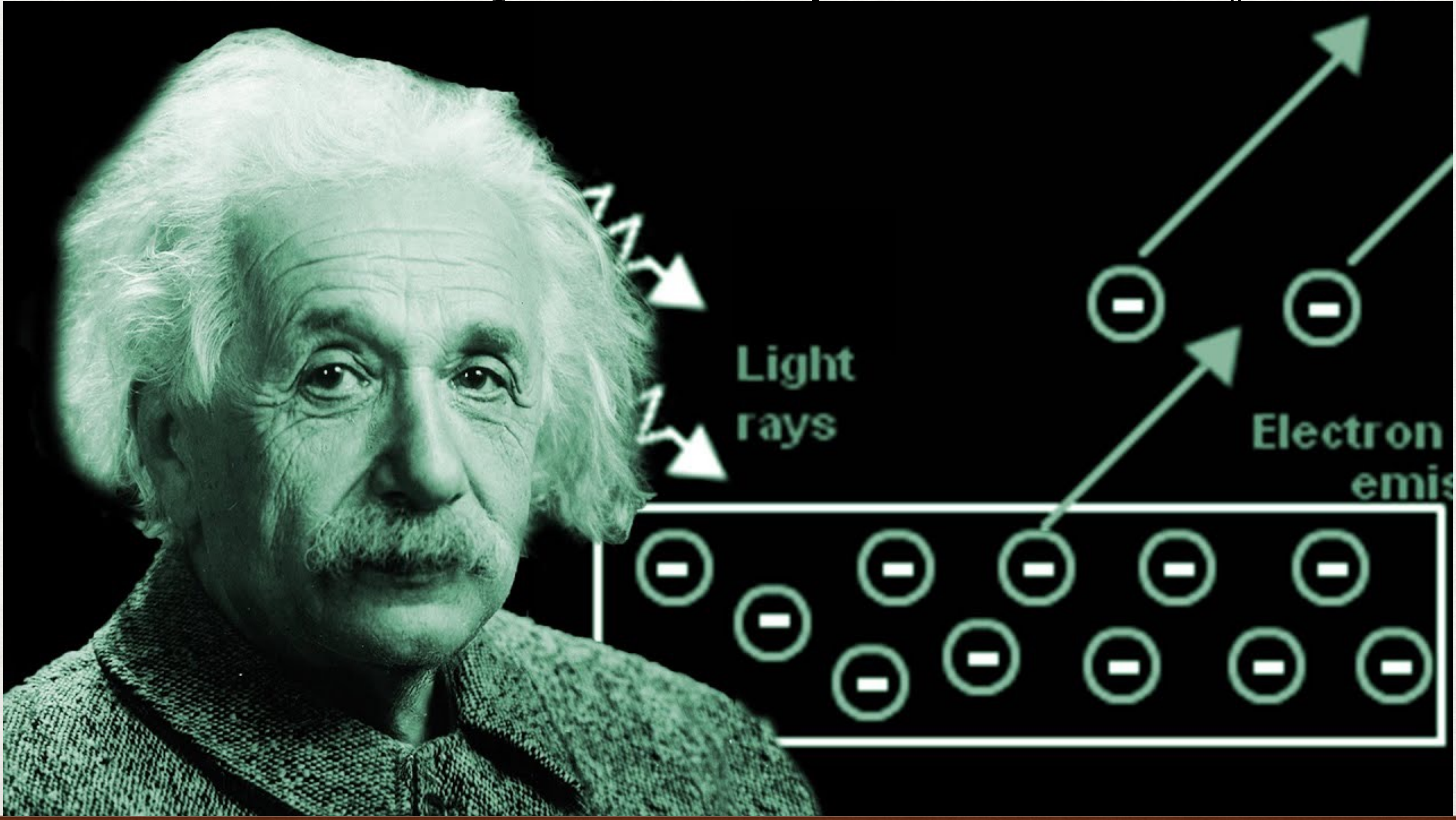
a quick recap of Lecture 1

PHOTOELECTRIC EFFECT: ENERGY & MOMENTUM

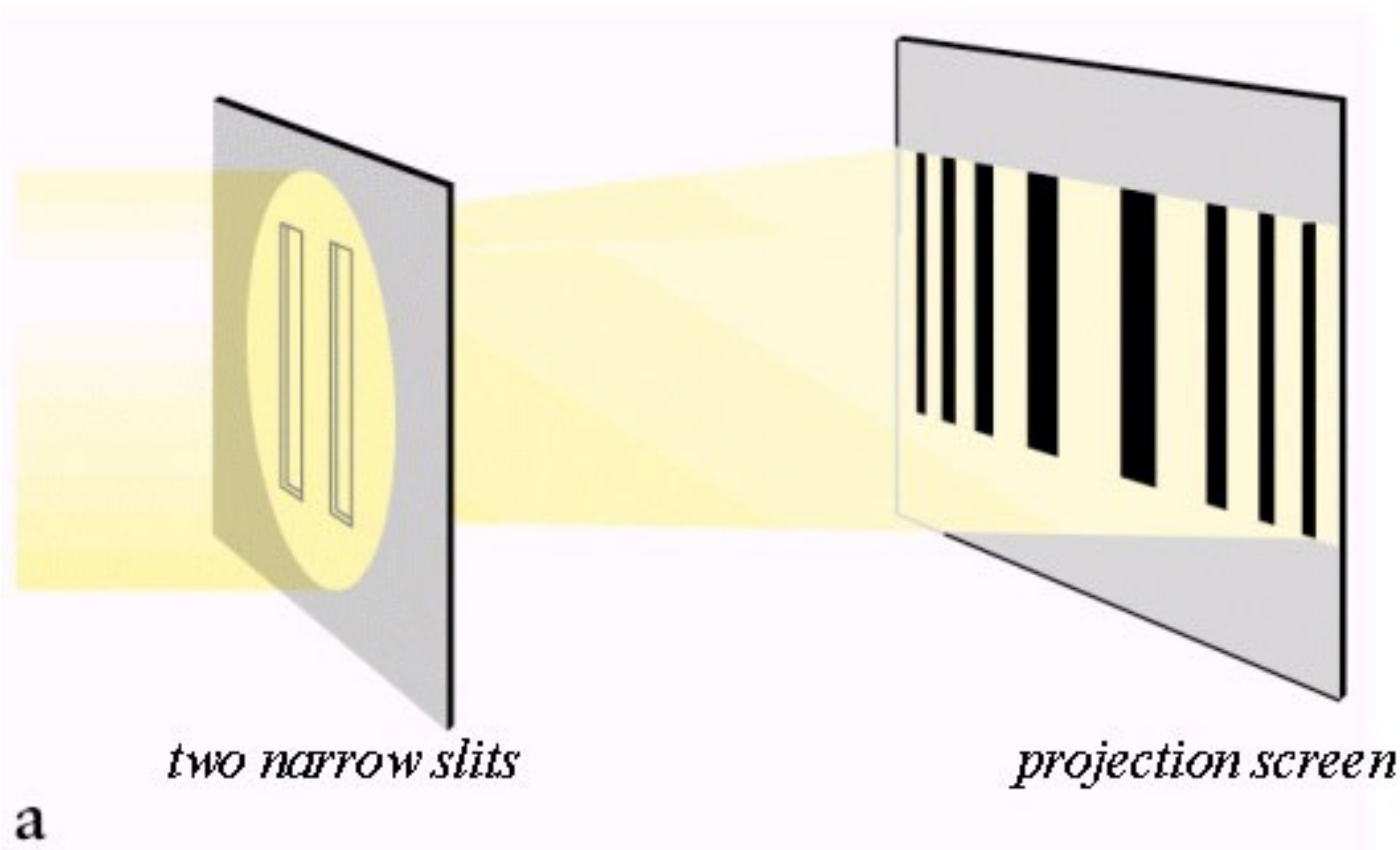
Energy of photons: $E = h\nu$

Momentum of photons: $p = h\nu/c = h/\lambda$

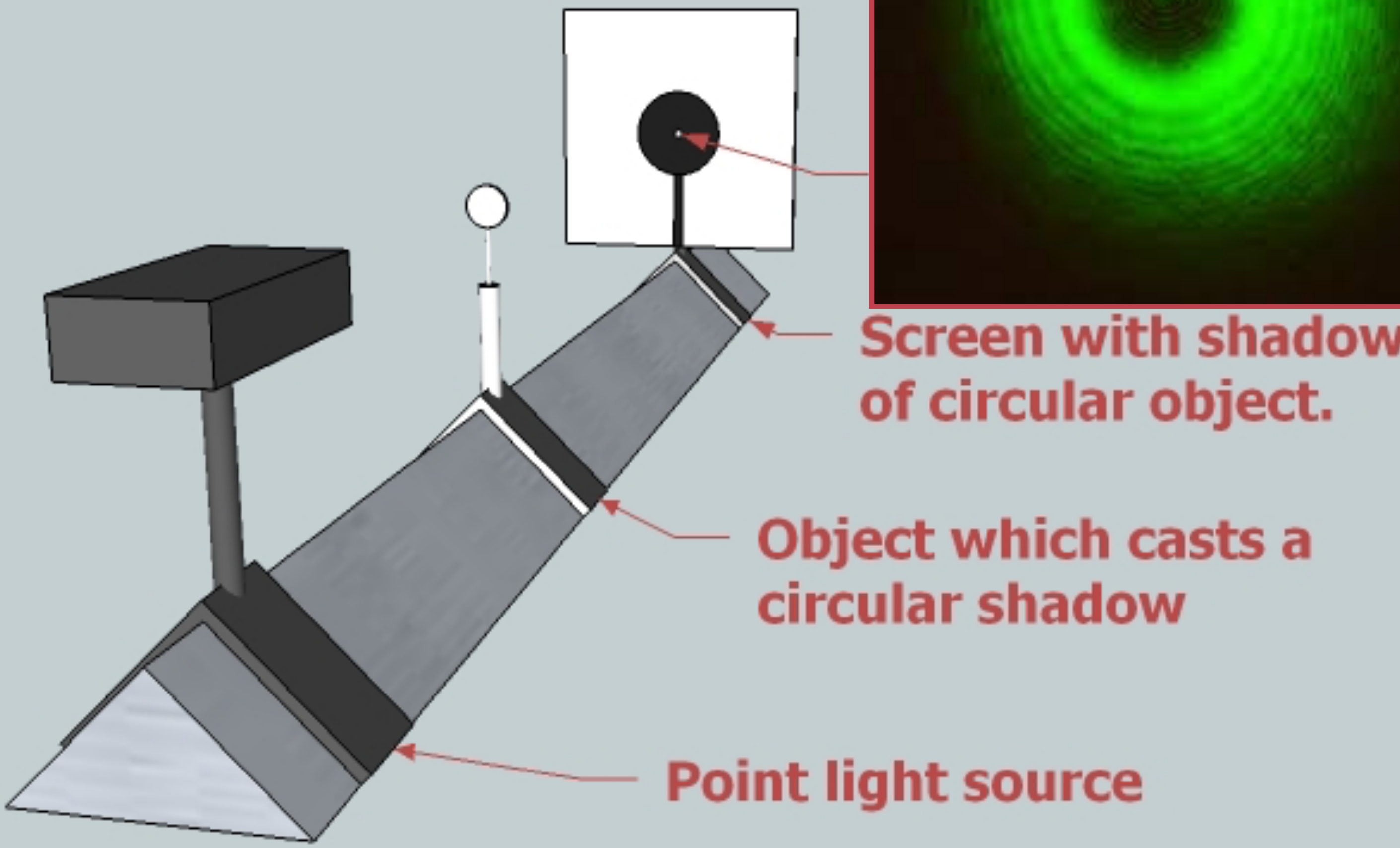
derived from special relativity: $E^2 = (pc)^2 + (m_0c^2)^2$

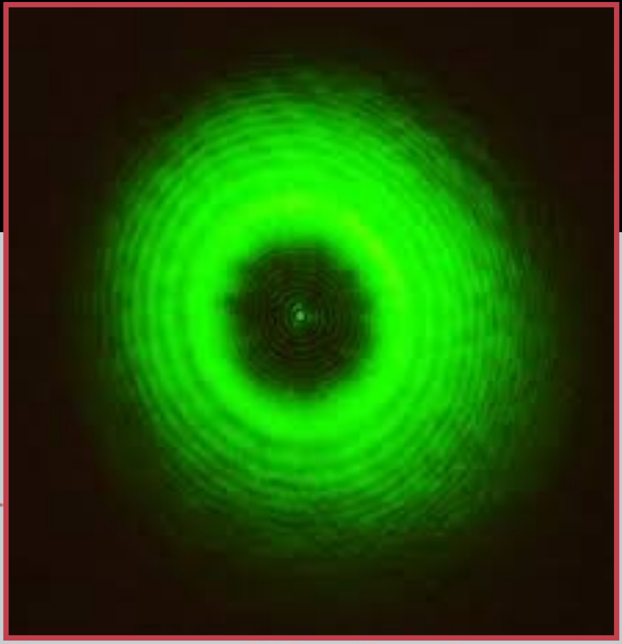


Wavelike: Diffraction & Interference

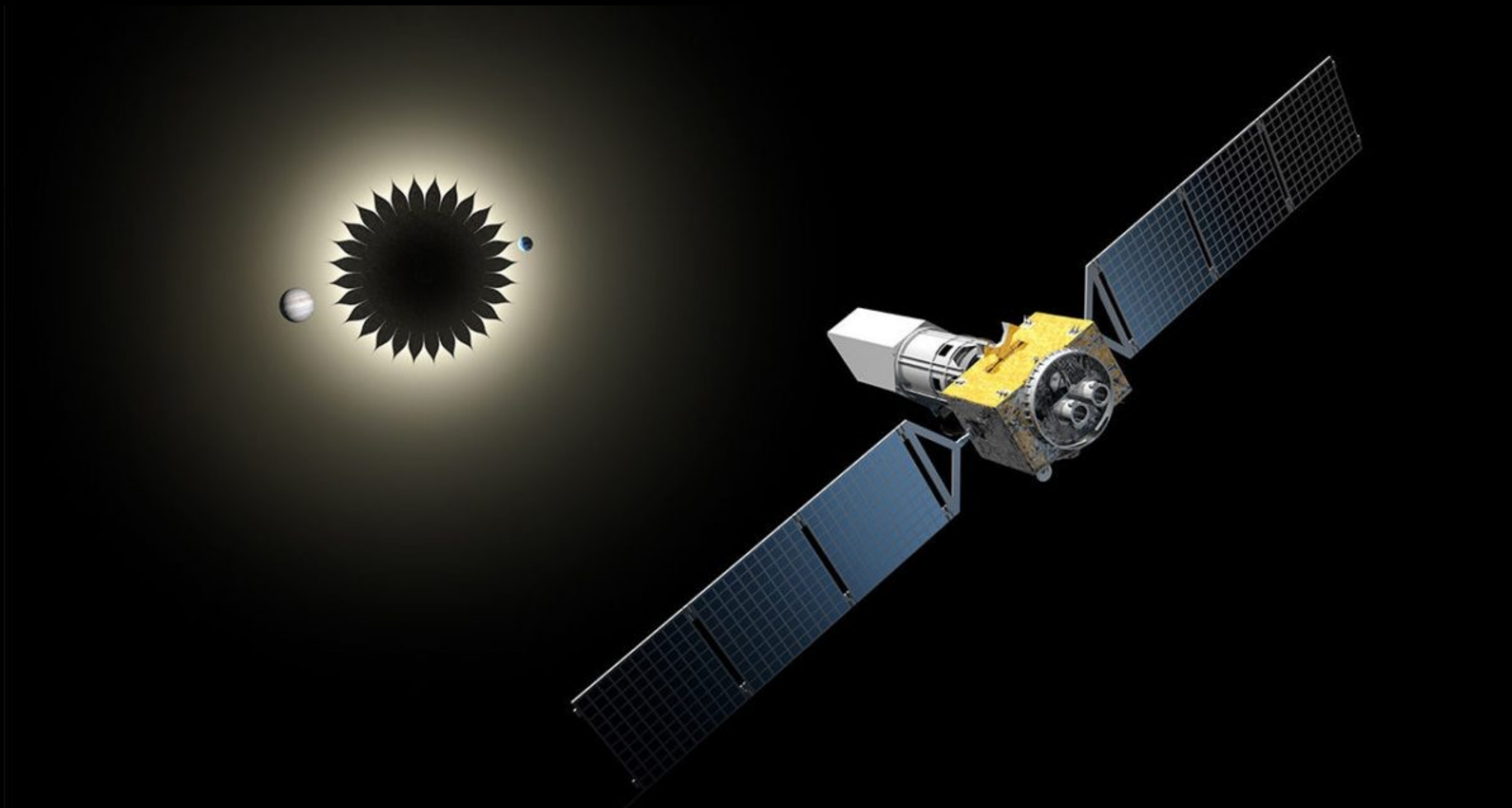


Diffraction: Poisson Spot

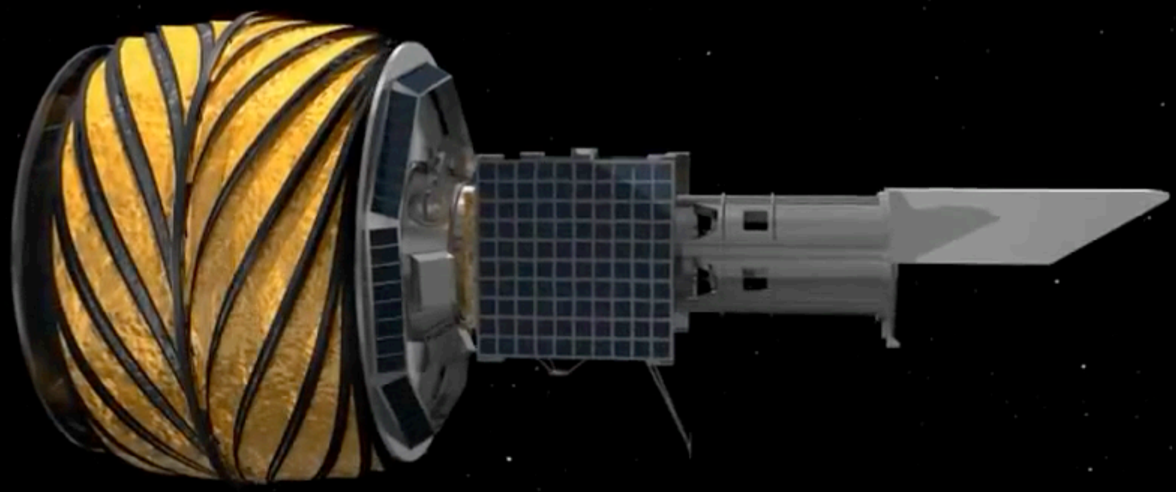




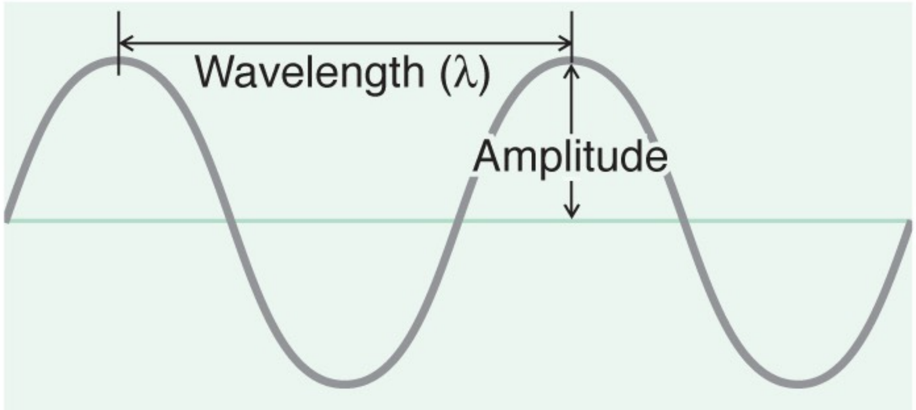
Use a flower-shaped coronagraph to reduce undesired diffraction pattern (Poisson spot)



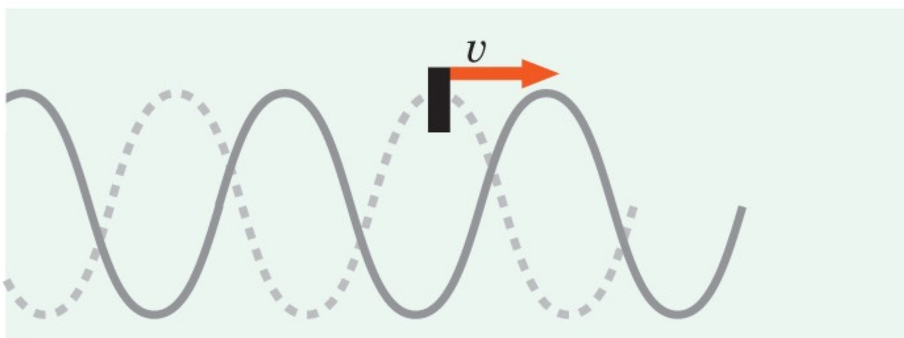
NASA's Starshade



Wavelength, frequency, and speed of light in Vacuum



- *Wavelength* (λ): length between crests (unit: nm, um, …)

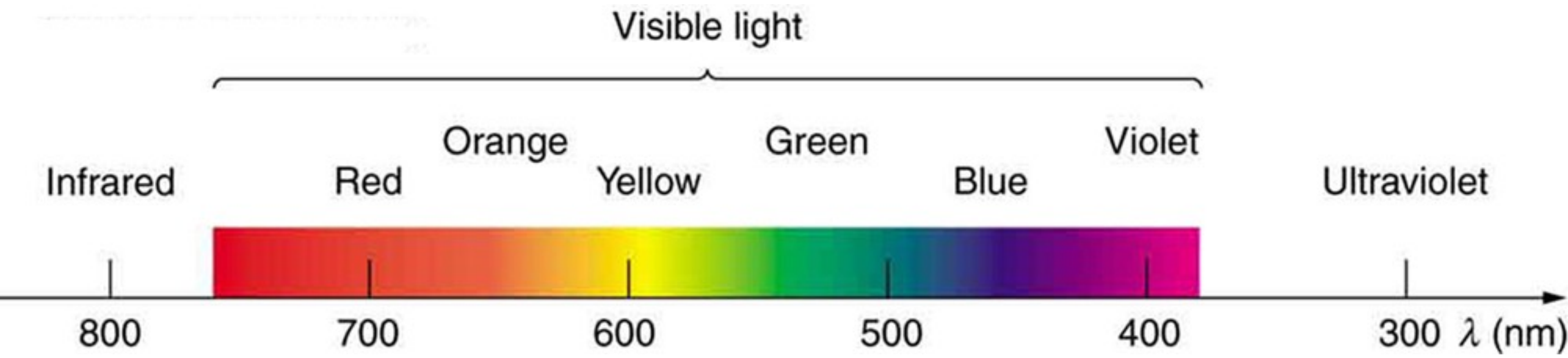


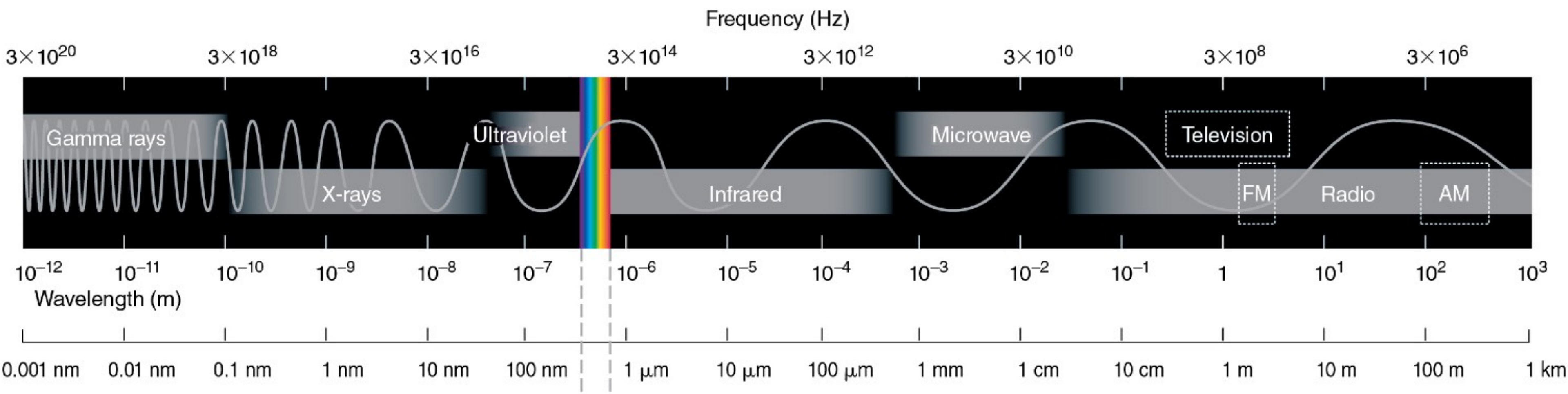
- *Frequency* (ν): number of waves that pass by per unit time (unit: Hz, MHz, …)

Wave Speed = Wavelength x Frequency

$$c = \lambda\nu$$

The wide range in λ & ν: from gamma rays to radio waves





Decimal Prefixes in astropy.units

https://docs.astropy.org/en/stable/units/standard_units.html

https://kbarbary-astropy.readthedocs.io/en/latest/units/index.html

Available decimal prefixes		
Symbol	Prefix	Value
P	peta-	1E+15
T	tera-	1E+12
G	giga-	1E+09
M	mega-	1E+06
k	kilo-	1E+03
h	hecto-	1E+02
da	deka-, deca	1E+01
d	deci-	1E-01
c	centi-	1E-02
m	milli-	1E-03
u	micro-	1E-06
n	nano-	1E-09
p	pico-	1E-12

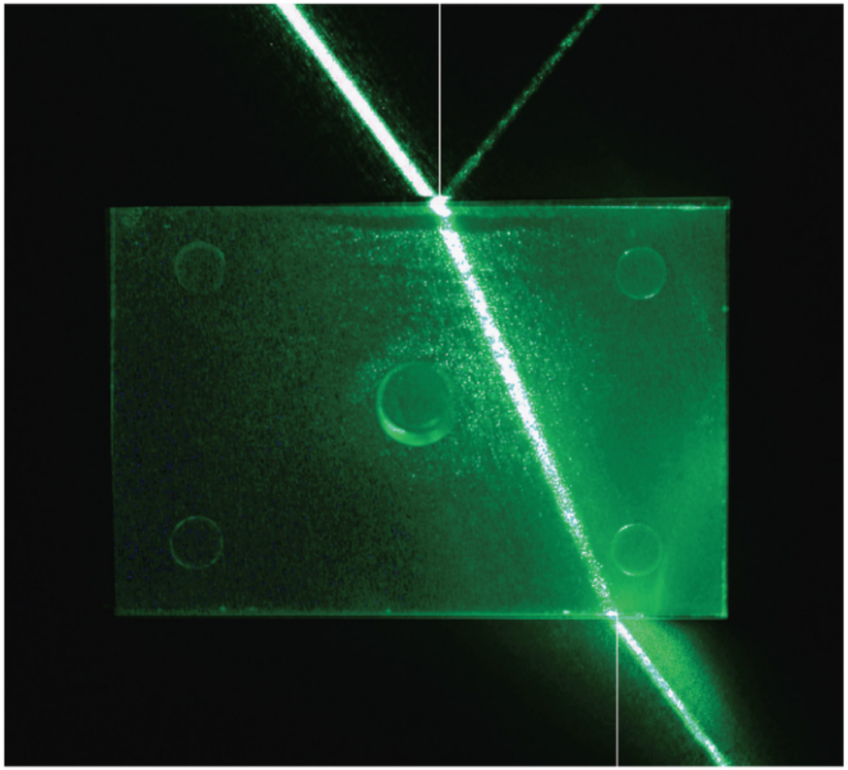
```
>>> from astropy import units as u
>>> # Convert from parsec to meter
>>> u.pc.to(u.m)
3.0856776e+16
>>> cms = u.cm / u.s
>>> mph = u.mile / u.hour
>>> cms.to(mph, 1)
0.02236936292054402
>>> cms.to(mph, [1., 1000., 5000.])
array([  2.23693629e-02,   2.23693629e+01,
1.11846815e+02])

>>> # Convert from nanometer to Hz
>>> u.nm.to(u.Hz, [1000, 2000], equivs=u.spectral())
array([  2.99792458e+14,   1.49896229e+14])
```

Wavelength and Speed of Light in vacuum vs. in medium

Light travels at a constant speed in vacuum

c = 299,792.458 km/s

Light travels slower in a medium (e.g., air, water, glass)



speed of light in medium

$$c_{\text{medium}} = c/n_{\text{medium}}$$

wavelength of light in medium

$$\lambda_{\text{medium}} = \lambda_{\text{vacuum}}/n_{\text{medium}}$$

The Law of Refraction & The Refractive Index of Medium

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

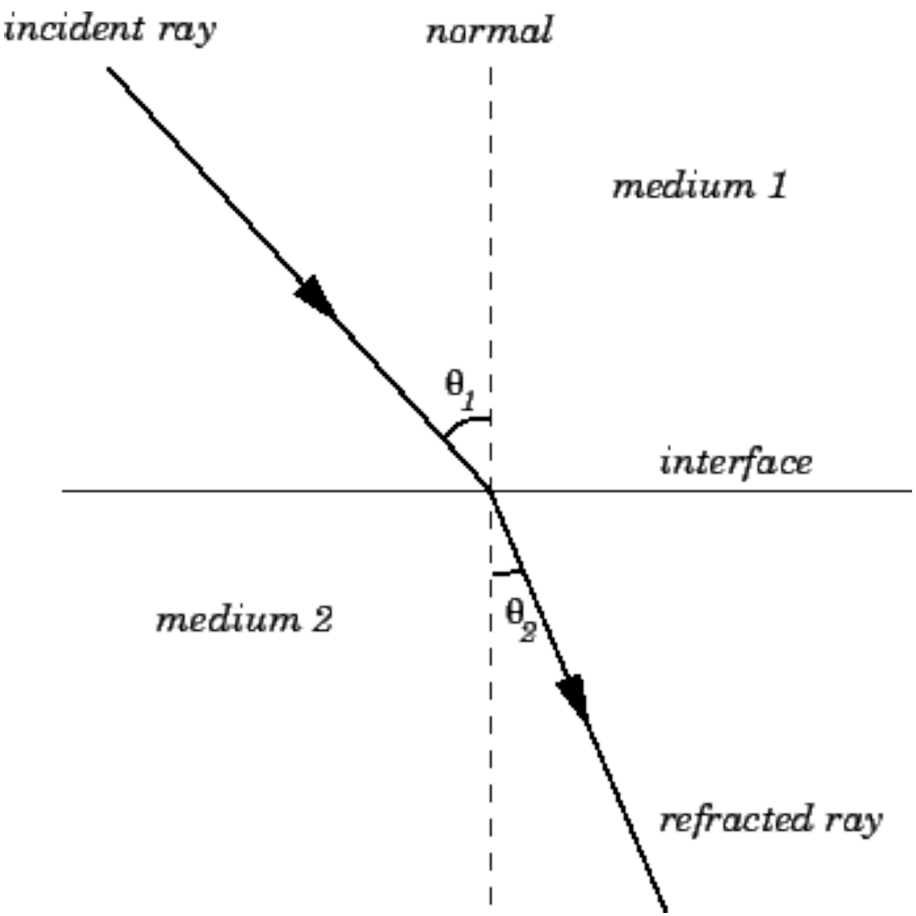


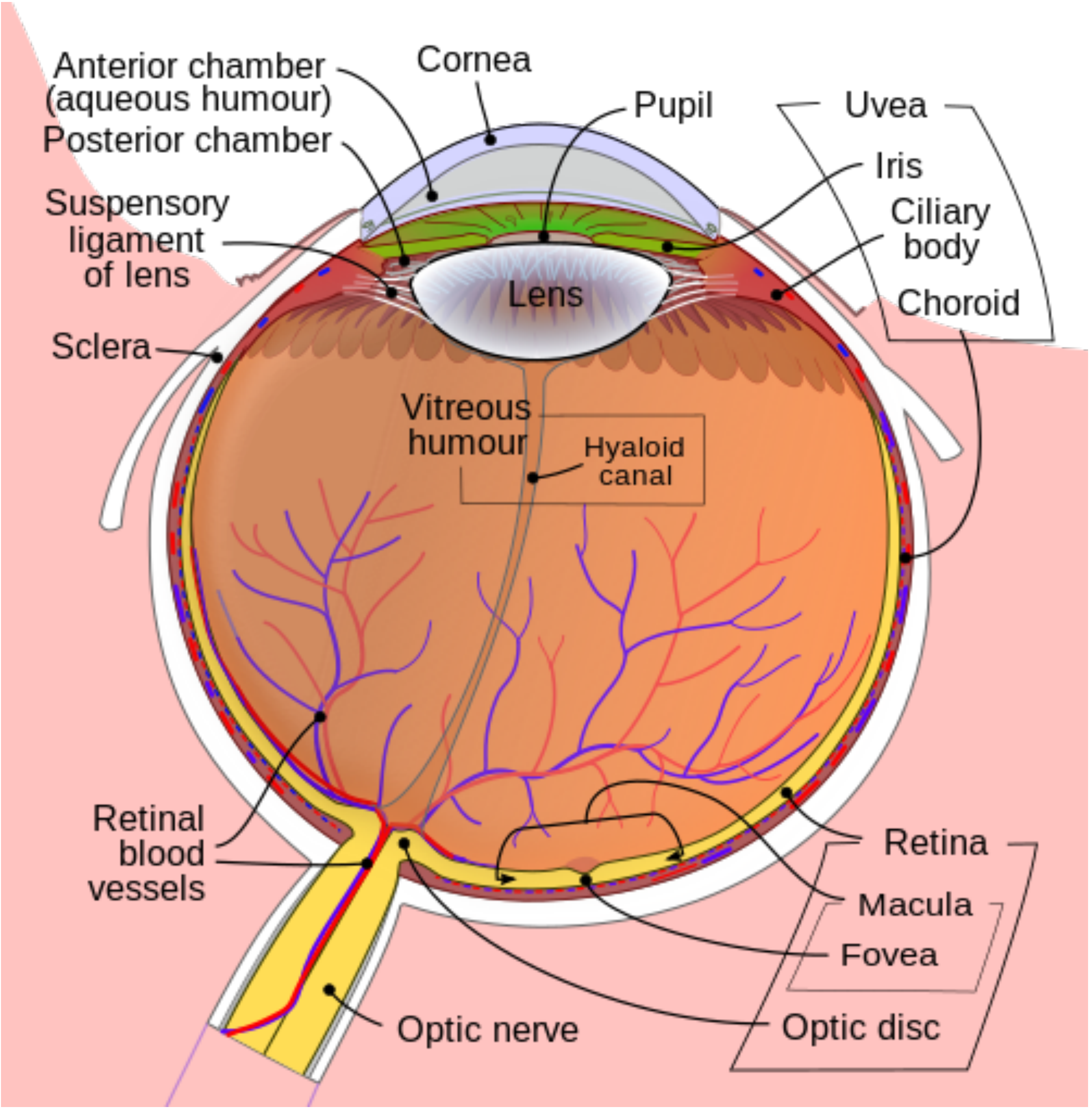
Table 4: *Refractive indices of some common materials at* $\lambda = 589$ *nm.*

Material	n
Air (STP)	1.00029
Water	1.33
Ice	1.31
Glass:	
Light flint	1.58
Heavy flint	1.65
Heaviest flint	1.89
Diamond	2.42

$$\lambda_{\text{medium}} = \lambda_{\text{vacuum}} / n_{\text{medium}}$$

Why color doesn't change under water?

Refractive index of Vitreous humor: n = 1.336 (similar to H_2O)

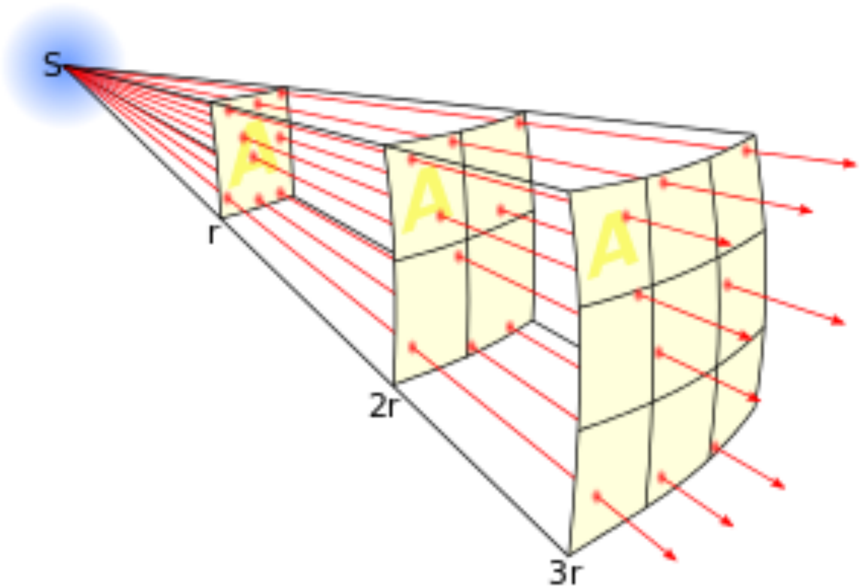


Luminosity vs. Flux

another inverse distance square law

The Inverse Square Law of EM Radiation Flux

- **Luminosity** is the total amount of energy per unit time (i.e., power) emitted by the source (unit: Watt = Joule/s)
- **Flux** is the amount of arriving energy per unit time per unit area (unit: Watt/m^2) at a distance *d* from source
- **Flux** decreases as the **distance** from the source increases, obeying an **inverse square law,** *if the source emits isotropically*:

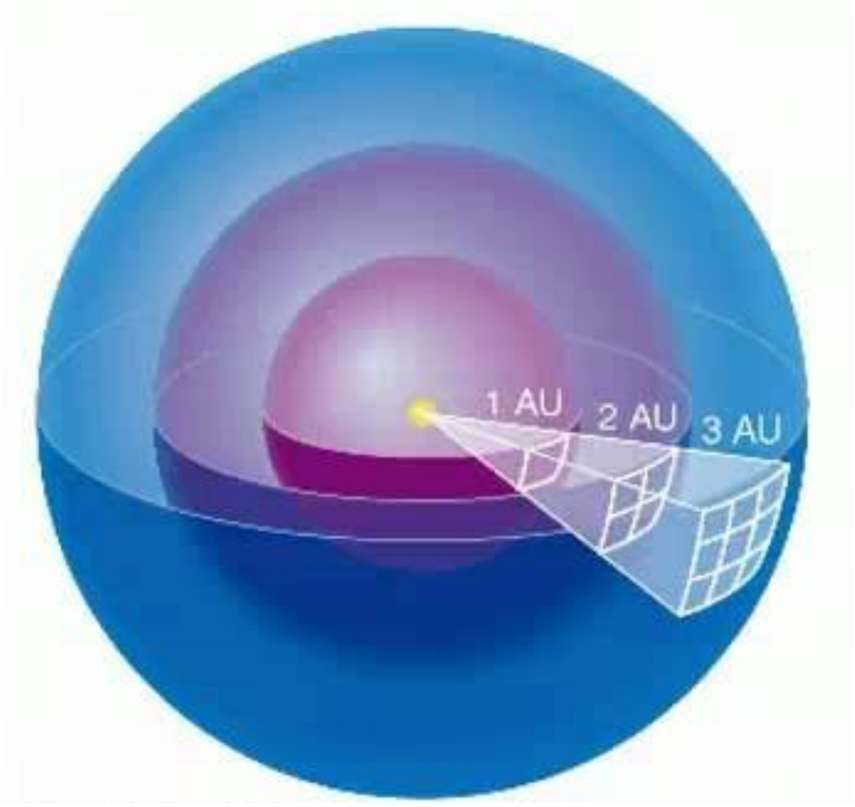


$$F = \frac{L}{4\pi d^2}$$

That is because luminosity is an intrinsic property of the source, so it does NOT change with distance

- **Luminosity** is the total power **(energy per unit time)** emitted by the source (SI unit: Watt = Joule/s; cgs unit: erg/s)
- Equations below assume the source emit light ***isotropically***

$$L = F(d) \times 4\pi d^2 = F(\text{surface}) \times 4\pi R^2$$



Practice: solar constant calculation

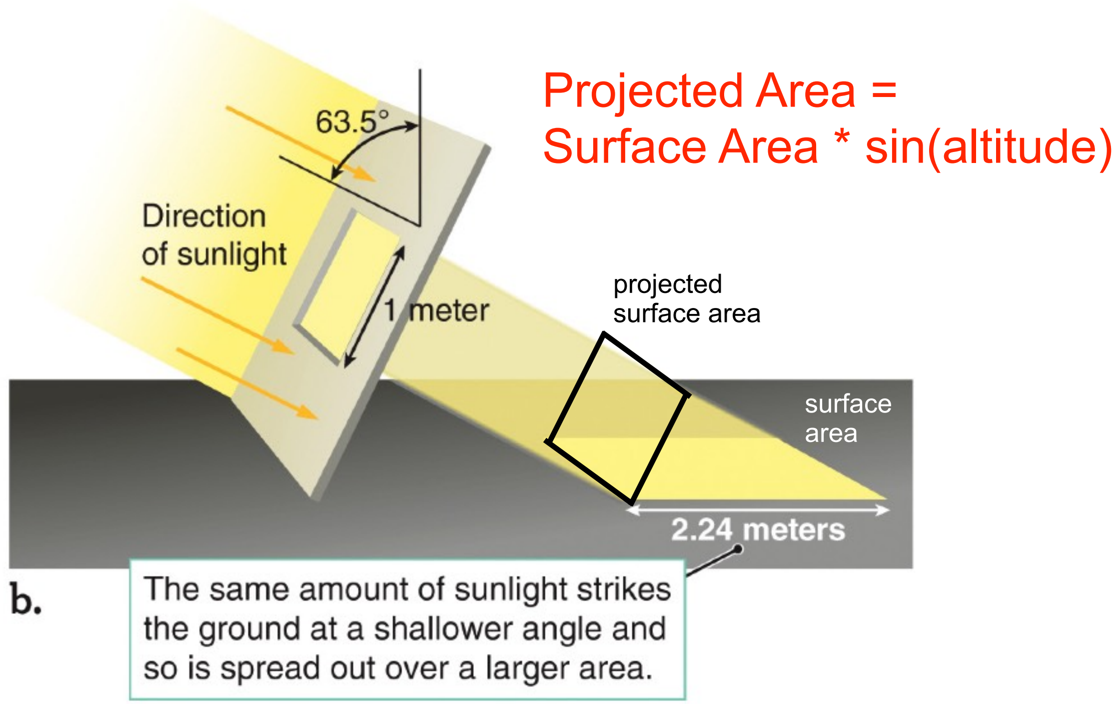
- The Sun has a **bolometric luminosity** of 3.86e26 Watts (L_{bol}: luminosity integrated over all wavelengths)
- The Earth is at the mean **distance** of 1.5e11 m (1 AU)
- What is the flux at the distance of the Earth for an area ***perpendicular*** to the Sun direction?

$$F = \frac{L}{4\pi d^2}$$

Flux & Received Power

- **Flux** is the amount of arriving energy per unit time per unit area (flux = power per unit area; unit: Watt/m²)
- What's the total amount of received power in a given area?

$$P = F \times \text{Projected Area}$$



Practice: Total Arrived Power from the Sun

- The flux of the Sun at the Earth's distance is 1.4 kW/m^2
- The Earth has a radius of 6400 km
- What is the total solar power intercepted by the Earth?
- US energy consumption: 400,000 MegaWatts

$$P = F \times \text{Projected Area}$$

Answer: 1.8e17 W = 180 Billion MegaWatts

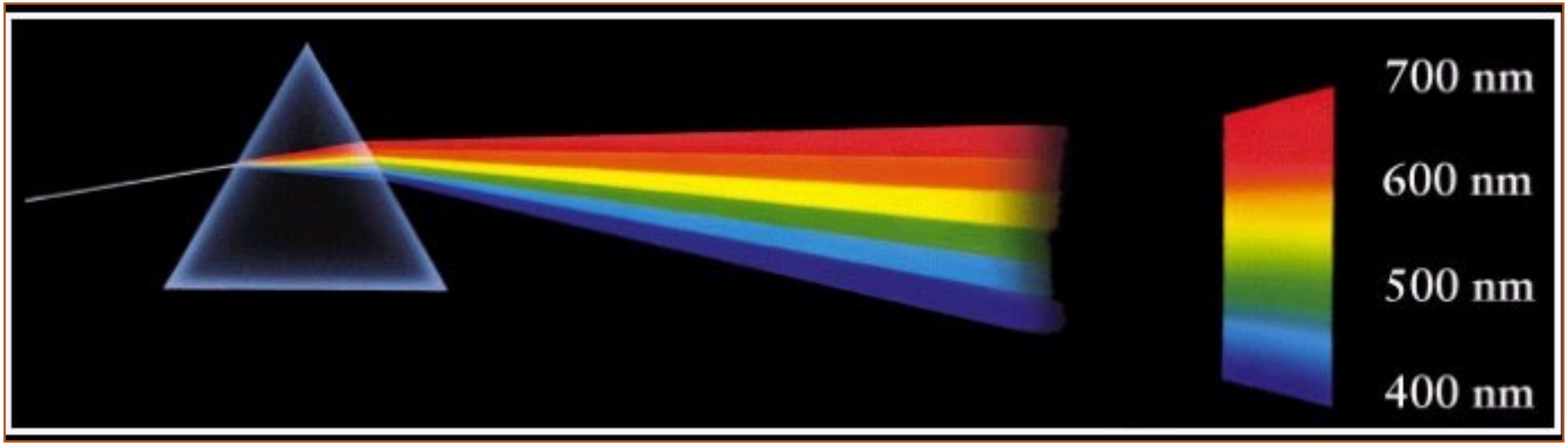
Palo Verde Generation Station (largest nuclear power plant in US): 3.8 MegaWatt, located in Arizona

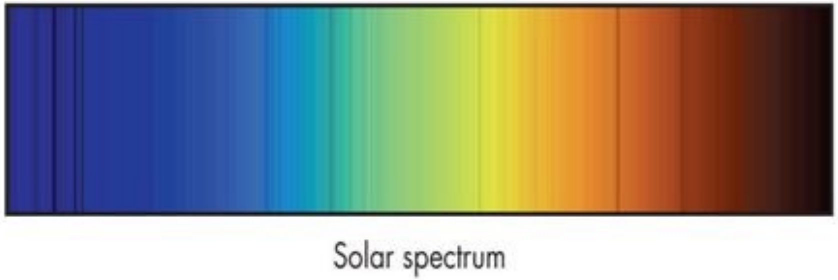
The Three Gorges Dam Power Station: 22,500 MegaWatts

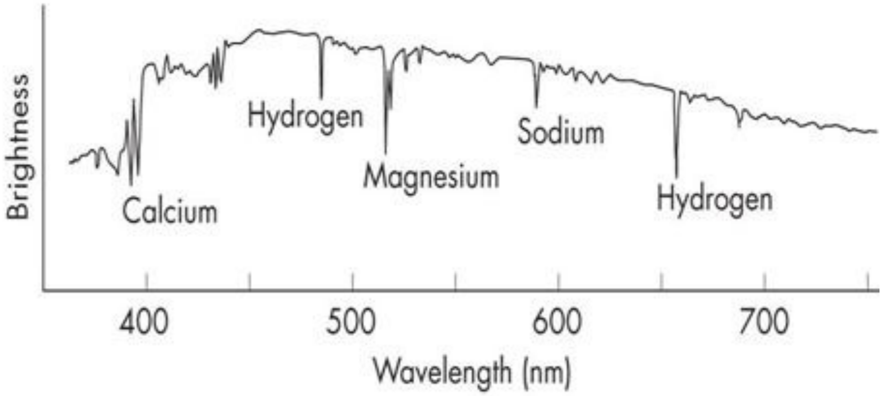
EM power as a function of wavelength or frequency

Basic Spectroscopy

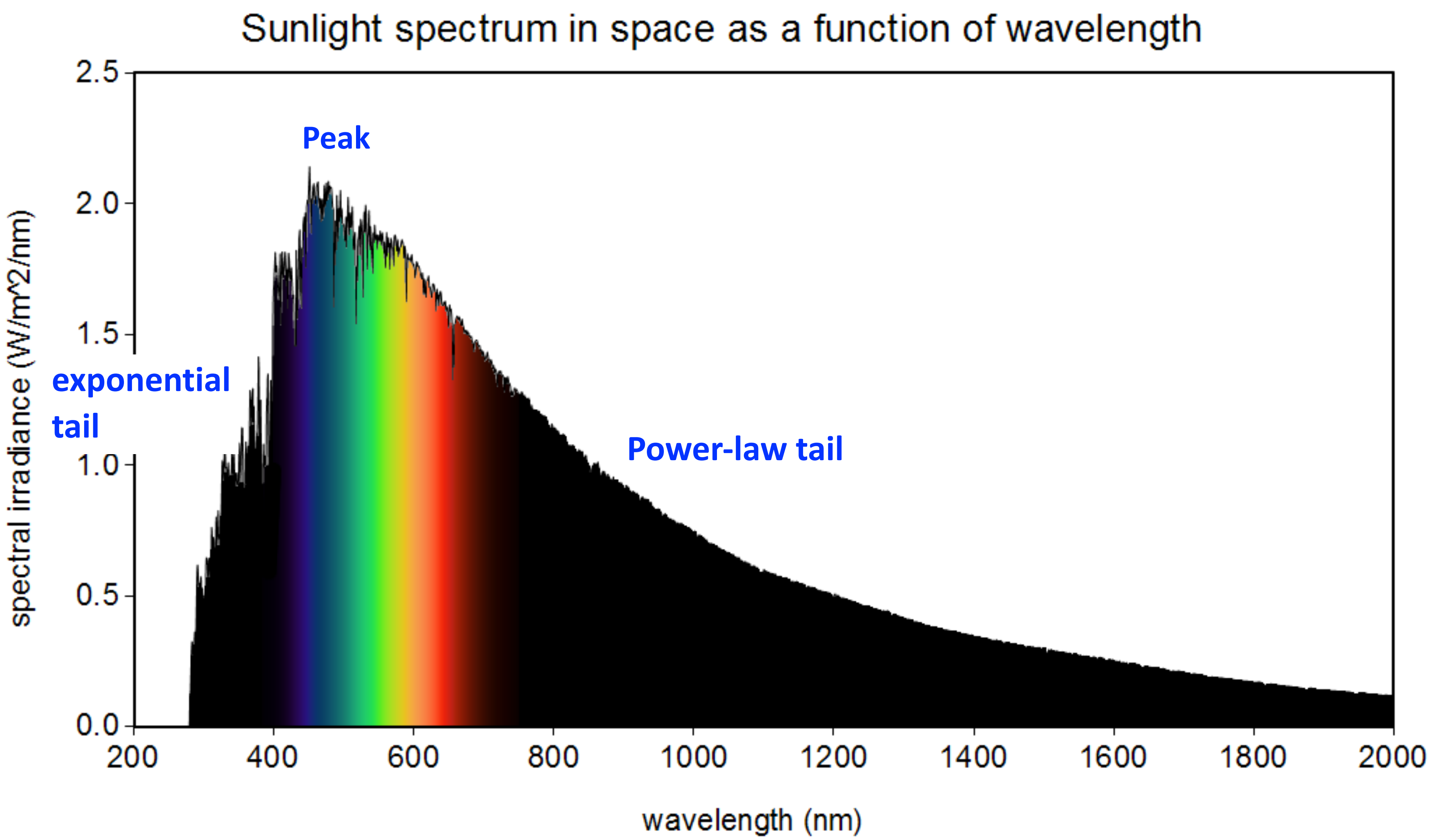
A Spectrum is a plot of
Flux Density (*flux per unit wavelength range*) vs. Wavelength







The Spectrum of the Sun from UV to IR



Chap 5 Part I: Equations of EM Radiation

speed-wavelength-frequency relation

$$c = \lambda \nu$$

Energy of a photon

$$E = h\nu$$

speed of light in medium

$$c_{\mathrm{medium}} = c/n_{\mathrm{medium}}$$

Inverse distance square law of received flux

$$L = F(d) \times 4\pi d^2$$

Total received power from radiative flux

$$P = F \times \mathrm{Projected\ Area}$$

Part II: Blackbody Emission

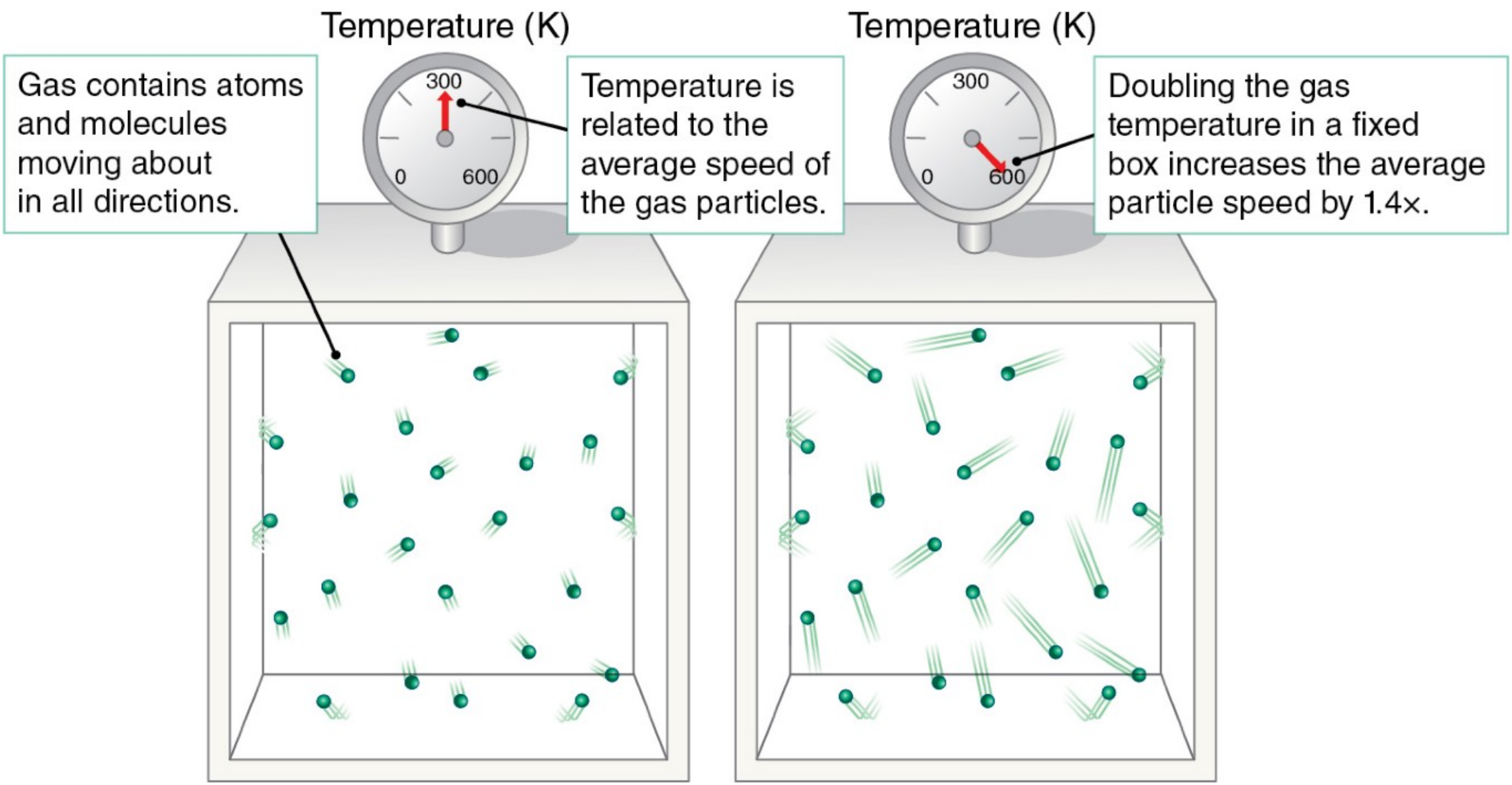
- **Blackbody emission**
  - Planck Function, determined by **Temperature (*T*)**
  - Surface flux vs. *T*: **Stefan-Boltzmann law**
  - Peak Radiation vs. *T*: **Wien's displacement law**
  - Application: **Equilibrium temperature of planets**

Blackbody Emission

Planck Function

What is Temperature (T)?

- **Temperature** is a measure of internal energy—and the average kinetic energy (speed) of atoms and molecules.
- **Kelvin scale**: water freezes/boils at 273 K/373 K
- **Absolute zero: thermal motion** stops



Different dense objects at the same temperature emit similar spectra of light

molten rock: 1,200 °C

molten steel: 1,200 °C

The hotter the object, the bluer its spectrum becomes

The Planck Function: Intensity (aka Brightness) vs. Temperature

$$B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc/\lambda}{kT}}-1}$$

h: Planck constant (J s)
k: Boltzmann constant (J/K)

this gives power ***emitted*** per unit projected surface area per unit solid angle per unit wavelength

Intensity is a.k.a.
Surface Brightness

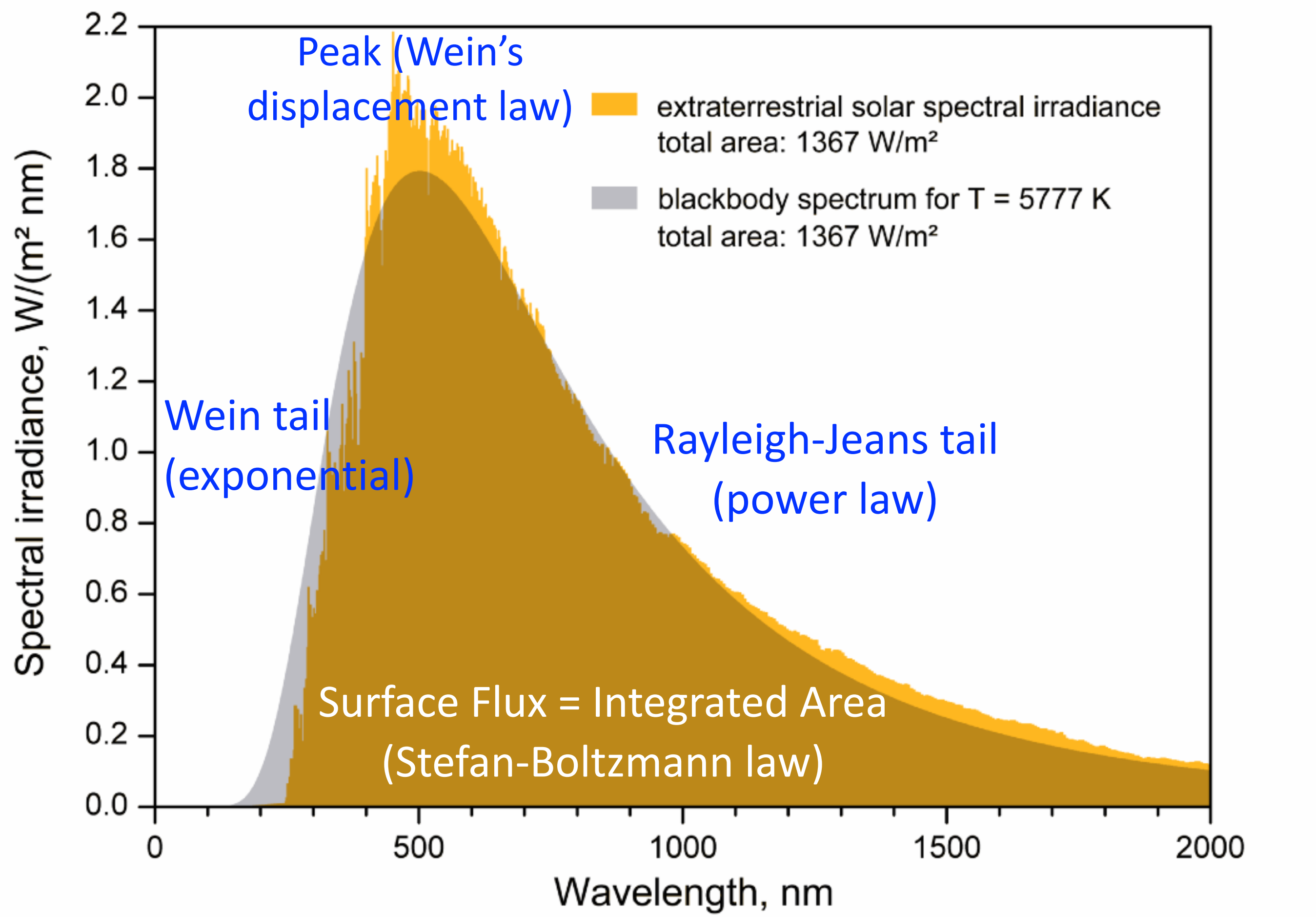
SI Unit of B_λ:
W/m2/m/sr

sr: steradian

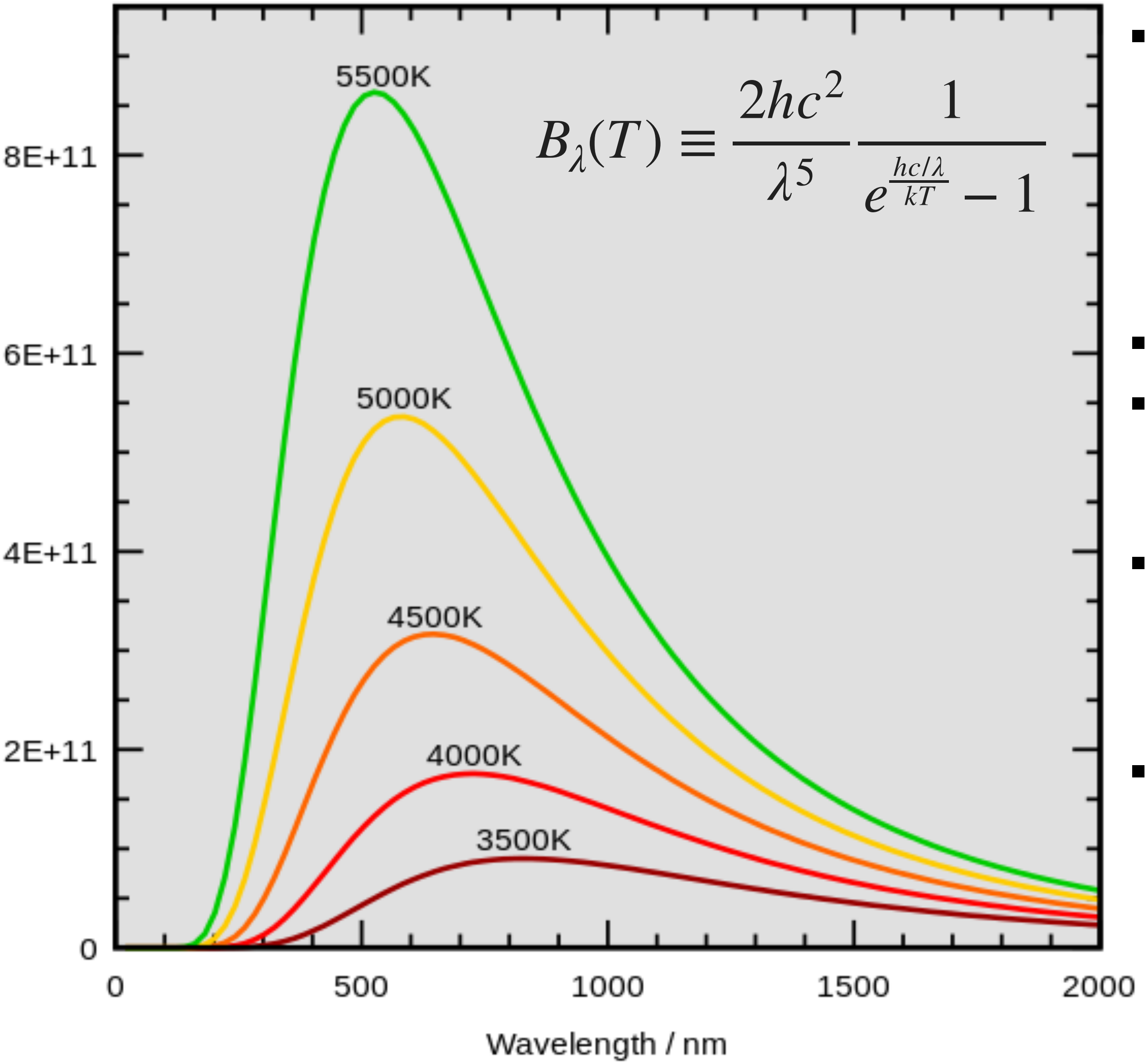
1 sr = $(180/\pi)^2$ $\deg^2$

whole sky = 4π sr = 41253 $\deg^2$

Spectral Irradiance =
$B_\lambda \times \Omega_{\text{unit}}$



The Planck Curves Describe the Spectra of Blackbody Emission



- It is determined by ***Temperature***, so blackbody emission is also called **thermal emission**

- **When *T* increases:**
- **Peak** shifts to shorter wavelength - **Wien's Displacement Law**
- **Surface Flux,** the total area under each Planck curve, increases rapidly - **Stefan-Boltzmann Law**
- **Surface brightness** increases at all wavelengths - **infrared thermometer**

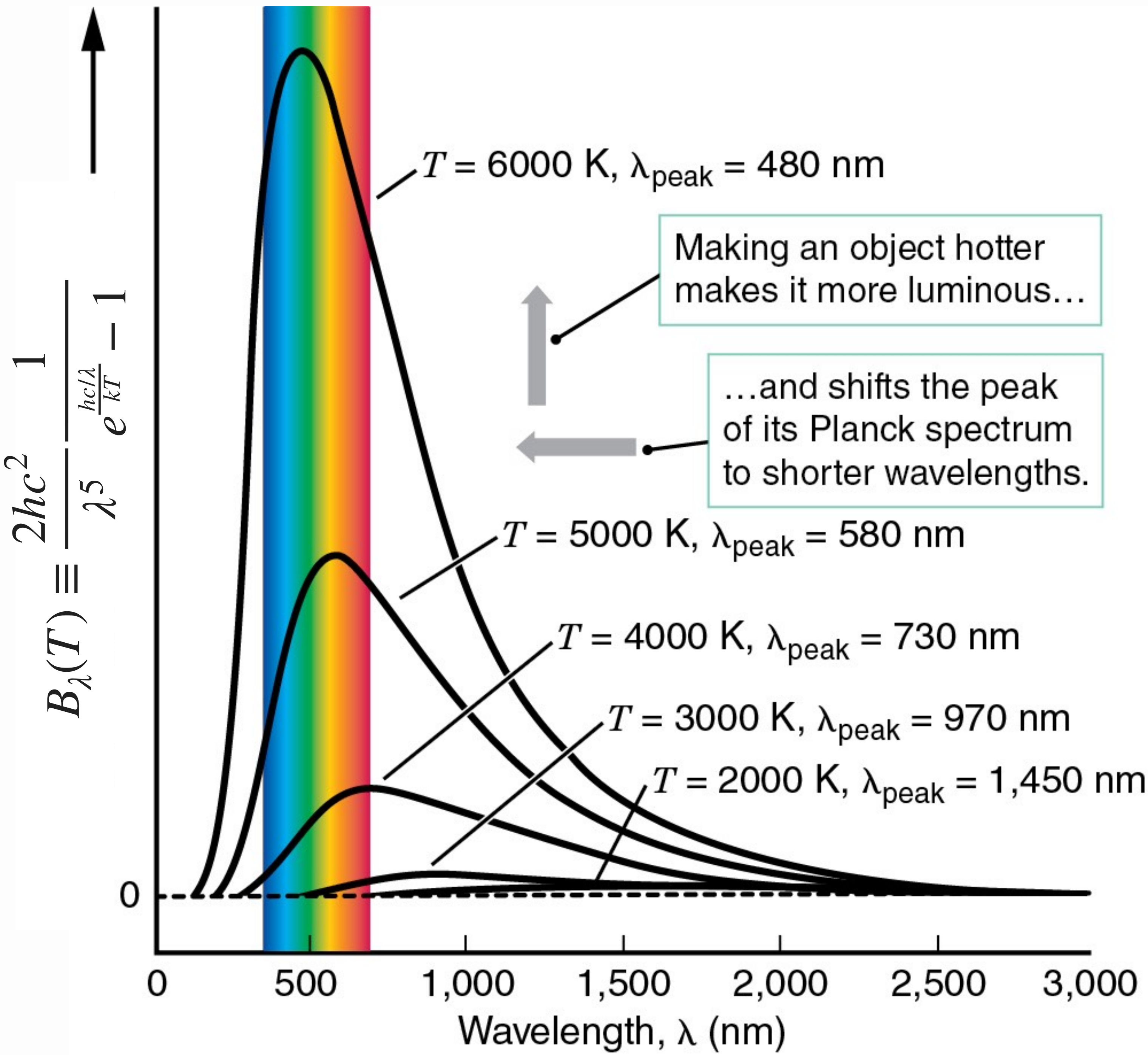
Using an infrared camera to measure temperature:

It measures surface brightness of the thermal emission at a fixed wavelength

Blackbody Emission

Peak Wavelength

The hotter the object, the bluer its spectrum becomes



$B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc/\lambda}{kT}}-1}$
T = 6000 K, λpeak = 480 nm
Making an object hotter makes it more luminous…
…and shifts the peak of its Planck spectrum to shorter wavelengths.
T = 5000 K, λpeak = 580 nm
T = 4000 K, λpeak = 730 nm
T = 3000 K, λpeak = 970 nm
T = 2000 K, λpeak = 1,450 nm
0
0
500
1,000
1,500
2,000
2,500
3,000
Wavelength, λ (nm)

Wien's Displacement Law

- The peak wavelength of a blackbody is inversely proportional to its temperature.

$$\lambda_{\text{peak}} = \frac{2.9 \text{ mm K}}{T}$$

- Peak wavelength λ_{peak} is the wavelength of light of a blackbody that is emitted the most.
- Here the wavelength is in nanometers and the temperature is in kelvins.
- "Hotter means bluer."

Wien's Displacement Law - Derivation

- Start with Planck function, do derivative over lambda (this is the gradient of the curve), find where the gradient equals zero

$$u_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}.$$

Differentiating $u(\lambda, T)$ with respect to λ and setting the derivative equal to zero gives:

$$\frac{\partial u}{\partial \lambda} = 2hc^2\left(\frac{hc}{kT\lambda^7}\frac{e^{hc/\lambda kT}}{\left(e^{hc/\lambda kT} - 1\right)^2} - \frac{1}{\lambda^6}\frac{5}{e^{hc/\lambda kT} - 1}\right) = 0,$$

which can be simplified to give:

$$\frac{hc}{\lambda kT}\frac{e^{hc/\lambda kT}}{e^{hc/\lambda kT} - 1} - 5 = 0.$$

By defining:

$$x \equiv \frac{hc}{\lambda kT},$$

the equation becomes one in the single variable *x*:

$$\frac{xe^x}{e^x - 1} - 5 = 0.$$

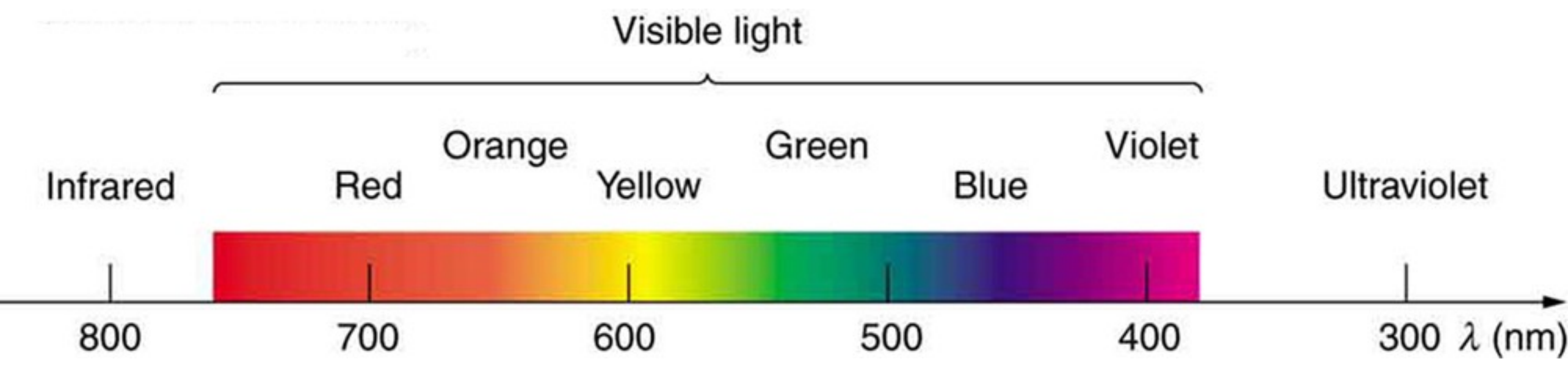
$\lambda_{\text{peak}} = hc / xkT = (2.897\,\text{mm}\cdot\text{K}) / T.$

Practice: Wien's Displacement Law

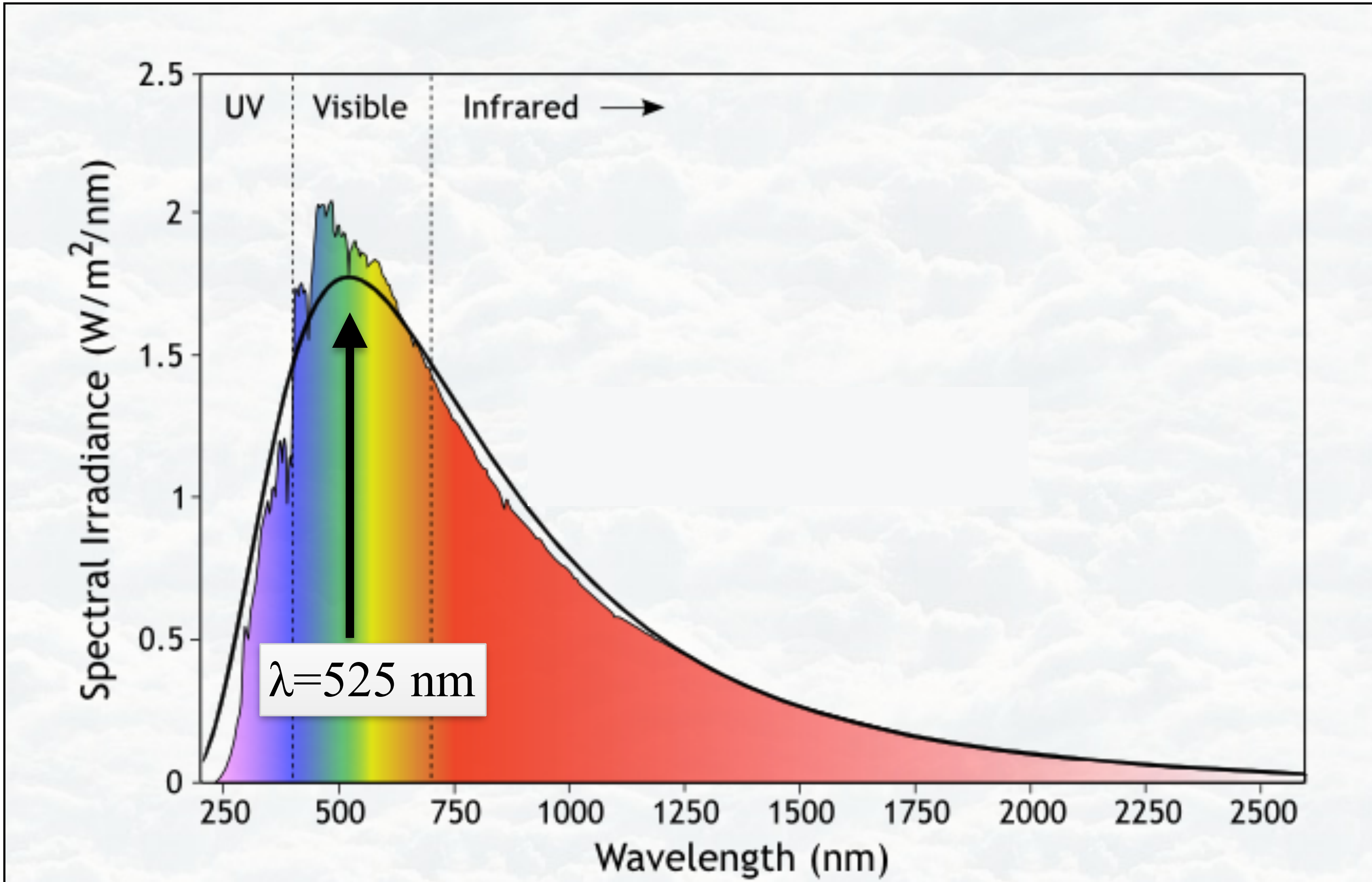
- The Sun has a mean surface temperature of 5800 K, calculate the wavelength at which the emission peaks. Give your answer in nanometer (nm).

$$\lambda_{\text{peak}} = \frac{2.9\ \text{mm K}}{T}$$

Answer: 500 nm



The Spectrum of the Sun outside of the Earth's atmosphere

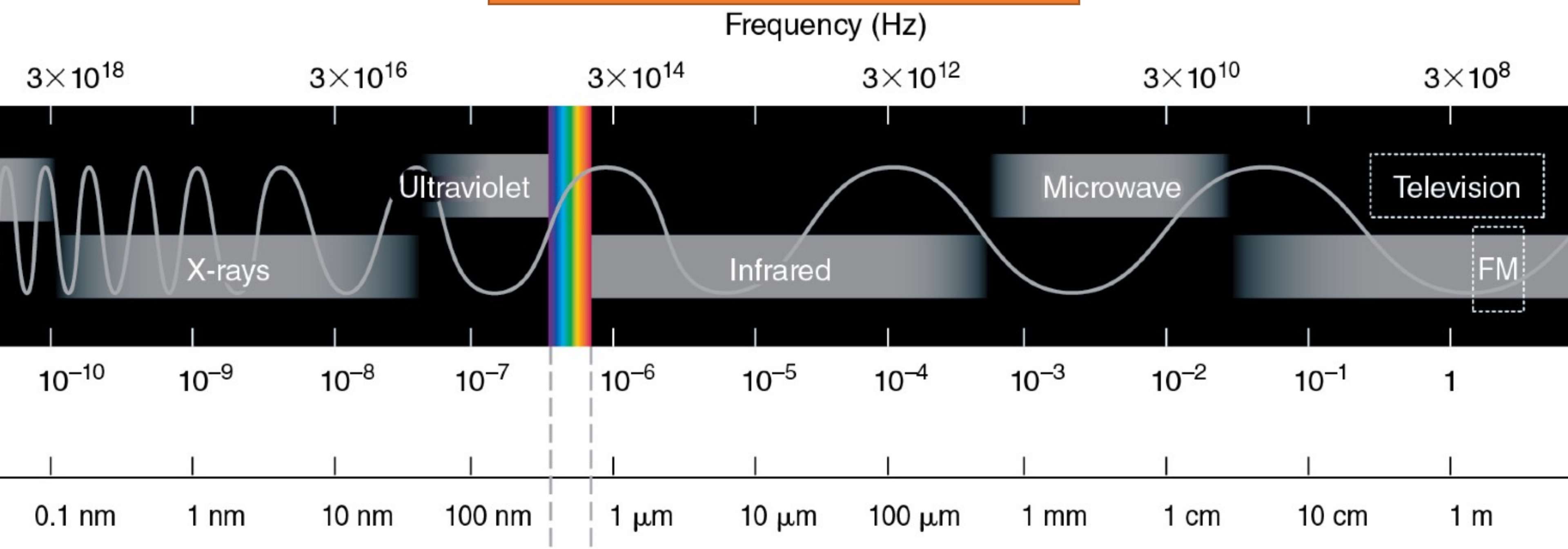


Practice: Wien's Displacement Law

- The Earth has a mean surface temperature of 288 K, calculate the wavelength at which the emission peaks. Give your answer in micron (um).

$$\lambda_{\text{peak}} = \frac{2.9 \text{ mm K}}{T}$$

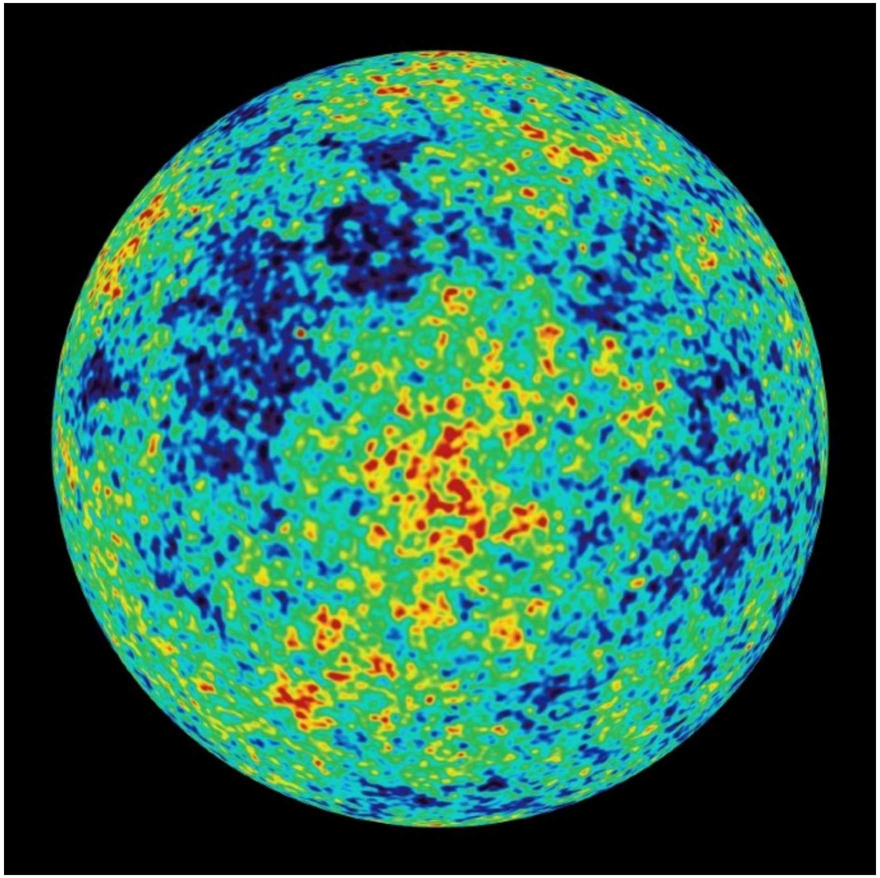
Answer: 10,000 nm = 10 um

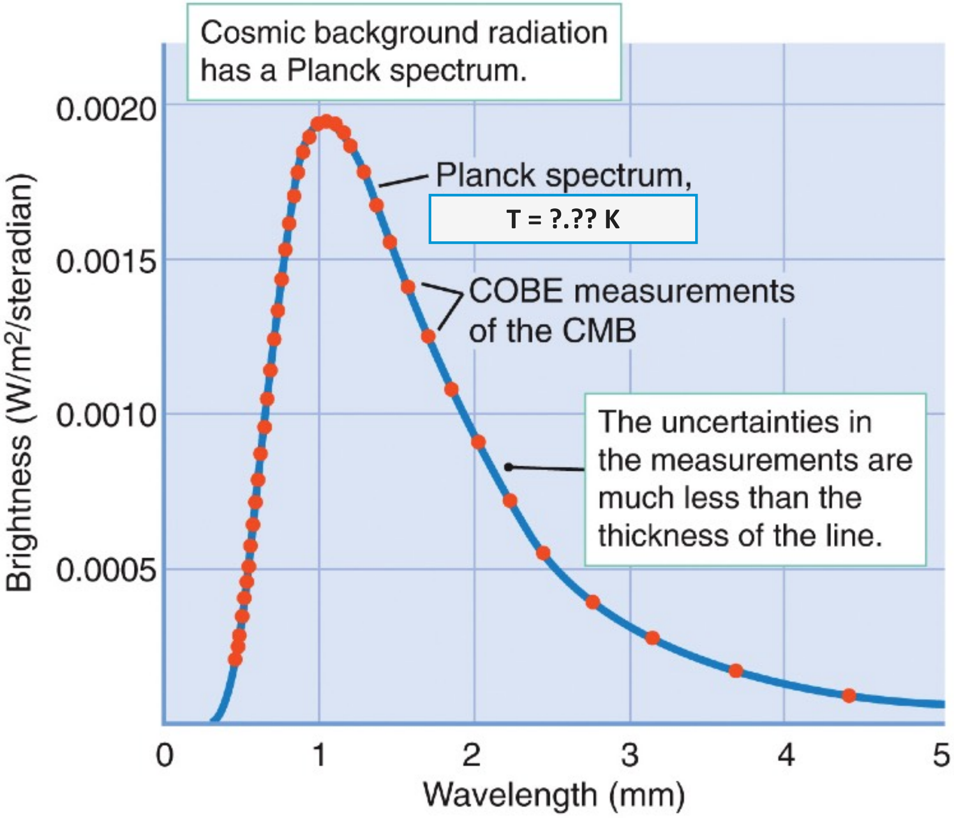


Measure the Temperature of the Universe with the Cosmic Microwave Background

- The CMB spectrum peaks at 1 mm, what's its temperature?

$$\lambda_{\text{peak}} = \frac{2.9 \text{ mm K}}{T}$$

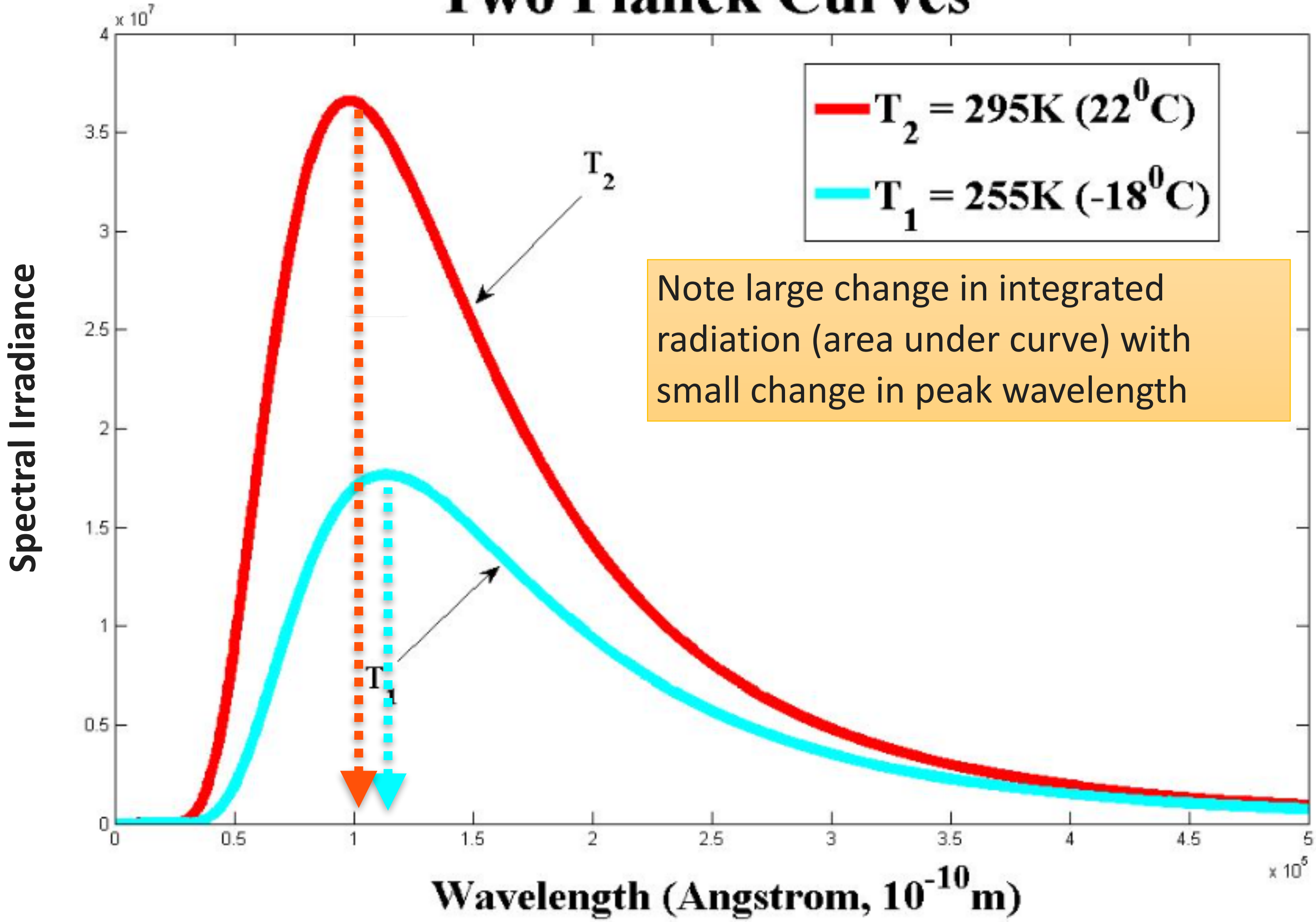




Blackbody Emission

Surface Flux

Two Planck Curves



Surface Flux – Stefan-Boltzmann Law

- **Surface Flux** is the total amount of energy emitted per square meter every second (the luminosity per area).

$$F = \sigma_{\mathrm{SB}} T^4$$

$$T(K) = T(C) + 273.15$$

$$\sigma_{\mathrm{SB}} = \frac{2\pi^5 k^4}{15 c^2 h^3} = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$$

where *T* is the temperature in K, *F* is the flux, and σ (sigma) is called the **Stefan-Boltzmann constant.**

- Hotter objects emit *much* more energy (per square meter per second) than cool objects: ***surface flux ~ T^4***

Stefan-Boltzmann Law - Derivation

- Start with intensity given by the Planck function, first integrate over 2PI solid angle, then integrate over frequency from zero to infinity, and

Intensity: power emitted per unit projected surface area per unit solid angle per unit frequency

$$I_\nu = B(\nu, T) \equiv \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

Surface Flux: power emitted per unit surface area

$$F = \int_{\nu=0}^{\infty} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} B(\nu, T) \; \cos\theta \; \sin\theta \; d\theta \; d\phi \; d\nu$$

To evaluate this integral, do a substitution,

$$u = \frac{h\nu}{kT}$$

$$du = \frac{h}{kT} d\nu$$

$$F = \frac{2\pi h}{c^2} \left(\frac{kT}{h} \right)^4 \int_0^\infty \frac{u^3}{e^u - 1} du.$$

Practice: Stefan-Boltzmann law

- With the Stefan-Boltzmann law, find Earth's surface flux using its average temperature of +15 C.

$$F = \sigma_{\mathrm{SB}} T^4$$

$$T(K) = T(C) + 273.15$$

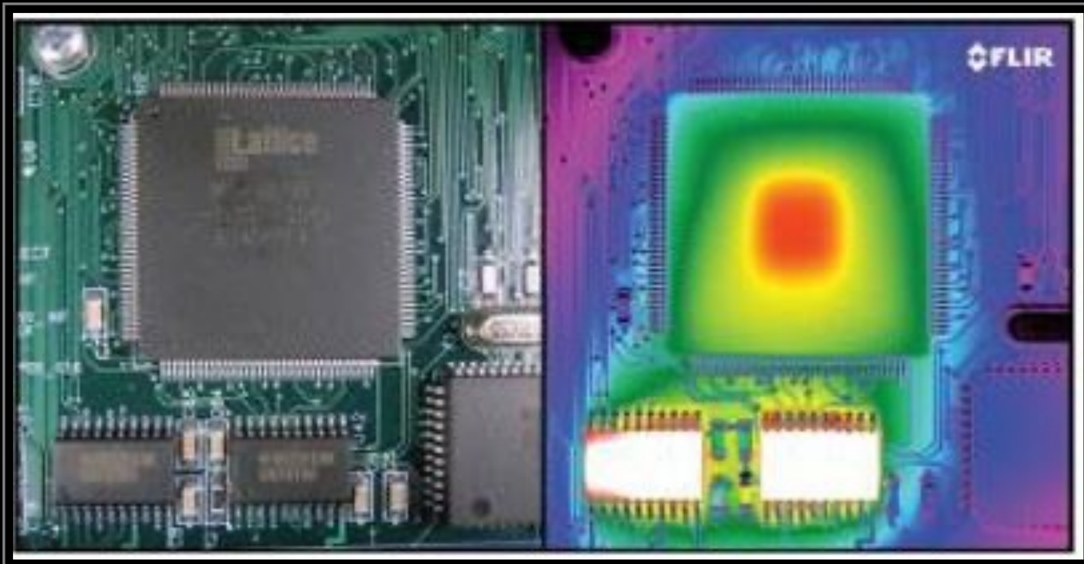
$$\sigma_{\mathrm{SB}} = \frac{2\pi^5 k^4}{15 c^2 h^3} = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$$

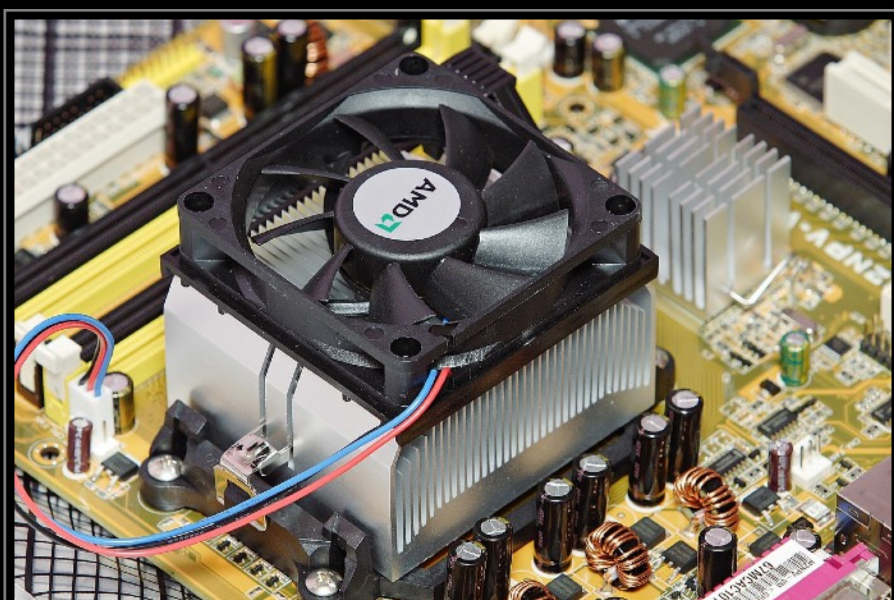
Answer: 390 W/m^2
About a quarter of the Solar constant (1366 W/m^2)
Is this a coincidence?

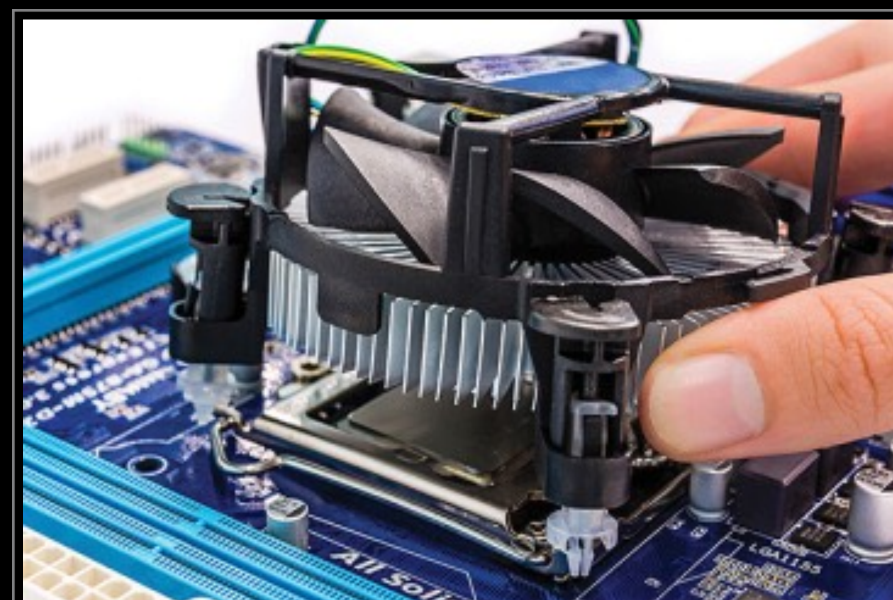
Application of Blackbody Theory

Equilibrium Temperatures of Planets in the absence of atmosphere

heating & cooling - CPU temperature

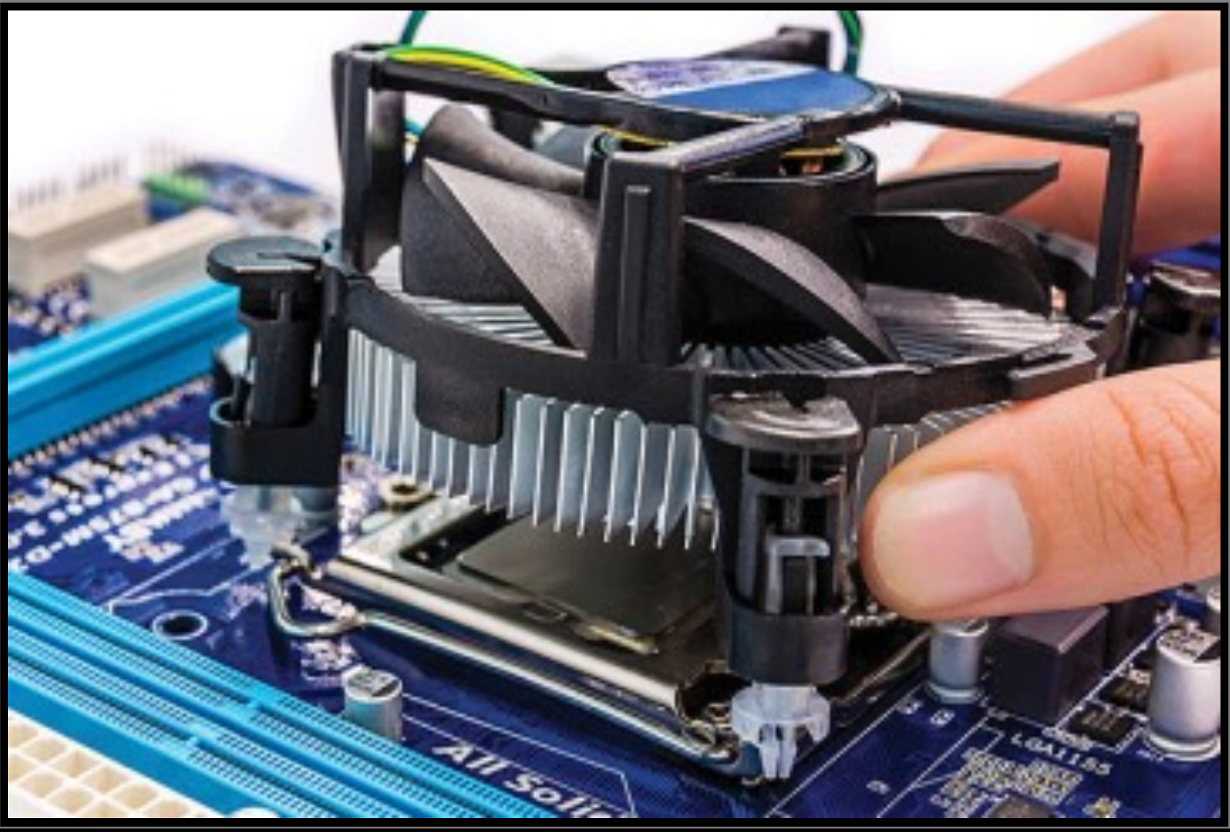






what happens to the CPU when the cooling fan stops working?

Its temperature increases until it stabilizes again (reaching a new equilibrium)



Calculating a Planet's Equilibrium Temperature

- **Heating rate**: The solar power received by the planet depends on its heliocentric distance, modulated by its **albedo** (***a***; *which* measures its reflectivity, the higher the albedo, the less energy it absorbs).

$$\begin{pmatrix}\text{Energy absorbed}\\ \text{by the planet}\\ \text{each second}\end{pmatrix} = \begin{pmatrix}\text{Absorbing}\\ \text{area of}\\ \text{the planet}\end{pmatrix} \times \begin{pmatrix}\text{Solar Flux}\end{pmatrix} \times \begin{pmatrix}\text{Fraction}\\ \text{of sunlight}\\ \text{absorbed}\end{pmatrix}$$

$$= \pi R_{\text{planet}}^2 \times \frac{L_{\text{Sun}}}{4\pi d^2} \times (1 - a)$$

Calculating a Planet's Equilibrium Temperature

- **Cooling Rate**: The power radiated away from the planet depends on its surface area and its surface temperature.

$$\begin{pmatrix}\text{Energy radiated}\\ \text{by planet each}\\ \text{second}\end{pmatrix} = \begin{pmatrix}\text{Surface}\\ \text{area of}\\ \text{planet}\end{pmatrix} \times \begin{pmatrix}\text{Energy radiated}\\ \text{per square meter}\\ \text{per second}\end{pmatrix}$$

$$= 4\pi R^2 \times \sigma T^4$$

- Homework question. Here we had assumed *uniform* surface temperature, what if this wasn't the case?

Calculating a Planet's Equilibrium Temperature

- The **equilibrium temperature** of a planet is the temperature at which the cooling rate balances the heating rate:

$$\begin{pmatrix}\text{Energy radiated}\\ \text{by the planet}\\ \text{each second}\end{pmatrix} = \begin{pmatrix}\text{Energy absorbed}\\ \text{by the planet}\\ \text{each second}\end{pmatrix}$$

$$4\pi R_{\text{planet}}^2 \sigma T^4 = \pi R_{\text{planet}}^2 \frac{L_{\text{Sun}}}{4\pi d^2}(1 - a)$$

- Solving for T, we get the equilibrium temperature:

$$T = \left(\frac{L_{\text{Sun}}(1 - a)}{16\sigma\pi d^2}\right)^{1/4}$$

Blackbody Emission

quick recap of previous lecture

How to convert frequency to wavelength

Chat-GPT version 3.5

The relationship between frequency and wavelength is defined by the speed of light. In a vacuum, the speed of light is a constant, denoted by c, which is approximately 3.00×10^8 meters per second. The formula for the relationship between frequency (f) and wavelength (λ) is as follows:

$$c = \lambda \times f$$

where:

- c is the speed of light in a vacuum (approximately 3.00×10^8 meters per second).
- λ is the wavelength in meters.
- f is the frequency in hertz (Hz).

To convert frequency to wavelength, you can use the formula:

$$\lambda = \frac{c}{f}$$

For example, if you have a frequency of 500 megahertz (MHz), you would convert it to hertz by multiplying by 10^6 (since 1 MHz = 10^6 Hz), and then plug it into the formula to find the

Write me a python code to calculate equilibrium temperature of planets

Chat-GPT version 3.5

Calculating the equilibrium temperature of a planet involves considering several factors, including the incoming solar flux, albedo (reflectivity), greenhouse effect, and the Stefan-Boltzmann law, among others. Here's a simple Python script that calculates the equilibrium temperature of a planet, assuming it behaves as a black body and neglecting any atmosphere effects:

python Copy code

```python
import math

# Constants
solar_constant = 1361  # Solar constant in W/m^2
albedo = 0.3  # Albedo of the planet
stefan_boltzmann_constant = 5.67e-8  # Stefan-Boltzmann constant in W/m^2K^4

# Function to calculate equilibrium temperature
def calculate_equilibrium_temperature(solar_constant, albedo):
    temperature = ((1 - albedo) * solar_constant / (4 * stefan_boltzmann_con
    return temperature
```

Tell me about blackbody emission

Chat-GPT version 3.5

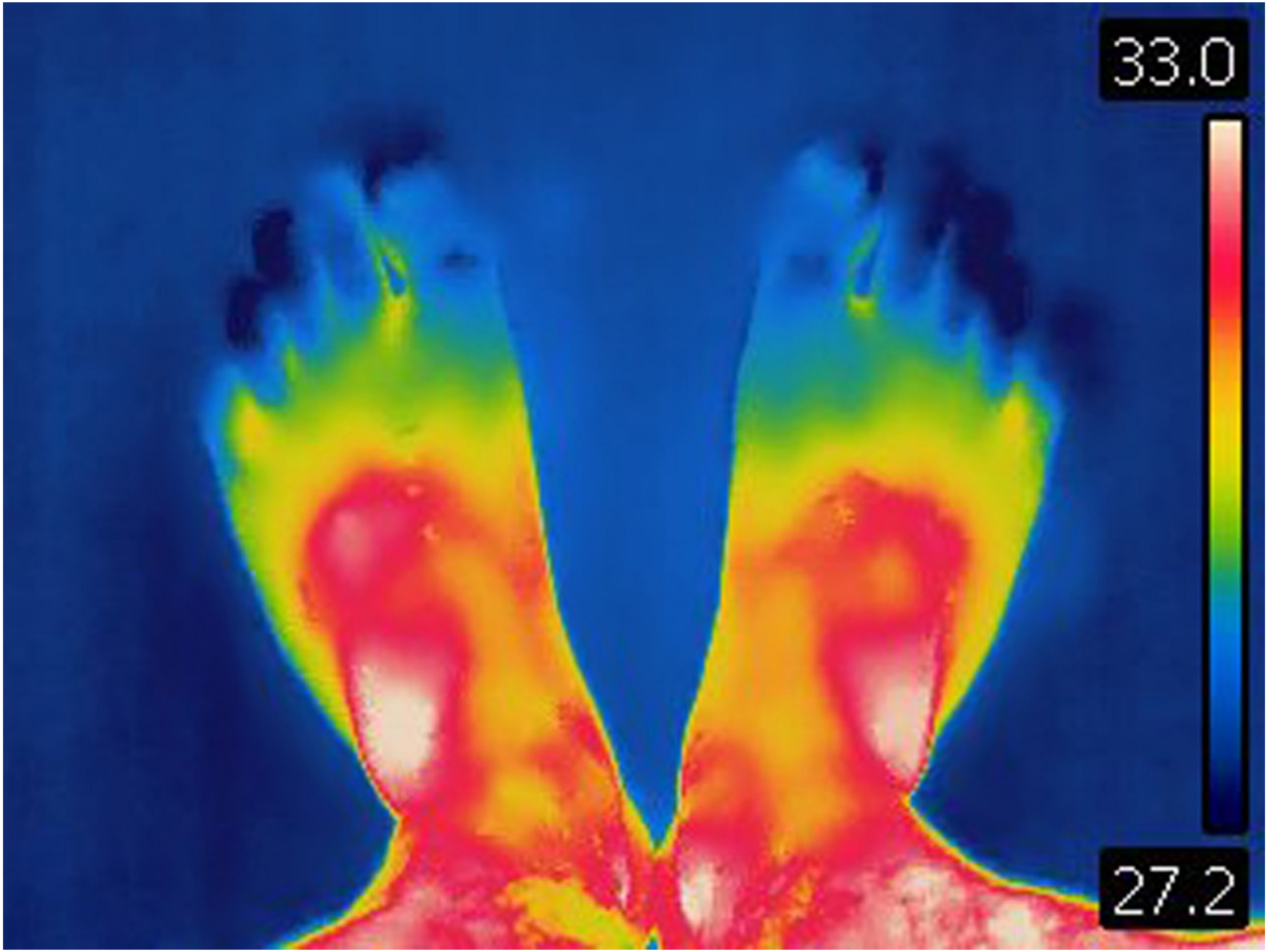
Blackbody radiation, also known as blackbody emission, is the electromagnetic radiation emitted by a blackbody, which is an idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence. Blackbody radiation has been a crucial concept in the development of modern physics and plays a significant role in fields such as astrophysics, thermodynamics, and quantum mechanics.

The fundamental principles of blackbody radiation were first formulated by the German physicist Max Planck in 1900 as part of his successful attempt to derive a formula for the spectral distribution of energy emitted by a blackbody at a given temperature. Planck's work marked the beginning of quantum theory.

Key characteristics of blackbody radiation include:

1. **Spectrum:** The spectral distribution of energy emitted by a blackbody depends solely on its temperature. At any given temperature, a blackbody emits radiation across a continuous spectrum of wavelengths, and the intensity of the emitted radiation at each wavelength follows a specific pattern, as described by Planck's law.
2. **Temperature Dependence:** As the temperature of a blackbody increases, the total amount of radiation it emits increases, and the peak of the emission shifts to shorter wavelengths,

Blackbody Emission = *Thermal* Emission, is determined by *Temperature*



Examples of Blackbody Emitters and non-Blackbody Emitters

- **Blackbody emission** is determined by the temperature of the emitter
  - Incandescent light bulbs
  - The reddish glow from oven, lava, blast furnace
  - Stars of all masses
  - Mid-infrared emission of planets
  - Far-infrared emission of dusts in the interstellar medium
  - Microwave emission of the Universe
  - Thermal emission of any objects (e.g., human body)

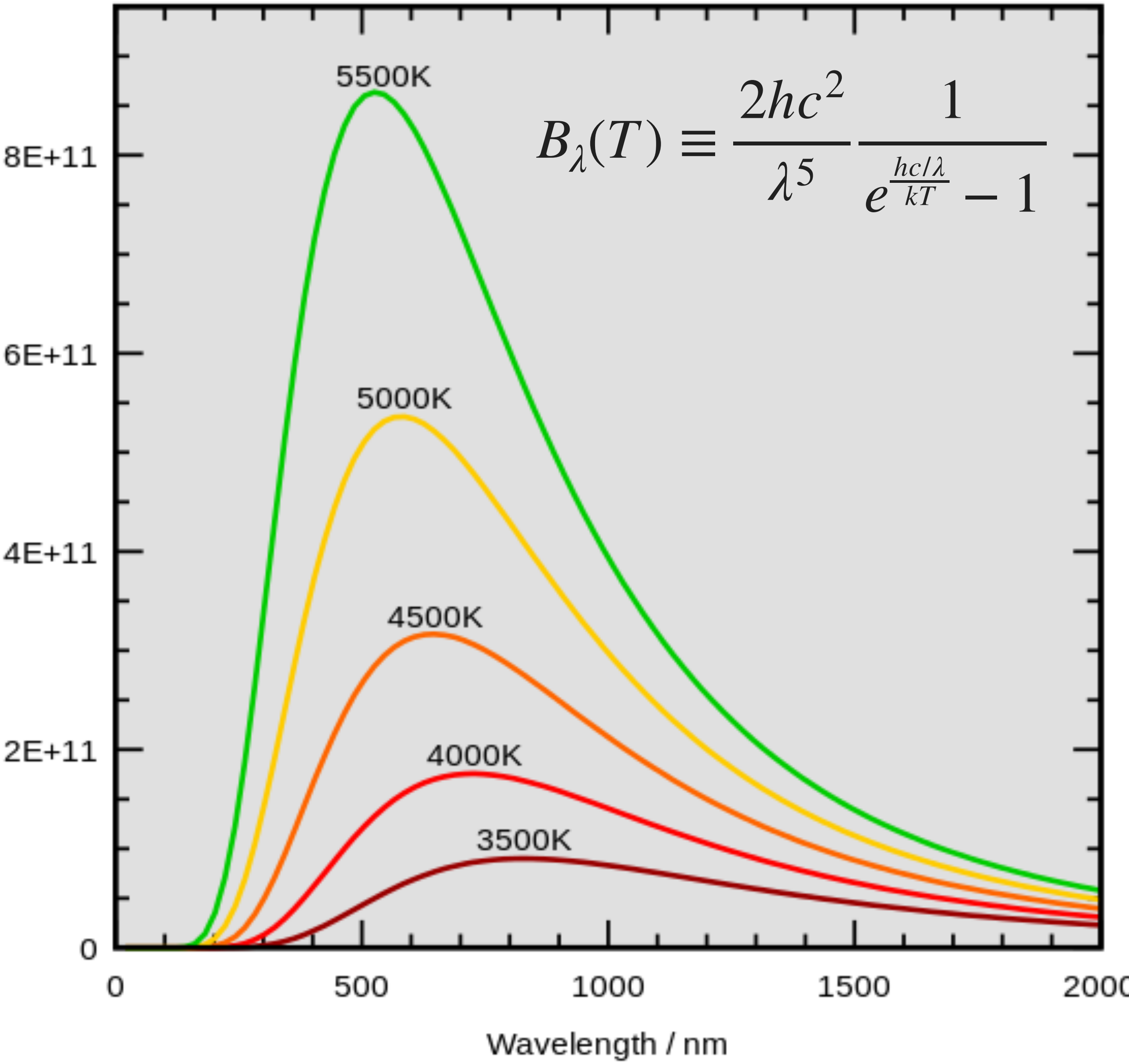
- **non-Blackbody emission** is *NOT* determined by the temperature of the emitter
  - Light-emitting diode (LED)
  - Phone / TV screens
  - Fluorescent light bulbs
  - Nebular emission from low density gas (e.g., Ring nebula)
  - jets from black holes
  - X-ray emission from the intra-cluster medium
  - Reflected light from the surface of any objects (e.g., planets in visible light)

Received Flux, Surface Flux, Surface Brightness, & Luminosity

- *Received Flux* is an *apparent* property that depends on the distance to the emitter:
  - $$F = \frac{L}{4\pi d^2}$$

- *Luminosity (L), Surface Flux (F_S), and Surface Brightness (I_λ)* are all *intrinsic* properties of the emitter. For **blackbody emitters** (i.e., thermal emitters), we have the following relations:
  - $F_S = \sigma_{SB} T^4$ (Stefan-Boltzmann Law)
  - $I_\lambda = B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc/\lambda}{kT}} - 1}$ (Planck Function)
  - $L = \int F_S(T) dA = F_S(T) \times 4\pi R^2$ (for uniform T sphere)

The Planck Curves Describe the Spectra of Blackbody Emission



$$B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc/\lambda}{kT}} - 1}$$

- It is determined by ***Temperature***, so blackbody emission is also called **thermal emission**

- **When *T* increases:**

1. **Peak** shifts to shorter wavelength - **Wien's Displacement Law**
2. **Surface Flux,** the total area under each Planck curve, increases rapidly - **Stefan-Boltzmann Law**
3. **Surface brightness** increases at all wavelengths - **infrared thermometer**

Calculating a Planet's Equilibrium Temperature

- The **equilibrium temperature** of a planet is the temperature at which the **cooling rate** balances the **heating rate**:

$$\begin{pmatrix}\text{Energy radiated}\\ \text{by the planet}\\ \text{each second}\end{pmatrix} = \begin{pmatrix}\text{Energy absorbed}\\ \text{by the planet}\\ \text{each second}\end{pmatrix}$$

$$4\pi R_{\text{planet}}^2 \sigma T^4 = \pi R_{\text{planet}}^2 \frac{L_{\text{Sun}}}{4\pi d^2}(1 - a)$$

- Solving for T, we get the equilibrium temperature:

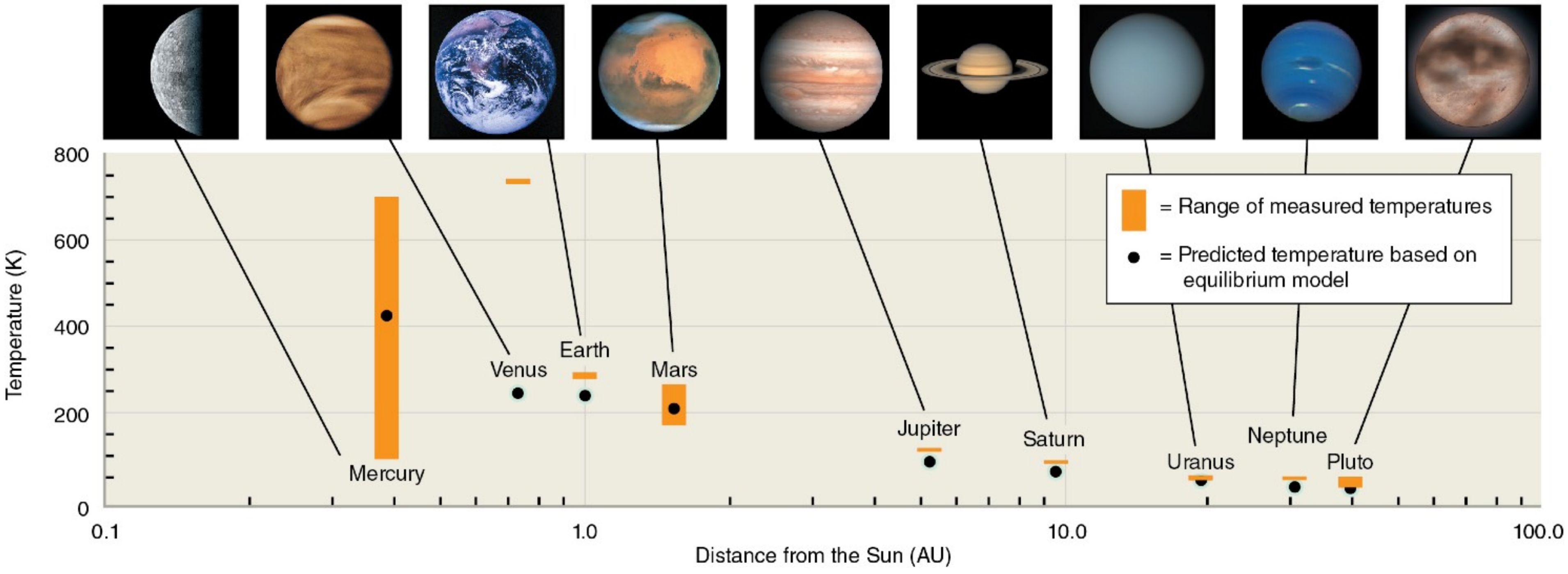
$$T = \left(\frac{L_{\text{Sun}}(1 - a)}{16\sigma\pi d^2}\right)^{1/4}$$

Equilibrium Temperature of Fast-Rotating Planets

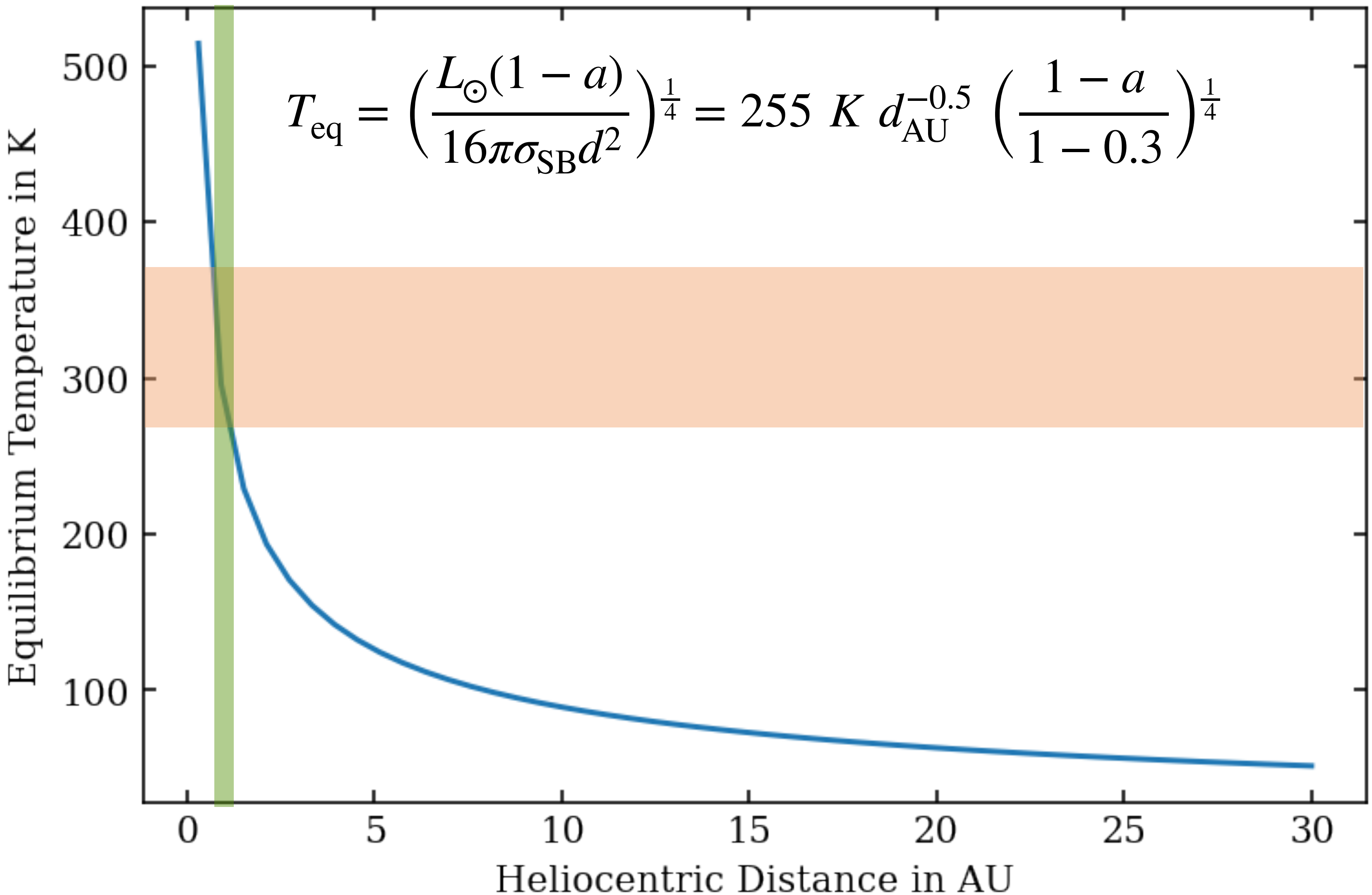
- For fast-rotating planets (uniform surface *T*):

$$(T_{\rm eq})^4 = \frac{L_{\rm star}(1-a)}{16\pi\sigma_{\rm SB}d^2}$$

- The **surface temperature** of the planet *increases* as (a) the luminosity of the Sun *increases*, (b) the albedo of the planet *decreases*, and/or (c) the distance to the Sun *decreases.*

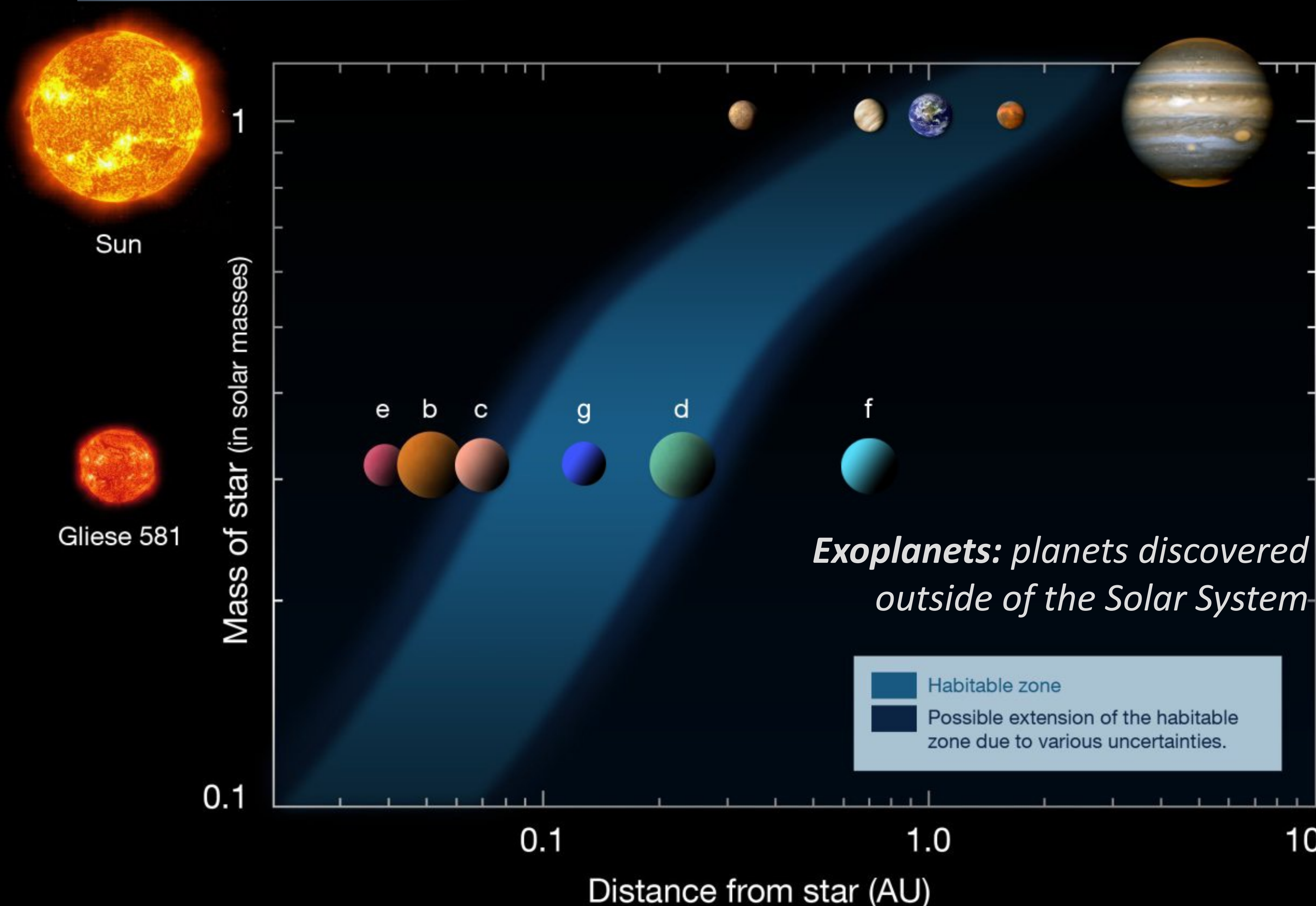


Circumstellar Habitable Zone: ***the distance range that could support liquid water under sufficient atmospheric pressure (at 1 atm, 273 K < T < 373 K)***



See code in provided python notebook "Blackbody.ipynb" on github

Circumstellar Habitable Zone: *the distance range that could support liquid water under sufficient atmospheric pressure (at 1 atm, 273 K < T < 373 K)*



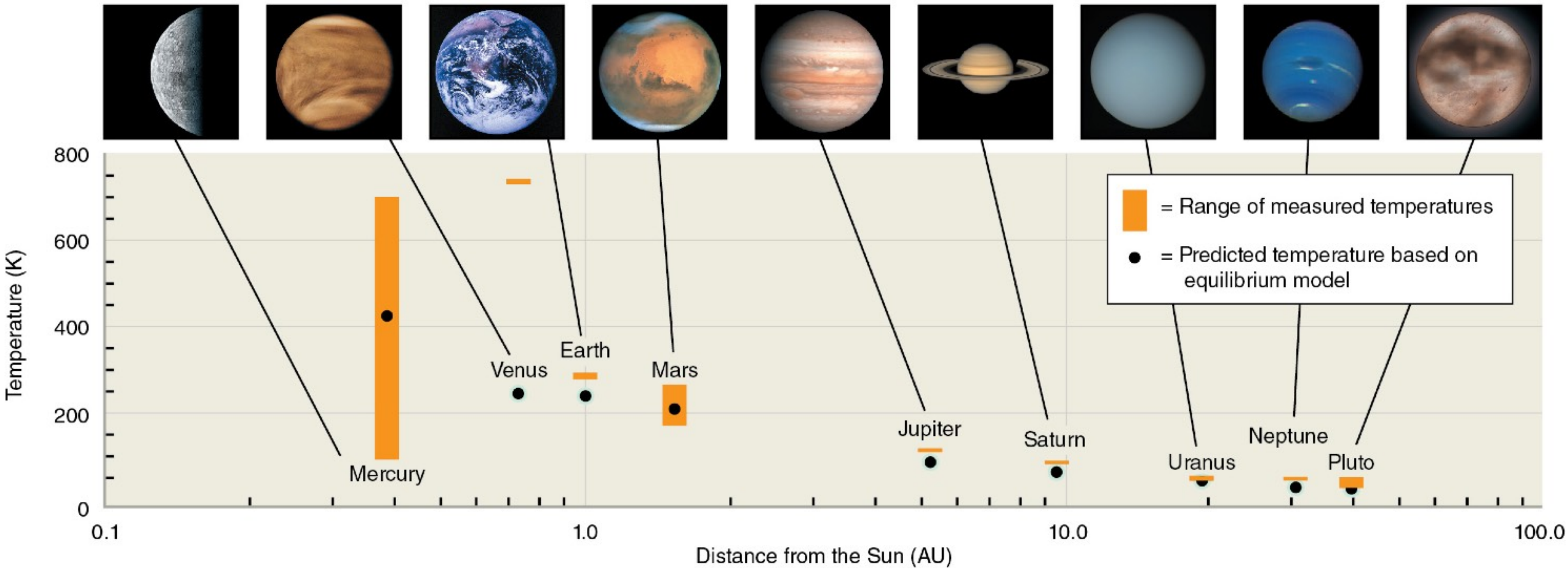
Exoplanets: *planets discovered outside of the Solar System*

Application of Blackbody Theory

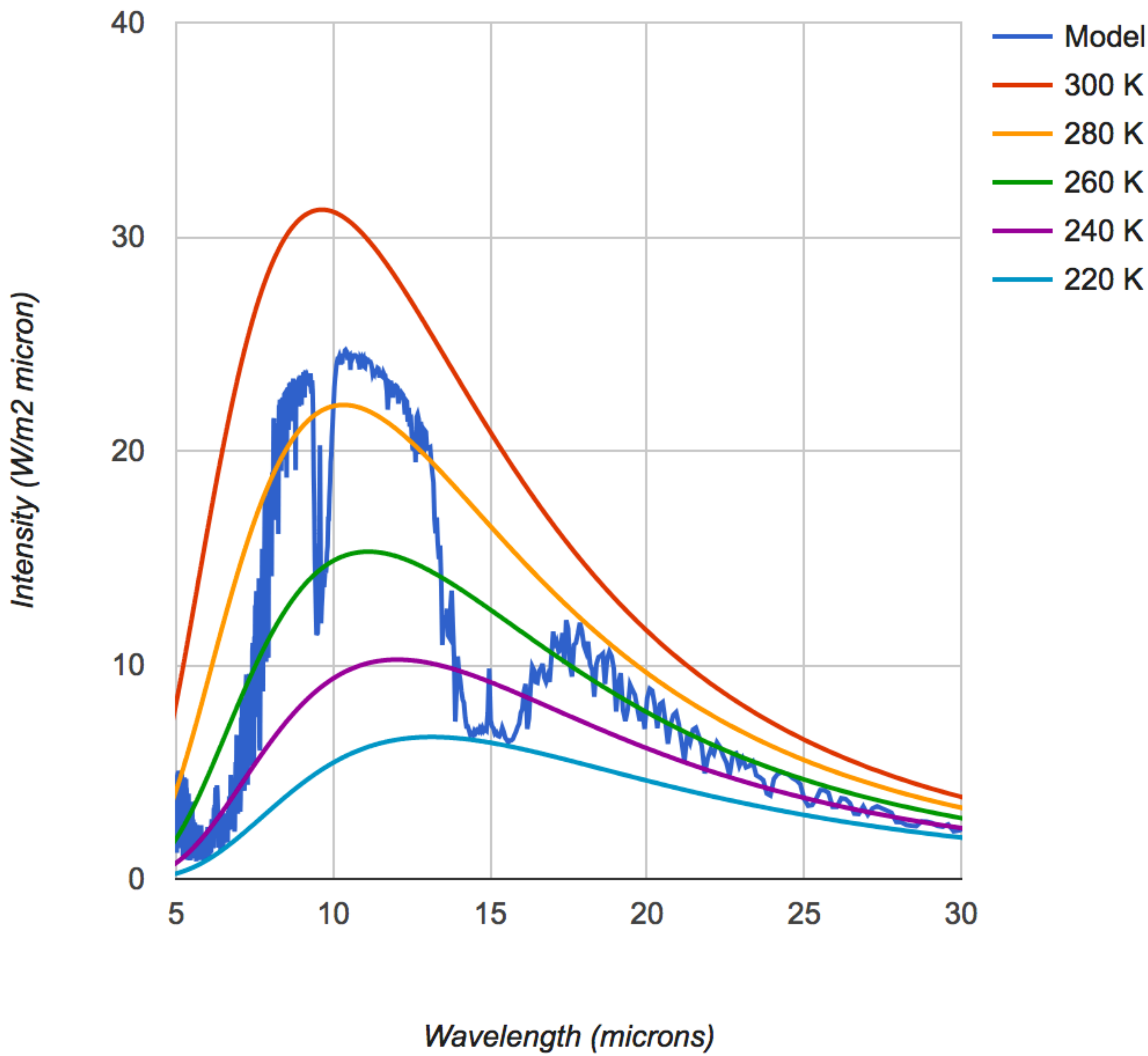
Equilibrium Temperatures of Planets when there are atmospheres

Planetary Temperatures: Predictions vs. actual values

- **The balance between <u>solar heating</u> and <u>radiative cooling</u>** predicts the equilibrium surface temperature of planets without atmospheres.
- The presence of **atmosphere** increases planet's surface temperature via **greenhouse effect**
- *Does this imply that heating ≠ cooling for planets with atmosphere?*



The Earth's Thermal Emission Seen from Space



Equilibrium Temperature with Atmosphere

- Greenhouse gases are nearly transparent to incoming solar radiation (mostly optical light), but they strongly absorb the planet's reemitted thermal emission (mid-IR light).
- They reduce the ability of the planet to cool, by a factor of $1/(1+\tau)$, where τ is the mean opacity of the atmosphere in mid-IR

$$\begin{pmatrix} \text{Energy radiated} \\ \text{by the planet} \\ \text{each second} \end{pmatrix} = \begin{pmatrix} \text{Energy absorbed} \\ \text{by the planet} \\ \text{each second} \end{pmatrix}$$

$$\frac{4\pi R_{\text{planet}}^2 \sigma T^4}{1+\tau} = \pi R_{\text{planet}}^2 \frac{L_\odot}{4\pi d^2}(1-a)$$

- Solving for T, we get the equilibrium temperature w/ atmosphere:

$$T^4 = \frac{L_\odot (1-a)}{16\pi\sigma d^2}(1+\tau)$$

Practice: Greenhouse effect

- For fast-rotating planets without greenhouse gas:

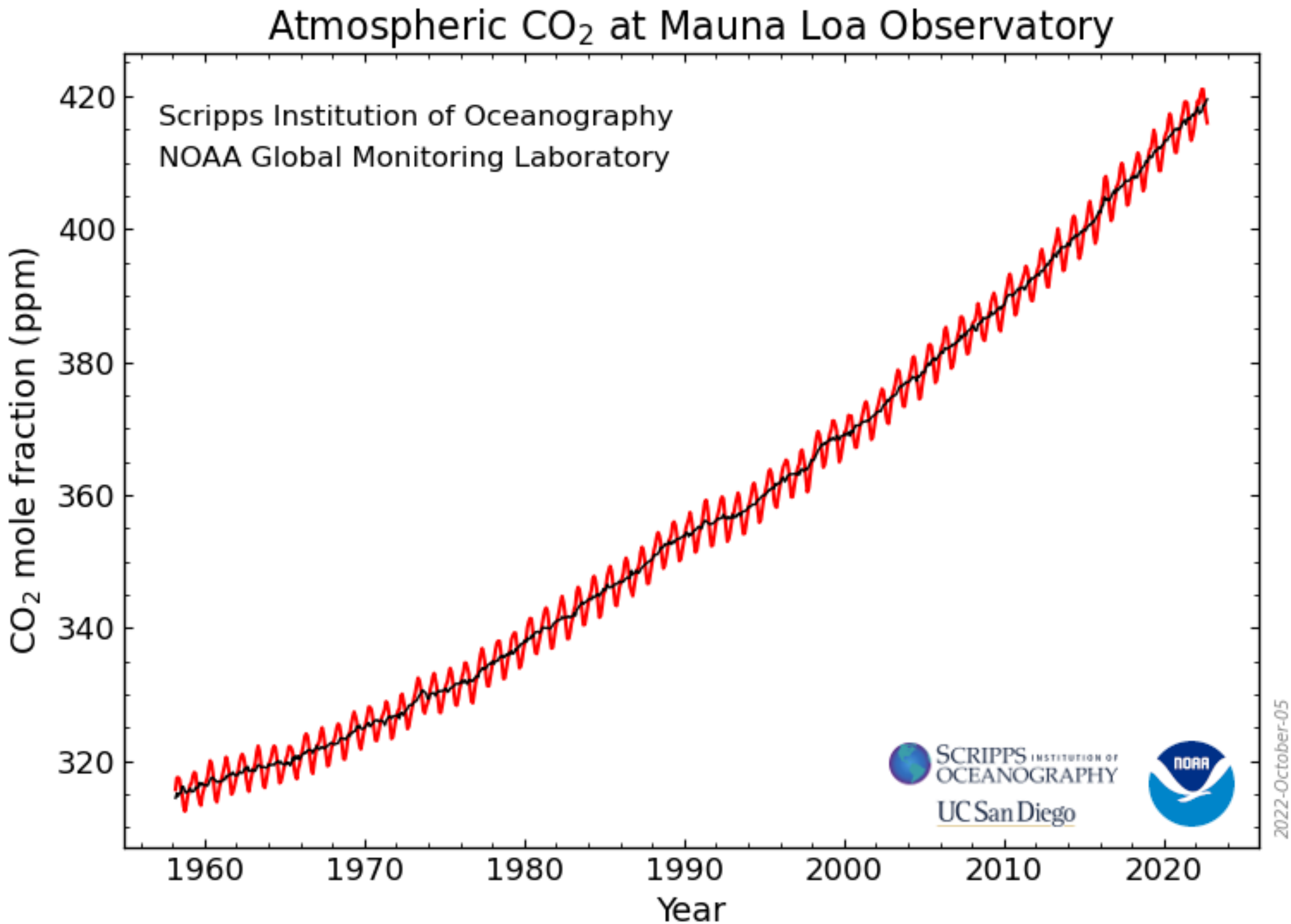
$$(T_{\rm eq})^4 = \frac{L_\odot(1-a)}{16\pi\sigma_{\rm SB}d^2}$$

- For fast-rotating planets with greenhouse gas:

$$(T_{\rm eq}^{\rm atm})^4 = \frac{L_\odot(1-a)}{16\pi\sigma_{\rm SB}d^2}(1+\tau)$$

  where tau is the mean opacity of the atmosphere in mid-IR

- Suppose you calculated the Earth's equilibrium temperature of 260 K (-13 C) using the first equation, but the actual mean temperature is 288 K (+15 C). What is the mean mid-IR opacity of the Earth's atmosphere?
- When the Earth is 2 deg C hotter than today (290 K), what will be the mean mid-IR opacity of the atmosphere? How much percent was the increase compared to today's opacity?

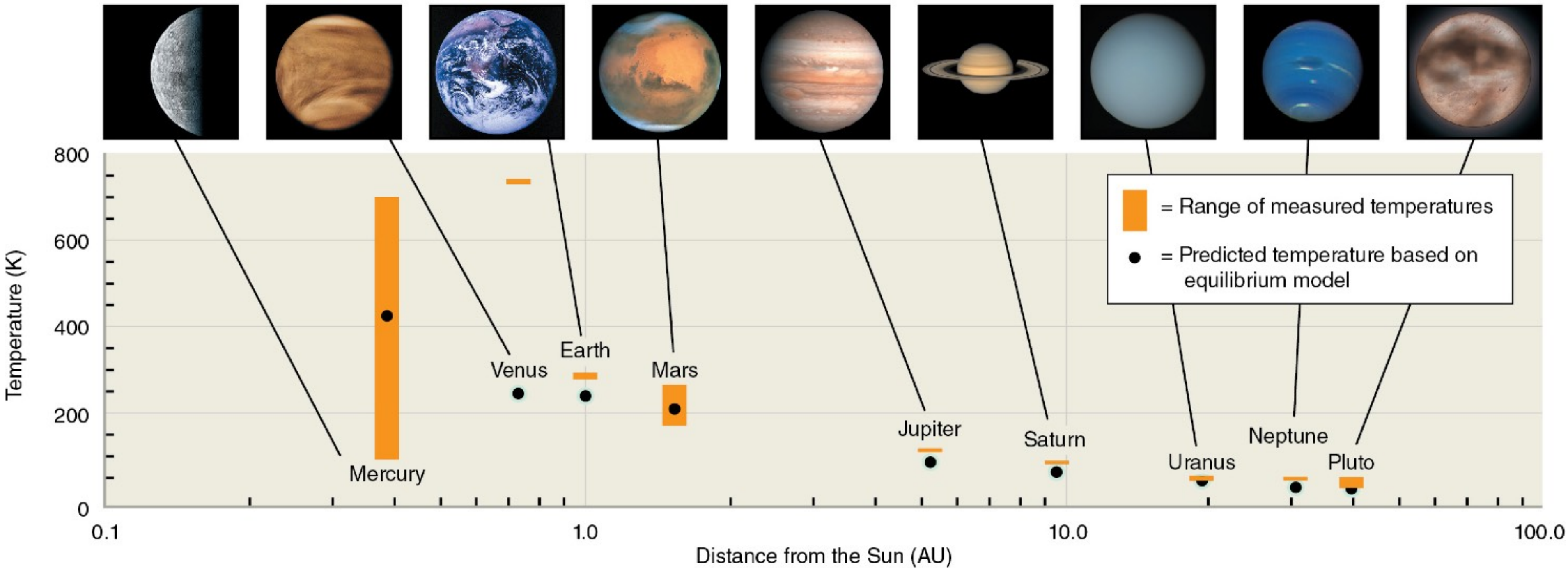

Atmospheric CO_2 at Mauna Loa Observatory
Scripps Institution of Oceanography
NOAA Global Monitoring Laboratory
CO_2 mole fraction (ppm)
420
400
380
360
340
320
1960
1970
1980
1990
2000
2010
2020
Year
SCRIPPS INSTITUTION OF OCEANOGRAPHY
UC San Diego
NOAA
2022-October-05

Equilibrium Temperature of Fast-Rotating Planets w/ Atmosphere

- For fast-rotating planets with greenhouse gas:

$$(T_{\rm eq}^{\rm atm})^4 = \frac{L_\odot(1-a)}{16\pi\sigma_{\rm SB}d^2}(1+\tau)$$

- The **surface temperature** of the planet *increases* as (a) the luminosity of the Sun *increases*, (b) the albedo of the planet *decreases*, (c) distance *decreases* (d) greenhouse *increases.*



Chap 5 Part II: Equations of Blackbody Emission

Planck's Function: $$I_\lambda = B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc/\lambda}{kT}} - 1}$$

Wien's Displacement Law: $$\lambda_{\text{peak}} = \frac{2.9 \text{ mm K}}{T}$$

$$F = \sigma_{\text{SB}} T^4$$

Stefan-Boltzmann Law

$$T(K) = T(C) + 273.15$$

$$\sigma_{\text{SB}} = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

For fast-rotating planets without greenhouse gas:

$$T_{\text{eq}} = \left(\frac{L_\odot(1-a)}{16\pi\sigma_{\text{SB}}d^2}\right)^{\frac{1}{4}} = 255\ K\ d_{\text{AU}}^{-0.5} \left(\frac{1-a}{1-0.3}\right)^{\frac{1}{4}}$$

For fast-rotating planets with greenhouse gas:

$$T_{\text{eq}}^{\text{atm}} = T_{\text{eq}}(1+\tau)^{1/4}$$