Chap 7: The Formation of the Solar System

“From time immemorial, man has desired to comprehend the complexity of nature in terms of as few elementary concepts as possible”
- Abdus Salam (Pakistani Physicist, Nobel Laureate 1979)

“Nature is capable of building complex structures by processes of self-organization; simplicity begets complexity”
- Victor Stenger (American Physicist)

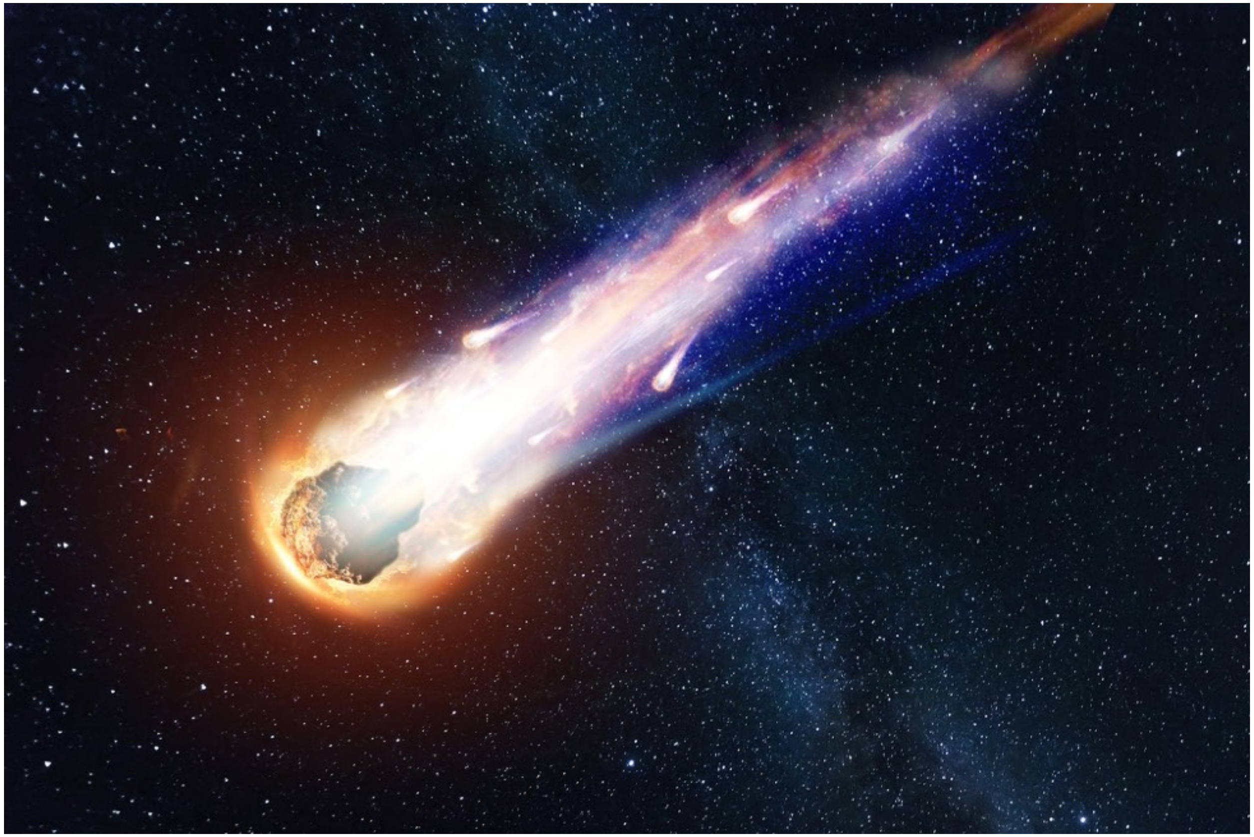
“Dealing with complexity is an inefficient and unnecessary waste of time, attention and mental energy.
- Edward de Bono (Maltese psychologist)

Part I: Meteorites - Our source of information

Different types of meteorites

Meterors vs. Meteorites

- A **meteorite** is a solid piece of debris that originates in outer space and **survives its passage** through the atmosphere to reach the surface of the Earth or other planets.
- Meteorites contain some of the most accurate information about the Solar System's formation.



Where do meteorites come from?

Most meteorites found on Earth come from shattered asteroids, although some come from Mars or the Moon.

Alex Moreno


Alex Moreno

Meteorite Hunting

Meteorites on the surface of Mars

The Most Primitive Meteorites: Chondrites

- **Chondrites** are stony meteorites that, after been formed, were never melted, thus preserving mineralogy from the days of their formation.
- **Chondrites** are dominated by **chondrules,** which are roughly millimeter-sized silicate-rich spherules.
- **Achondrites** are stony meteorites that do not contain **chondrules**

Evidence of aggregation from meteorites: inclusions & chondrules

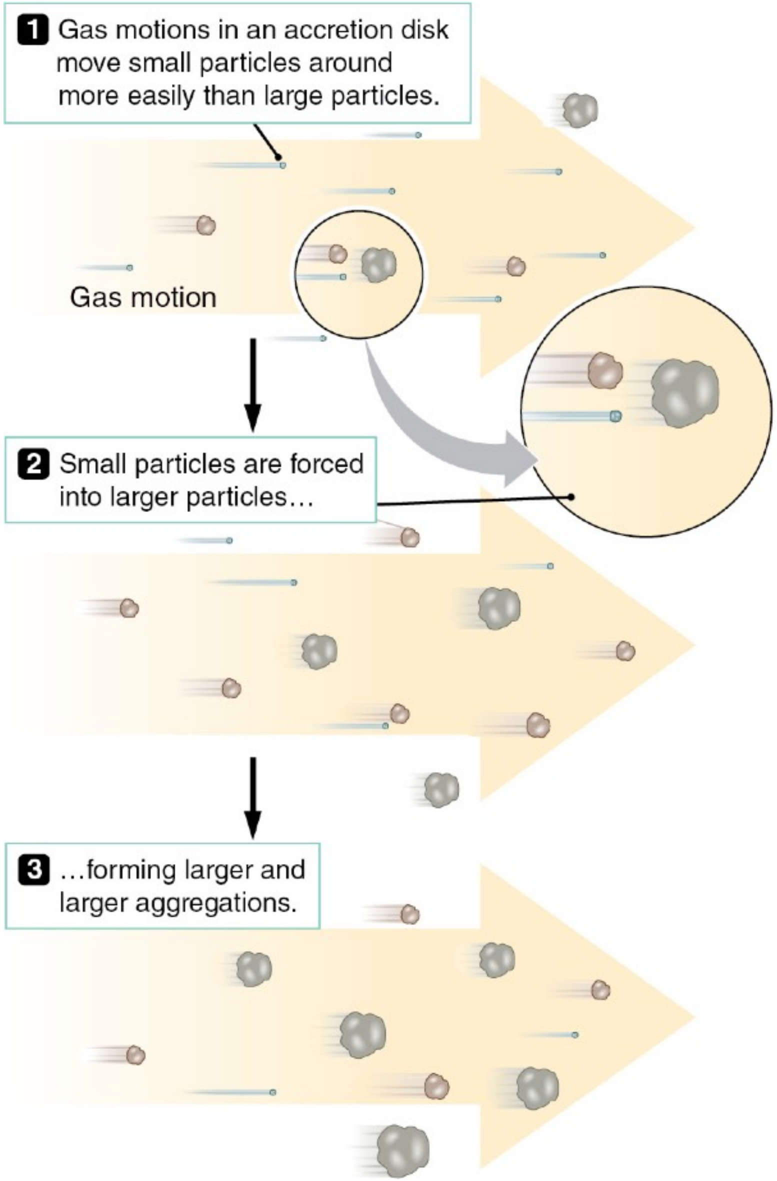
- Some meteorites show mixtures of minerals that formed at distinctly different temperatures, suggesting formation through a process of aggregation.

Piece of the **Allende meteorite**, a **carbonaceous chondrite**, which fell as a shower of numerous fragments in Mexico in 1969.

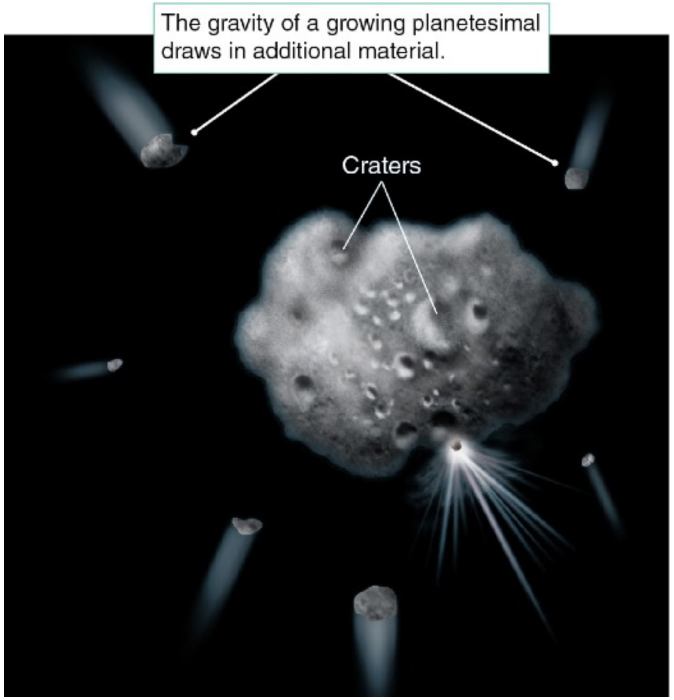
The **large light spots** are **calcium**- and **aluminum**-rich **refractory inclusions**; along with many rounded **chondrules** (containing **olivine** and **pyroxene**).

These **nuggets** of inclusions and chondrules, which condensed at high temperatures, are embedded in a dark gray matrix containing fine-grained minerals (e.g., silicates) that formed at much lower temperatures.

Aggregation of dust to form rocks and planetesimals



- Within the disk, small particles will collide and stick (**aggregation**)
- This leads to pebble-sized rocks, and eventually, really large bodies (~1 km in size) called **planetesimals.**



To understand different types of meteorites, we first need to understand why larger planetesimals tend to melt and differentiate at formation

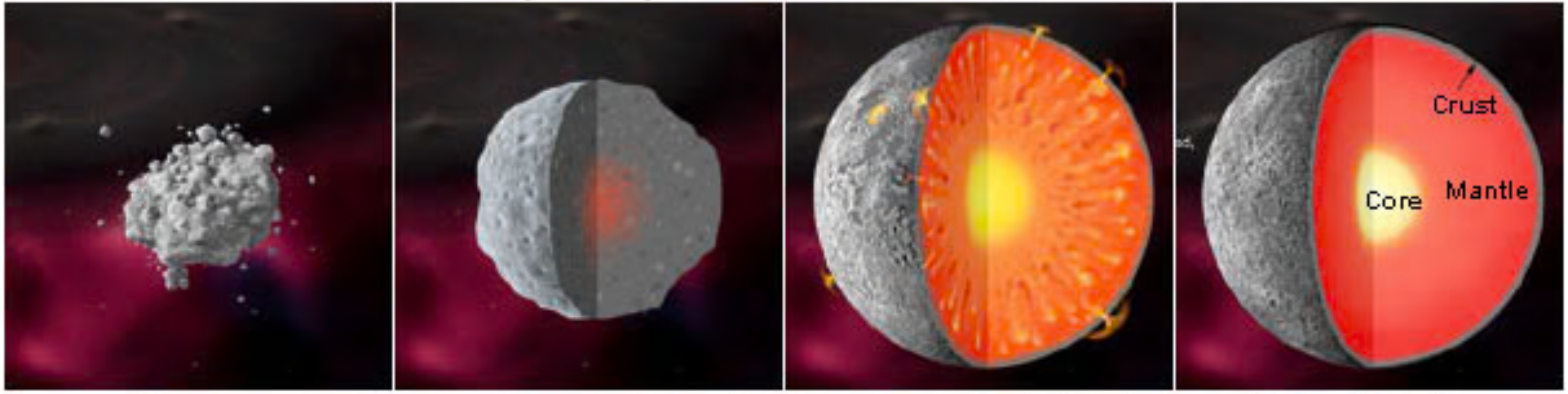
When planets begin to **melt**, the materials in them begin to separate from one another. The heaviest materials, such as metallic iron, sink to form cores. Low-density magmas rise, forming crusts. This process is called **differentiation**.

The key to understand this is to consider the total energy change during **aggregation of solid materials**:

$$E = K + U = U/2 = -GM^2/2R$$

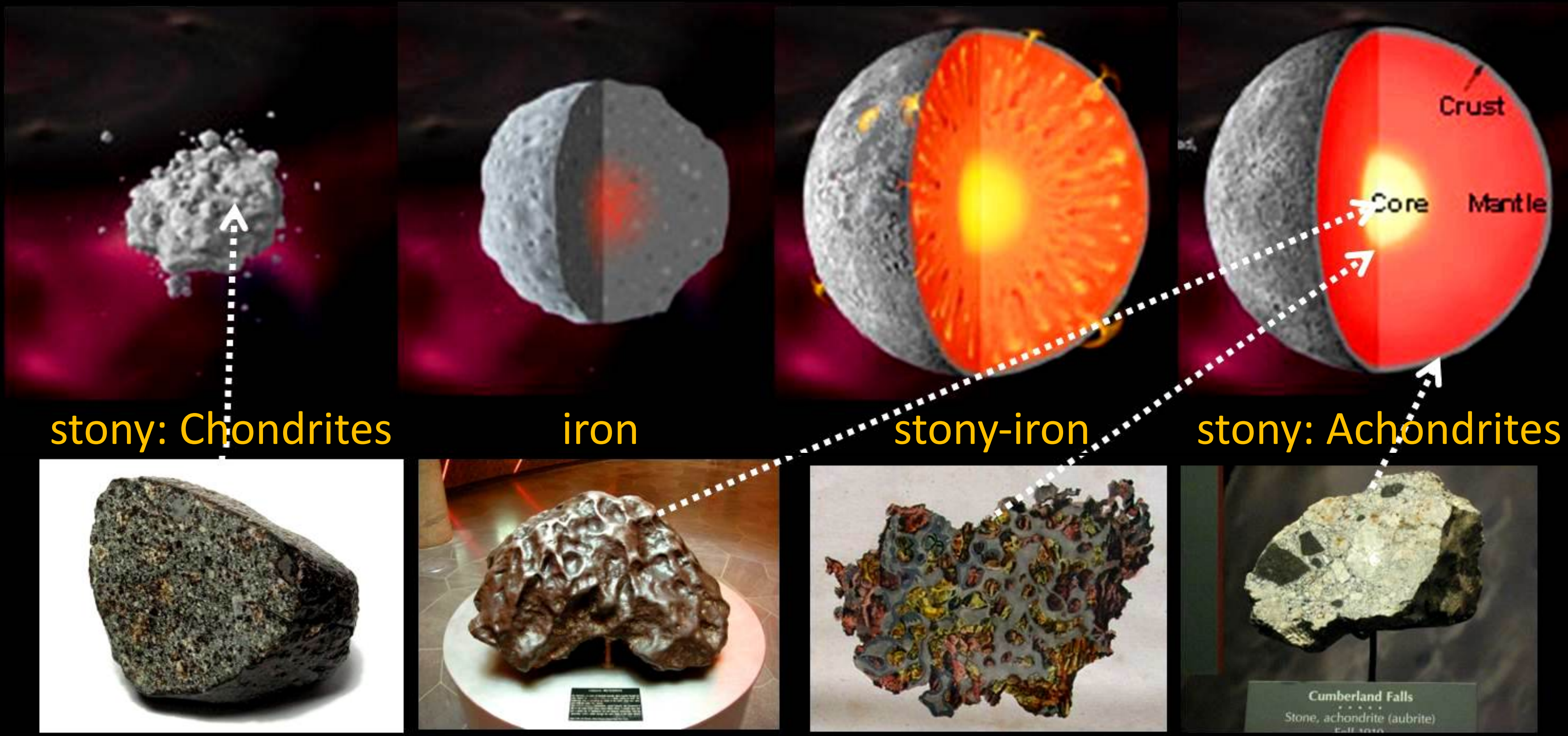
$$\Delta E = \frac{GM^2}{2}\left(\frac{1}{R_p} - \frac{1}{R_c}\right) \approx \frac{GM^2}{2R_p} \Rightarrow \Delta T = \frac{GM^2}{2R_p c_p M} \propto \frac{M}{R_p}$$

A Rocky Body Forms and Differentiates



Different Asteroid & Meteorite Types

Source: Smithsonian Museum of Natural History http://www.mnh.si.edu/earth/text/5_1_4_0.html



stony: Chondrites

iron

stony-iron

stony: Achondrites

Chondritic Stony Meteorite

Iron Meteorite

Pallasite Meteorite

Achondritic Stony Meteorite

License: Wikimedia Creative Commons

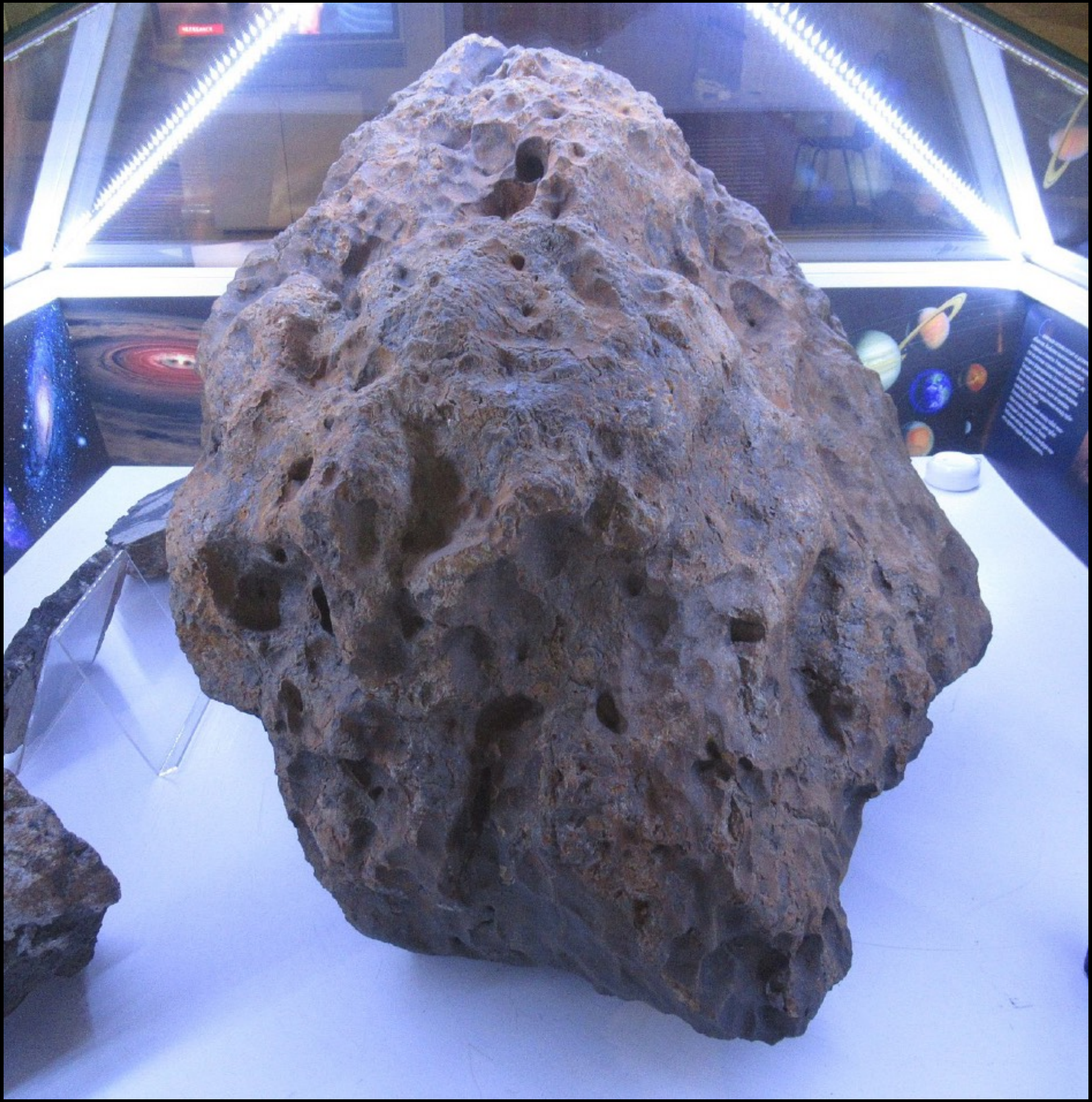
Asteroid Type C

Asteroid Type M

Asteroid Type S

Chelyabinsk Meteor Asteroid: 18-m, 10k ton, 19 km/s

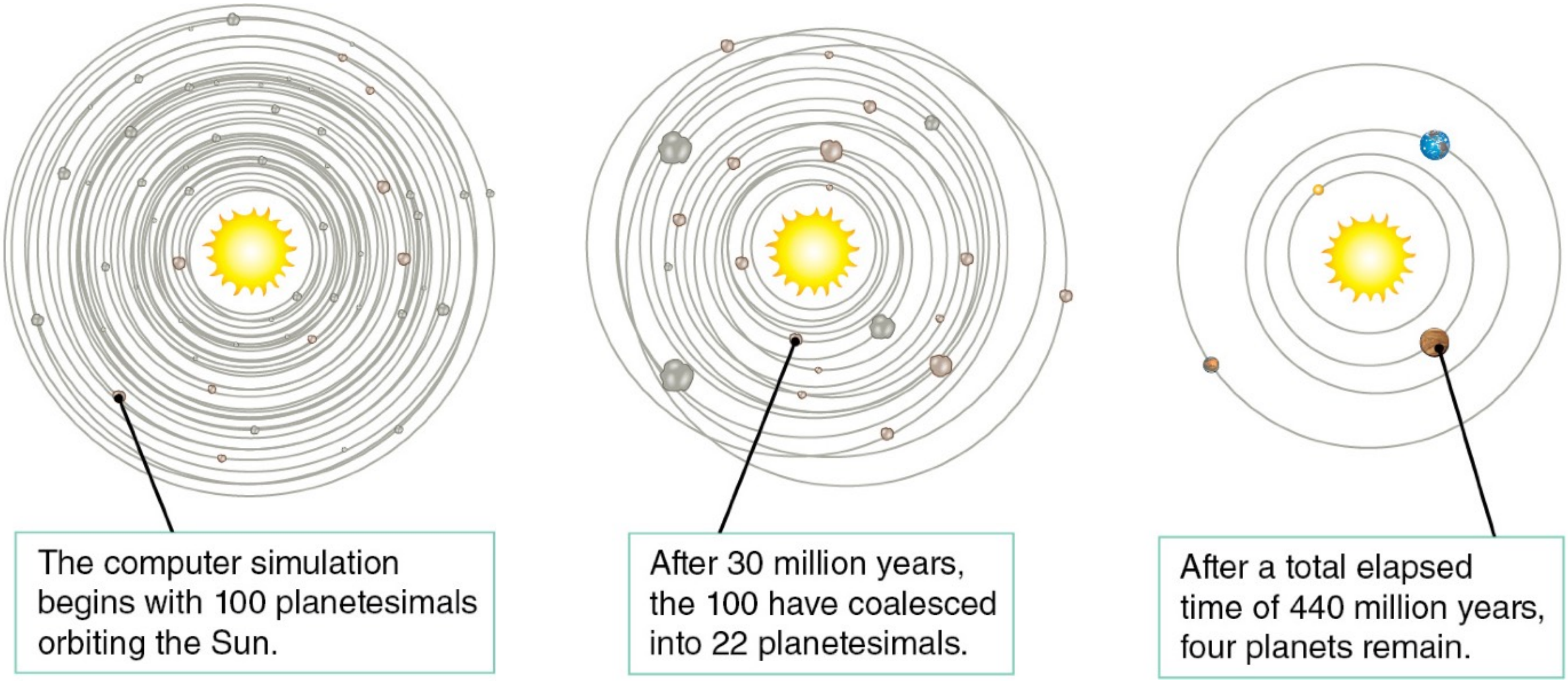
Chelyabinsk meteorite: 2 ft, 540 kg



Theia-Earth Collision

Consolidation: Growth of Planetesimals to Planets

- At this size, the **planetesimals** combine easily to form **planets**.
- Today's remaining planetesimals are **asteroids** in the asteroid belt
- Because **meteorites** are fragments of asteroids, this explains why meteorites differ in chemical properties from all known terrestrial and lunar rocks.



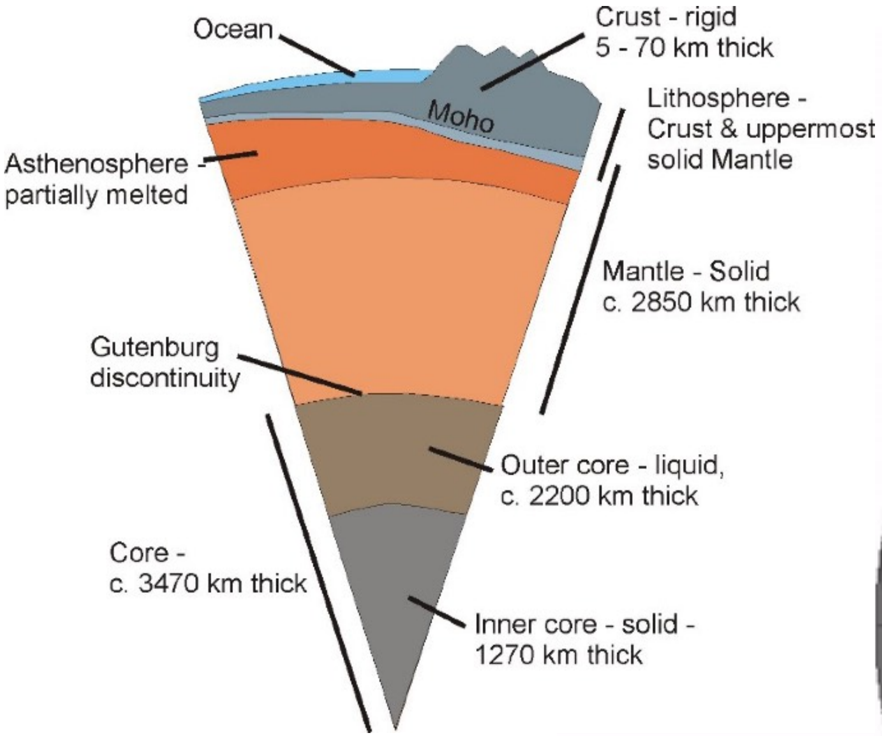
The computer simulation begins with 100 planetesimals orbiting the Sun.

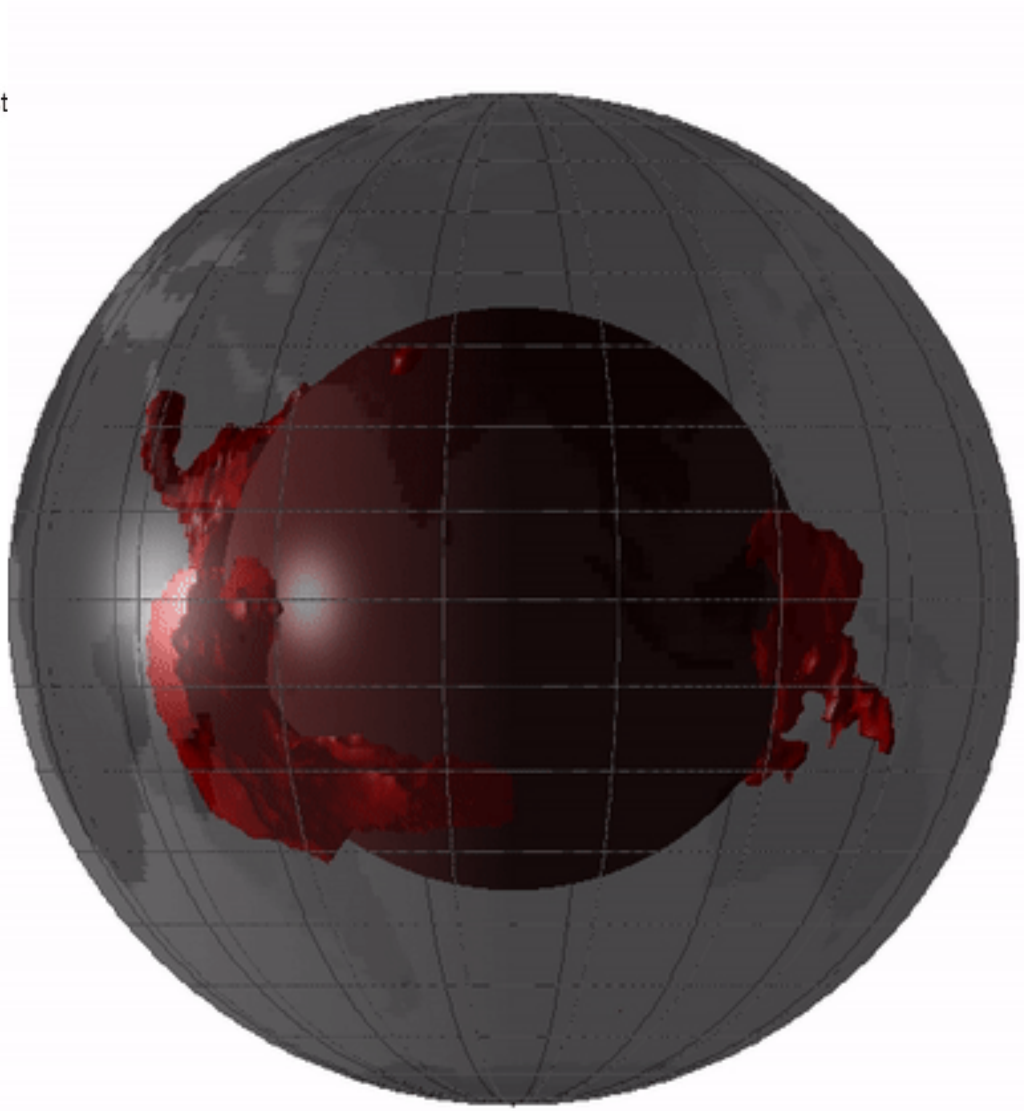
After 30 million years, the 100 have coalesced into 22 planetesimals.

After a total elapsed time of 440 million years, four planets remain.

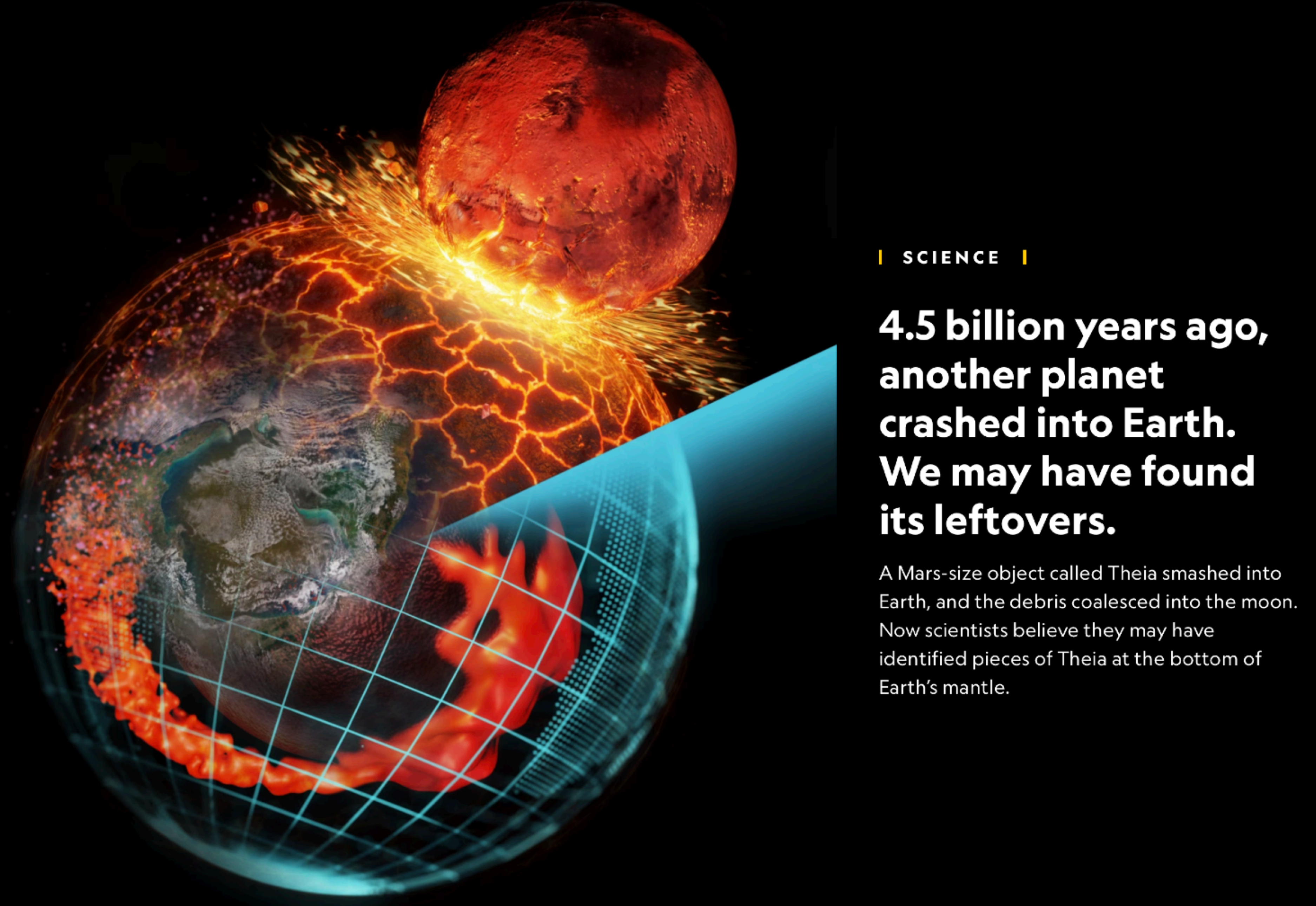
The Formation of the Moon 4.5 Billion Years Ago Through a Collision

Large Low-Shear-Velocity Provinces from Seismic Tomography





New Development published in November, 2023



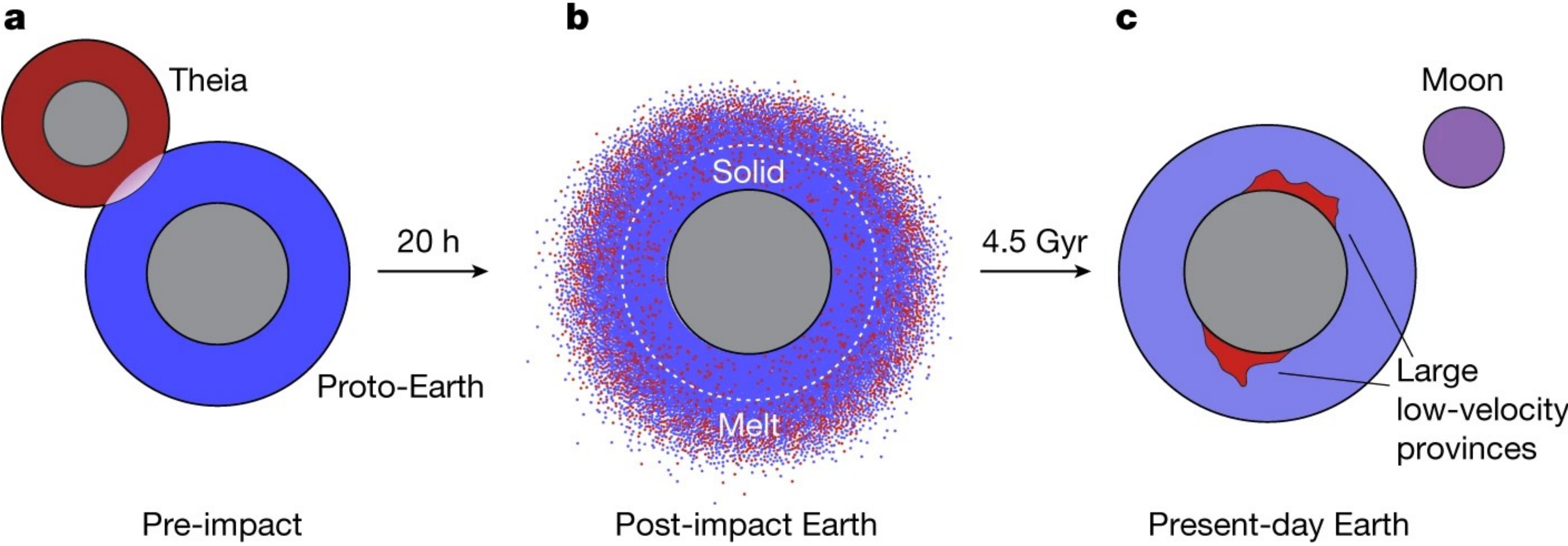
SCIENCE

4.5 billion years ago, another planet crashed into Earth. We may have found its leftovers.

A Mars-size object called Theia smashed into Earth, and the debris coalesced into the moon. Now scientists believe they may have identified pieces of Theia at the bottom of Earth's mantle.

Moon-forming impactor as a source of Earth's basal mantle anomalies

Abstract: Seismic images of Earth's interior have revealed two continent-sized anomalies with low seismic velocities, known as the large low-velocity provinces (LLVPs), in the lowermost mantle1. The LLVPs are often interpreted as intrinsically dense heterogeneities that are compositionally distinct from the surrounding mantle2. Here we show that LLVPs may represent buried relics of Theia mantle material (TMM) that was preserved in proto-Earth's mantle after the Moon-forming giant impact.



https://www.nature.com/articles/s41586-023-06589-1

Radiometric dating of meteorites

Radiometric Dating - Half-Life of Caffeine

How long will caffeine be in my system?
When did I drink my last cup of coffee?

- The half-life of caffeine in healthy adults is about 6 hours, due to breaking down by metabolism. This means if you consume 200 mg of caffeine at noon, you will still have
  - 100 mg at 6PM, 50 mg at 12AM, 25 mg at 6AM.
- Expressed in an equation, this is:

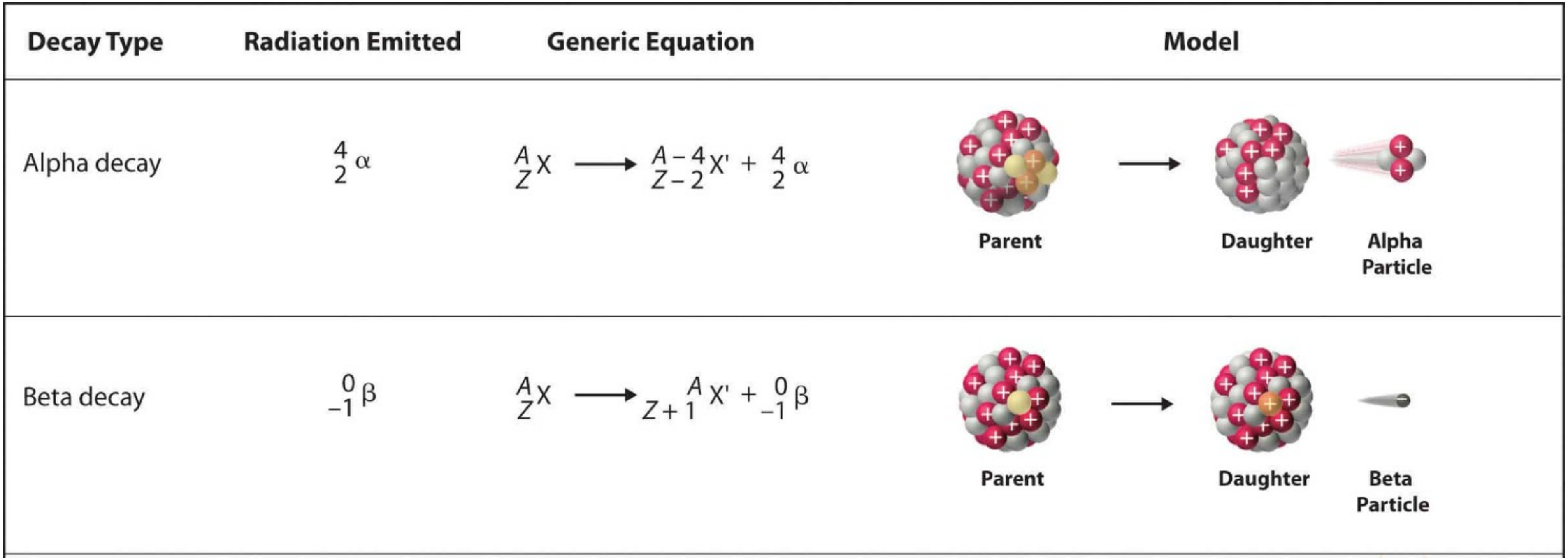
$$m(t) = m_0 \cdot 0.5^{t/\tau}$$

where m_0 is the initial caffeine mass, m(t) is the mass at time t, and τ is the half-life (6 hours)

- We can determine when you drank the last cup of coffee if we can measure how much you still have in your body:

$$t = \tau \log \frac{m(t)}{m_0} / \log(0.5) = 3.32\ \tau\ \log \frac{m_0}{m(t)}$$

Radiometric Dating - Radioactive Decay



Decay Type	Radiation Emitted	Generic Equation	Model
Alpha decay	${}^{4}_{2}\alpha$	${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A-4}_{Z-2}\mathrm{X'} + {}^{4}_{2}\alpha$	Parent → Daughter Alpha Particle
Beta decay	${}^{0}_{-1}\beta$	${}^{A}_{Z}\mathrm{X} \longrightarrow {}_{Z+1}^{A}\mathrm{X'} + {}^{0}_{-1}\beta$	Parent → Daughter Beta Particle

Example beta decay important in meteorites, half-life of 16 Myr:

$$ {}^{129}_{53}\mathrm{I} \rightarrow {}^{129}_{54}\mathrm{Xe} + e^{-} $$

53 **I** Iodine 126.90	54 **Xe** Xenon 131.29

129 - atomic weight ($n_P + n_N$), 53 or 54 - atomic number (n_P), 126.90 - mean atomic weight

Radiometric Dating - Half-Life of Iodine-129

When did the rock solidify and locked in the Iodine-129?

- The beta decay: $^{129}_{53}\mathrm{I} \rightarrow ^{129}_{54}\mathrm{Xe} + e^-$
- Expressing the number of atoms in equations, we have:

$$N_I(t) = N_I(0) \cdot 0.5^{t/\tau}$$

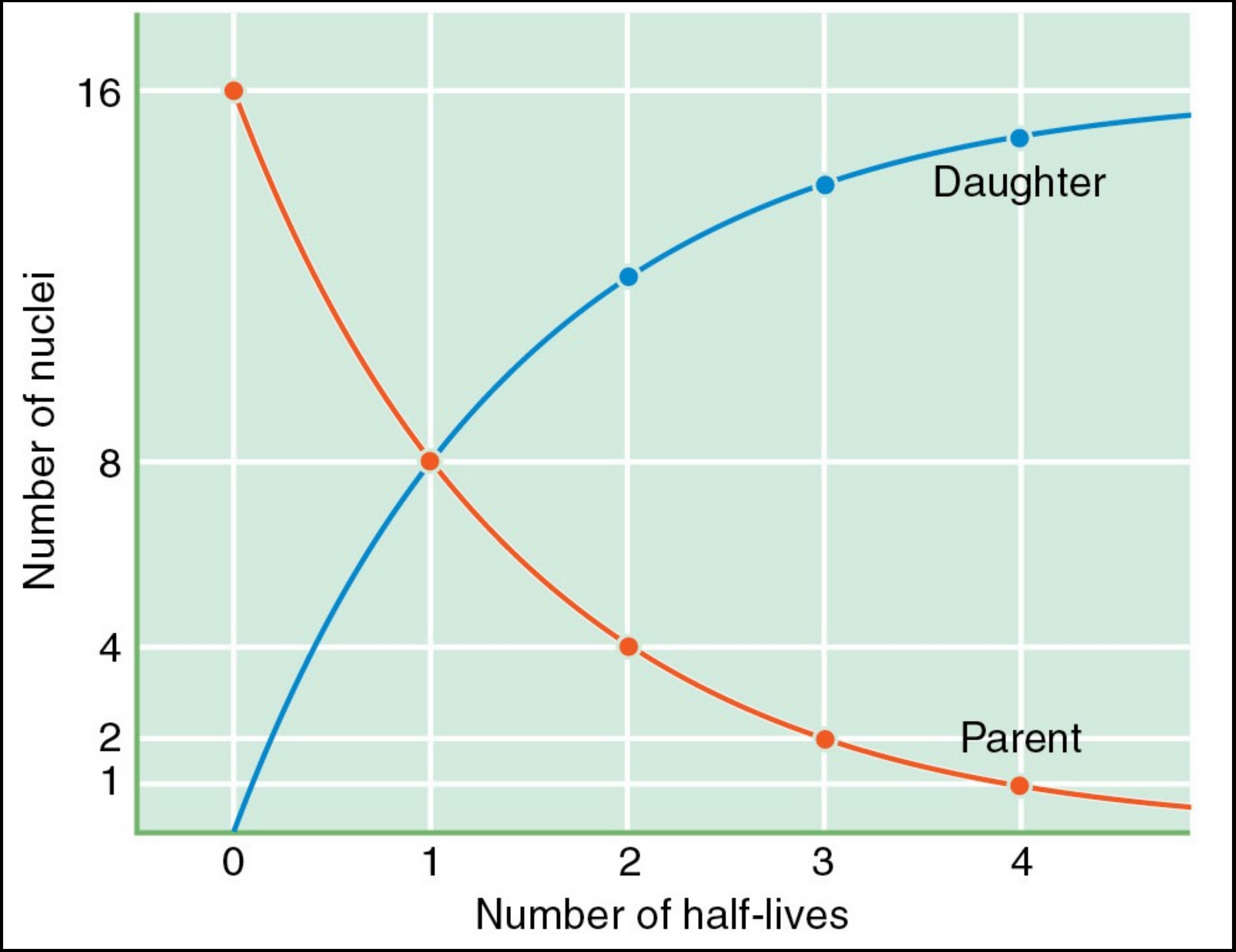
  where t₀ is the initial solidification time and τ is the half-life (16 Myrs)

- Just like we can determine when you drank the last cup of coffee, we can determine the **solidification time**:

$$t = \tau \log \frac{N_I(t)}{N_I(0)} / \log(0.5) = 3.32\ \tau\ \log \frac{N_I(0)}{N_I(t)}$$

- *But how do we know the initial amount of Iodine,* $N_I(t_0)$*?*

- After each half-life, the number of parent element has decreased by 50% of the previous number; so it follows an exponential curve
- Because the disappeared parents have become daughters, the total number of parents + daughters remain the same



Radiometric Dating - IF there were no Xenon-129 at t=0

- The beta decay: ${}^{129}_{53}\mathrm{I} \rightarrow {}^{129}_{54}\,\mathrm{Xe} + e^{-}$, the number of **Iodine** decreases over time exponentially: $N_I(t) = N_I(0) \cdot 0.5^{t/\tau}$
- The number of **Xenon** at time *t* equals the number of **decayed Iodine**:

$$N_{Xe}(t) = N_I(0) - N_I(t) = N_I(0)(1 - 0.5^{t/\tau})$$

- Take the ratio between daughter and parent to cancel out the unknown amount of Iodine at time 0, we get:

$$\frac{N_{Xe}(t)}{N_I(t)} = 2^{t/\tau} - 1$$

- Solving for *t* by taking log on both side, we get the age:

$$t = \tau \frac{\log(N_{Xe}/N_I + 1)}{\log 2} = 3.32\ \tau\ \log(N_{Xe}/N_I + 1)$$

Radiometric Dating - IF there were some Xenon-129 at t=0

- The beta decay: ${}^{129}_{53}\mathrm{I} \rightarrow {}^{129}_{54}\,\mathrm{Xe} + e^{-}$
- If we define the **initial ratio** between the two species as:

$$r_0 \equiv N_{Xe}(0)/N_I(0)$$

- The number of Xenon-129 at time t is **the sum of two parts** - the initial number $N_{Xe}(0)$ and the number produced by Iodine $N_I(0)(1-0.5^{t/\tau})$:

$$N_{Xe}(t) = N_{Xe}(0) + N_I(0)(1-0.5^{t/\tau}) = N_I(0)\ (r_0 + 1 - 0.5^{t/\tau})$$

- The number density ratio between the daughter and parent:

$$\frac{N_{Xe}(t)}{N_I(t)} = (1 + r_0)\cdot 2^{t/\tau} - 1$$

- Solving for t in the above equation, we get the age:

$$t = 3.32\ \tau\ \log\left(\frac{N_{Xe}(t)/N_I(t) + 1}{1 + r_0}\right)$$

Solidification Age and Formation Age (~4.6 Gyr)

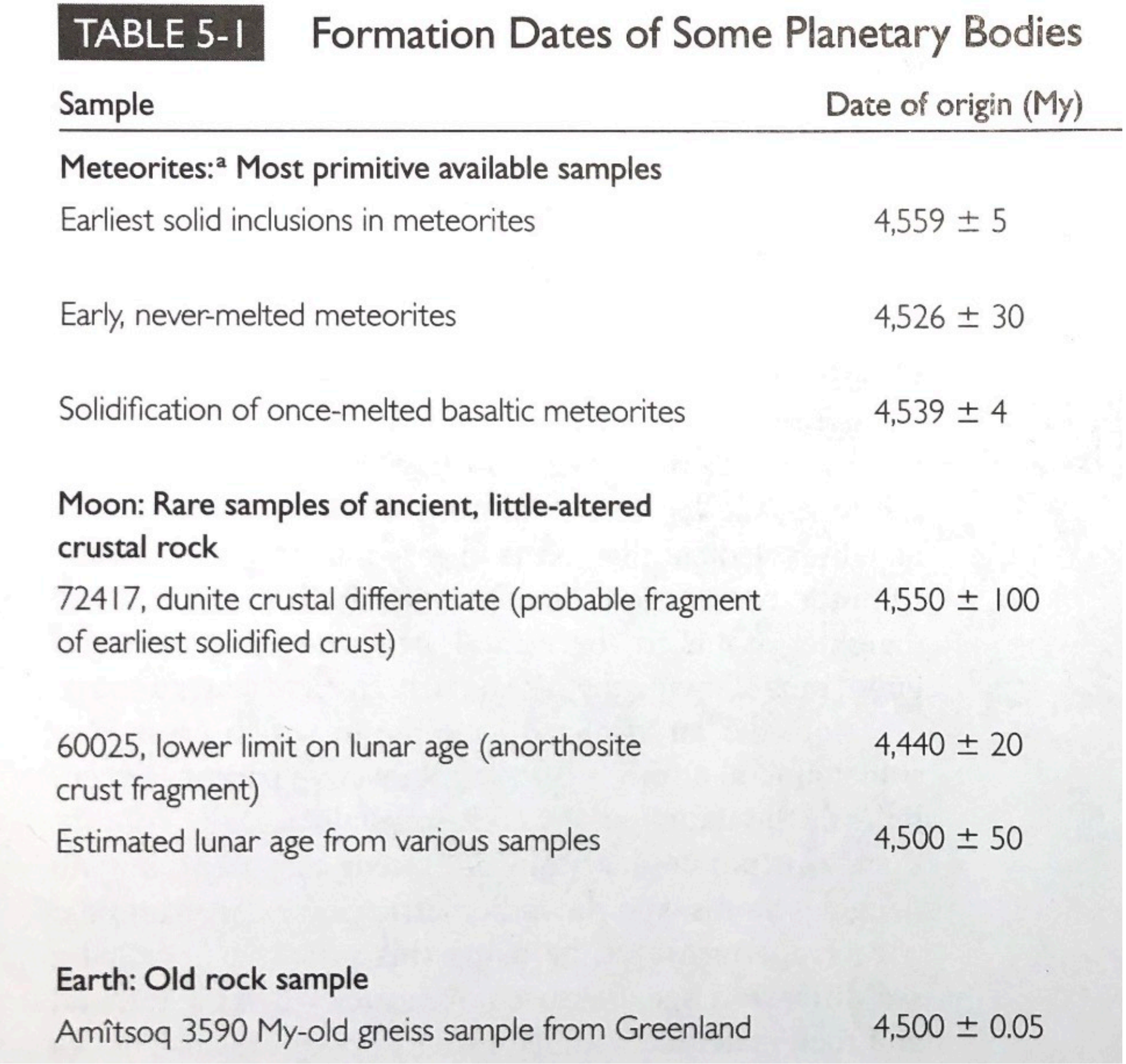


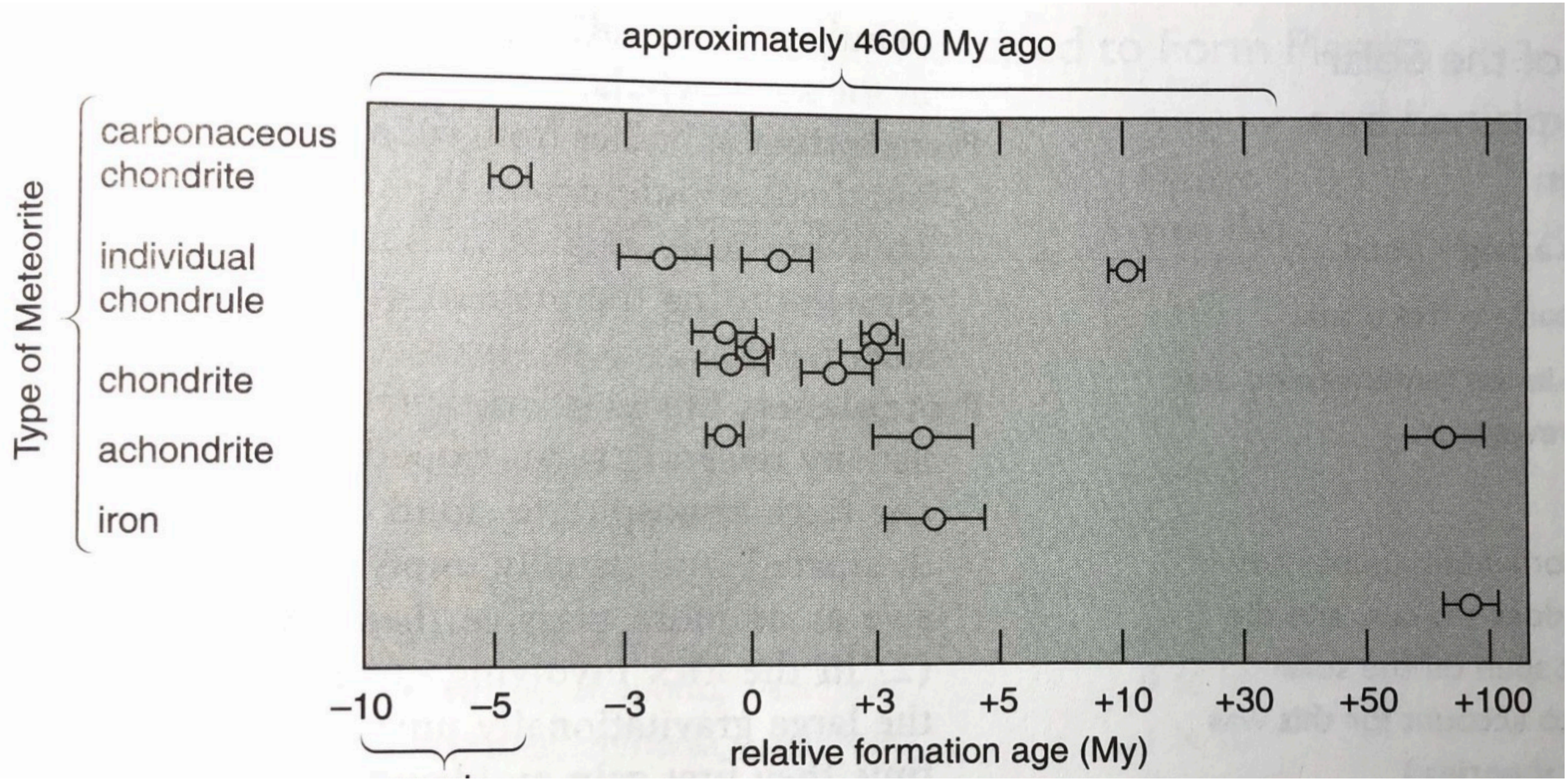
TABLE 5-1 Formation Dates of Some Planetary Bodies

Sample	Date of origin (My)
Meteorites:[a] Most primitive available samples	
Earliest solid inclusions in meteorites	4,559 ± 5
Early, never-melted meteorites	4,526 ± 30
Solidification of once-melted basaltic meteorites	4,539 ± 4
Moon: Rare samples of ancient, little-altered crustal rock	
72417, dunite crustal differentiate (probable fragment of earliest solidified crust)	4,550 ± 100
60025, lower limit on lunar age (anorthosite crust fragment)	4,440 ± 20
Estimated lunar age from various samples	4,500 ± 50
Earth: Old rock sample	
Amîtsoq 3590 My-old gneiss sample from Greenland	4,500 ± 0.05

Hartmann 2005

Taylor 1992

Formation Interval (~100 Myr)



Hartmann 2005

Why are there short-lived radioactive isotopes at the time of formation?

- $^{87}_{37}\mathrm{Rb} \rightarrow ^{87}_{38}\mathrm{Sr} + e^{-}$ (half-life = **49 Gyr**; Rubidium -> Strontium)
- $^{129}_{53}\mathrm{I} \rightarrow ^{129}_{54}\mathrm{Xe} + e^{-}$ (half-life = **16 Myr**)
- $^{26}_{13}\mathrm{Al} \rightarrow ^{26}_{12}\mathrm{Mg} + e^{+}$ (half-life = **0.72 Myr**)
- Both Xe-129 and Mg-26 are found in meteorites. This implies the presence of large quantities of their highly unstable parents (I-129 and Al-26) in the Solar nebula.
- **What created these unstable isotopes (I-129 and Al-26)?** The creation must have happened shortly before the formation of the system given their short half-lives, because otherwise there wouldn't have much left to be incorporated into the meteorites.

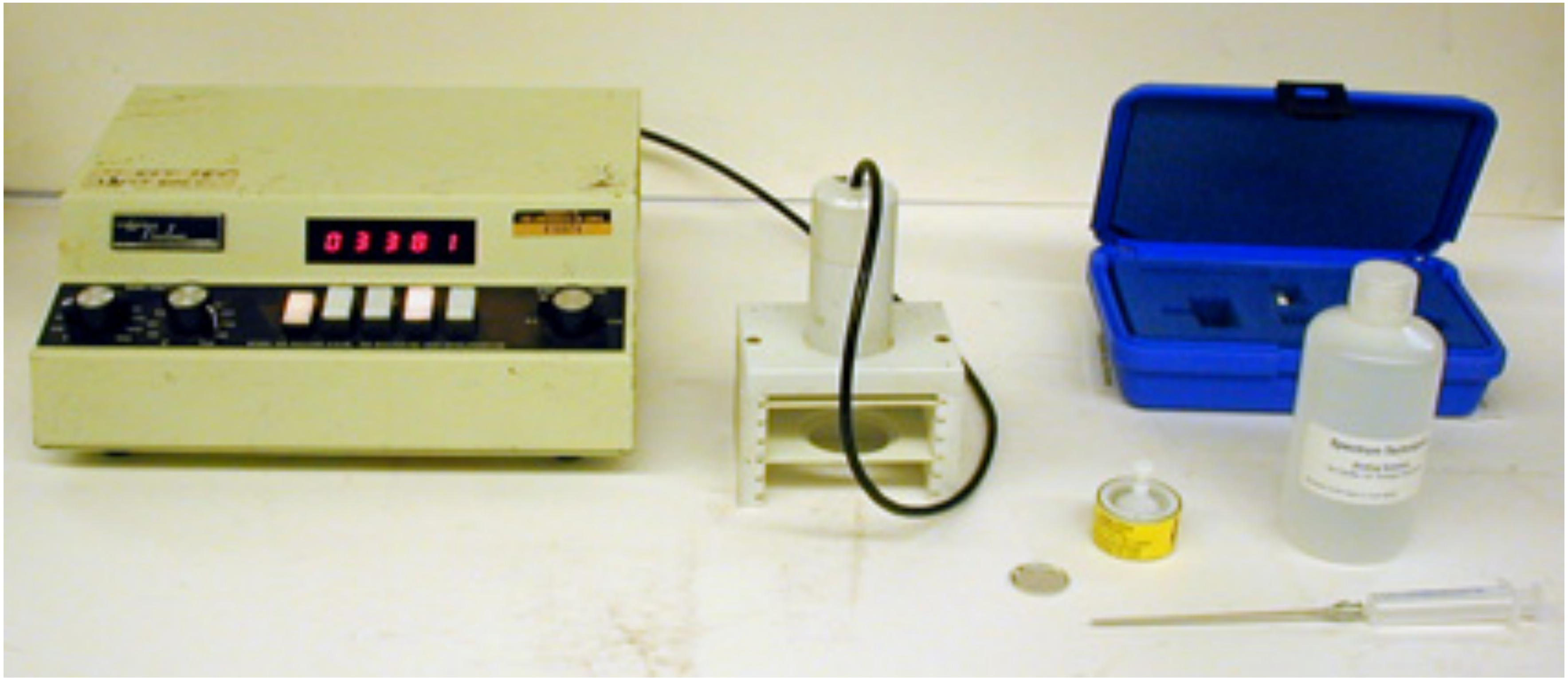
Did a nearby supernova explode just before the Solar system formed?

- Why did the two events (SN and solar system formation) happen so close in time (100-400 Myr separated in time)?
- Did the supernova actually triggered the formation of the Solar System?

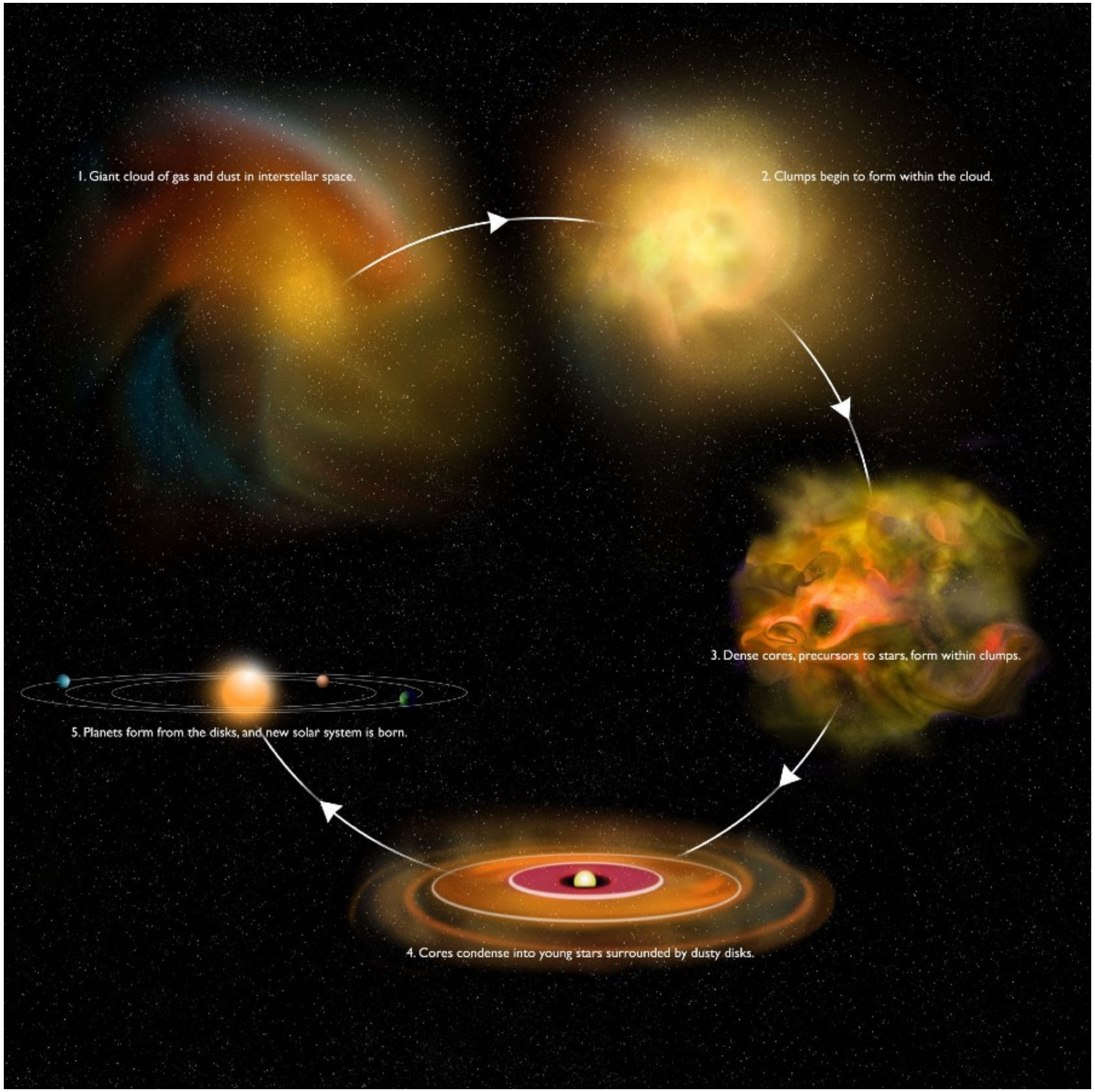
Practice Question

Suppose you were given a sample of radioactive materials in the lab. At 14:34, you measured that the sample showed an activity level of 300 counts/s, and one hour later at 15:34, its activity level dropped to 30 counts/s. What is the half life of your sample?

$$N_I(t) = N_I(0) \cdot 0.5^{t/\tau}$$



Part II: The Nebula Hypothesis



Key Formation Stages of Planetary Systems

- Gravitational collapse of dense and cold molecular clouds, forming cocoon nebulae (Kant (1755) & Laplace (1796))
- Protostars begins to form at the center of the cocoon
- Circumstellar disks start to form around protostars, increasing its mass and driving bipolar jets to reduce the angular momentum of the protostar
- As the disk cools, solid materials (dust) start to condense out of the circumstellar disk
- The star forms and drives stellar winds that sweeps away the remaining gas in the disk
- Aggregation of dust grains allows the formation of planetesimals and eventually planets.

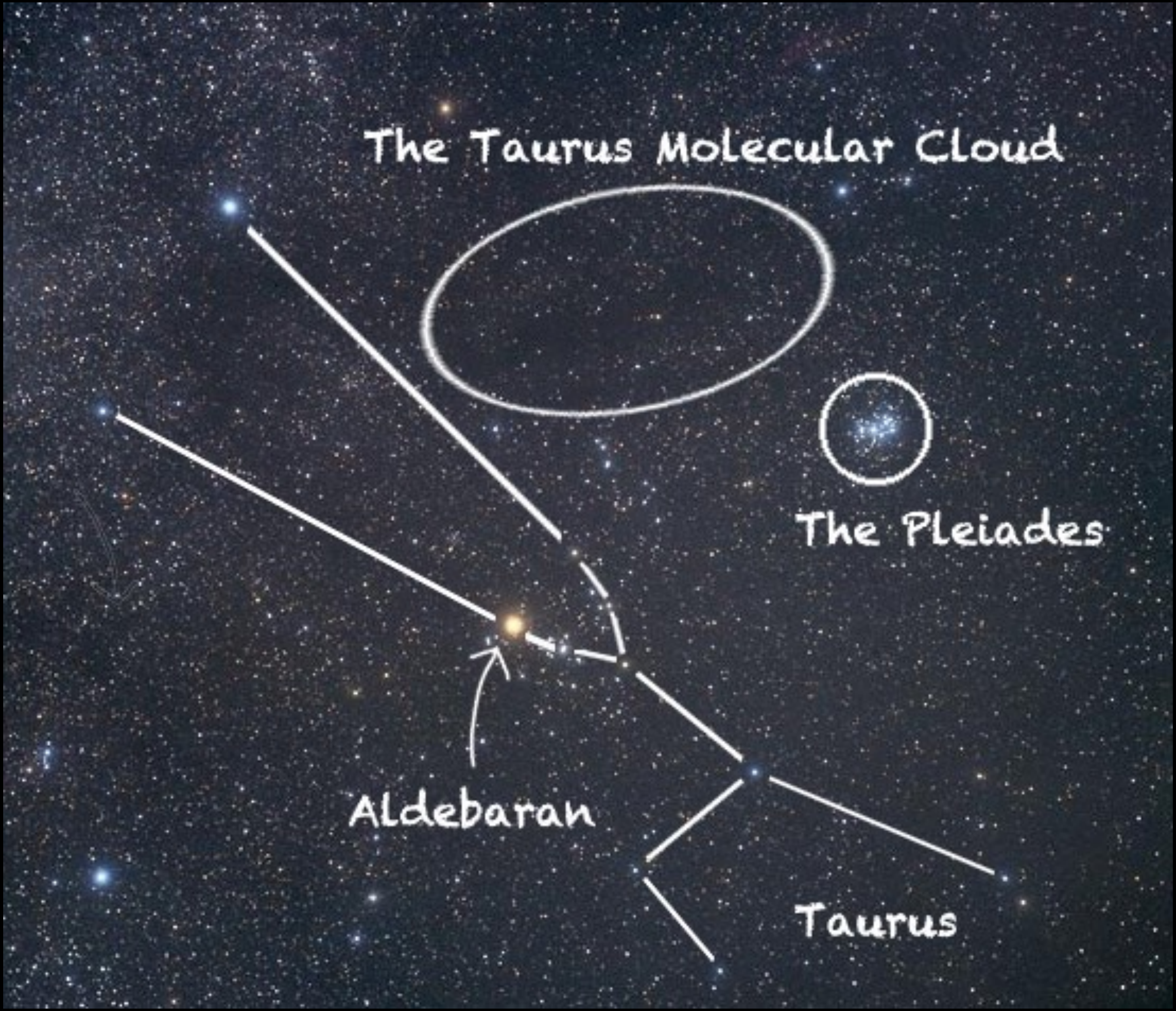
First, there was a cocoon nebula

gravitational collapse of
giant molecular clouds

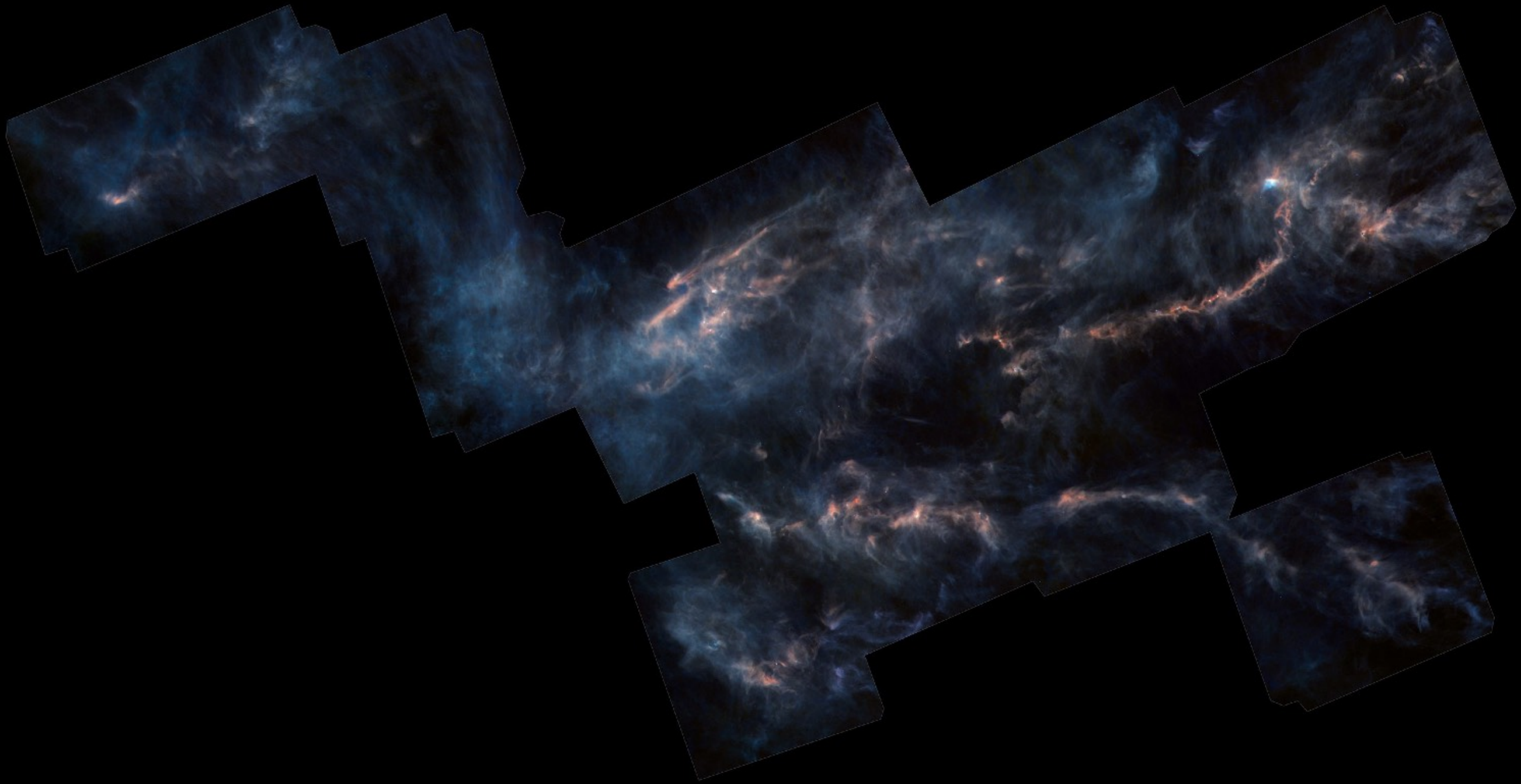
JWST’s near-to-mid-IR image of the Cosmic Cliffs
(the West section of the NGC 3324 star-forming region, 2.8 kpc away)

JWST - Pillars of Creation (Eagle Nebula)

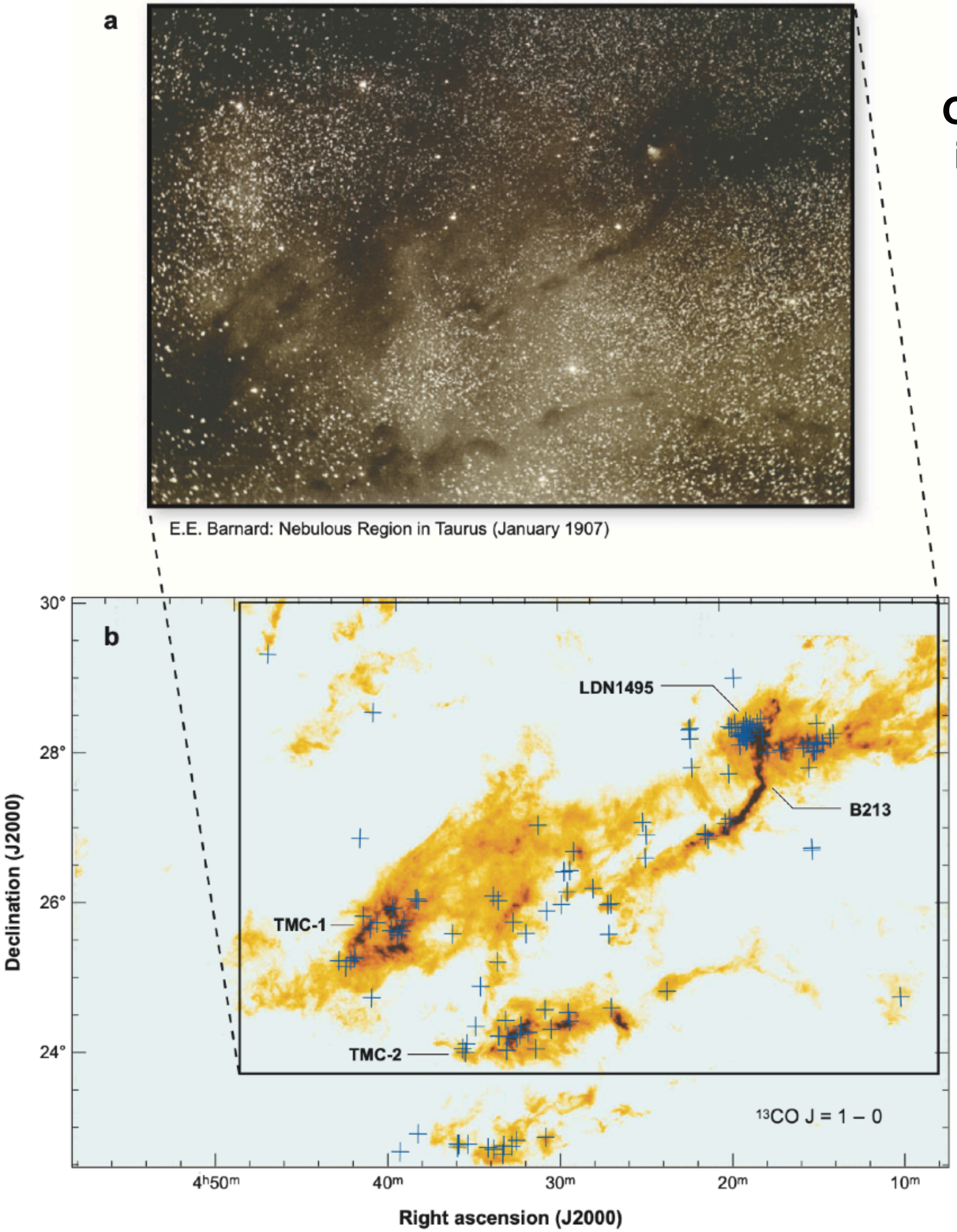
The Taurus molecular cloud, only 150 parsec away (1 parsec = 206265 AU)



Thermal dust emission from the Taurus molecular cloud
(*Herschel* Space Observatory, far-IR, 160-500 µm)



https://sci.esa.int/web/herschel/-/59536-herschel-s-view-of-the-taurus-molecular-cloud



Optical photographic image of the Taurus molecular cloud

only 140 pc away

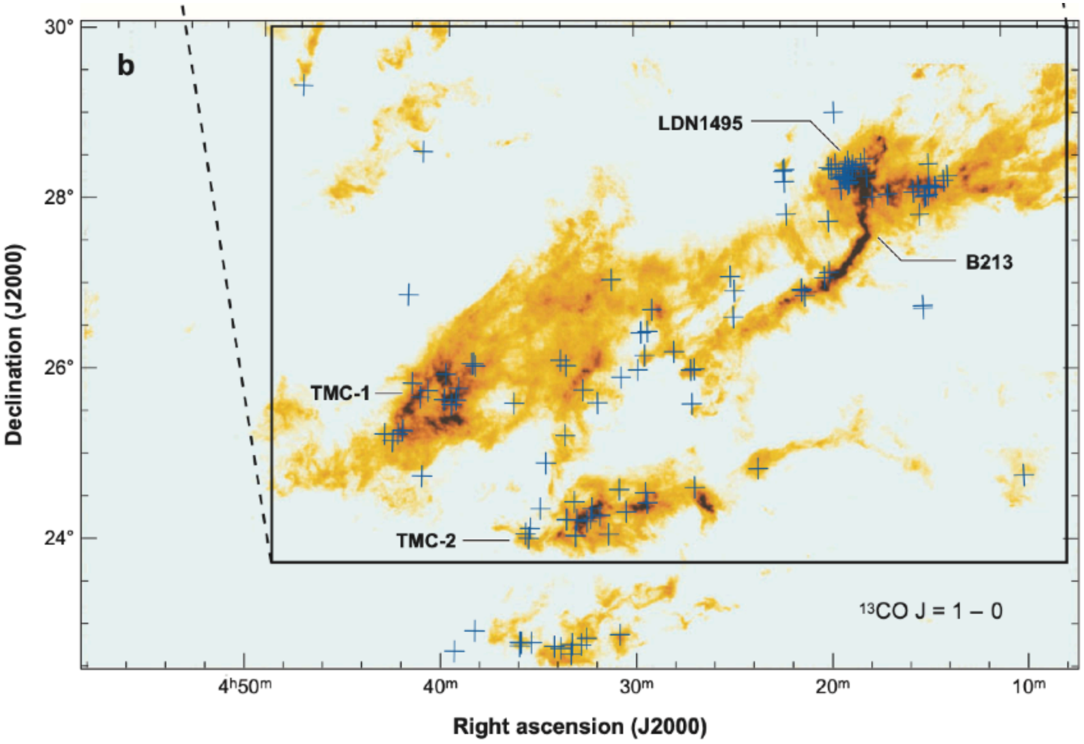
CO emission line (115 GHz, 2.6 mm)

crosses indicate known stellar and protostellar objects

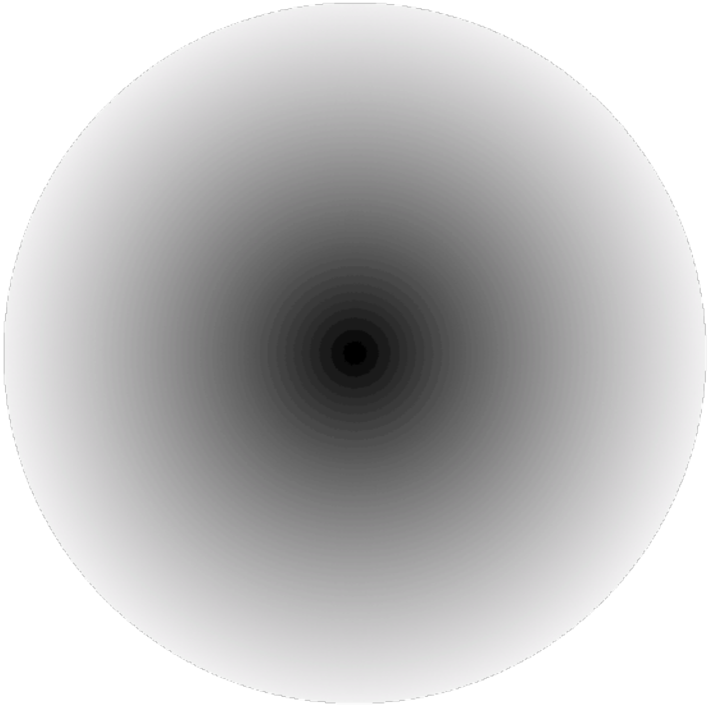
Bergin & Tafalla 2007

Simplify things until it cannot be simplified …

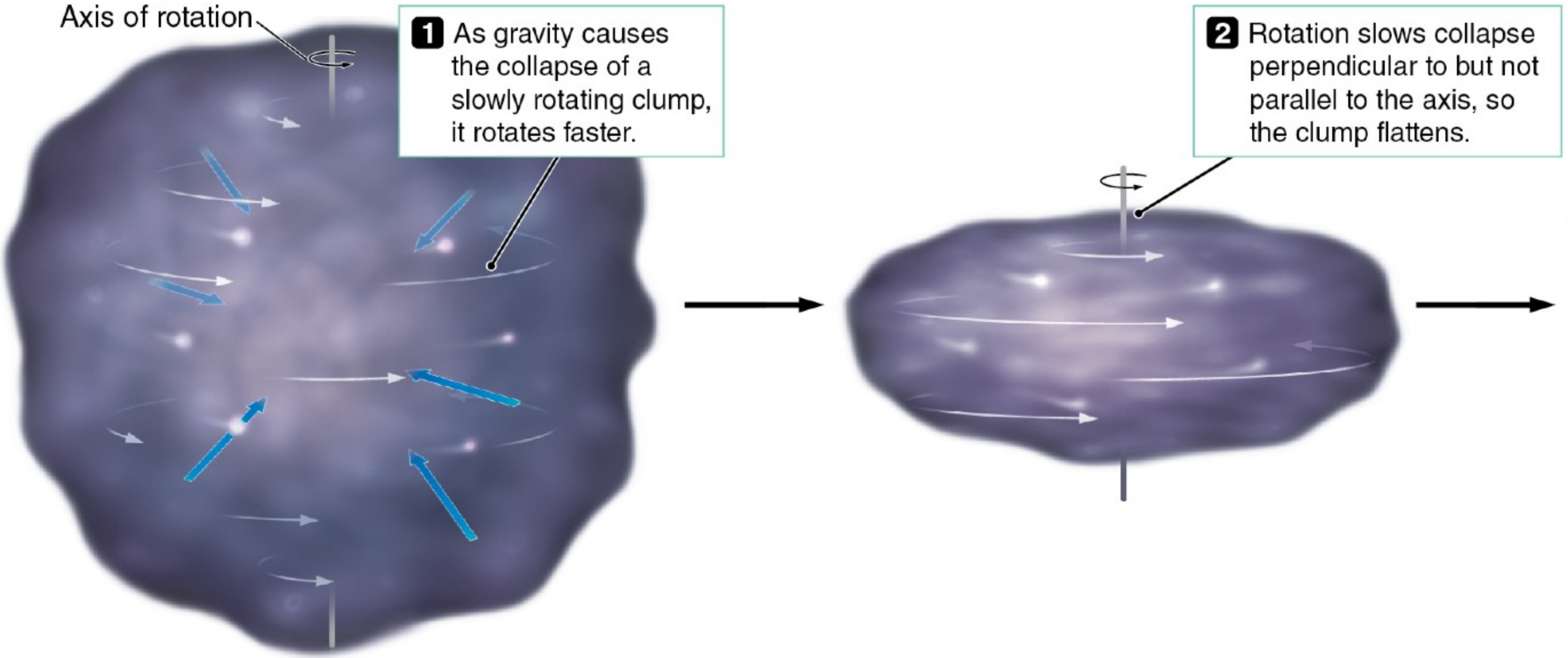
WHAT IS REALITY



What is in Astronomer's mind

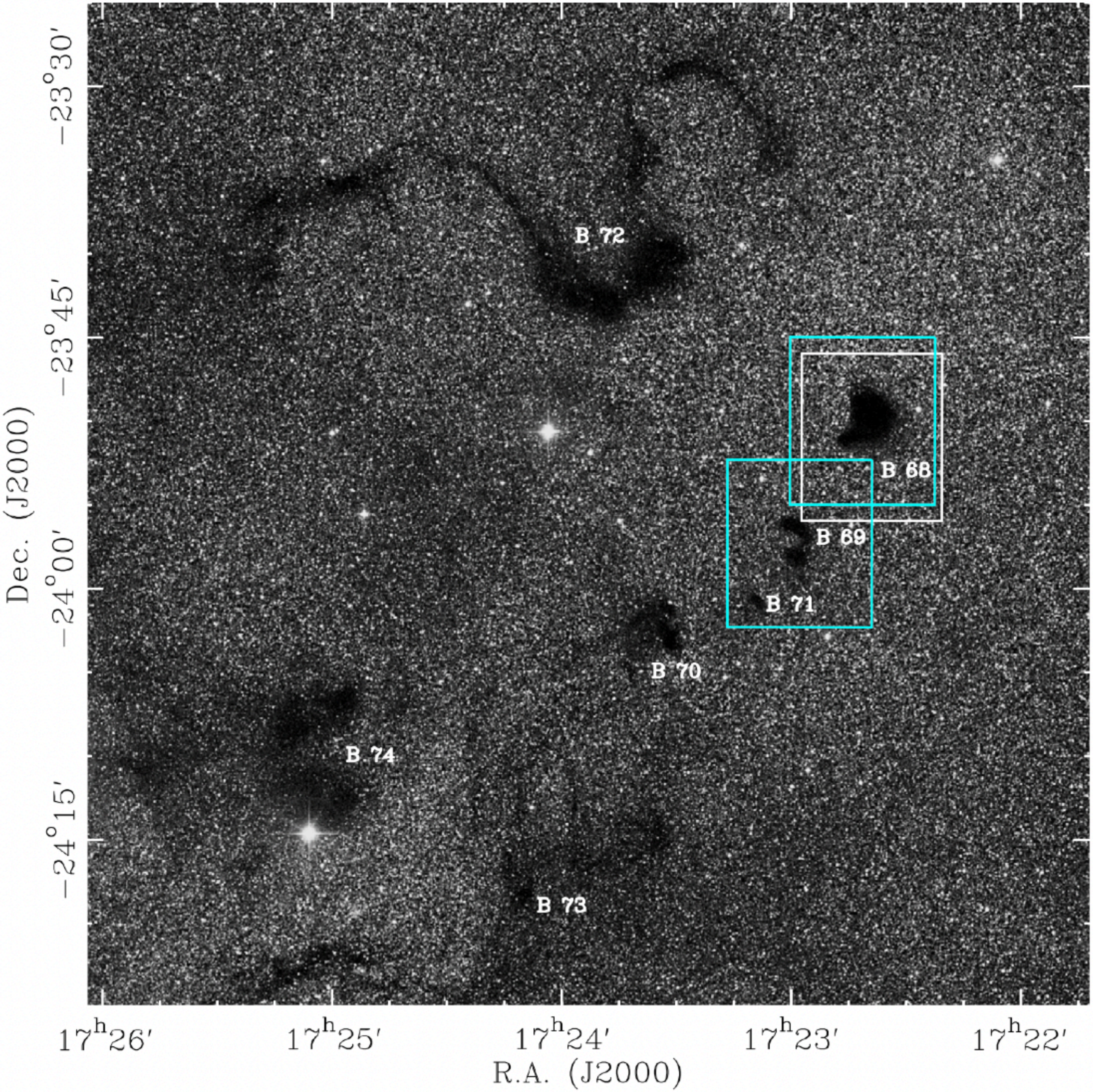


The Collapsing Cloud forms a Disk-Shaped Cocoon Nebula



1 As gravity causes the collapse of a slowly rotating clump, it rotates faster.

2 Rotation slows collapse perpendicular to but not parallel to the axis, so the clump flattens.


B 72
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−23°30′
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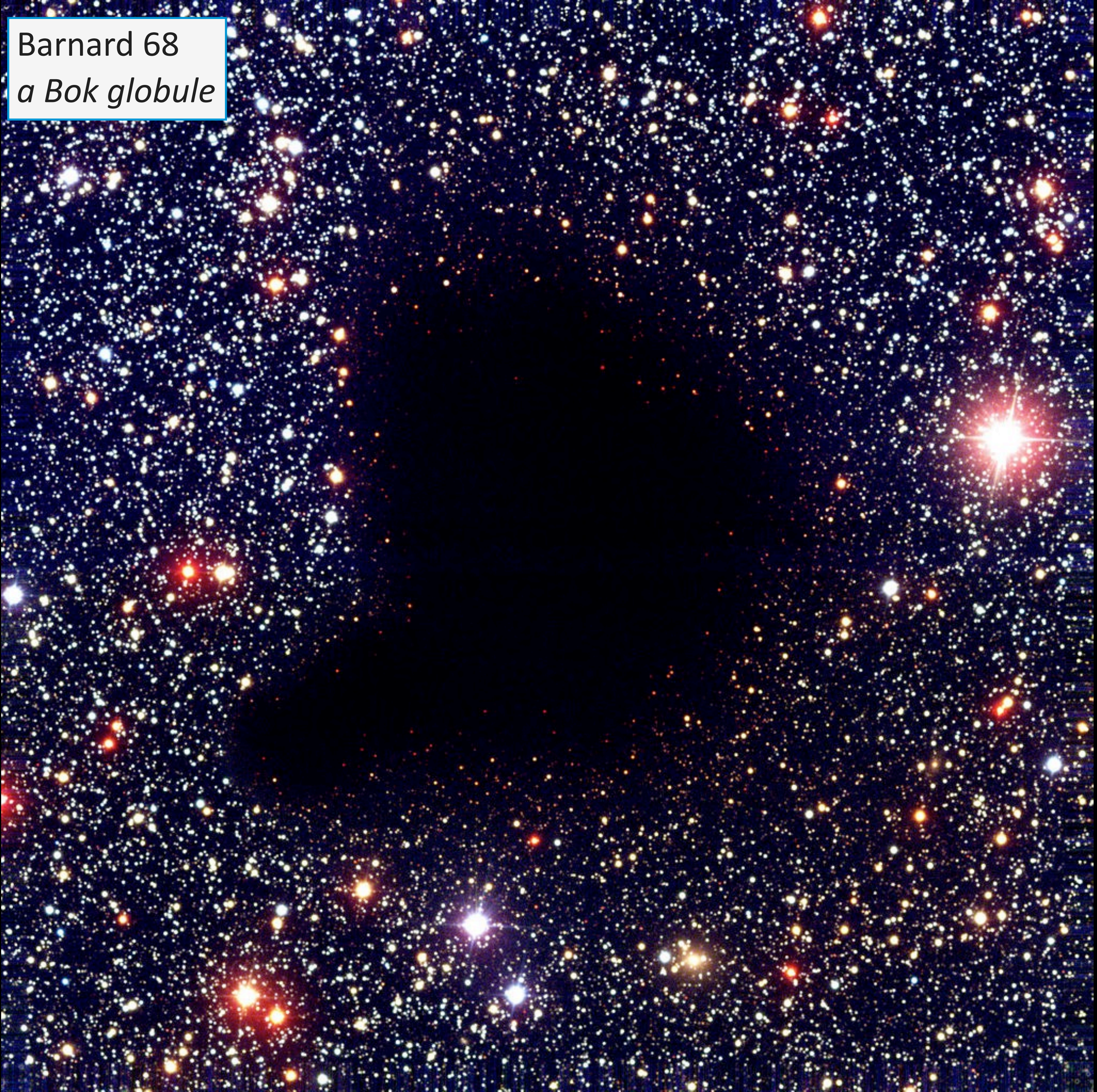


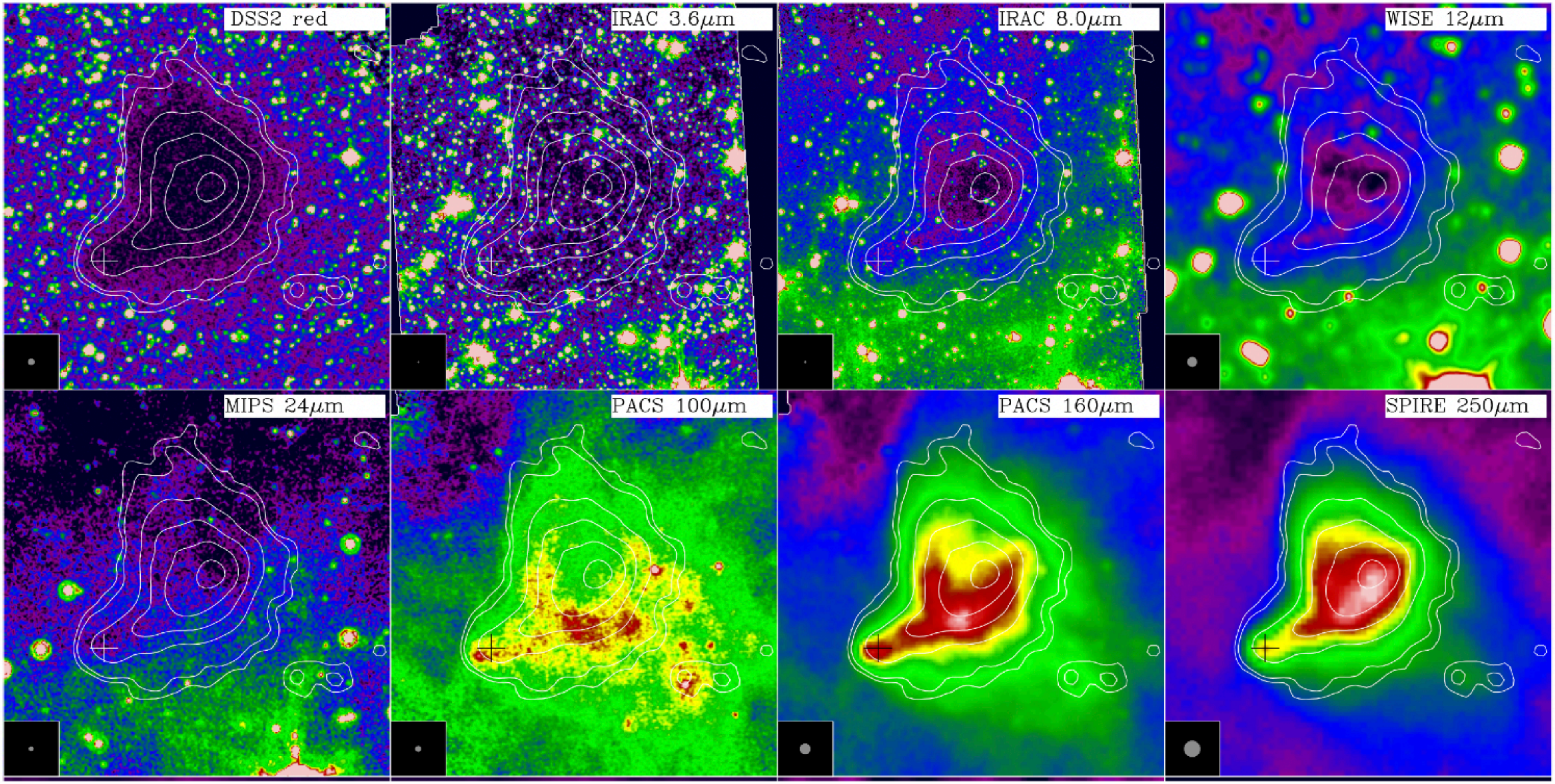
Barnard 68

a Bok globule

newly formed stars (proto-stars) are likely to be obscured by this envelope of dust and gas, coined the **cocoon nebulae**.

Kant (1755) and Laplace (1796)'s **nebular hypothesis** states that the early Sun was inside such a cocoon nebula, although neither of them ever saw an image like this.

Optical and infrared images of B68



Nielbock et al. 2012

Why Some Gas Clouds Collapse?

Jeans instability &
gravitational collapse

As N-body systems, stable gas clouds should follow the Virial Theorem

Virial theorem relates **the time-averaged kinetic energy** of a **stable system** of discrete particles, bound by potential forces, with that of **the time-averaged potential energy** of the system. For a **gravitationally bounded stable** system:

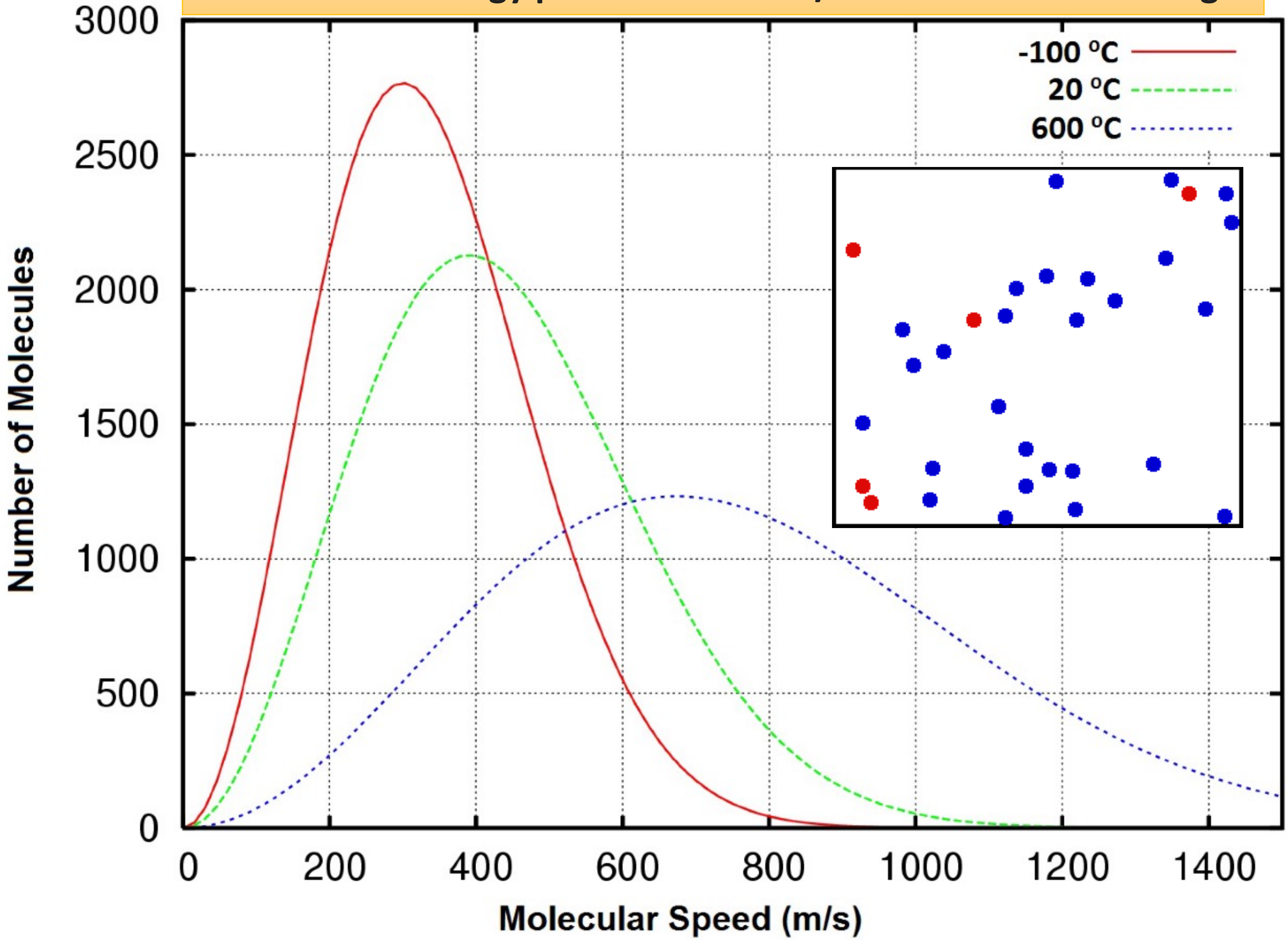
2 x Kinetic Energy + Gravitational Potential Energy = 0

- First, we need to know how to express the **total kinetic energy** of the gas particles.

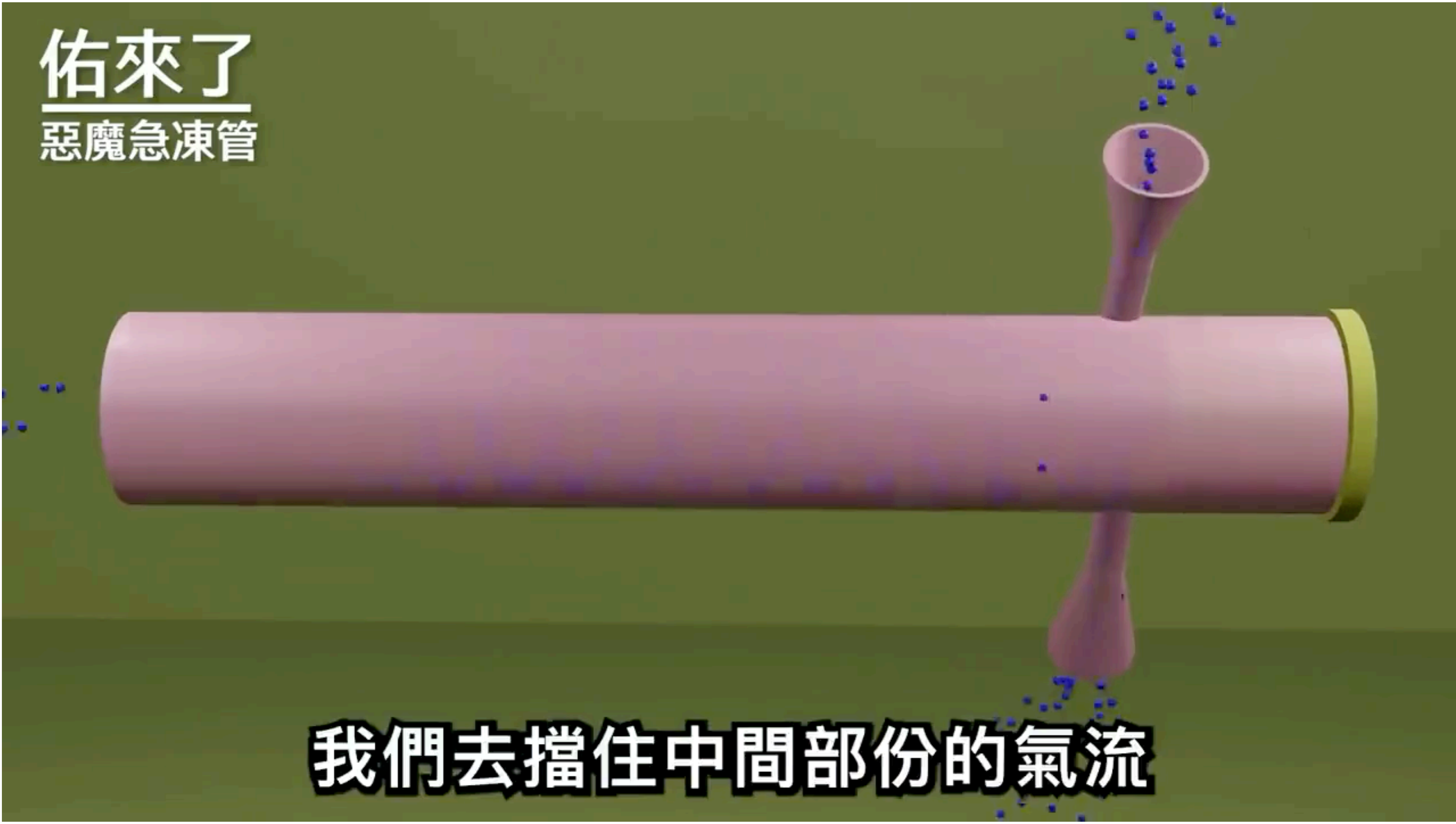
- Then, we need to know how to express the **potential energy** of a spherical gas cloud.

Random Motions of Gas Particles follow the Boltzmann Distribution

Mean Kinetic Energy per Particle = 3kT/2 for monatomic ideal gas



Vortex Tube Demo Explained



Internal Kinetic Energy of a Gas Cloud

Total Kinetic Energy is simply the mean multiplied by the number of gas particles:

$$KE = N_{\text{particle}} \cdot \frac{3}{2} kT = \frac{3}{2} \frac{M_{\text{gas}}}{\mu m_H} kT$$

where μm_H is the **average mass per particle**

Side note: recall the ideal gas law:

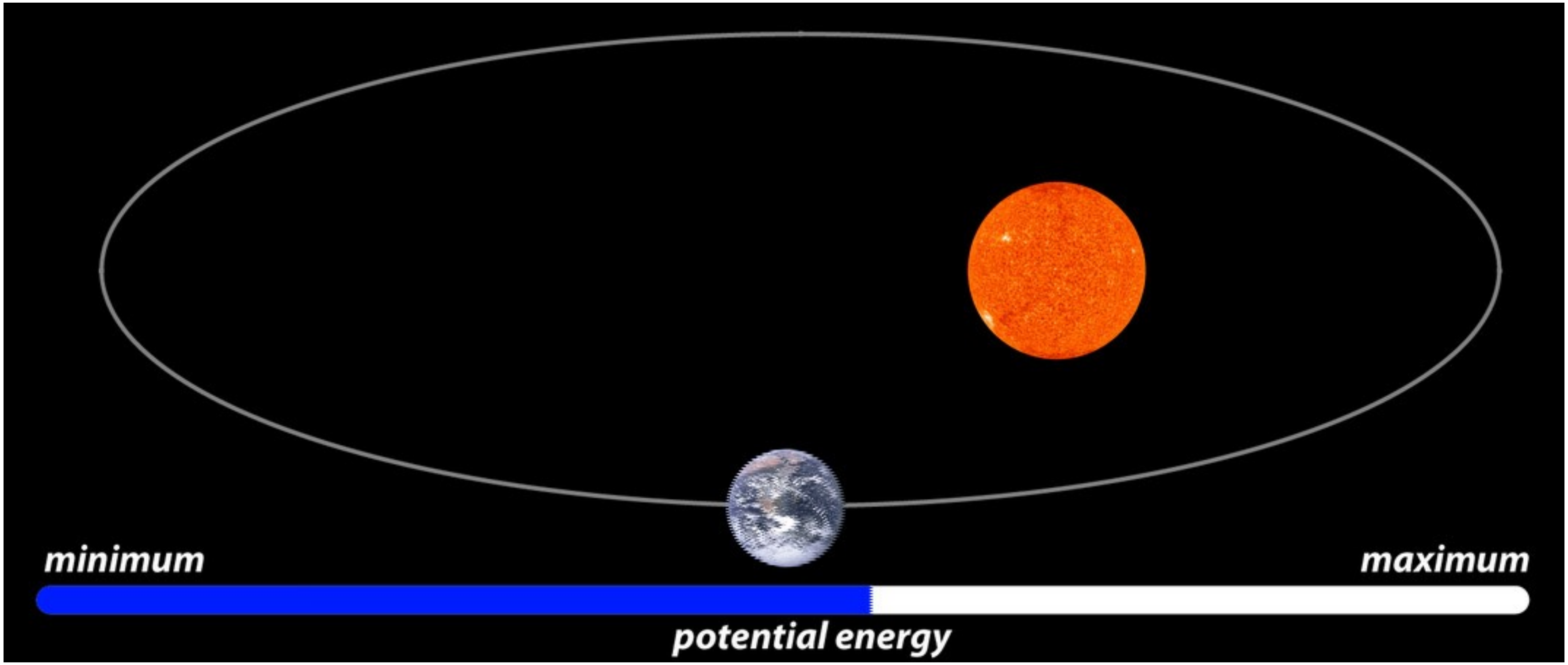
$$P = nkT = \frac{2}{3} KE/\text{Volume}$$

so, pressure is simply 2/3 of the kinetic energy density

Gravitational potential energy of a small mass m in the gravitational field of a larger mass M

Gravitational potential energy of mass *m* in *M*'s gravity

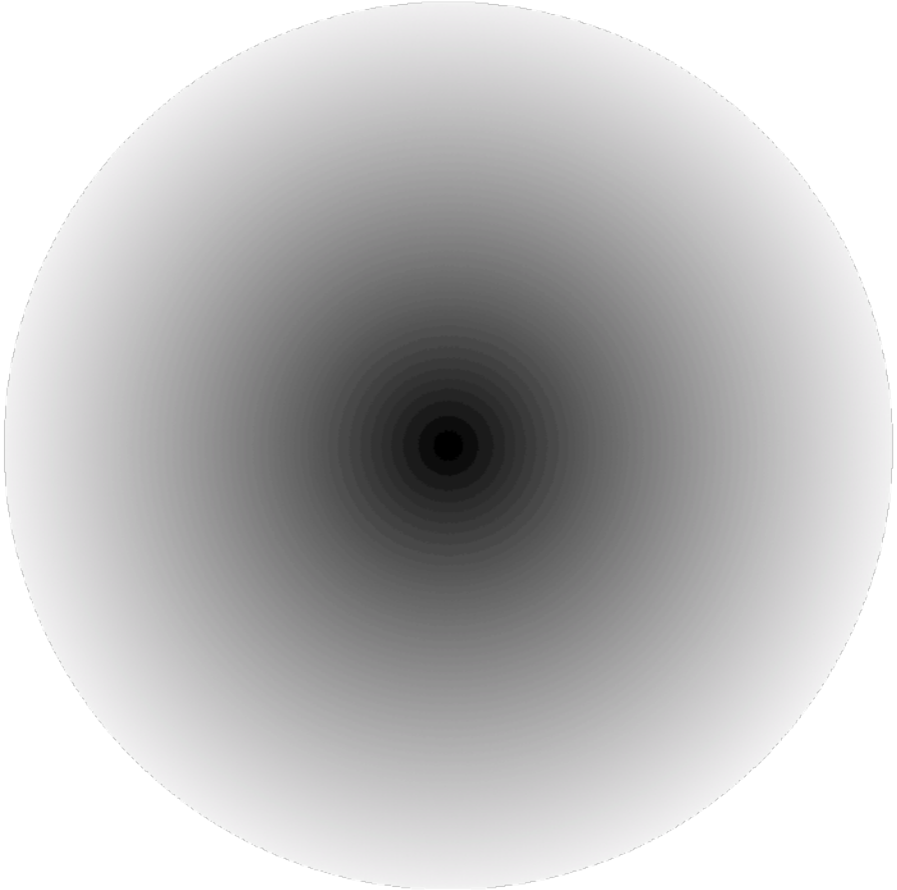
$$E_g = \frac{-GMm}{R}$$

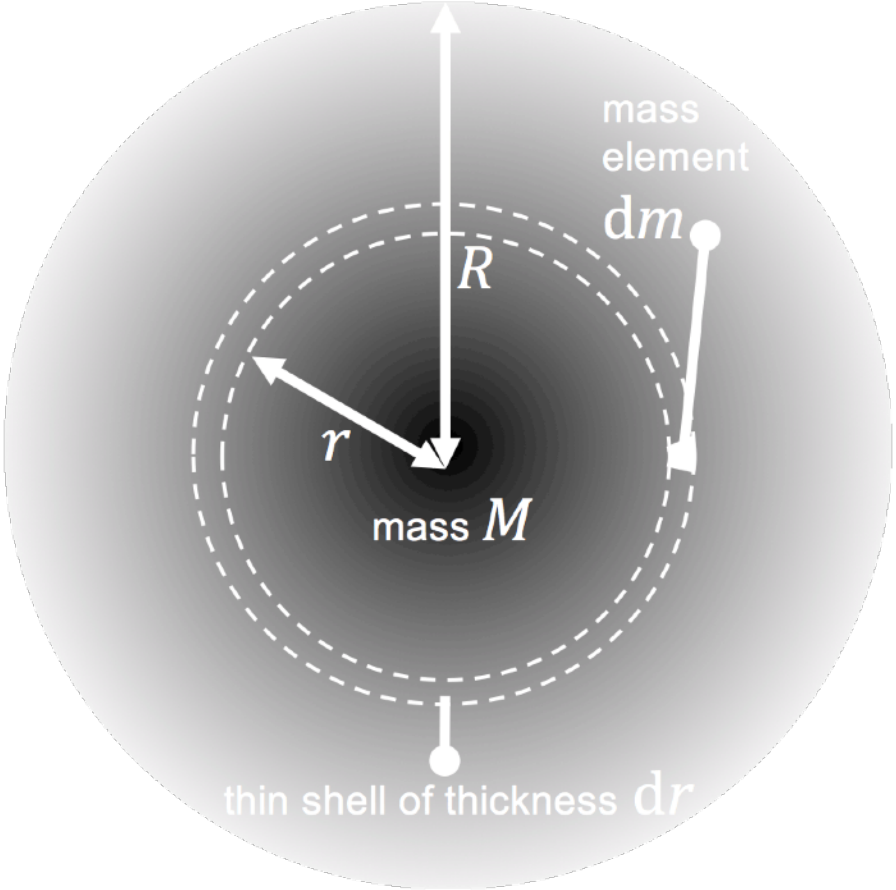


Now consider a spherical shell at distance r

Let's start by writing down the gravitational potential energy of a shell with mass of *dm* at radius of *r*

$$dU_r = -\frac{GM_r dm}{r} = -G(\rho \cdot \frac{4}{3}\pi r^3)(\rho \cdot 4\pi r^2 dr)/r$$

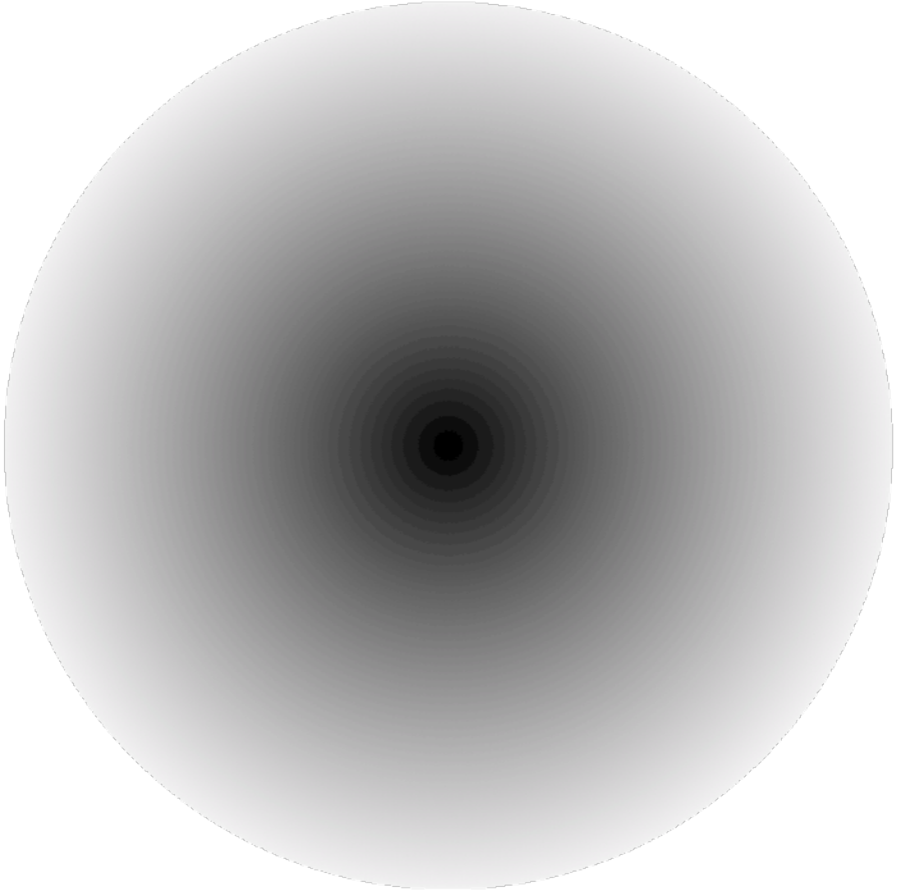


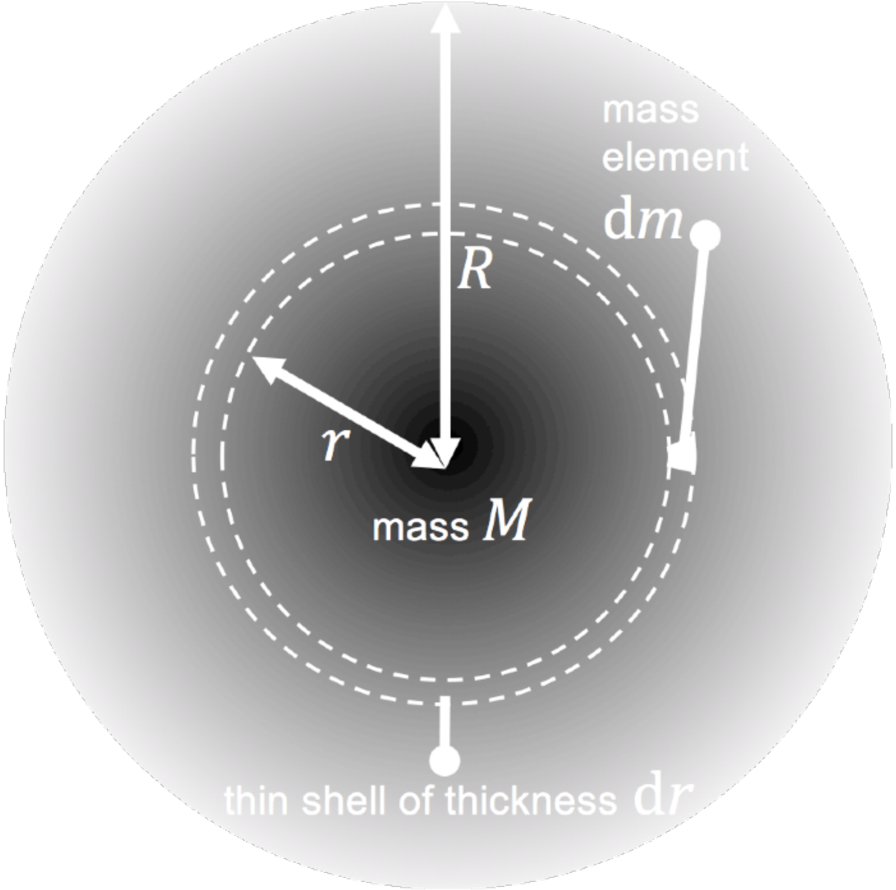


Now consider a spherical shell at distance r

Next, we sum up the potential energy of all shells up to radius of R, this operation is an integral:

$$U = \int_0^R dU_r = -\frac{16\pi^2}{3} G\rho^2 \int_0^R r^4 dr = -\frac{3}{5}\frac{GM^2}{R}$$





Virial Theorem applied to a spherical cloud

Now we can put both equations together and then write down the virial theorem for a uniform spherical gas cloud:

$$K = \frac{3}{2}\frac{M_{\mathrm{gas}}}{\mu m_H}kT \qquad\qquad U = -\frac{3}{5}\frac{GM^2}{R}$$

Virial theorem applies IF the cloud is stable:

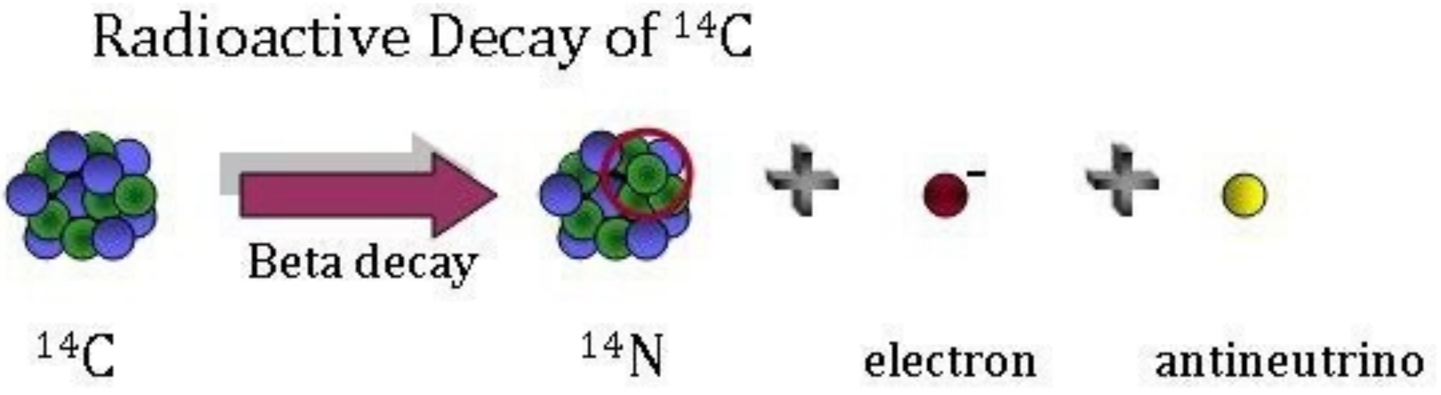
$$2K = -U \Rightarrow \frac{3MkT}{\mu m_H} = \frac{3GM^2}{5R}$$

Although 2K and -U both increase as R increases, they don't increase at the same rate ($K \sim R^3$, $U \sim R^5$). So beyond some point, the virial theorem is violated as $2K < -U$.

Note on Problem 1(b) in the Homework

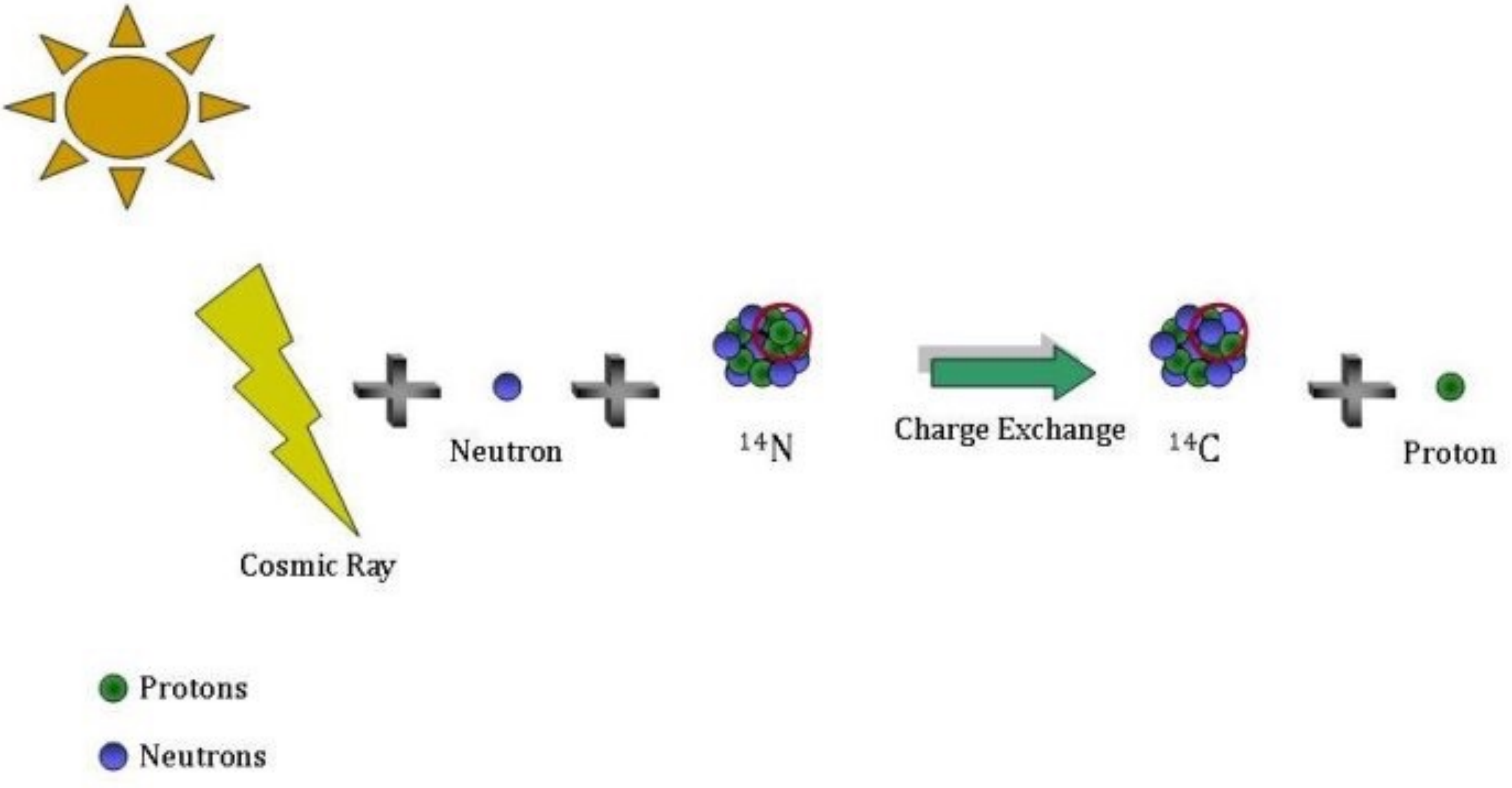
(b) In living things, the carbon isotope ratio between C^{14} and C^{12} is $C^{14}/C^{12} = 10^{-12}$. The half-life of C^{14} is 5730 yr. Suppose an archeologist finds that the charcoal in an ancient fire pit has a carbon isotope ratio of $C^{14}/C^{12} = 2 \times 10^{-14}$, estimate the age of the charcoal.

But what does C-14 decay into? C-12 or something else?



How do living things maintain a constant level of C-14/C-12 ratio?

Because there is a production mechanism of C-14 in the atmosphere.

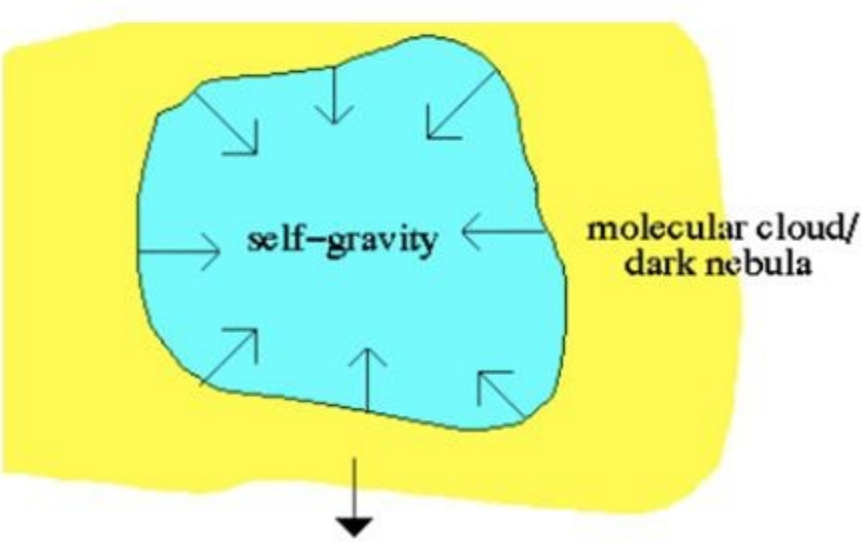


Jeans Instability (*Continued*)

Jeans Mass: Systems with *2K + U < 0* are unstable and will collapse

This is called the **Jeans instability criterion**:

$$2K < -U \Rightarrow \frac{3MkT}{\mu m_H} < \frac{3GM^2}{5R}$$



We can express *R* as a ratio of *M* and the mean mass density ρ_0:

$$R = \left(\frac{3M}{4\pi\rho_0}\right)^{\frac{1}{3}}$$

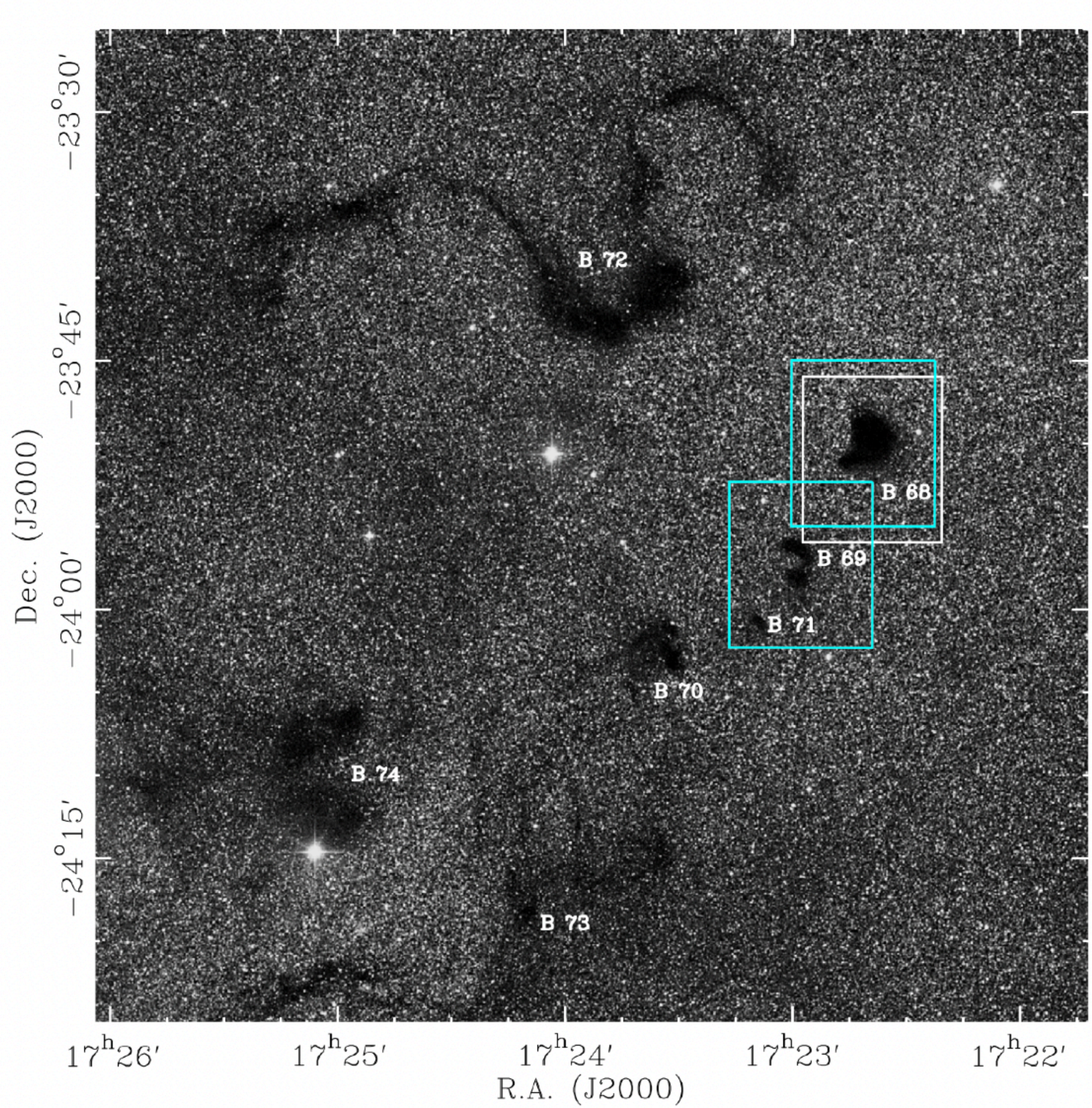
Replacing *R* in the Jeans criterion, we can derive a lower limit on the cloud mass *M*, above which it will become unstable:

$$M > \left(\frac{5kT}{G\mu m_H}\right)^{\frac{3}{2}}\left(\frac{3}{4\pi\rho_0}\right)^{\frac{1}{2}} = 8\ M_\odot \left(\frac{T}{10\ \mathrm{K}}\right)^{3/2}\left(\frac{n_0}{10^4\ \mathrm{cm}^{-3}}\right)^{-1/2}\left(\frac{\mu}{2}\right)^{-2}$$

i.e., for typical conditions in a dense molecular core, the **lower mass limit** (the **Jeans mass**) is about **8 M$_{\text{sun}}$**.

In the above, we used the density relation: $\rho_0 = n_0 \cdot \mu m_H$

Stars form in the coldest and densest gaseous regions in the Galaxy



Jeans Length: Estimating the initial radius of the parent cloud

We have derived the **Jeans mass**:

$$M_J = \left(\frac{5kT}{G\mu m_H}\right)^{\frac{3}{2}}\left(\frac{3}{4\pi\rho_0}\right)^{\frac{1}{2}} = 8\ M_\odot\left(\frac{T}{10\ \mathrm{K}}\right)^{3/2}\left(\frac{n_0}{10^4\ \mathrm{cm}^{-3}}\right)^{-1/2}\left(\frac{\mu}{2}\right)^{-2}$$

We also know the cloud radius *R* is simply a ratio between *M* and the mean mass density ρ_0:

$$R = \left(\frac{3M}{4\pi\rho_0}\right)^{\frac{1}{3}}$$

Thus, the Jeans Mass and the initial mean density gives us the **initial radius** of the unstable cloud (**Jeans length**):

$$R_J = \left(\frac{15kT}{4\pi G\mu m_H\rho_0}\right)^{\frac{1}{2}} = 0.1\ \mathrm{pc}\left(\frac{T}{10\ \mathrm{K}}\right)^{1/2}\left(\frac{n_0}{10^4\ \mathrm{cm}^{-3}}\right)^{-1/2}\left(\frac{\mu}{2}\right)^{-1}$$

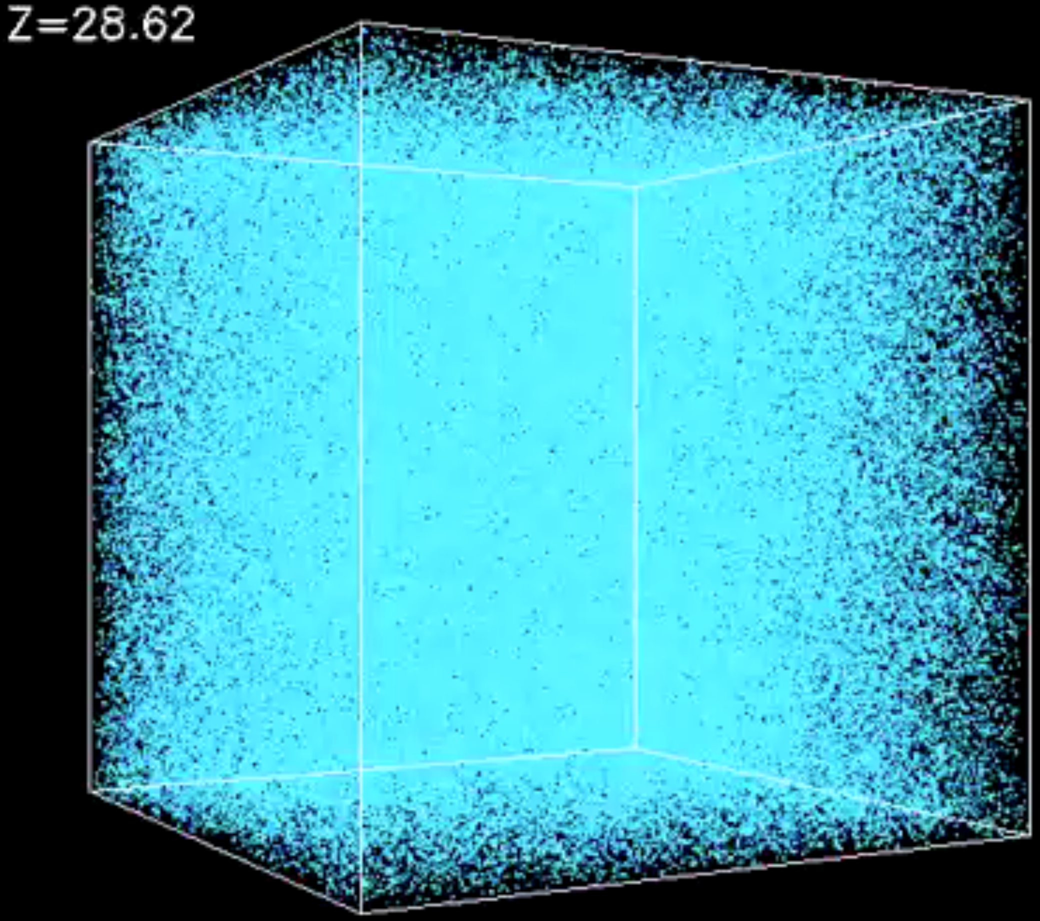
i.e., for typical conditions in dense molecular cores, the **Jeans length** is about **0.1 parsec** or **20,000 AU**.

In the above, we used density relation: $\rho_0 = n_0 \cdot \mu m_H$

Wikipedia: Barnard 68's interior is extremely cold, its temperature being about 16 K. Its mass is about twice that of the Sun and it measures about half a light-year across (0.15 parsec).

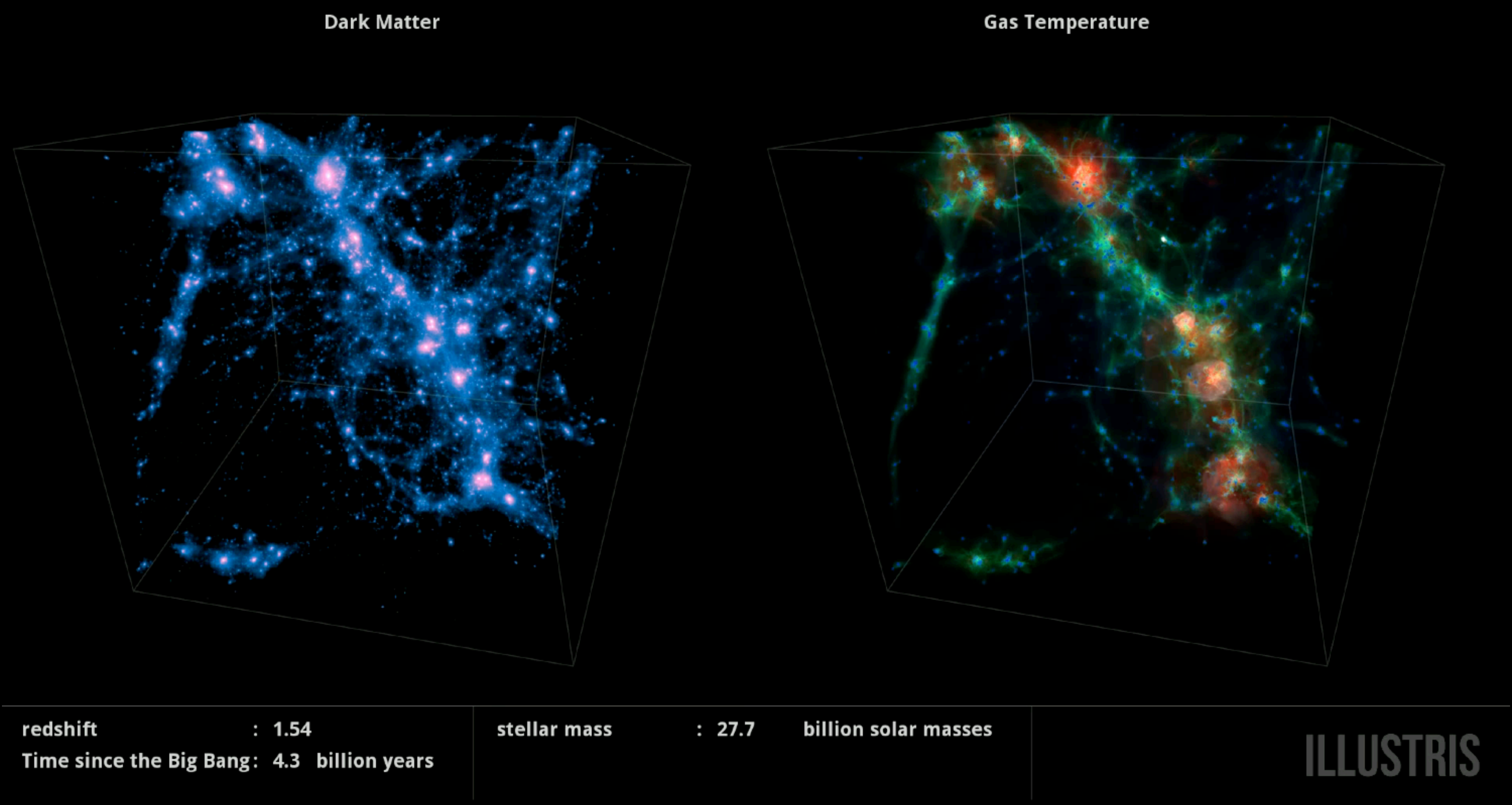
Barnard 68
a Bok globule

Gravitational instability occurs at all scales, from galaxies to stars. Here it's illustrated by a cosmological simulation of galaxy formation



simulation of a region in space that is 140 million light years across

Simulation of a Cube 30 Million Light Year Across



redshift : 1.54
Time since the Big Bang : 4.3 billion years

stellar mass : 27.7 billion solar masses

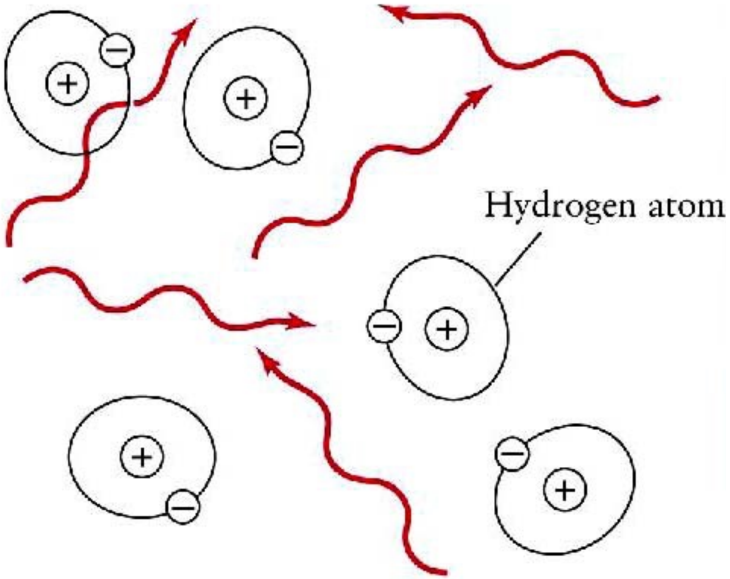
How to calculate μ ?

the Mean Molecular Weight

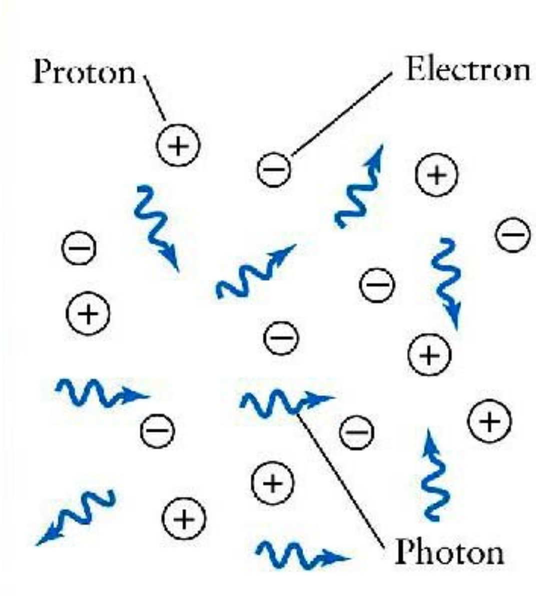
The Mean Molecular Weight

The mean molecular weight is a dimensionless quantity, and it is defined as the ratio between the mean mass per particle and the mass of a single hydrogen atom (1.67e-27 kg):

$$\mu \equiv \frac{\bar{m}}{m_H} = \frac{M_{\mathrm{gas}}/N_{\mathrm{particle}}}{m_H}$$



neutral atomic hydrogen



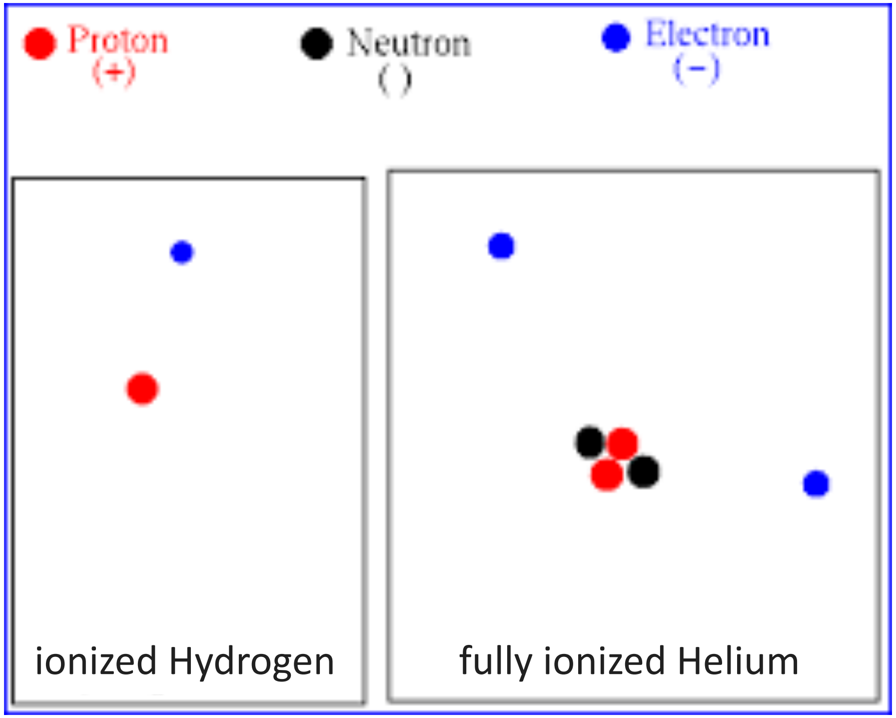
ionized hydrogen

The Mean Molecular Weight

The mean molecular weight is a dimensionless quantity, and it is defined as the ratio between the mean mass per particle and the mass of a single hydrogen atom (1.67e-27 kg):

$$\mu \equiv \frac{\bar{m}}{m_H} = \frac{M_{\text{gas}}/N_{\text{particle}}}{m_H}$$

- *What about neutral helium, partially ionized helium, and fully ionized helium gases?*
- $\mu = 4, 4/2, 4/3$, *respectively*



The Mean Molecular Weight of Mixed Gas

- What need to be known? the mean molecular weight of each component (μ_i), and their percentages by volume (f_i^v)

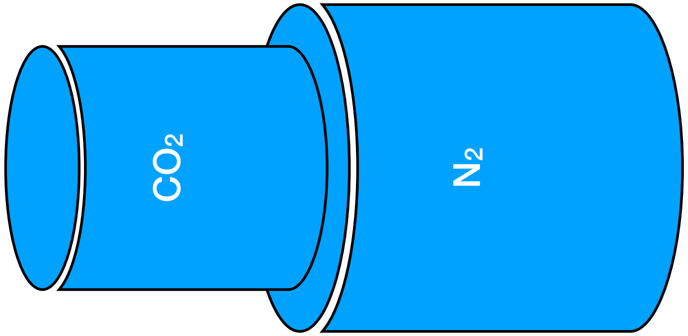
$$\mu = \sum_i \mu_i f_i^v$$

Practice:

- CO_2: μ = 44
- N_2: μ = 28

$$\mu m_H = \frac{\rho}{n}$$

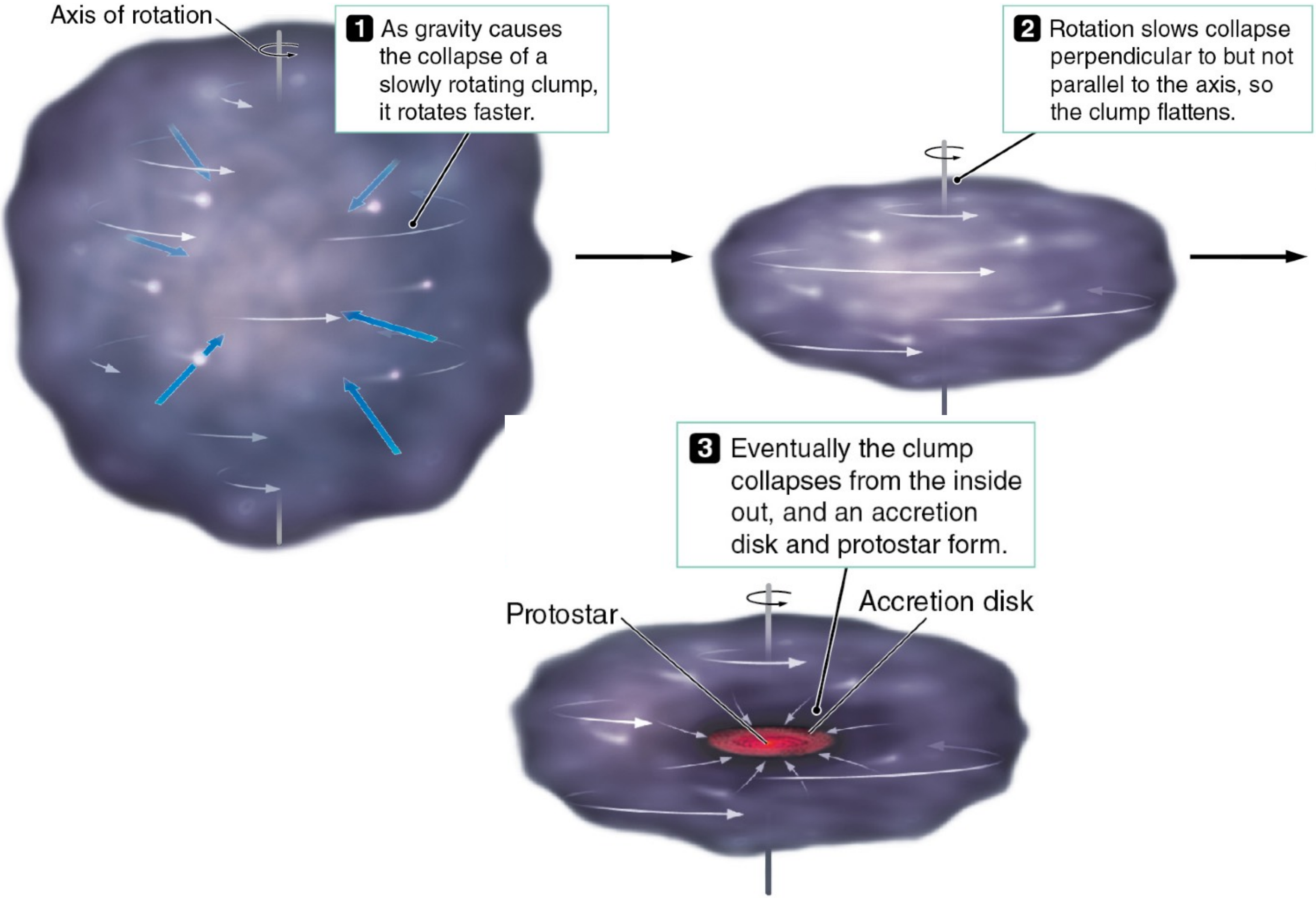
- What is the mean molecular weight of a mixed gas consisting of 40% CO_2 & 60% N_2 (by volume)?



The formation of proto-planetary disks

conservation of angular momentum

The Cocoon Nebula then forms an Accretion Disk around a Protostar



1 As gravity causes the collapse of a slowly rotating clump, it rotates faster.

2 Rotation slows collapse perpendicular to but not parallel to the axis, so the clump flattens.

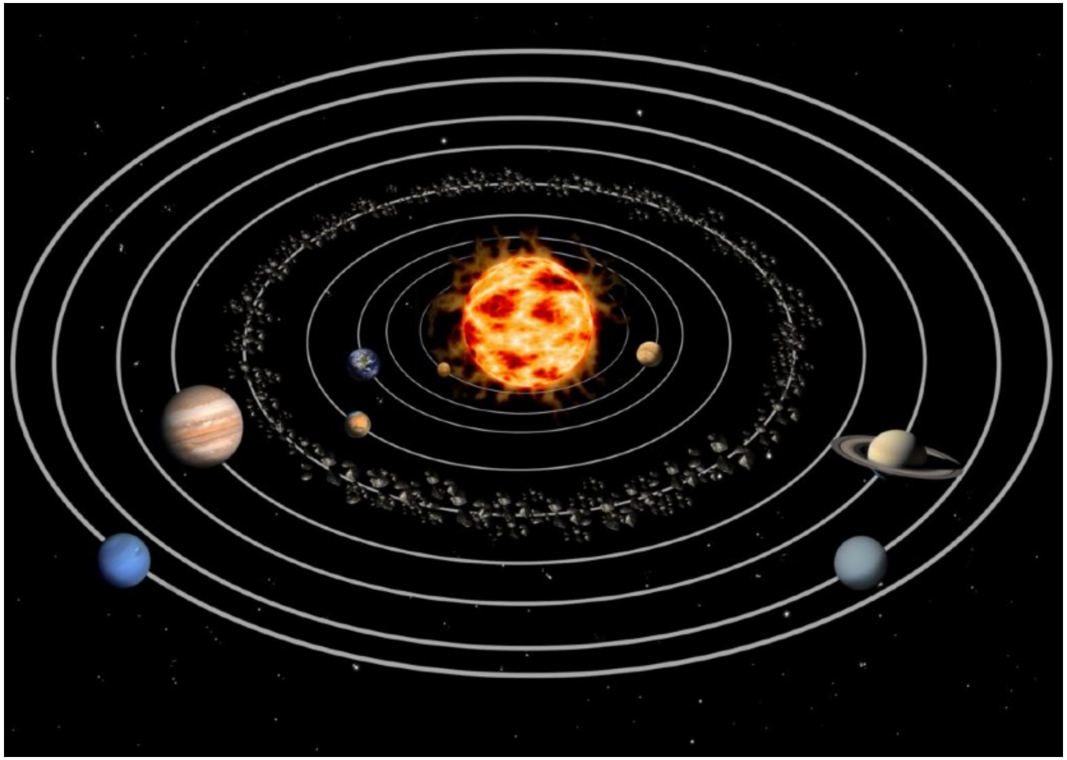
3 Eventually the clump collapses from the inside out, and an accretion disk and protostar form.

Orbital Angular Momentum

- The orbital angular momentum is the vector cross product of the momentum (***p*** = *m**v***) and the distance to the central attracting object (***r***):

$$\overrightarrow{L}_{\text{orbit}} = \vec{r} \times m\vec{v}_{\text{orbit}} = m r v_t \vec{k}$$

- For the case of a planet around the Sun: m is the mass of the planet, v_t its circular velocity (which equals the transverse velocity), and r its heliocentric distance
- Conservation of the orbital angular momentum gives Kepler's 2nd law

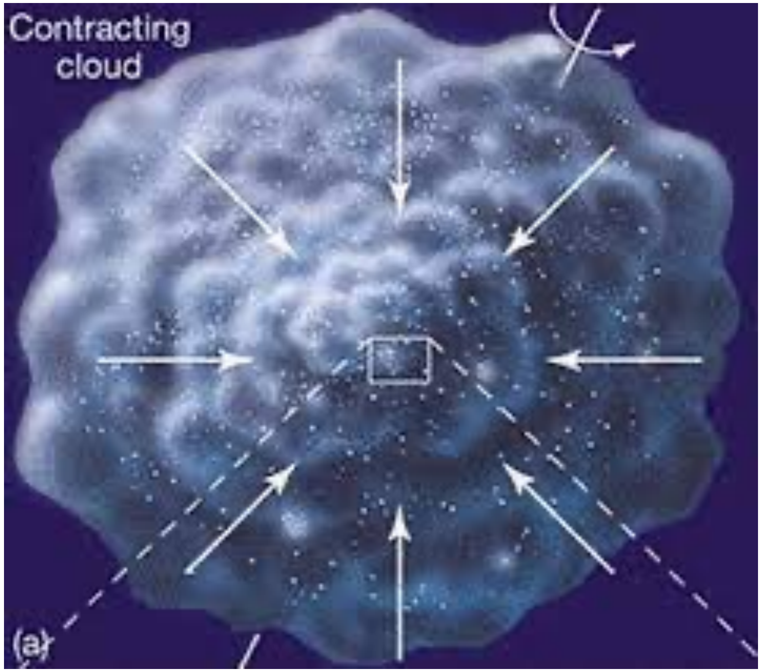


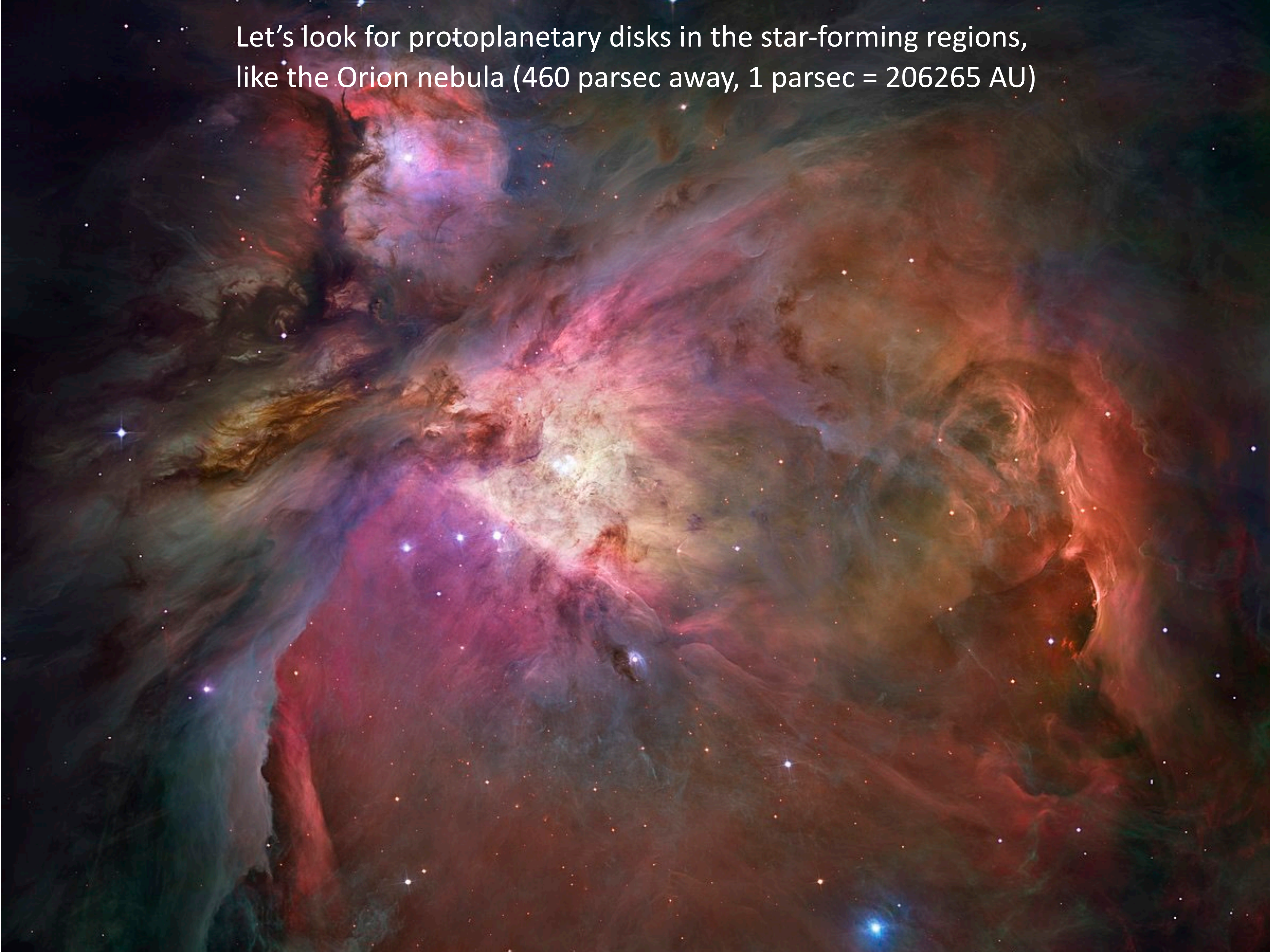
Spin Angular Momentum

- A spinning uniform sphere's angular momentum is:

$$L_{\text{spin}} = \frac{2}{5} M R^2 \omega \quad \text{where} \quad \omega = \frac{2\pi}{P}$$

- where M is the mass of the sphere, R its radius, ω its angular velocity, and P its rotation period
- As R decreases during a collapse, the ω must increase as $1/R^2$.





Let’s look for protoplanetary disks in the star-forming regions, like the Orion nebula (460 parsec away, 1 parsec = 206265 AU)

Comparison w/ Disk-shaped, dense protoplanetary clouds in the Orion nebula (HST images)

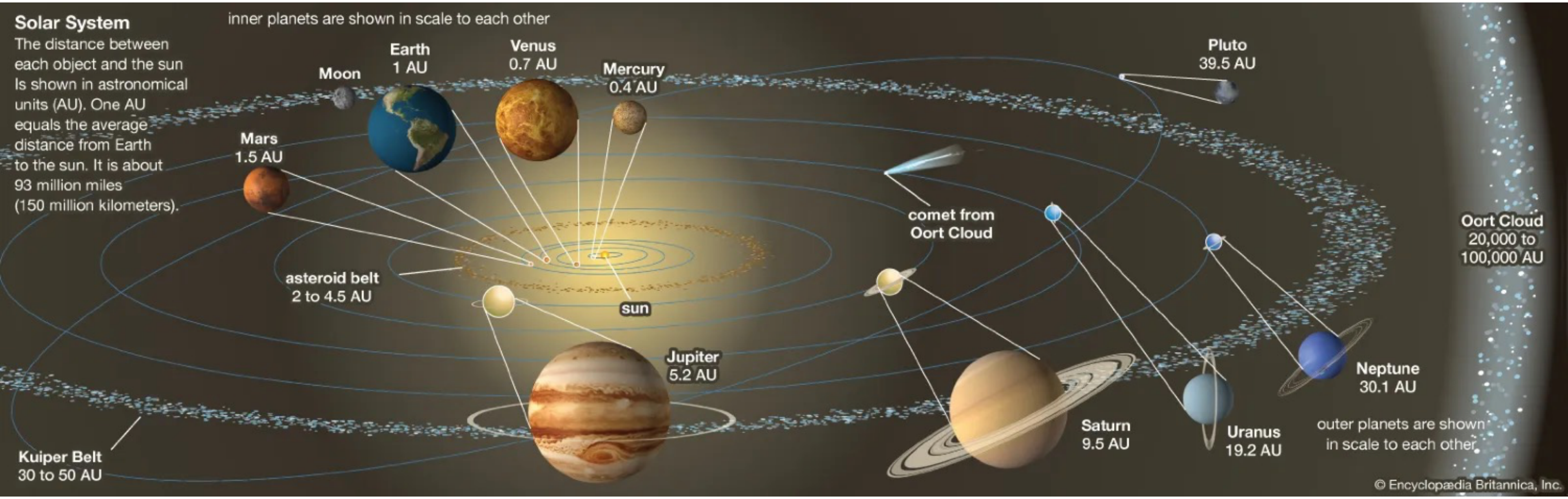
Proplyds stands for PROtoPLanetarY DiskS, typical diameter ~ 100s AU

Characteristics of the Solar System that Must Be Explained

√ ***All the planets' orbits lie roughly in a single plane. The Sun's rotational equator also lies nearly in this plane. The planets and the Sun all revolve in the same direction.***

? ***Planets differ in composition, roughly correlating with distance from the Sun.***

? ***Planets have much less mass than the Sun (planets contain only 0.14% of the total mass), but they have much more angular momentum.***



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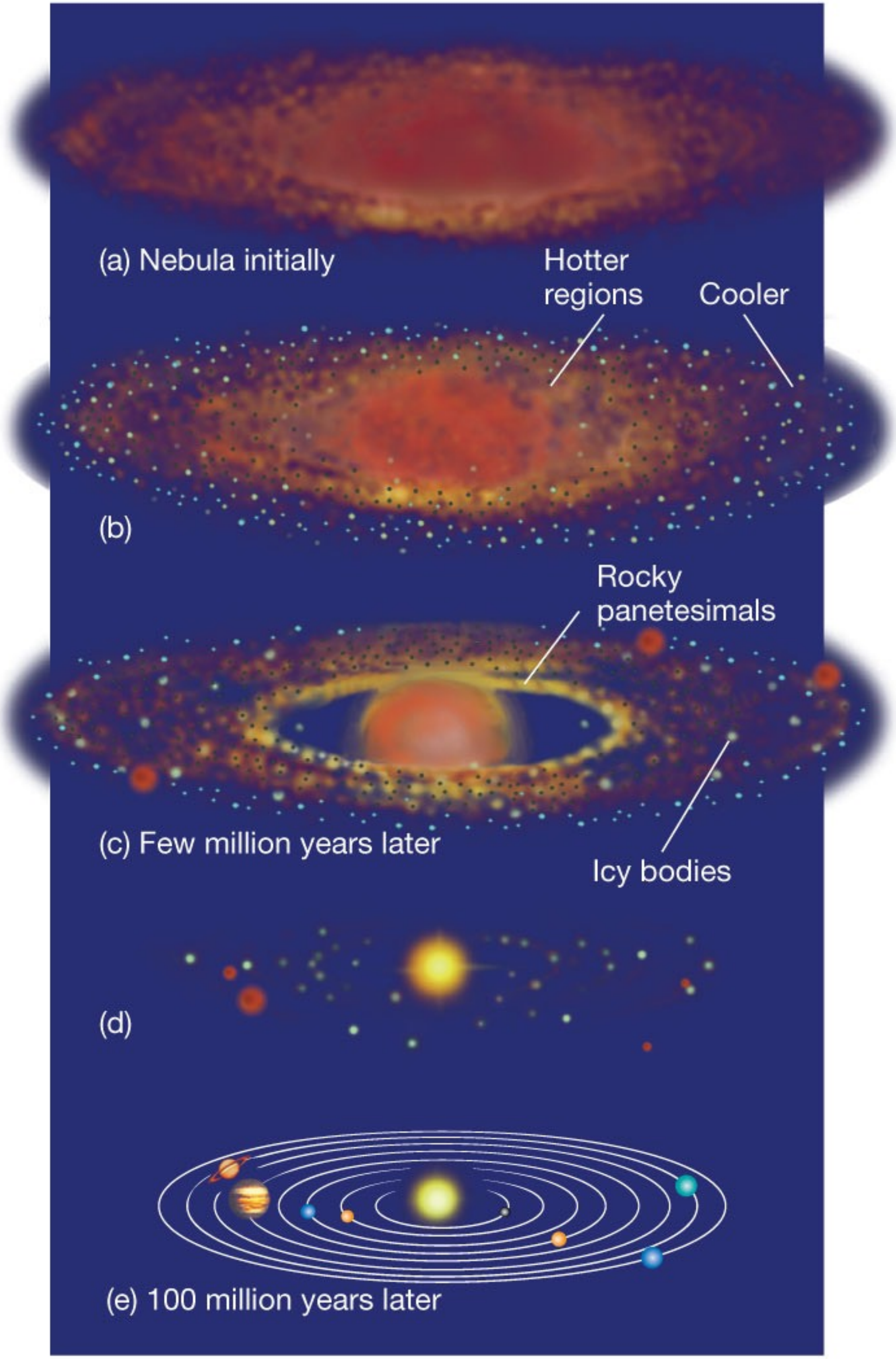
Part III: The Condensation Theory



Gravitational Heating at All Scales

Cloud -> Protostar -> Planetesimals

To understand different types of meteorites, we first need to understand why larger planetesimals tend to melt and differentiate at formation

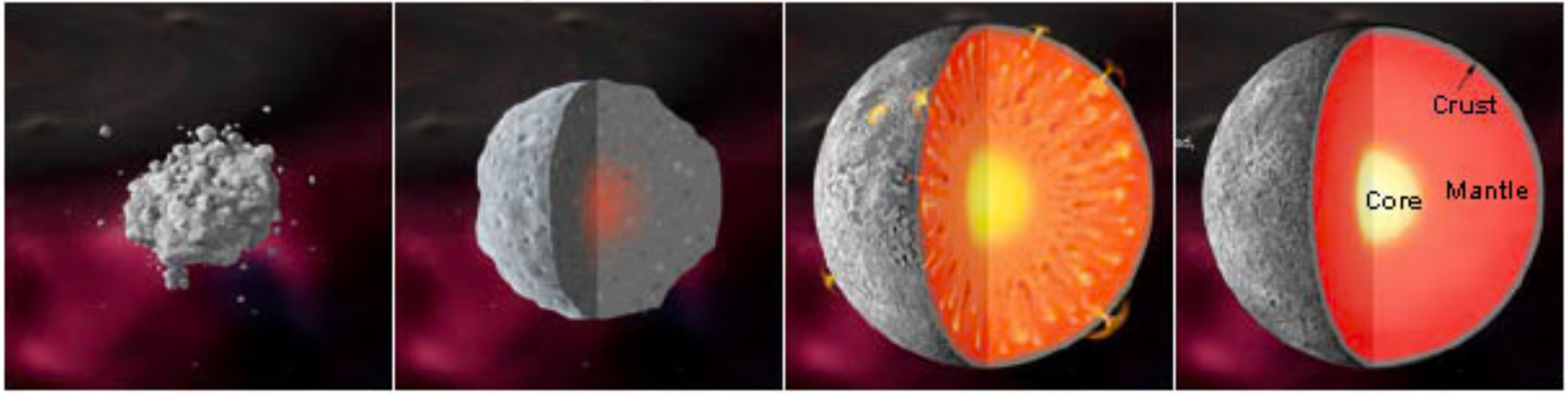
When planets begin to **melt**, the materials in them begin to separate from one another. The heaviest materials, such as metallic iron, sink to form cores. Low-density magmas rise, forming crusts. This process is called **differentiation**.

The key to understand this is to consider the total energy change during **aggregation of solid materials**:

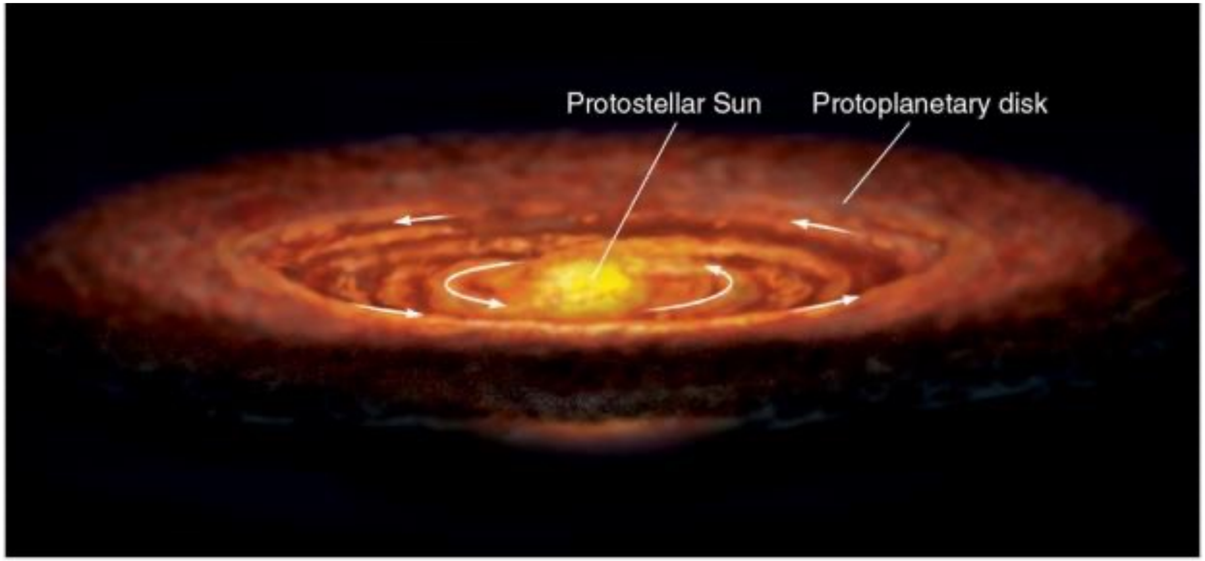
$$E = K + U = U/2 = -GM^2/2R$$

$$\Delta E = \frac{GM^2}{2}\left(\frac{1}{R_p} - \frac{1}{R_c}\right) \approx \frac{GM^2}{2R_p} \Rightarrow \Delta T = \frac{GM^2}{2R_p c_p M} \propto \frac{M}{R_p}$$

A Rocky Body Forms and Differentiates



A Protostar Powered by Gravitational Potential Energy



- **Protostar** forms at the center of a collapsing cloud.
- Its luminosity is not derived from nuclear fusion, but from the gravitational potential energy released during the collapse from R_0 to R_f:

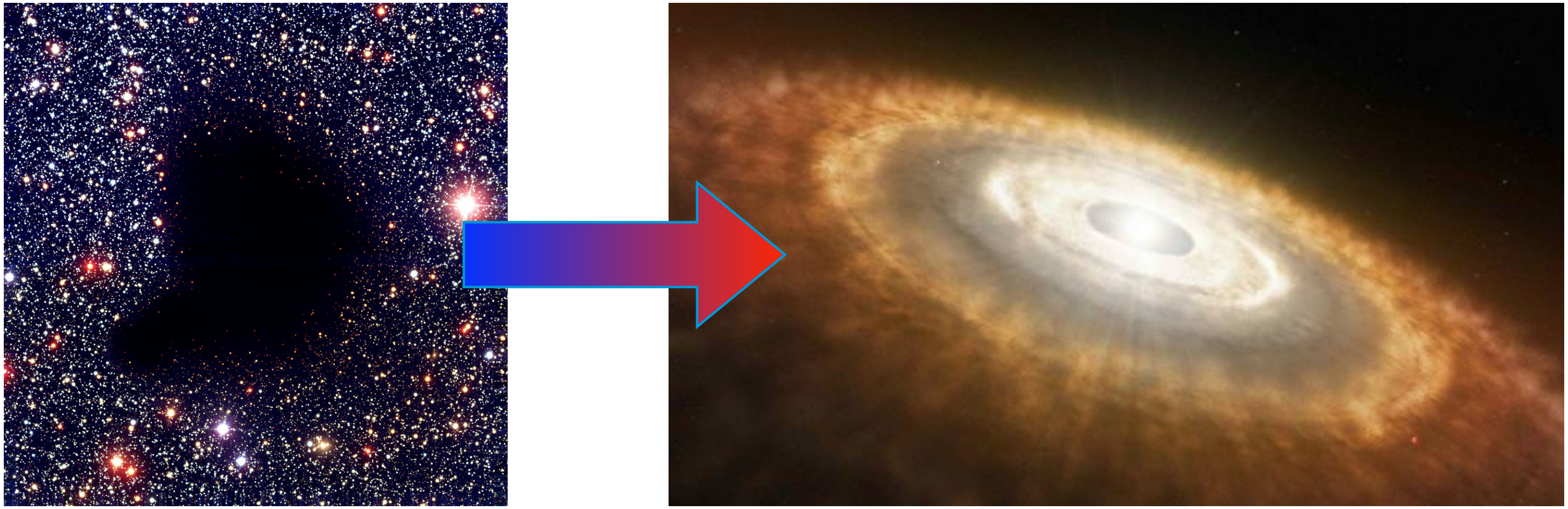
$$L = \frac{\Delta U/2}{\text{lifetime}} = \frac{3}{10}\frac{GM^2}{R_f \cdot \text{lifetime}} \text{ when } R_f << R_0$$

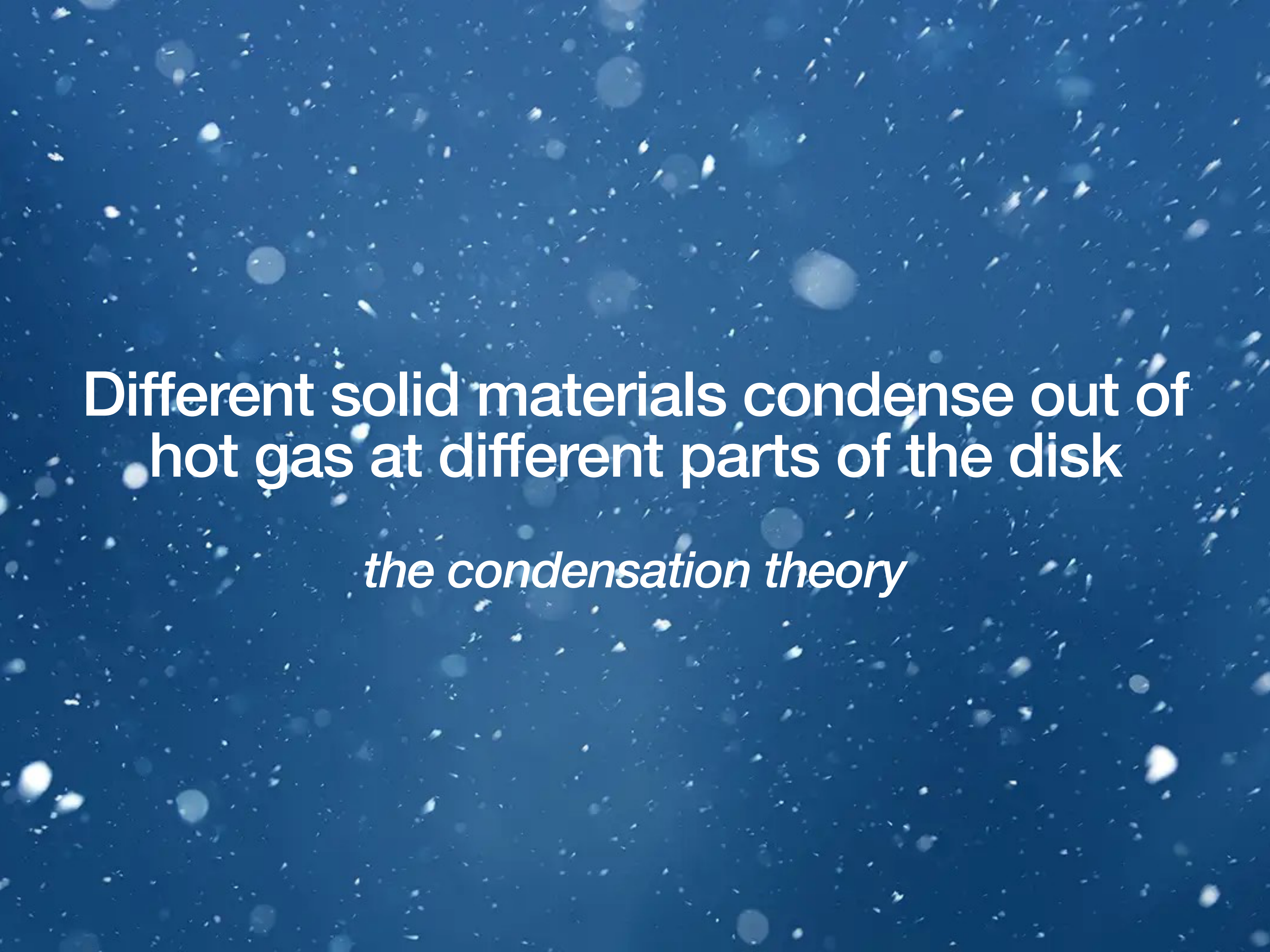
Temperature rises during collapse, vaporizing everything in the cloud

$$U = -\frac{3}{5}\frac{GM^2}{R} \quad \text{====>>>>} \quad K = \frac{3}{2}\frac{M}{\mu m_H}kT$$

As the cloud collapses, **R** decreases, and the gravitational potential energy **U** decreases as it becomes more negative, causing the internal kinetic energy **K** to rise, causing **T** to increase.

As a result, the early Solar nebula would have been heated to as hot as **2000K**, converting most of the elements, even the refractory elements like **W** and **Ti**, to **gas phase**.





Different solid materials condense out of hot gas at different parts of the disk

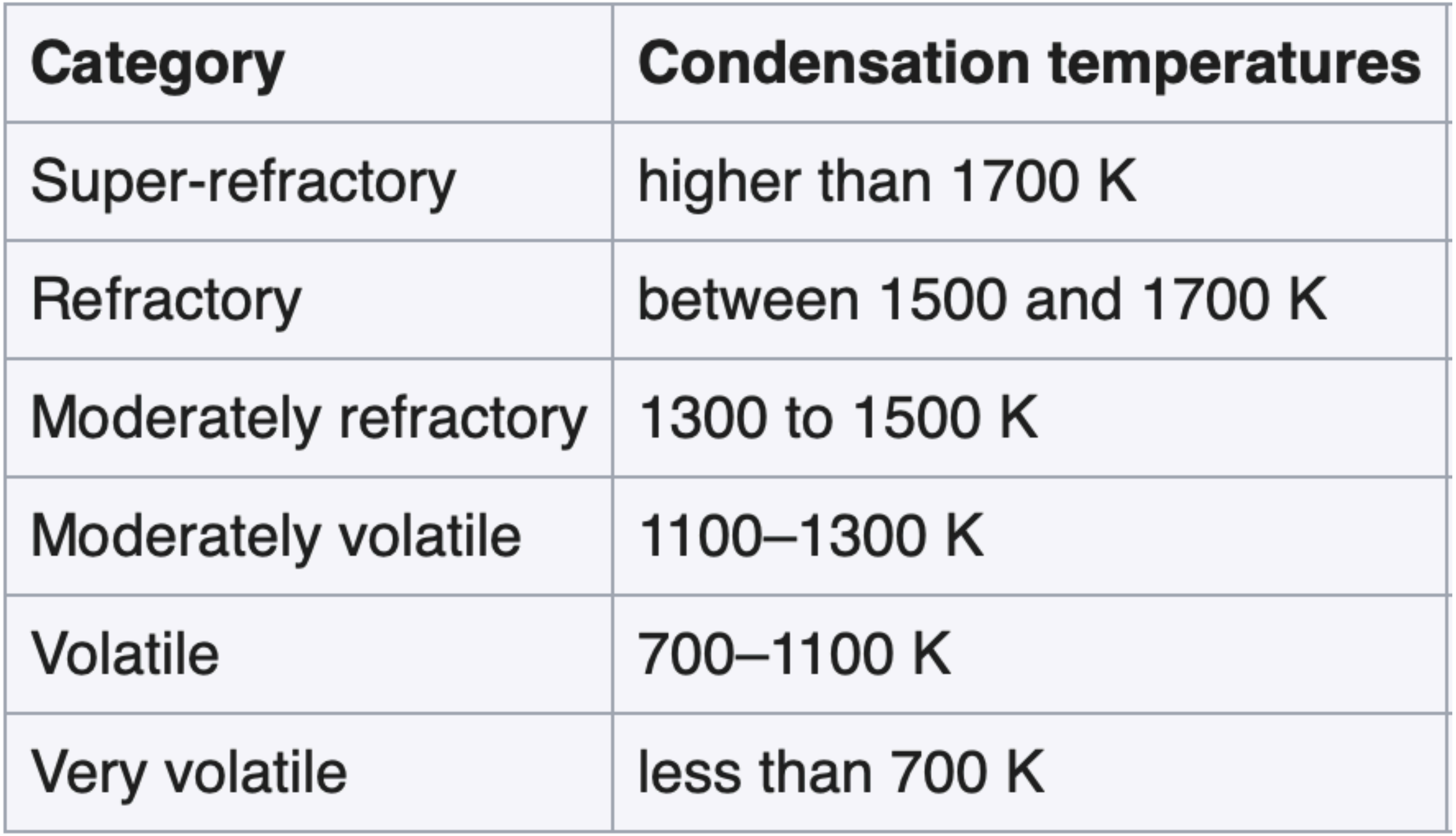
the condensation theory

Cooling of the gas disk causes “snow falls”

The protoplanetary disk **cools radiatively** from **~2000 K** to lower temperatures. Like snows, crystals of minerals begin to **condense** out of the cloud (***refractory first, volatiles last***), forming the initial micron-sized dust particles which then **aggregate** to form larger solid bodies.

Refractory vs. Volatile Elements and Minerals

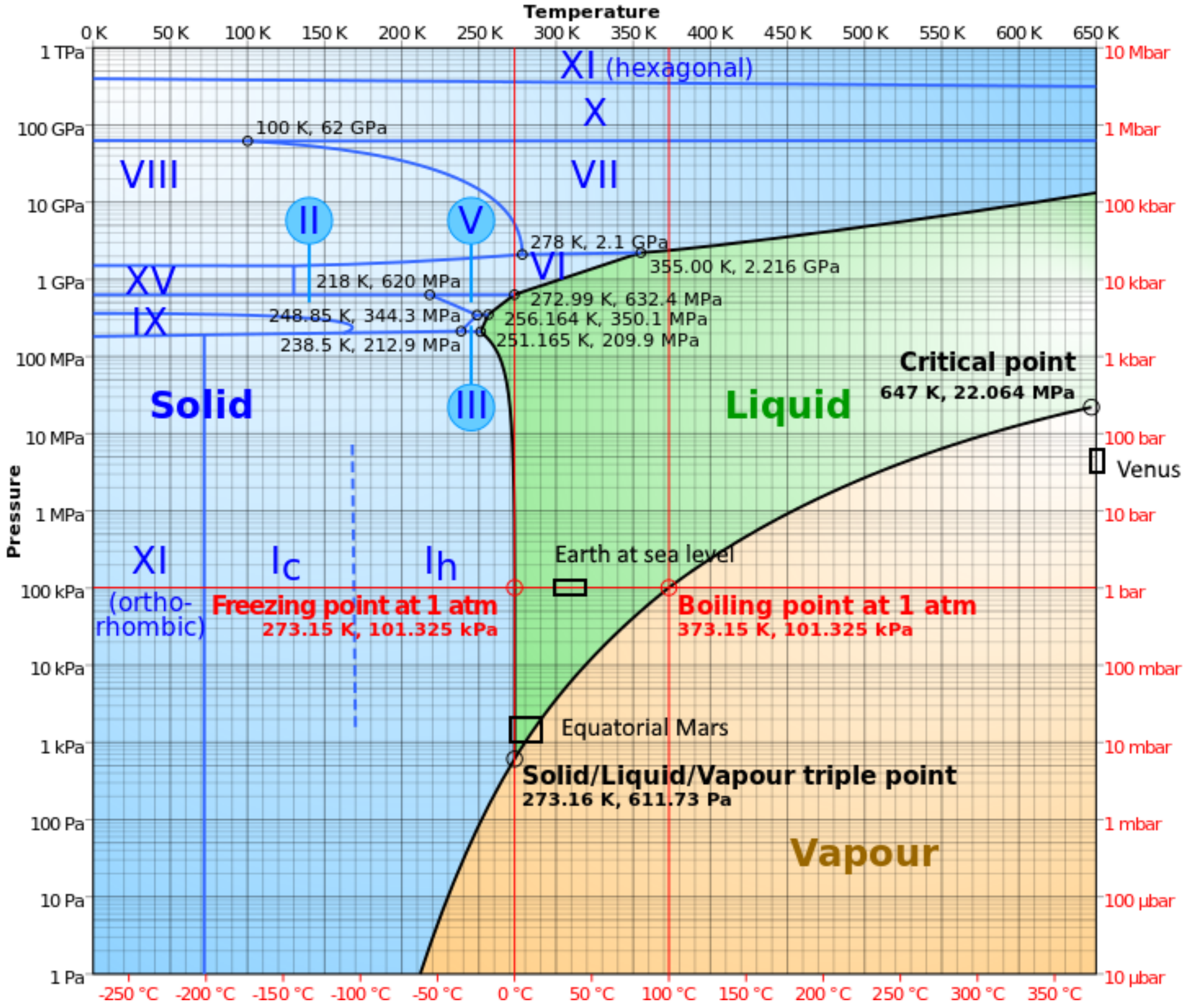
- **Refractory**: high melting/vaporizing temperature
- **Volatile**: low melting/vaporizing temperatures



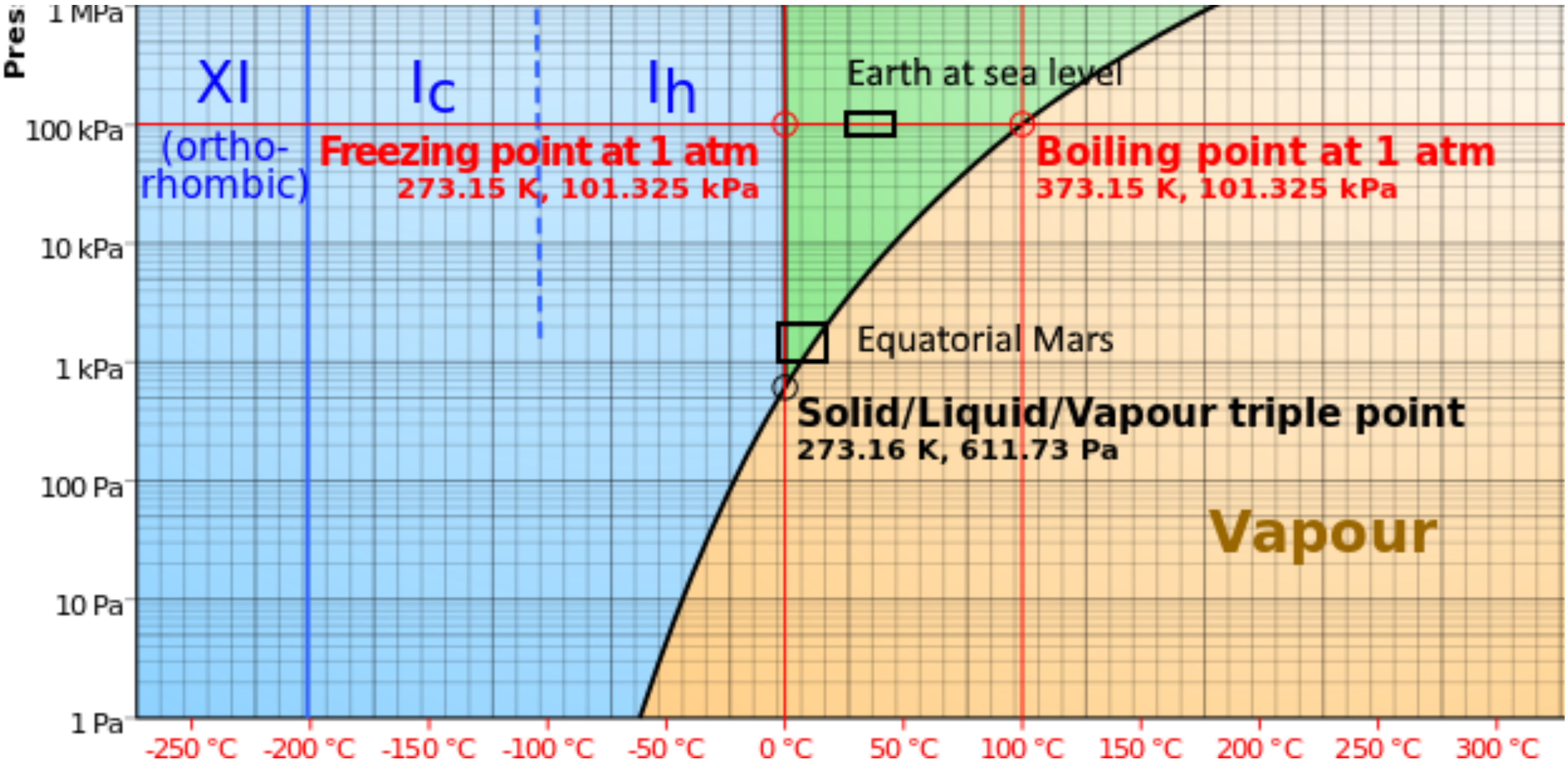
Category	Condensation temperatures
Super-refractory	higher than 1700 K
Refractory	between 1500 and 1700 K
Moderately refractory	1300 to 1500 K
Moderately volatile	1100–1300 K
Volatile	700–1100 K
Very volatile	less than 700 K

NOTE: Temperatures above are for 1 ATM of pressure condensation T decreases quickly with pressure

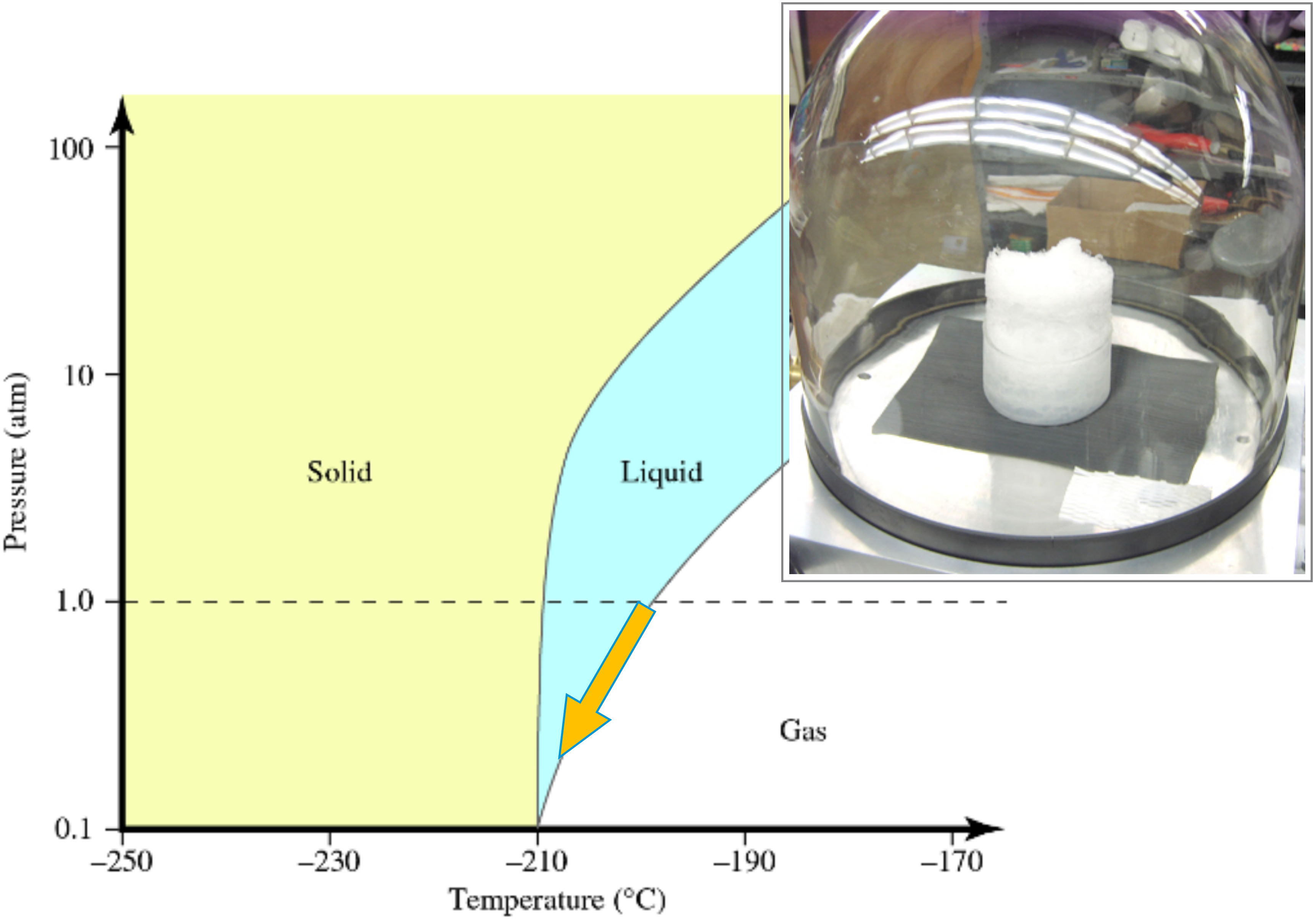
Phase Diagram of H_2O - phase in the plane of Pressure & Temperature



Below the triple point, condensation temperature decreases with pressure



Freezing Liquid Nitrogen by Decreasing Pressure



Example Refractory Minerals

Spinel - $MgAl_2O_4$

perovskite - $CaTiO_3$

Example Refractory Minerals

Olivine - Fe_2SiO_4 and Mg_2SiO_4

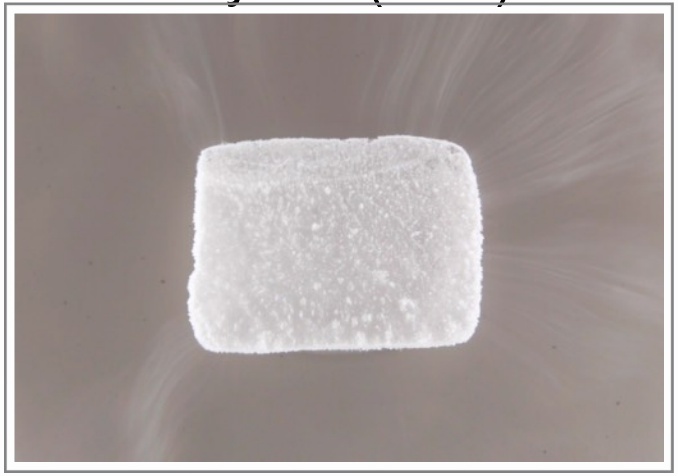
The green sand is created by a common mineral in Big Island lava called olivine, which stays deposited on this beach because it is heavier than the other components of the lava.

Example Volatile Minerals

methane (CH_4) hydrates

Most of the deposits are buried in or below **permafrost** and sediments in the ocean bottom along **continental margins**

Dry Ice (CO_2)



Water Ice (H_2O)

The Condensation Process (Minerals)

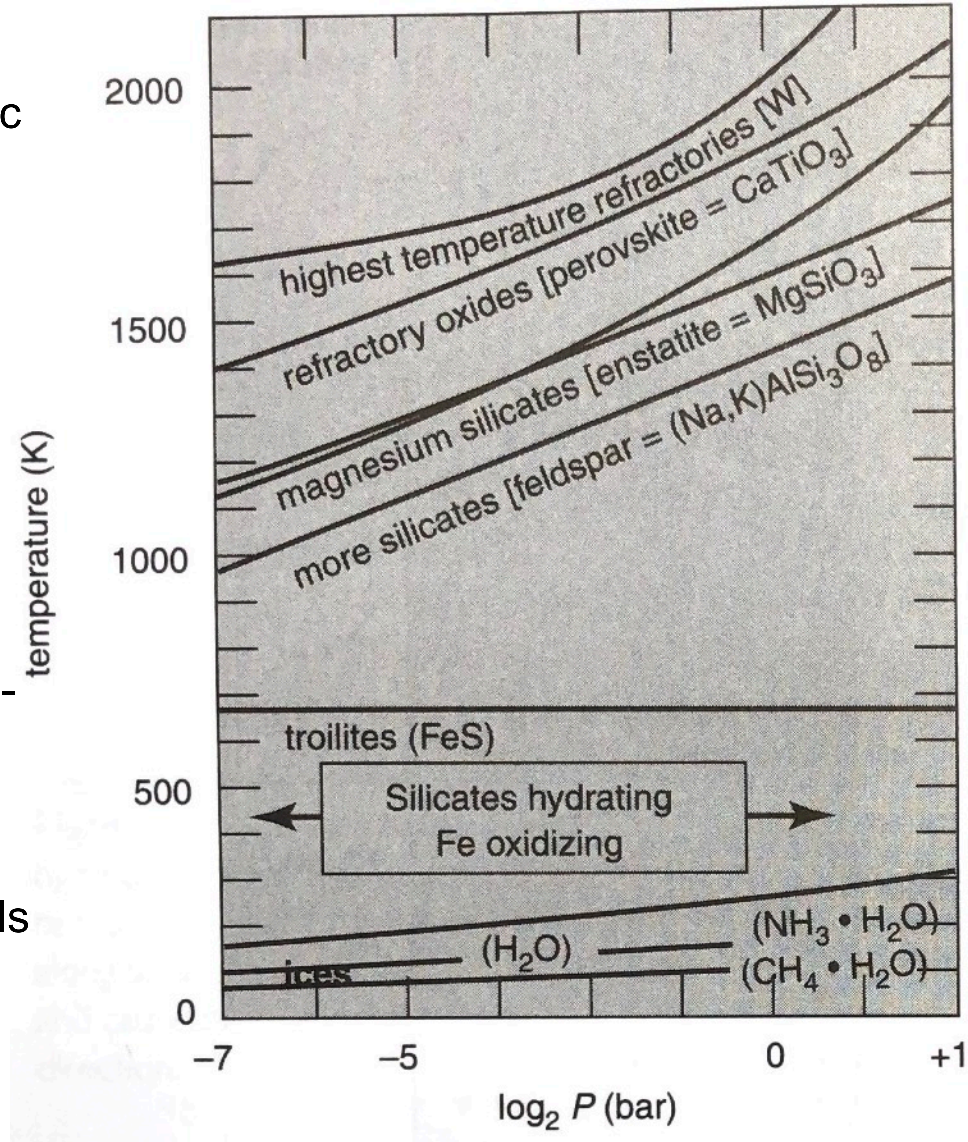
As elements condense out of the solar cocoon nebula, they form solid microscopic mineral grains in the following sequence:

1. **High-temperature refractories** (e.g., Tungsten [W], spinel [$MgAl_2O_4$], and perovskite [$CaTiO_3$])

2. **Nickel-Iron alloy, and Silicates** (e.g., olivine [Fe_2SiO_4 and Mg_2SiO_4]).

3. **Carbonaceous condensates** (graphite-like material mixed with organic molecules and silicates)

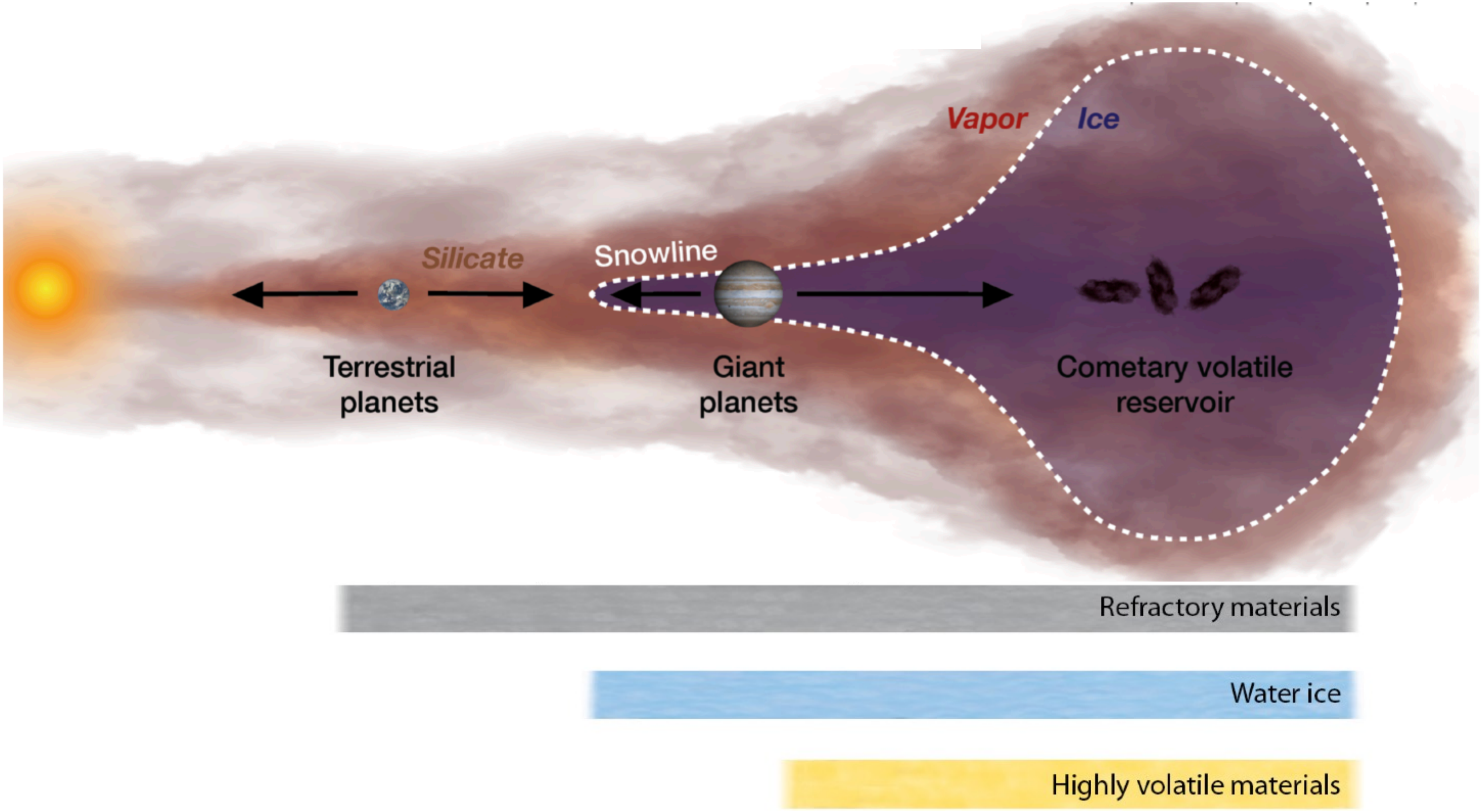
4. **Ices** (500 > T > 200 K, hydrated minerals [serpentine - hydrated olivine]; T < 200 K, ices of water, ammonia, methane, and hydrated ices; **T < 170 K, surface ice**)



Hartmann05 Fig 5-5: Condensation of minerals

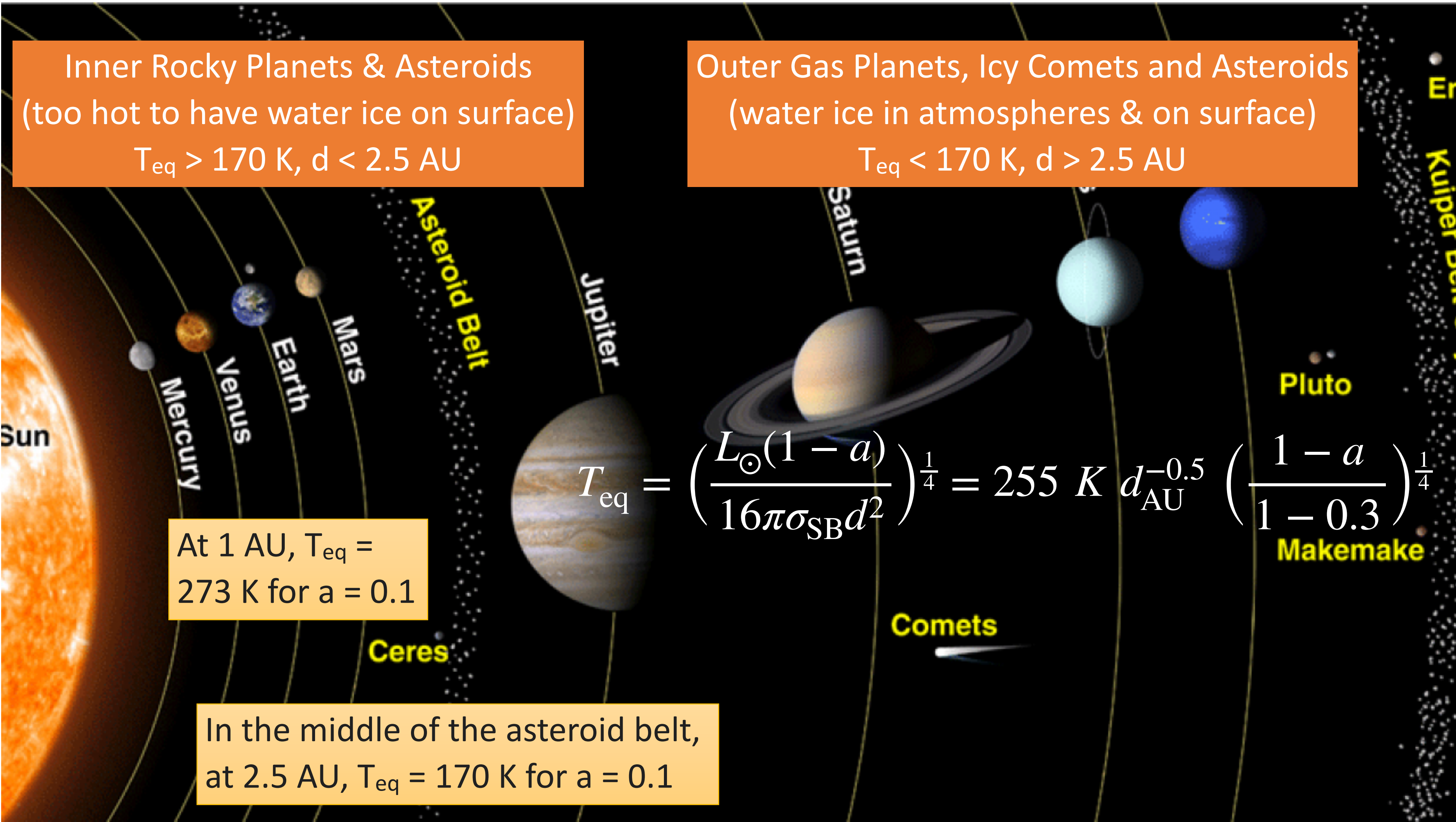
Due to the temperature gradient in the disk, the inner disk never reached the condensation temperature of volatile minerals *before the gas was blown out*

Pontoppidan, et al. 2019



Snow Line of the Solar System: The distance at which T_{eq} = -100 °C = 170 K

Note: The condensation temperature of H_2O in vacuum is 100 °C colder than that at 1 ATM of pressure (0 °C)



Inner Rocky Planets & Asteroids
(too hot to have water ice on surface)
$T_{eq} > 170$ K, $d < 2.5$ AU

Outer Gas Planets, Icy Comets and Asteroids
(water ice in atmospheres & on surface)
$T_{eq} < 170$ K, $d > 2.5$ AU

$$T_{eq} = \left(\frac{L_{\odot}(1-a)}{16\pi\sigma_{SB}d^2}\right)^{\frac{1}{4}} = 255\ K\ d_{AU}^{-0.5}\left(\frac{1-a}{1-0.3}\right)^{\frac{1}{4}}$$

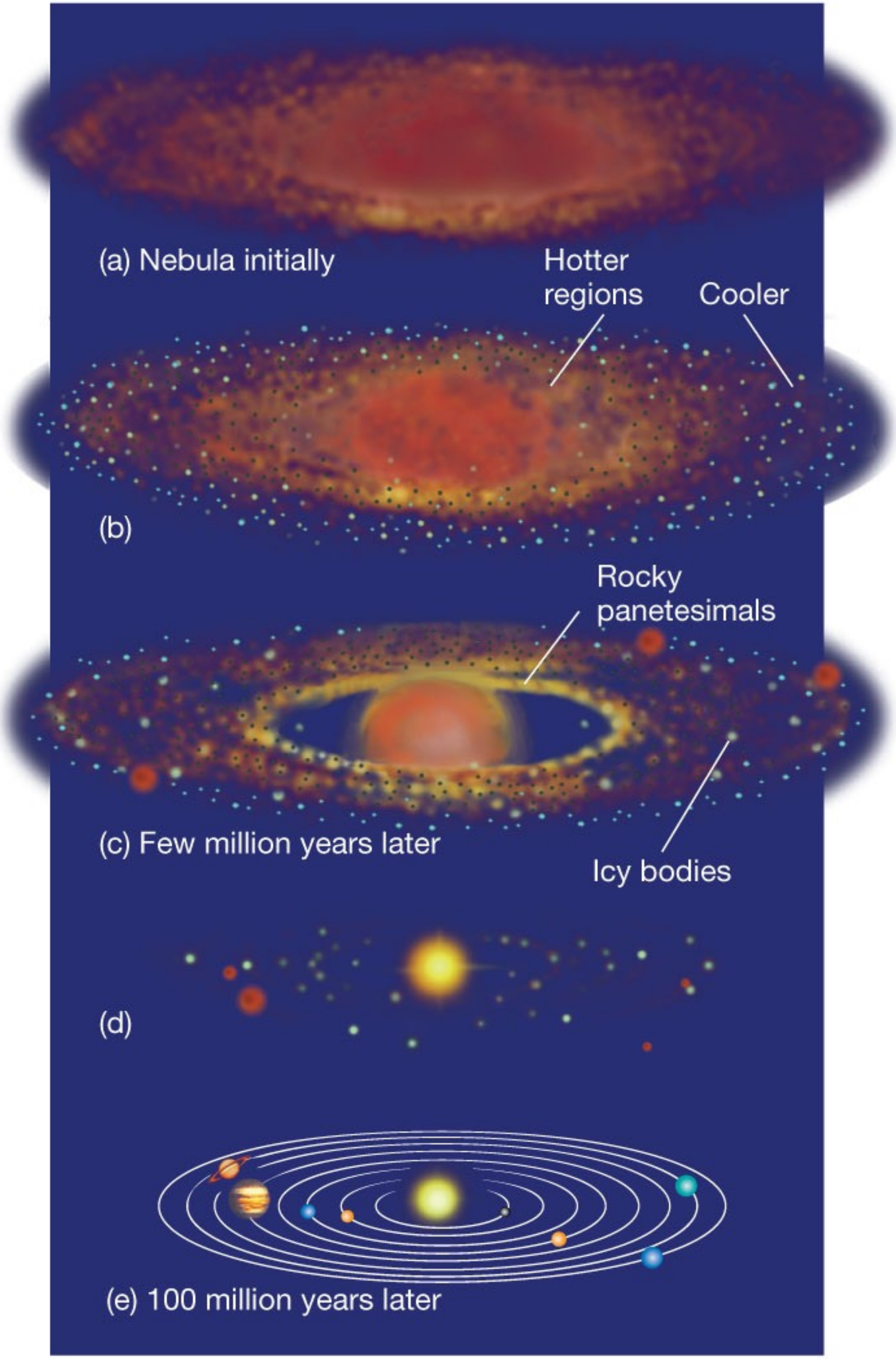
At 1 AU, T_{eq} = 273 K for a = 0.1

In the middle of the asteroid belt, at 2.5 AU, T_{eq} = 170 K for a = 0.1

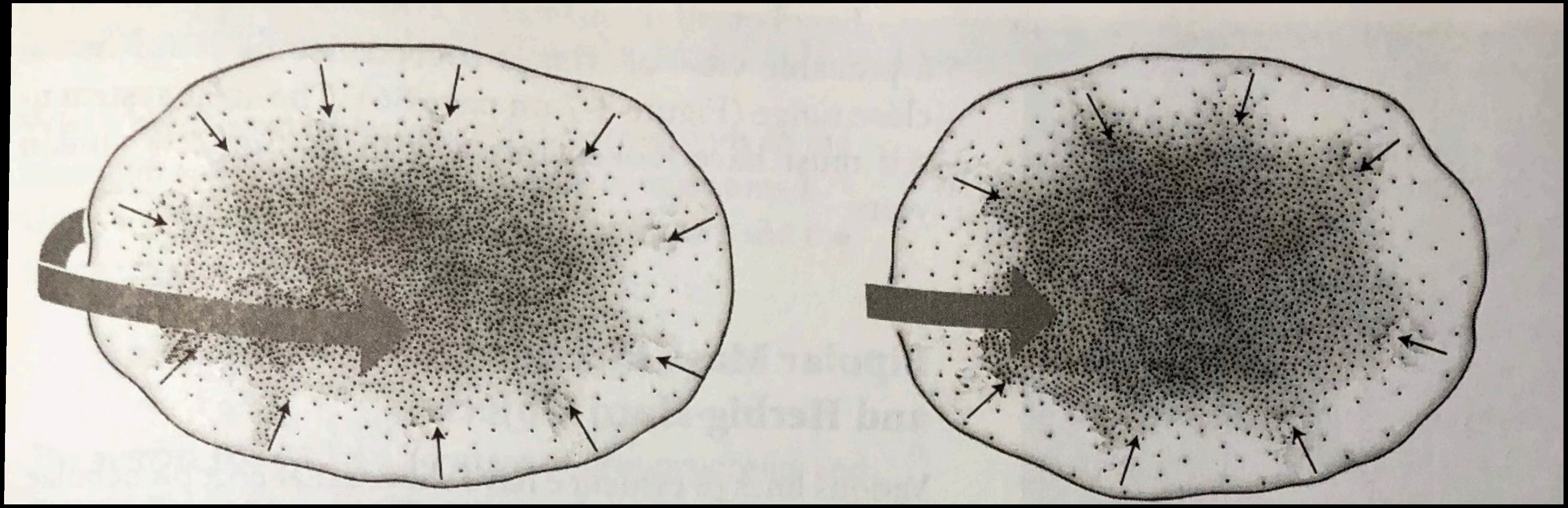
Interior to the snow line, water can hide in rocks (e.g., Serpentinite)

$Mg_3(Si_2O_5)(OH)_4$ or $(Mg^{2+}, Fe^{2+})_3Si_2O_5(OH)_4$

Summary

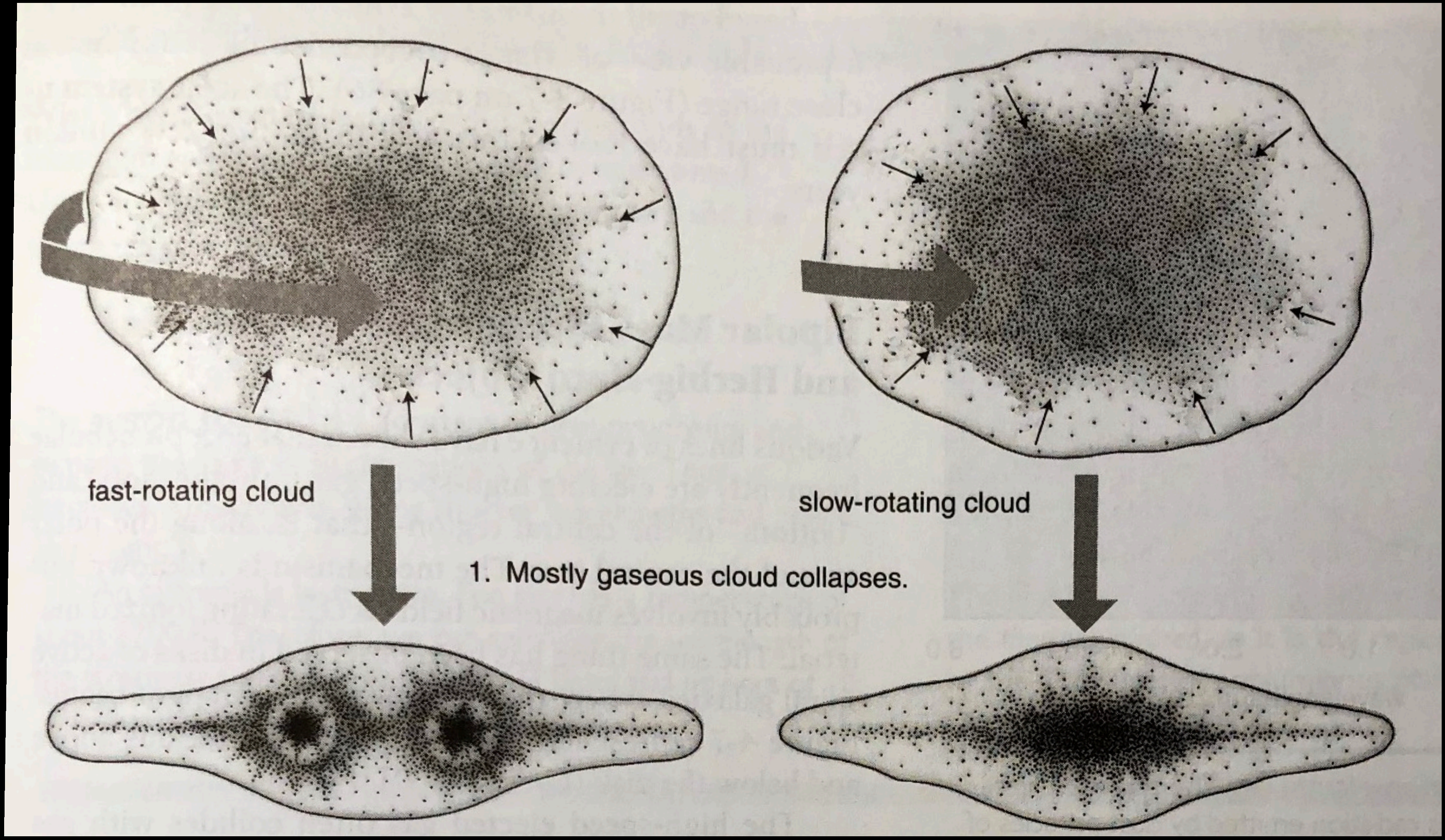

(a) Nebula initially
Hotter regions
Cooler
(b)
Rocky panetesimals
(c) Few million years later
Icy bodies
(d)
(e) 100 million years later

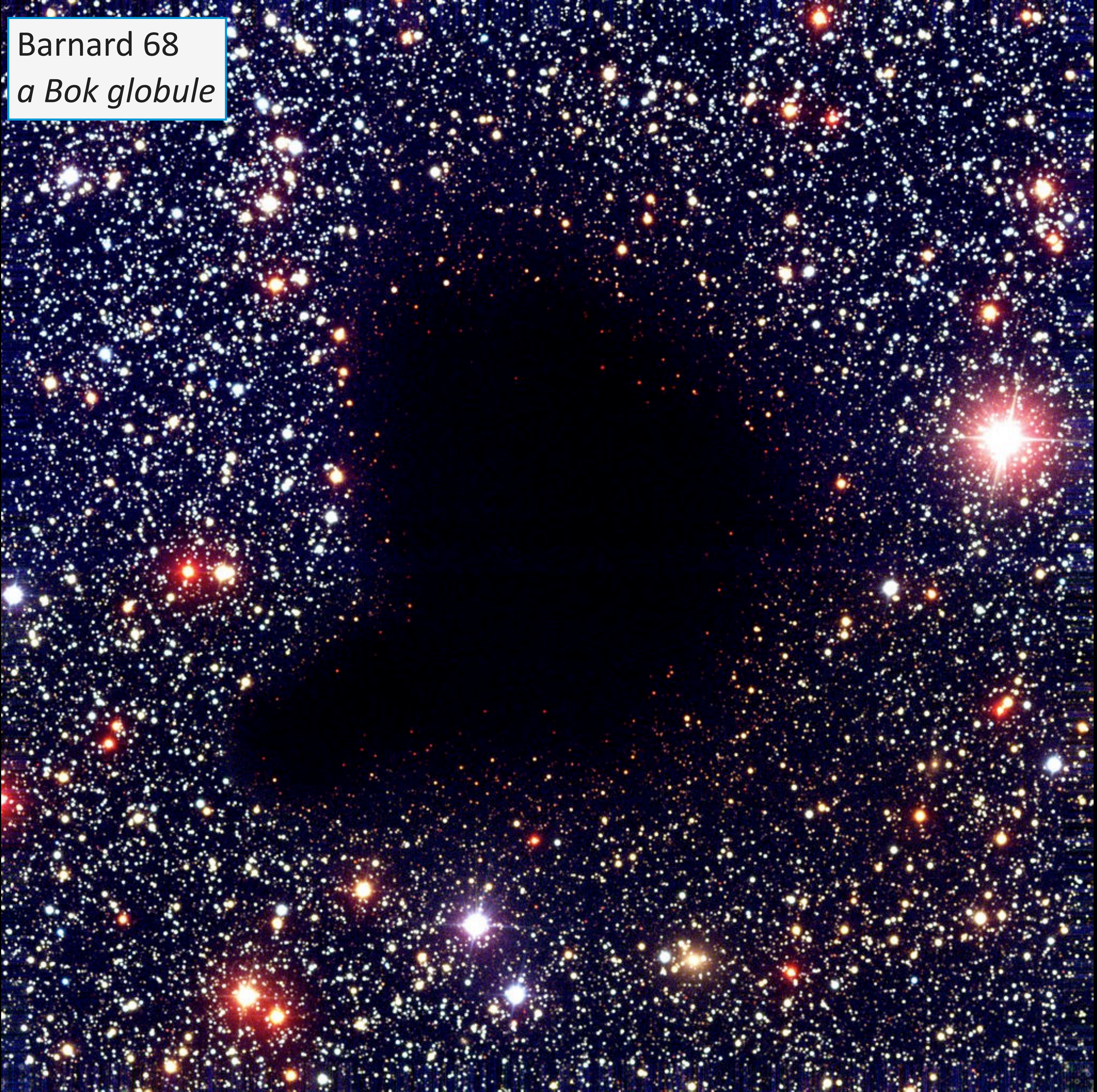
Stage 1: From molecular cloud to dense cores



JWST’s near-to-mid-IR image of the Cosmic Cliffs
(the West section of the NGC 3324 star-forming region, 2.8 kpc away)

Stage 2: From dense cores to a disk-shaped cocoon nebula
(the newly formed proto-star is obscured by dust)



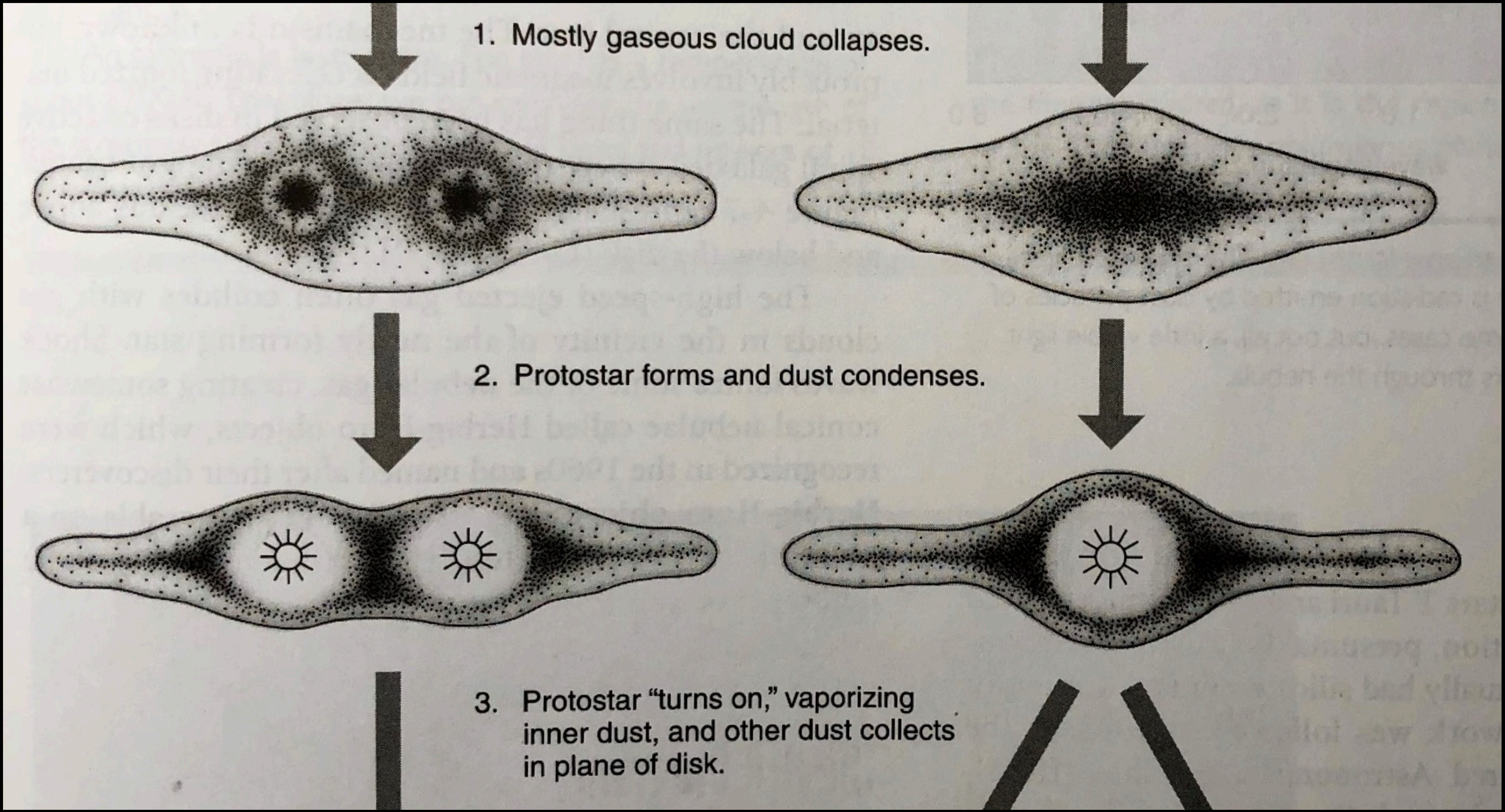

Barnard 68

a Bok globule

newly formed stars (proto-stars) are likely to be obscured by this envelope of dust and gas, coined the cocoon nebula.

Kant's nebular hypothesis in 1755 states that the early Sun was inside such a cocoon nebula

Stage 3: From disk-shaped cocoon nebula to protoplanetary disk (the central proto-star becomes visible because of dust aggregation and stellar winds)

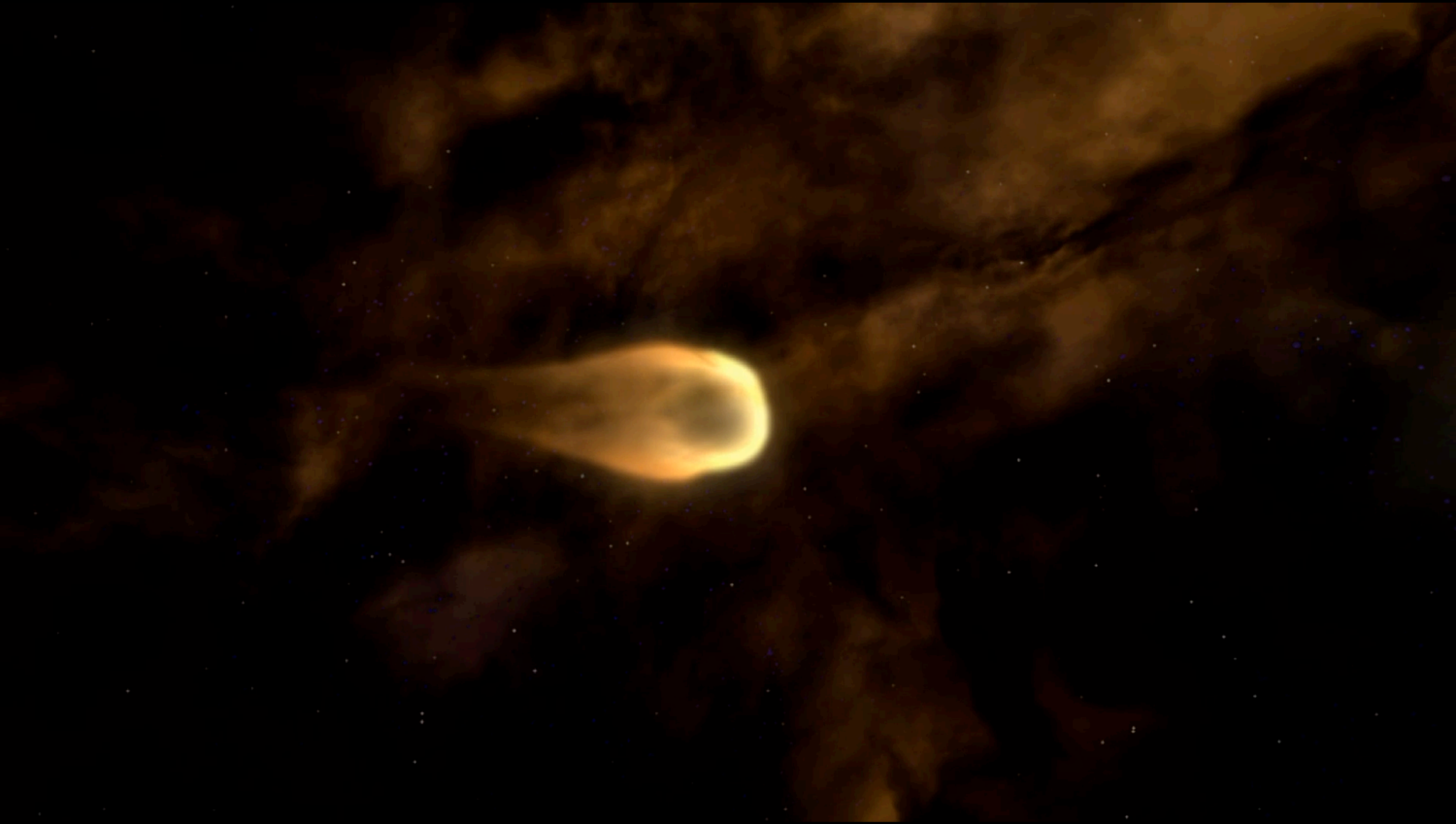


1. Mostly gaseous cloud collapses.
2. Protostar forms and dust condenses.
3. Protostar “turns on,” vaporizing inner dust, and other dust collects in plane of disk.

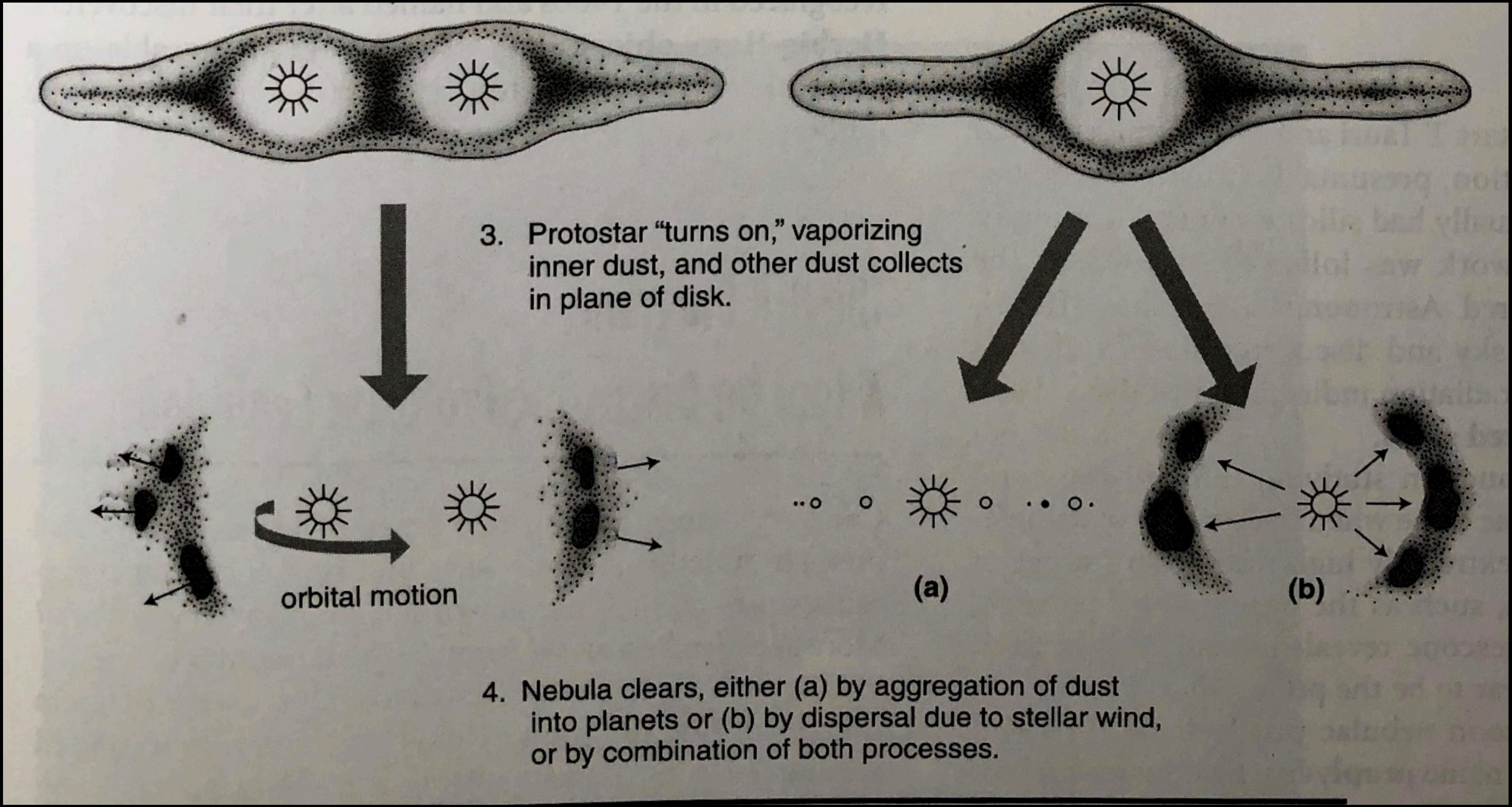
Proplyds (disk-shaped, dense protoplanetary clouds) in the Orion nebula

Proplyds stands for PROtoPLanetarY DiskS, typical diameter ~ 100s AU

3D animation of a Proplyd



Stage 4: From ***residual*** protoplanetary disk to planets (aggregation of µm-sized dust to km-sized planetesimals)

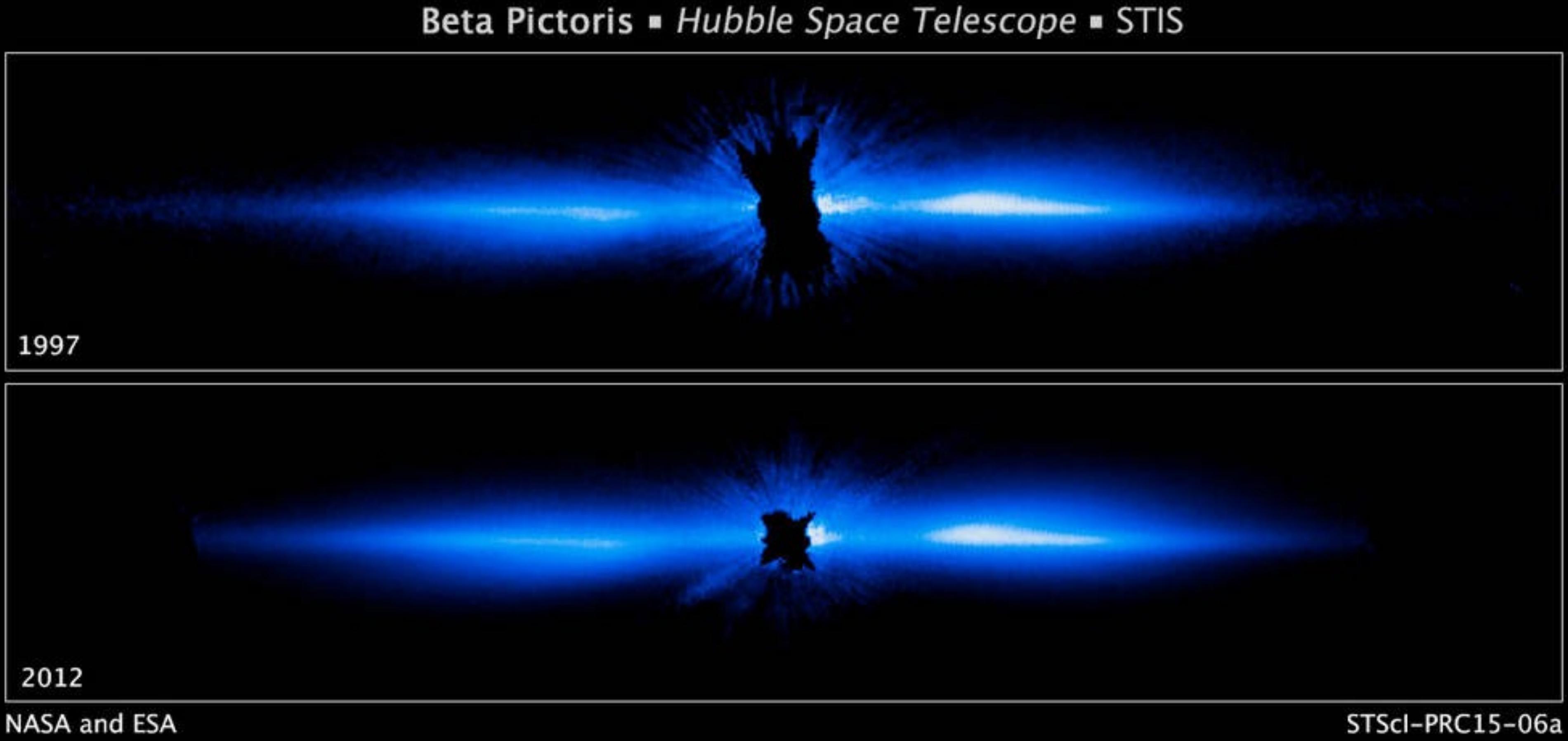


3. Protostar "turns on," vaporizing inner dust, and other dust collects in plane of disk.

4. Nebula clears, either (a) by aggregation of dust into planets or (b) by dispersal due to stellar wind, or by combination of both processes.

β Pictoris appears to be a normal star

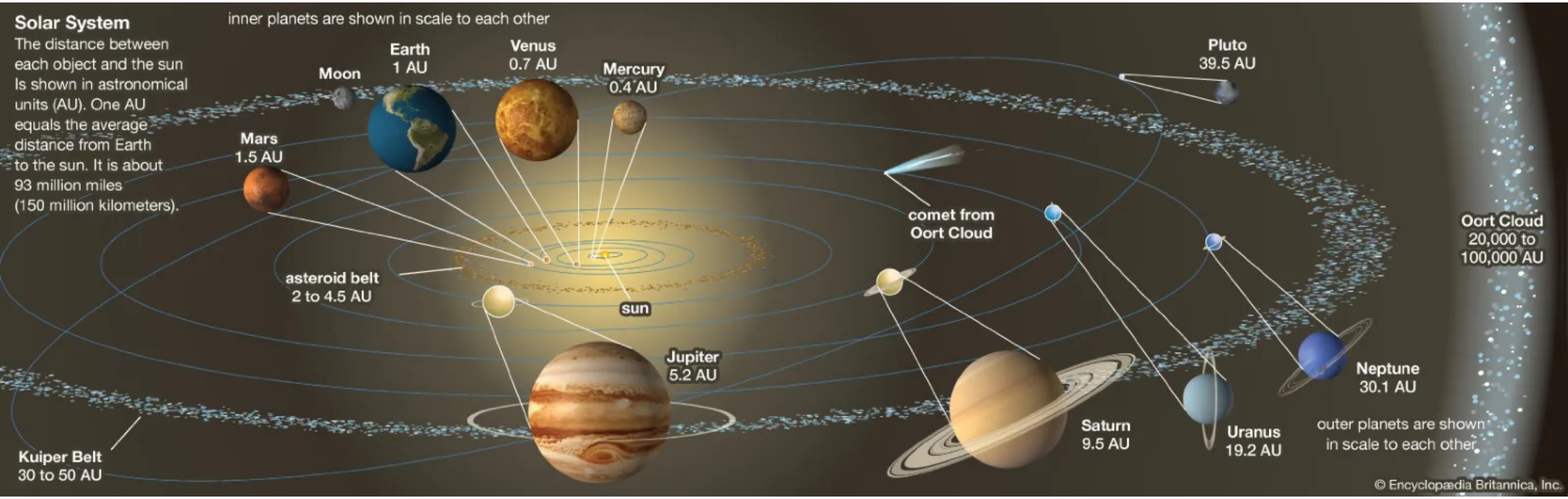
But it is surrounded by a thin disk of dust, comets, and asteroids
(reaching 400 AU from the star)



Part IV: More Loose Ends / Open Questions

Burning Questions #1 & #2

- *The protoplanetary disk contains ~50% of the total mass, why planets today contain only 0.14% of the mass?*
- *Why no giant planets formed in the inner solar system?*



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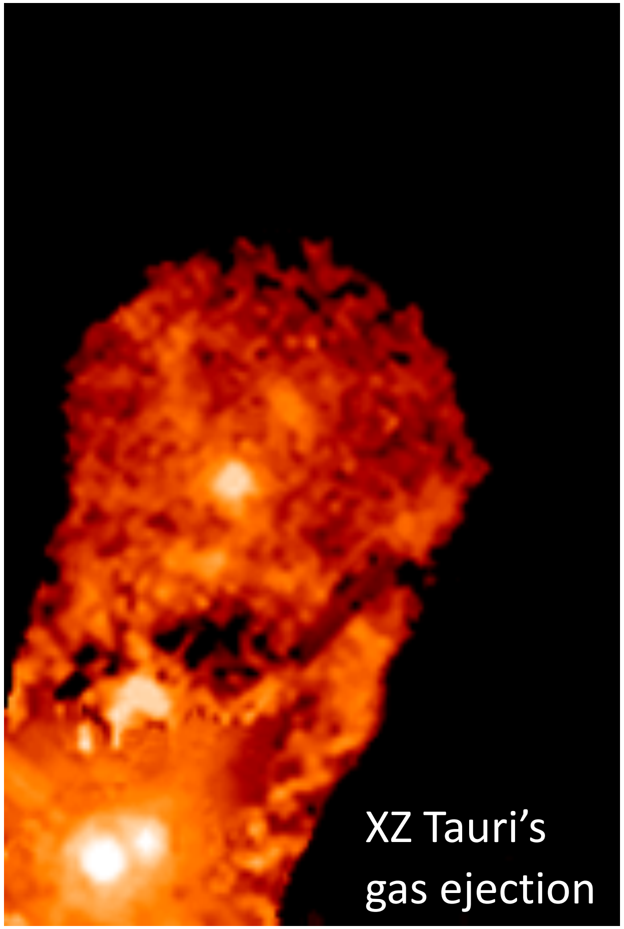
Before all minerals could condense at all distances, gas is swept away by stellar wind

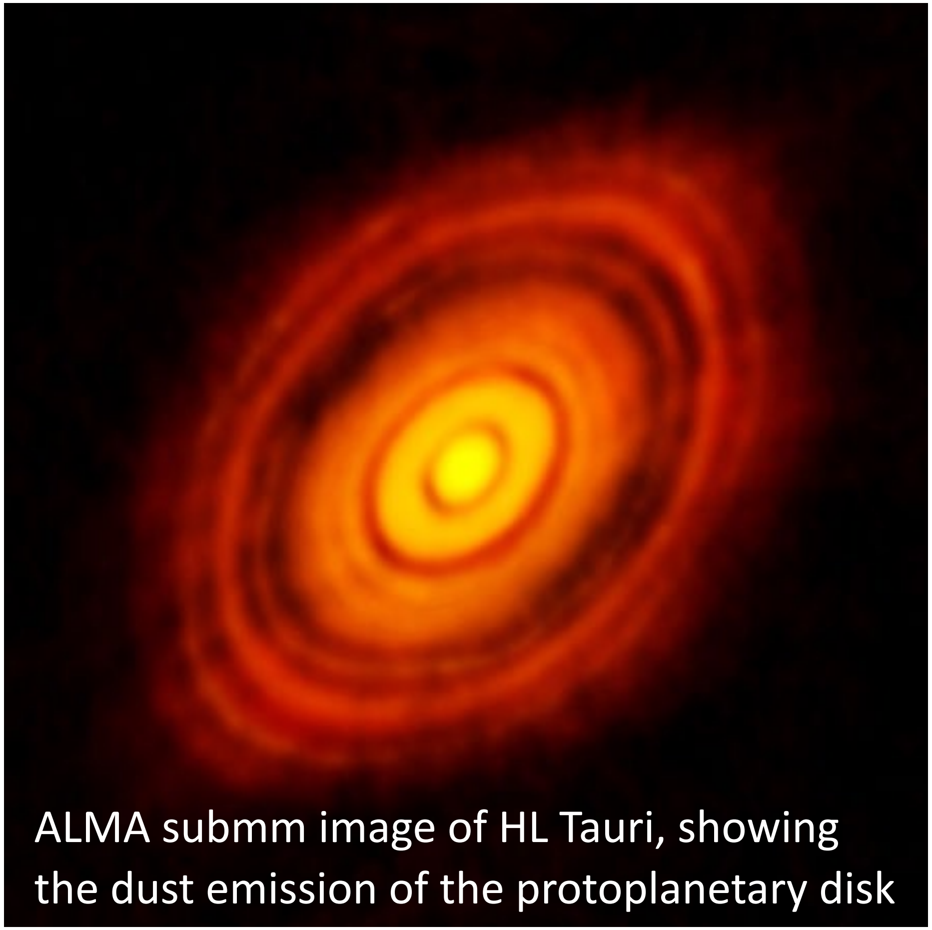
Freezing the condensation outcome in time

The Removal of Gas from the Disk by the Protostar

- During the proto-star phase, the gas in the solar nebula (~0.3 M_{sun}) is swept away by powerful stellar winds, leaving only the dust behind (~0.03 M_{sun})
- The composition of the remaining dust would be a function of distance to the Sun, because the inner disk is much hotter than the outer disk

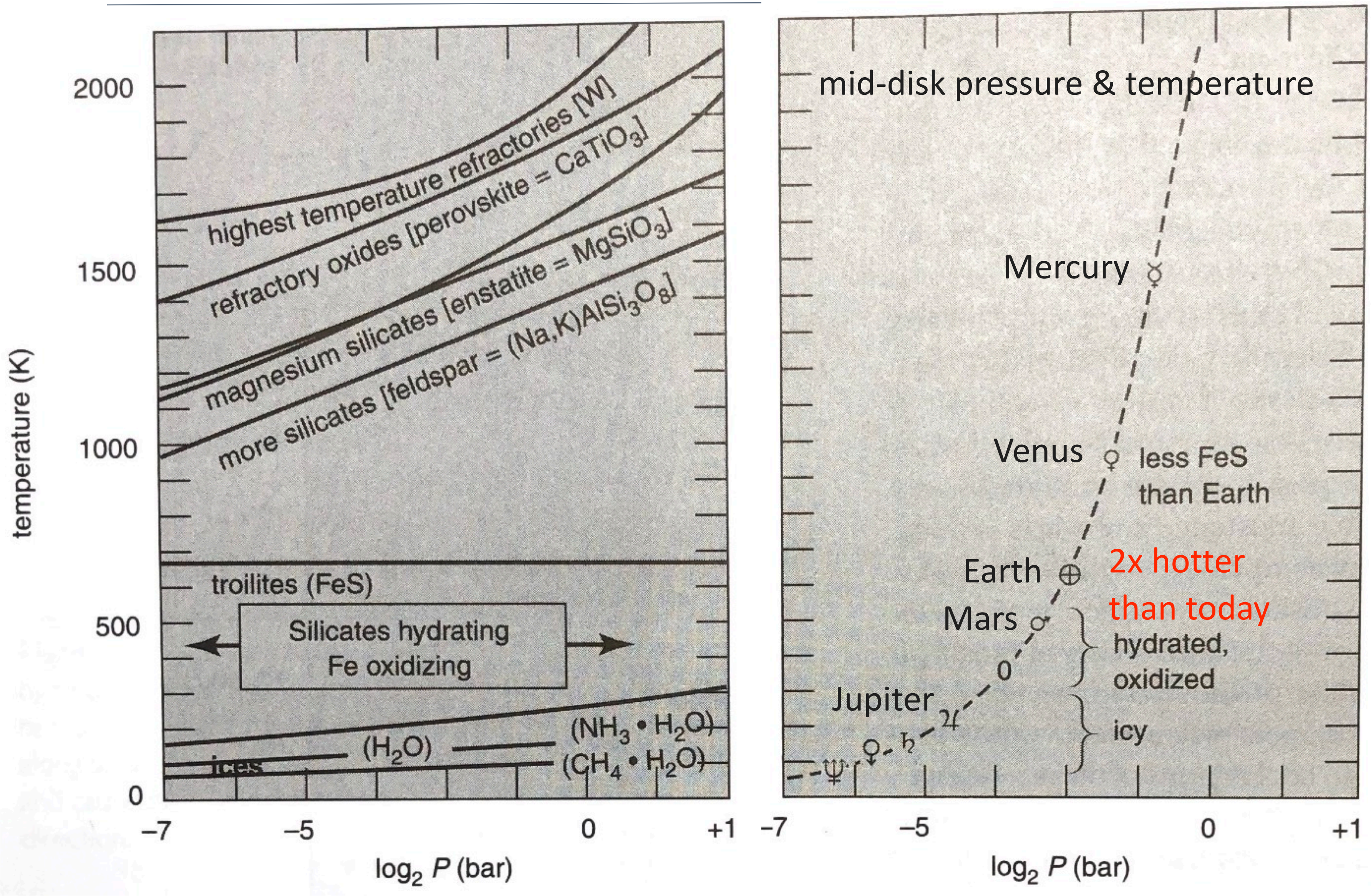


XZ Tauri's gas ejection



ALMA submm image of HL Tauri, showing the dust emission of the protoplanetary disk

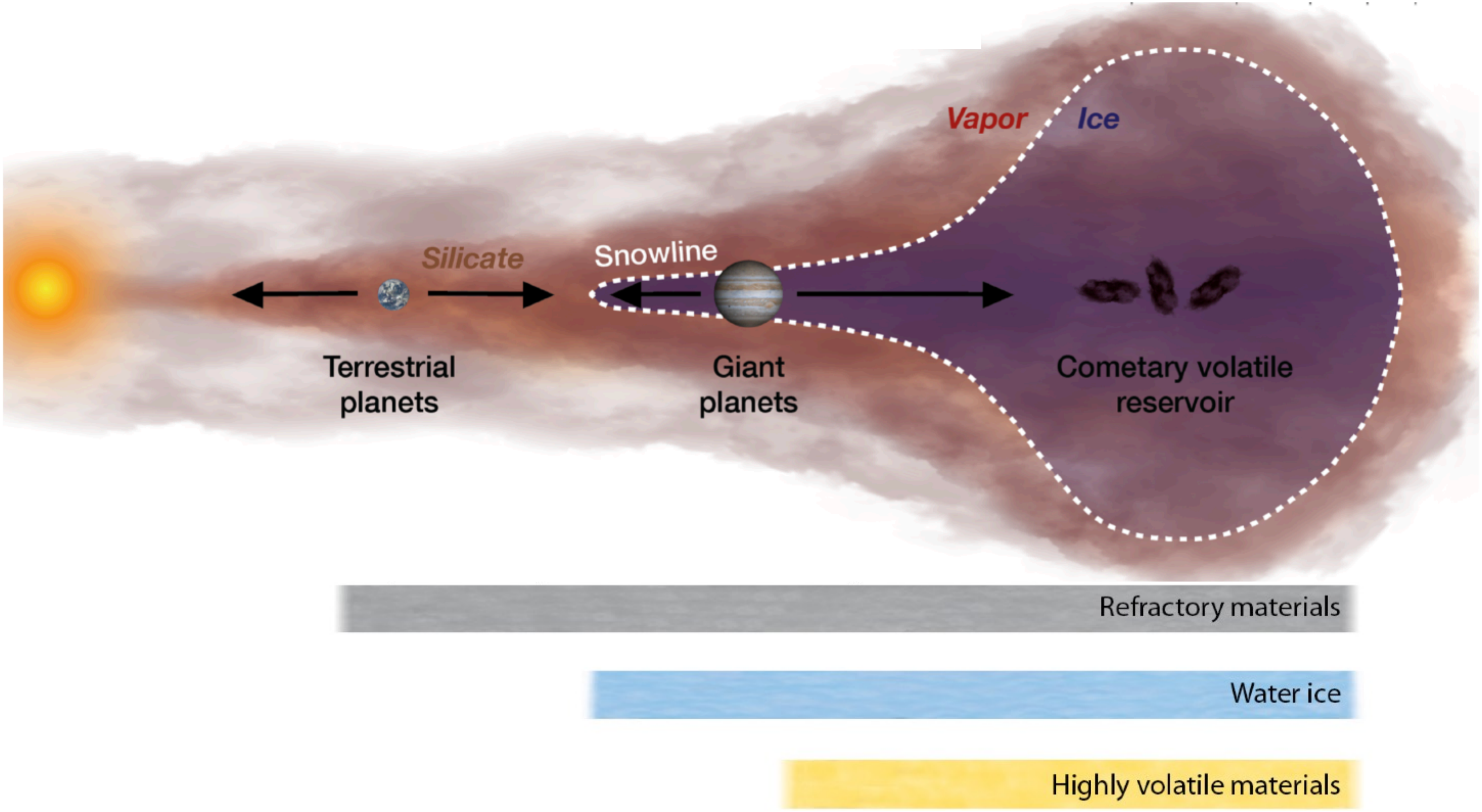
A model of Temperature Gradient + Condensation + Stellar Wind can explain the distance-dependency of the chemical composition



Hartmann05 Fig 5-5: Condensation of minerals

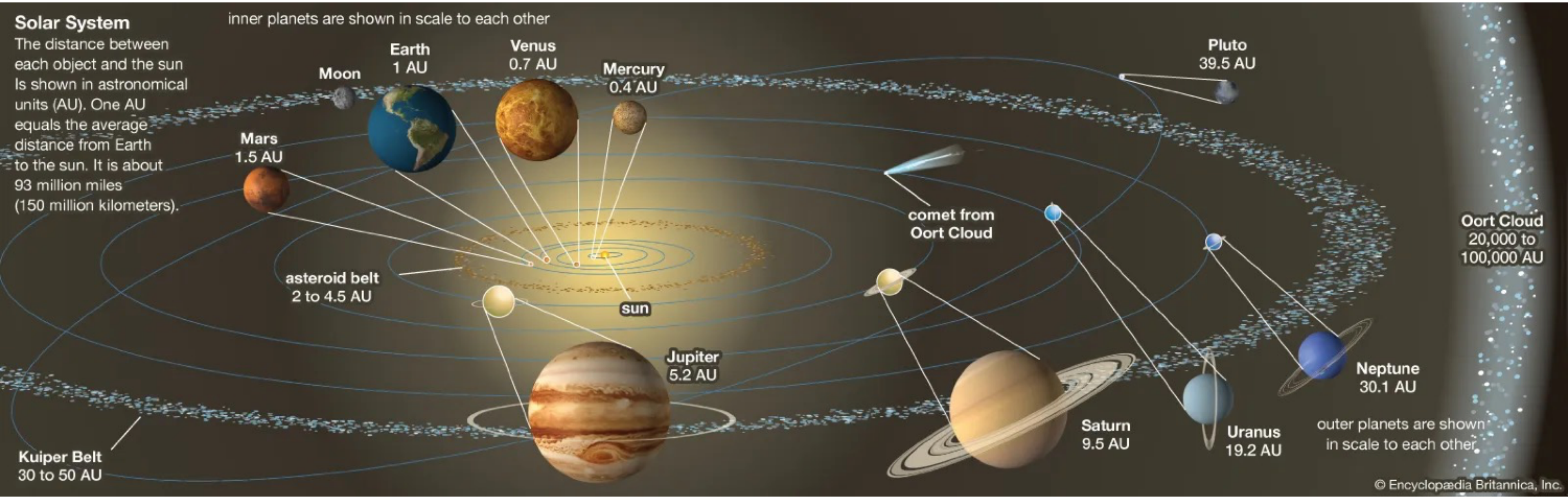
By removing most of the gas from the disk when T_{disk} ~ 650 K at 1 AU, the condensation outcome of the early temperature gradient is locked in the disk

Pontoppidan, et al. 2019



Burning Question #3

- ***Why the Sun's angular momentum is so small compared to that of the planets?***



The birth of proto-stars

non-conservation of angular momentum

Given angular momentum conservation, how fast do we expect the Sun to rotate?

Given an initial size of **20,000 AU** (Jeans Length) and a mild rotation velocity of **0.1 km/s** (comparable to sound speed)**,** can we estimate its rotation velocity when it collapses to the size of the Sun (r_{sun} **= 1/215 AU = 695,700 km**)?

Again, the angular momentum should be conserved between t and t=0:

$$L_{\text{spin}} = \frac{2}{5} M R^2 \omega \quad \text{where} \quad \omega = \frac{2\pi}{P}$$

$$R_0^2 \omega_0 = R_t^2 \omega_t \Rightarrow v_t r_t = v_0 R_0$$

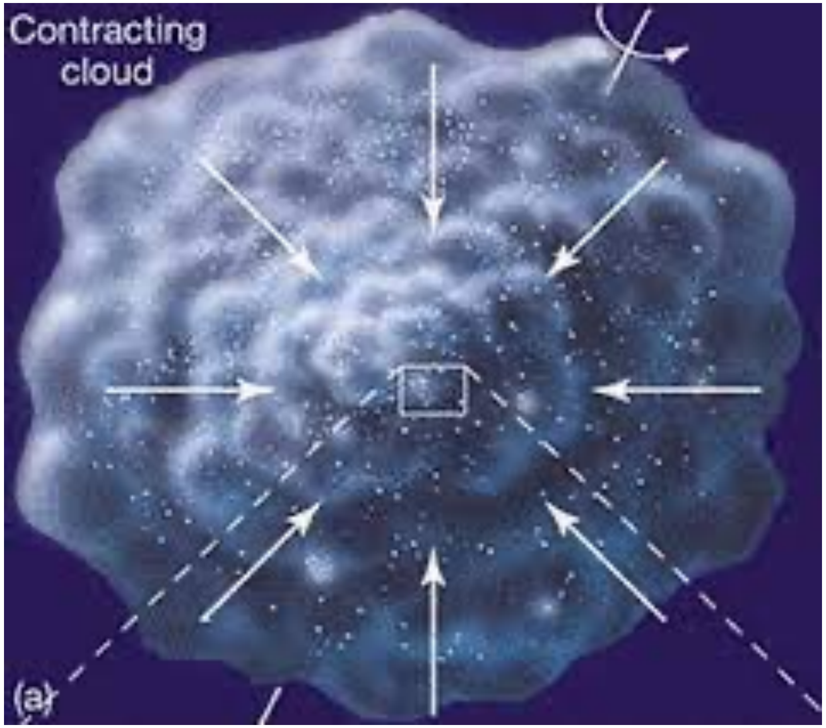
$$\Rightarrow v_t = v_0 R_0 / R_t$$

$$v_t = v_0 R_J / r_\odot = 4 \times 10^5 \text{ km/s}$$

(greater than speed of light)

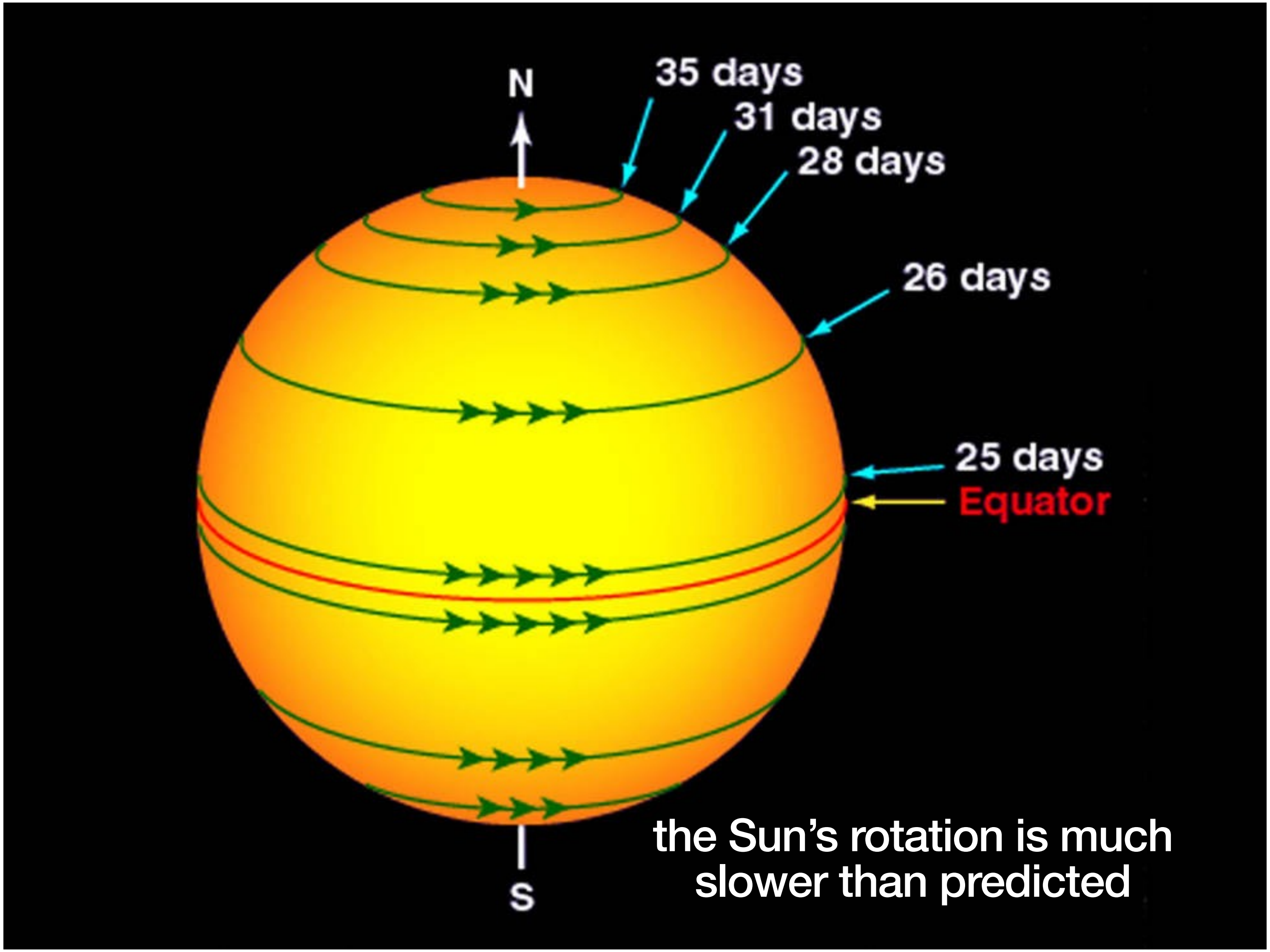
$$\Rightarrow P = \frac{2\pi r_\odot}{v_\odot} = 11 \text{ seconds}$$

(actual spin period is 25 days)



The Sun’s Rotation Periods: 25-35 days, depending on latitude

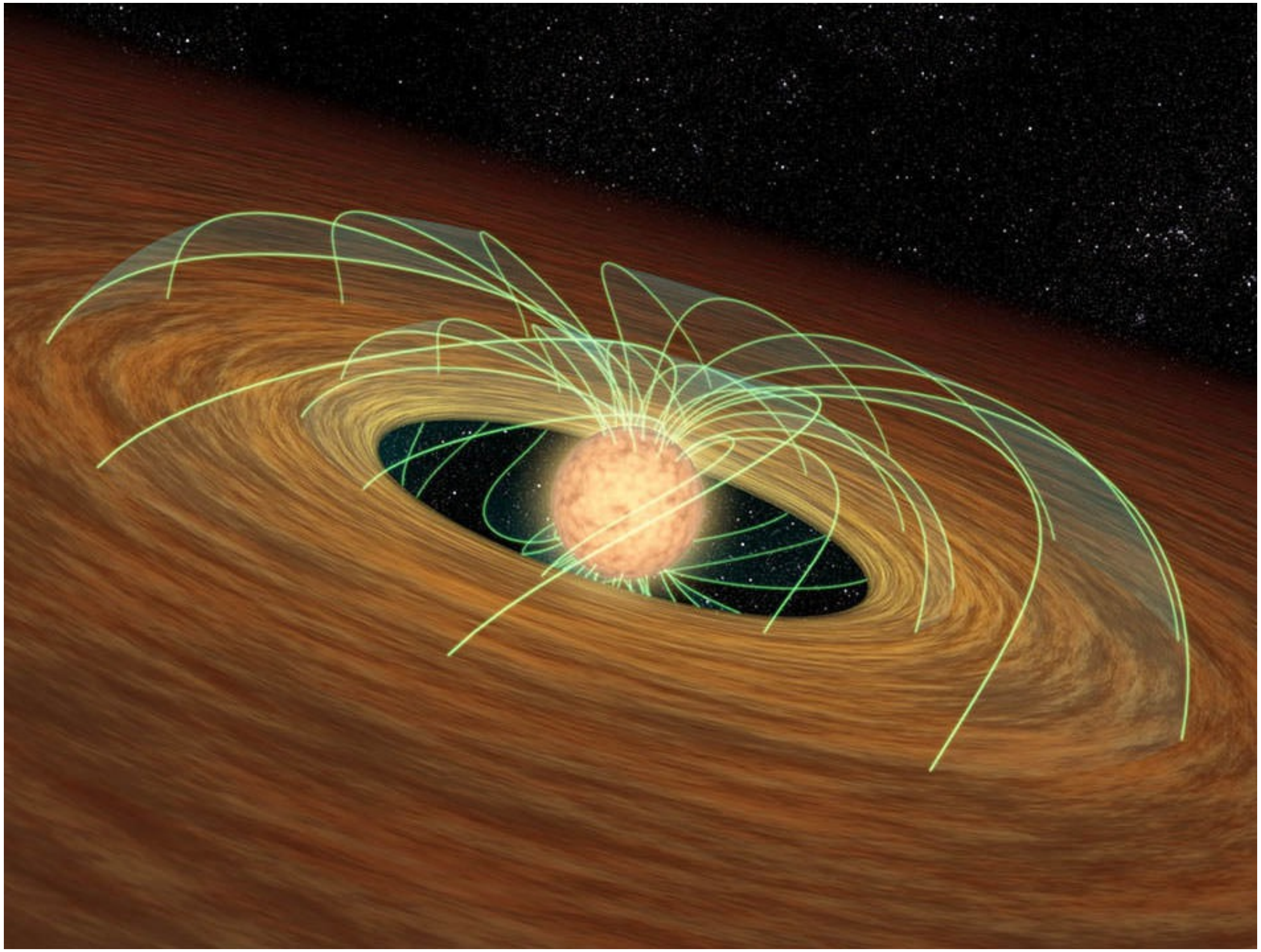
What’s the rotation velocity on the equator? $v_\odot = \frac{2\pi r_\odot}{P} = 2\ \mathrm{km/s}\ \text{for}\ P = 25\ \text{days}$



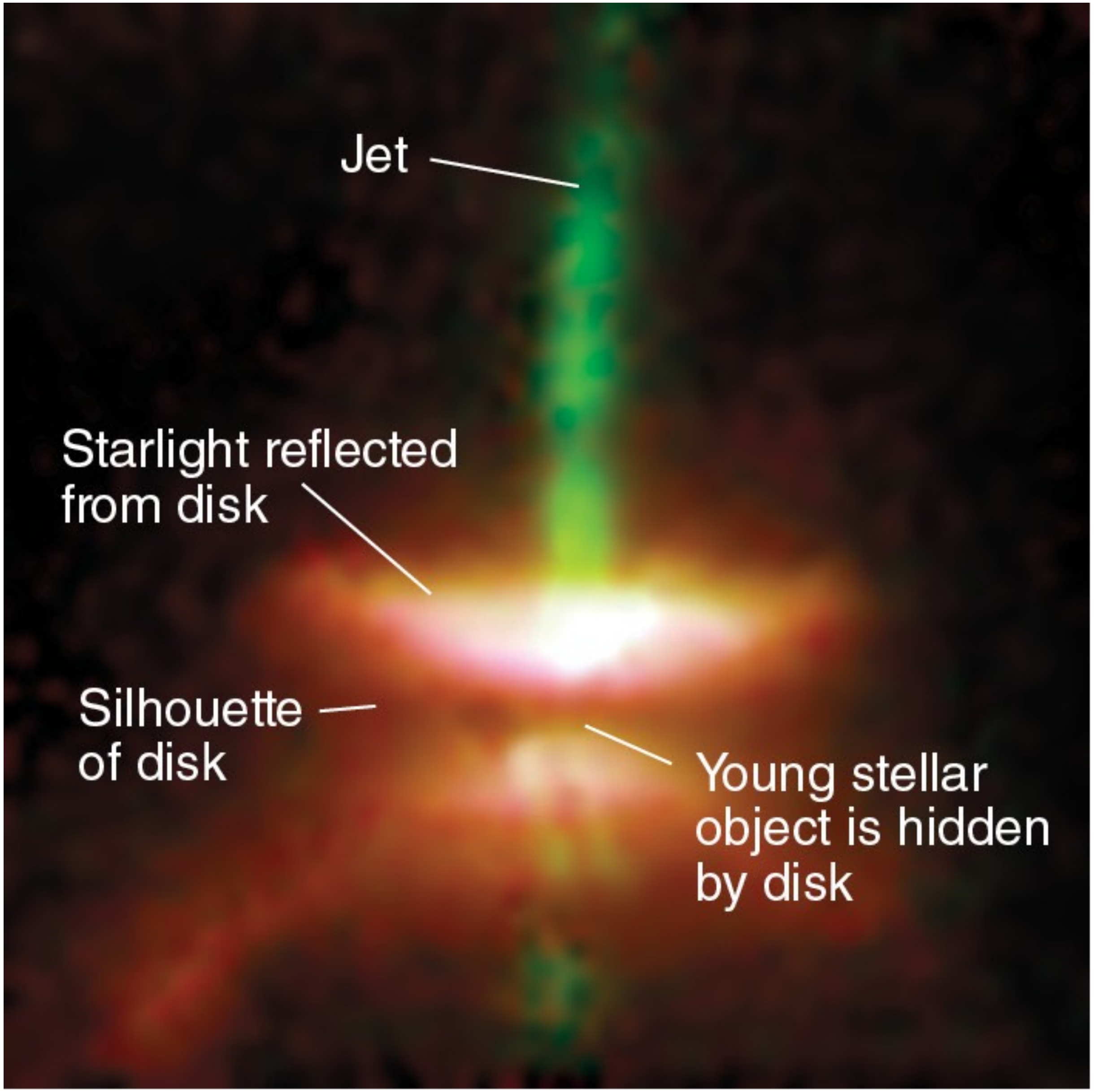
the Sun’s rotation is much slower than predicted

Magnetic braking of the Sun’s rotation

- The young Sun’s magnetic field would grip the partially ionized cocoon nebula, and the nebula in turn would cause a drag on the magnetic field, slowing the Sun’s spin.



Bipolar jet-like outflows from young stars



Chap 7: The Formation of Planetary Systems

- Physical framework
  - Gravitational instability (Jeans criterion)
  - Conservation of angular momentum & the formation of an accretion disk
  - Gravitational heating
  - Radiative cooling
  - Condensation of minerals

- Astrophysical patchworks
  - Stellar winds, bipolar jets, and magnetic breaking to remove angular momentum
  - A nearby supernova to enrich the cloud with short-lived radioactive elements
  - Stellar winds to remove gas from the proto-planetary disk so that the mineralogy is locked in place

Chap 7: The Formation of Planetary Systems - Equations

Solidification age estimate from isotope ratio:

$$t = 3.32\ \tau\ \log\left(\frac{N_d(t)/N_p(t) + 1}{1 + r_0}\right)$$

Virial theorem of spherical gas clouds:

$$2K = -U \Rightarrow \frac{3MkT}{\mu m_H} = \frac{3GM^2}{5R}$$

Luminosity of Protostar from Gravitational Heating:

$$L = \frac{\Delta U}{\text{lifetime}} = \frac{3}{5}\frac{GM^2}{R_f \cdot \text{lifetime}}$$ when $R_f << R_0$

Jeans Mass and Jeans Length:

$$M_J = \left(\frac{5kT}{G\mu m_H}\right)^{\frac{3}{2}}\left(\frac{3}{4\pi\rho_0}\right)^{\frac{1}{2}} = 8\ M_\odot\left(\frac{T}{10\ \text{K}}\right)^{3/2}\left(\frac{n_0}{10^4\ \text{cm}^{-3}}\right)^{-1/2}\left(\frac{\mu}{2}\right)^{-2}$$

$$R_J = \left(\frac{15kT}{4\pi G\mu m_H\rho_0}\right)^{\frac{1}{2}} = 0.1\ \text{pc}\left(\frac{T}{10\ \text{K}}\right)^{1/2}\left(\frac{n_0}{10^4\ \text{cm}^{-3}}\right)^{-1/2}\left(\frac{\mu}{2}\right)^{-1}$$

Spin angular momentum of uniform sphere:

$$L = \frac{2}{5}MR^2\omega \quad \text{where} \quad \omega = \frac{2\pi}{P}$$

Orbital angular momentum:

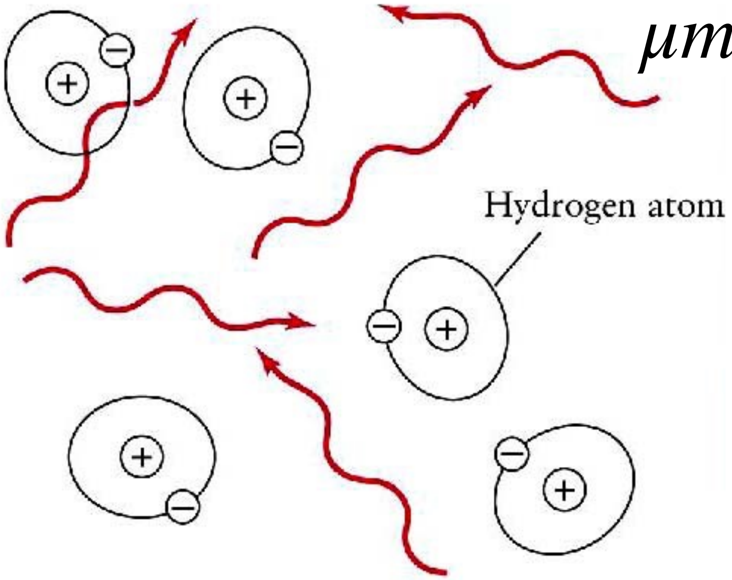
$$L = mv_t r$$

The Mean Molecular Weight

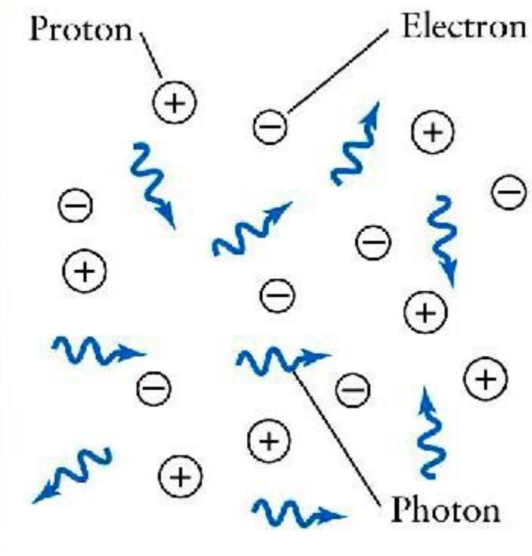
The mean molecular weight is a dimensionless quantity, and it is defined as the ratio between the mean mass per particle and the mass of a single hydrogen atom (1.67e-27 kg):

$$\mu \equiv \frac{\bar{m}}{m_H} = \frac{M_{\text{gas}}/N_{\text{particle}}}{m_H}$$

$$\mu m_H = \frac{\rho}{n}$$



neutral atomic hydrogen



ionized hydrogen