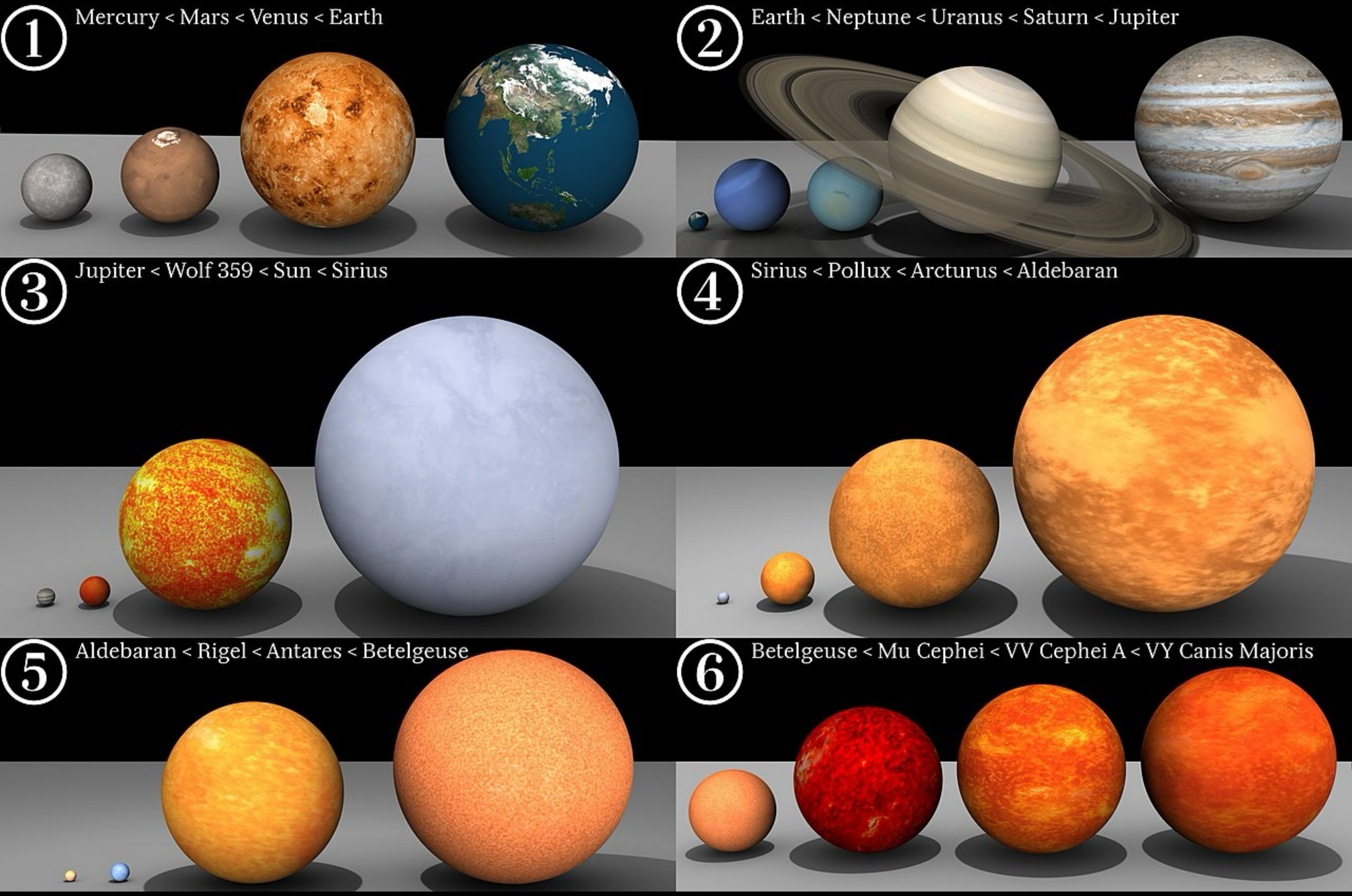
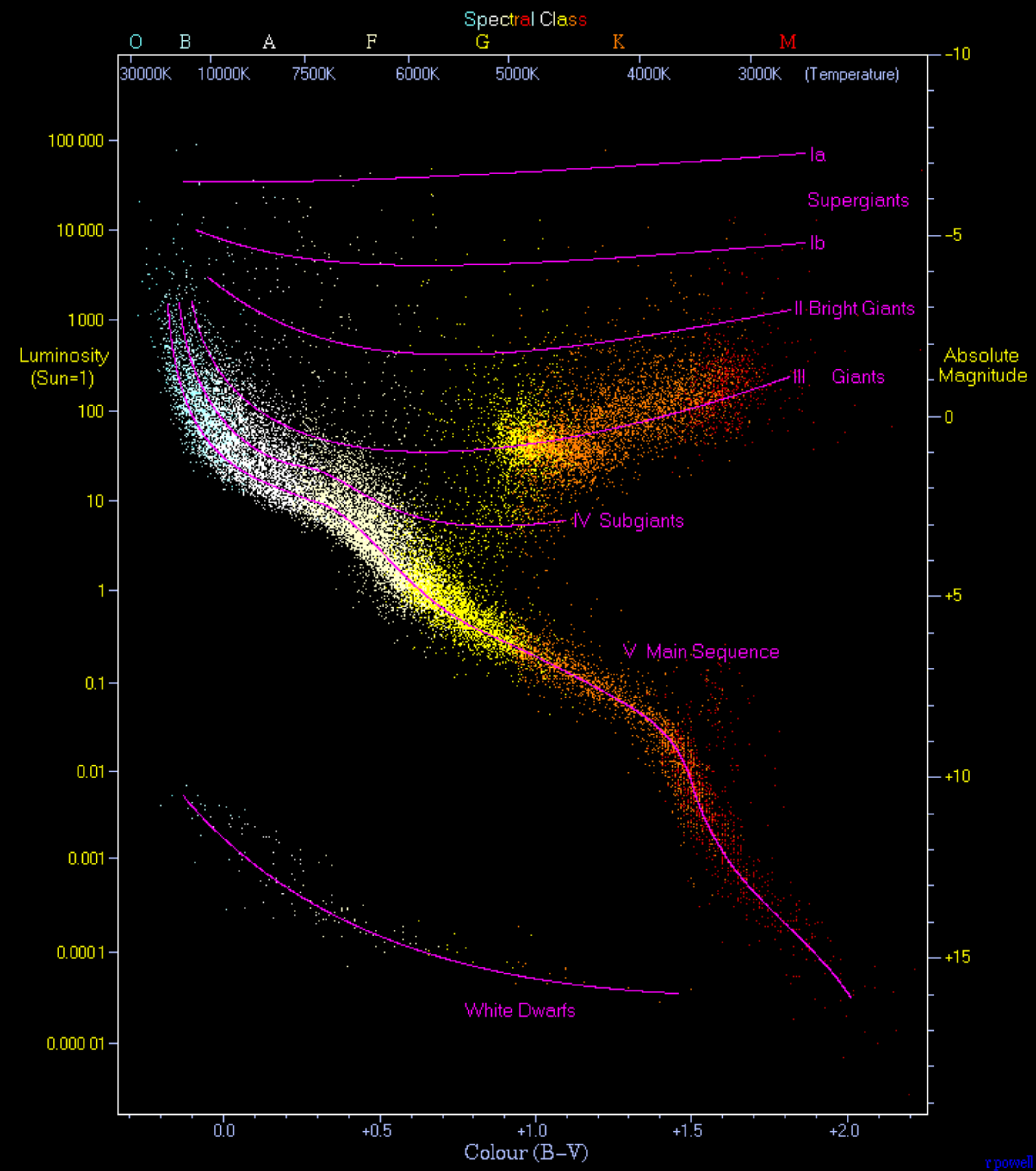
Chap 1: Taking the Measure of Stars



Chap 1: Key Diagram

log(luminosity) vs. log(flux ratio)

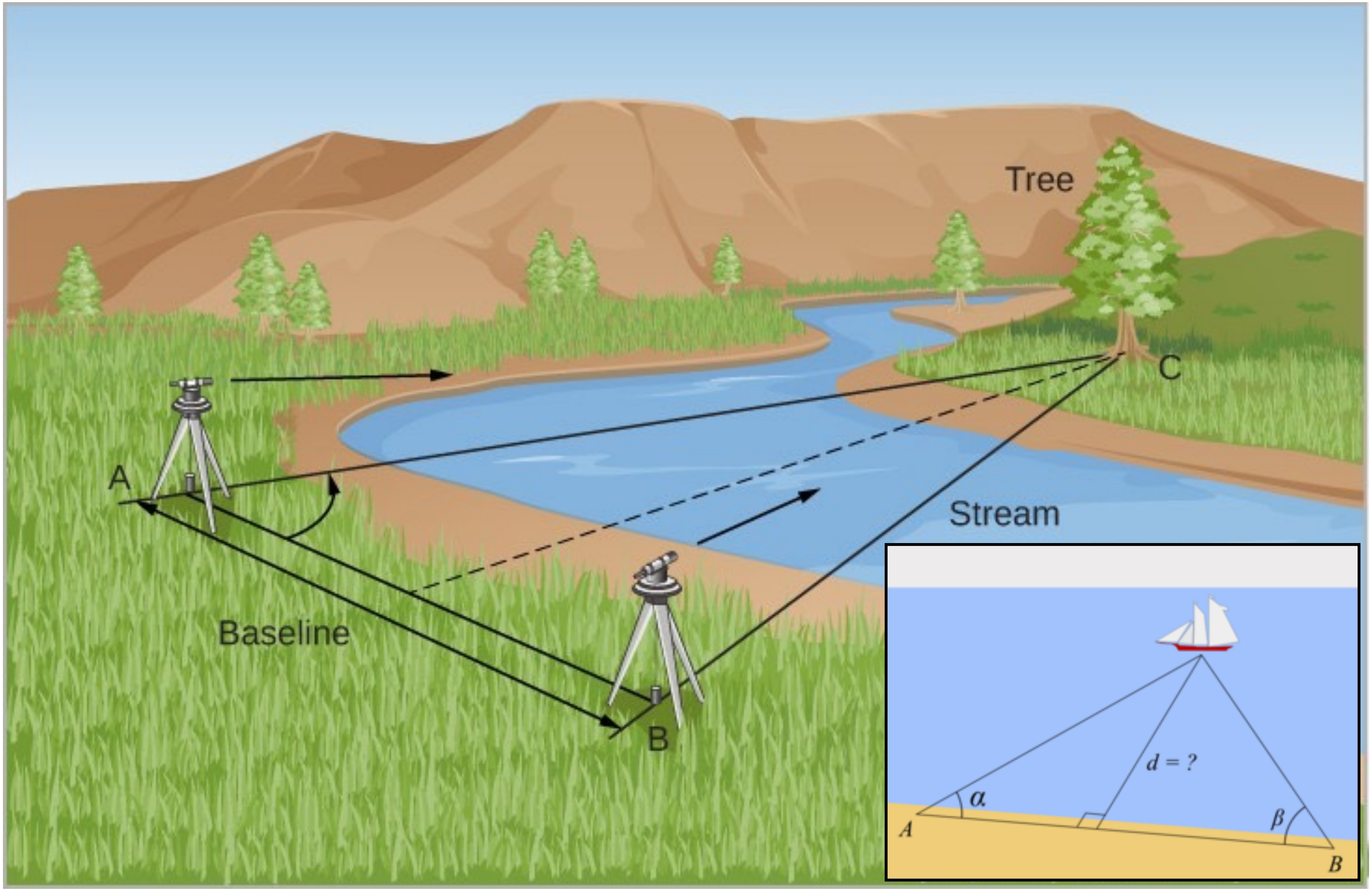


Chap 1: Taking the Measure of Stars

- How do we use parallax to determine distance? Astrometry.
- How do we measure brightness? Photometry.
- How do we combine distance (d) with brightness (apparent magnitude, m) to determine luminosity (absolute magnitude, M)?
- How do we measure temperature (T)? color index
- The Hertzsprung-Russell (H-R) diagram: M vs. color index
- Key concepts:
  - **parallax, magnitude system, distance modulus**
  - **H-R diagram and the distribution of stars on the diagram**
- Other measurements: size & mass of stars

Distance Measurements: Parallax

Geological Survey Method



Geological Survey Method: Theodolite measurements

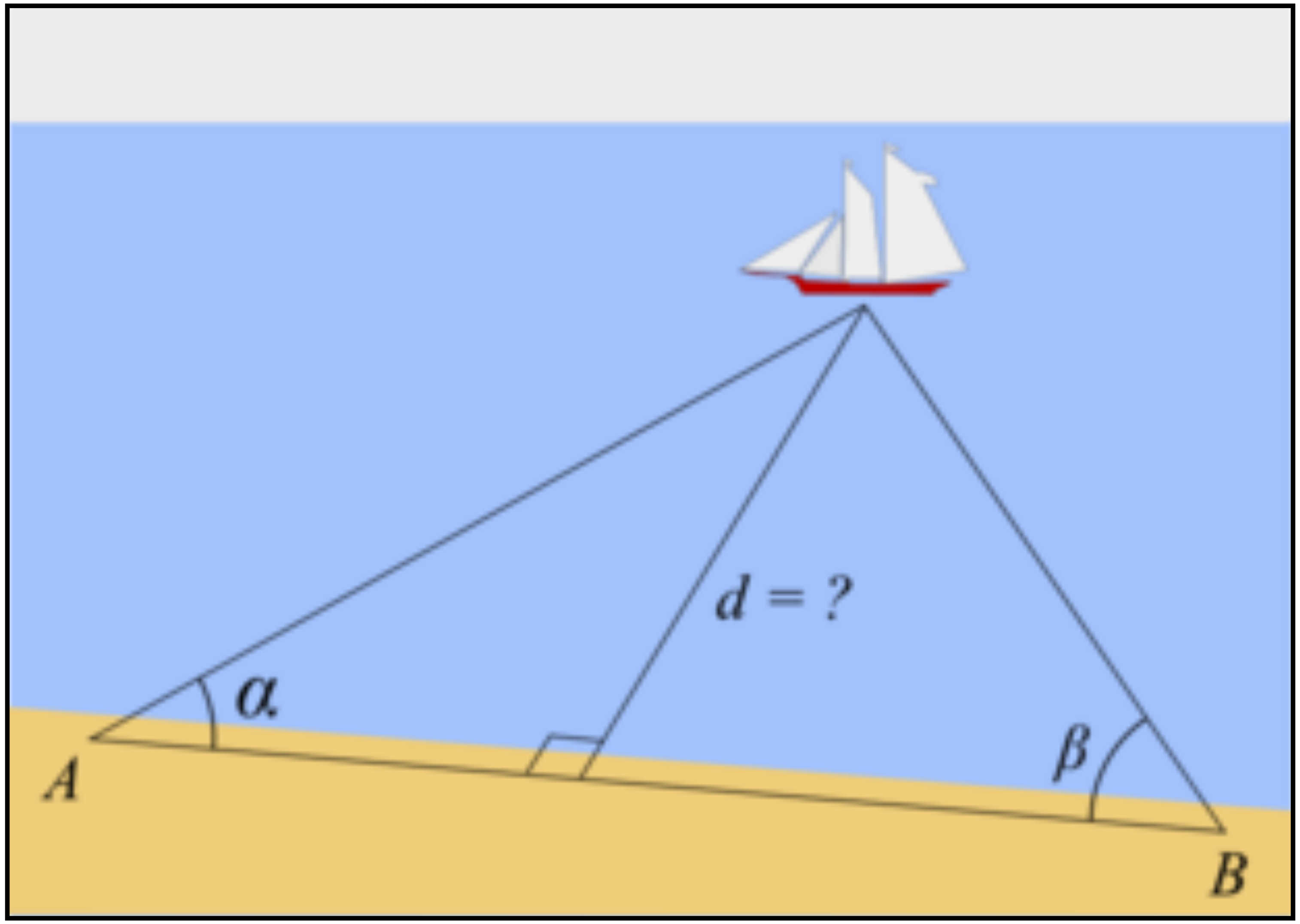
need to know the baseline length ($l = AB$) and the two angles (α, β)

$$\ell = d\left(\frac{\cos\alpha}{\sin\alpha} + \frac{\cos\beta}{\sin\beta}\right)$$

$$\ell = d\,\frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$$

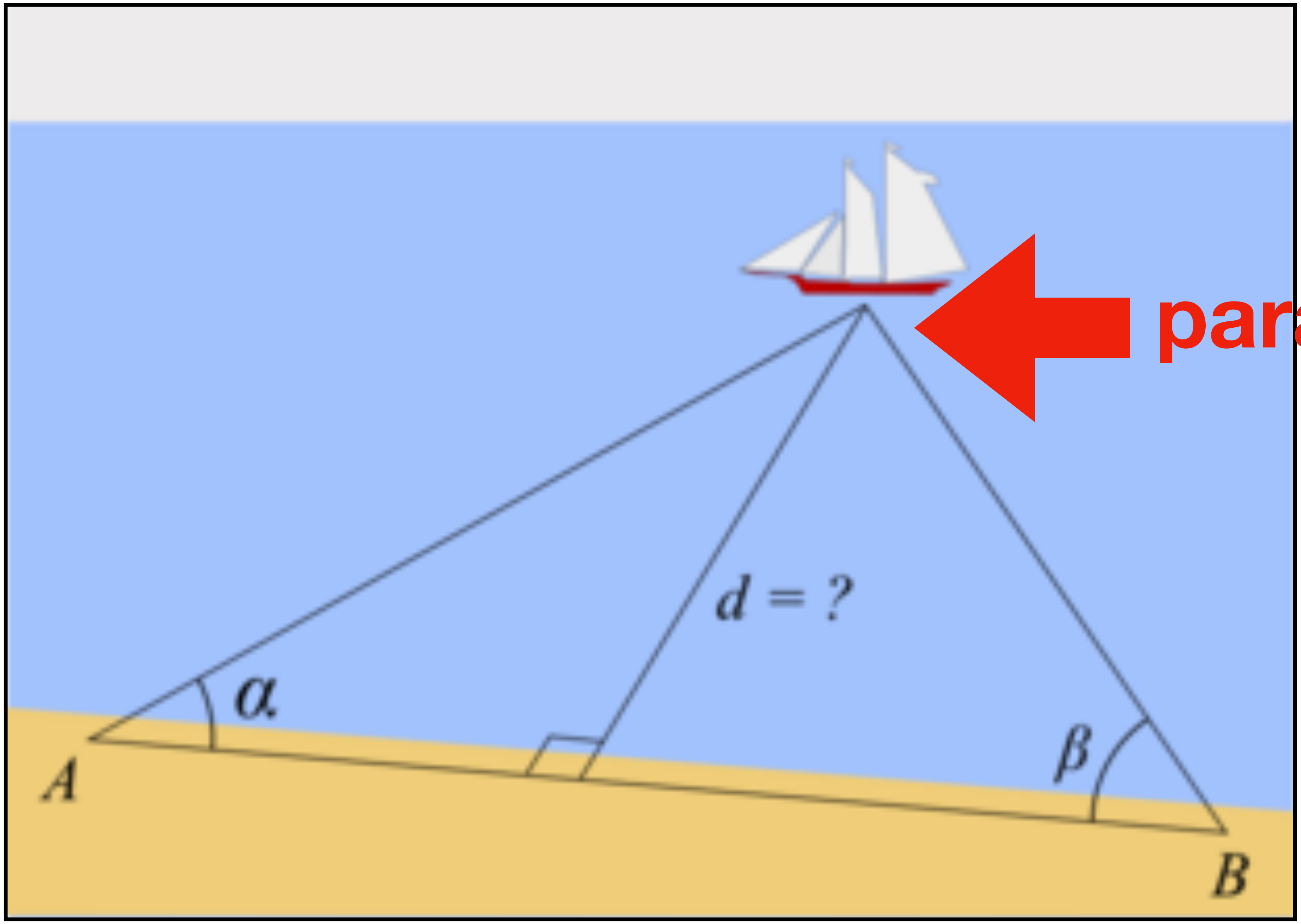
therefore:

$$d = \ell\,\frac{\sin\alpha\sin\beta}{\sin(\alpha+\beta)}$$



Geological Survey Method: Theodolite measurements

What would the angles become when d is much much greater than AB?



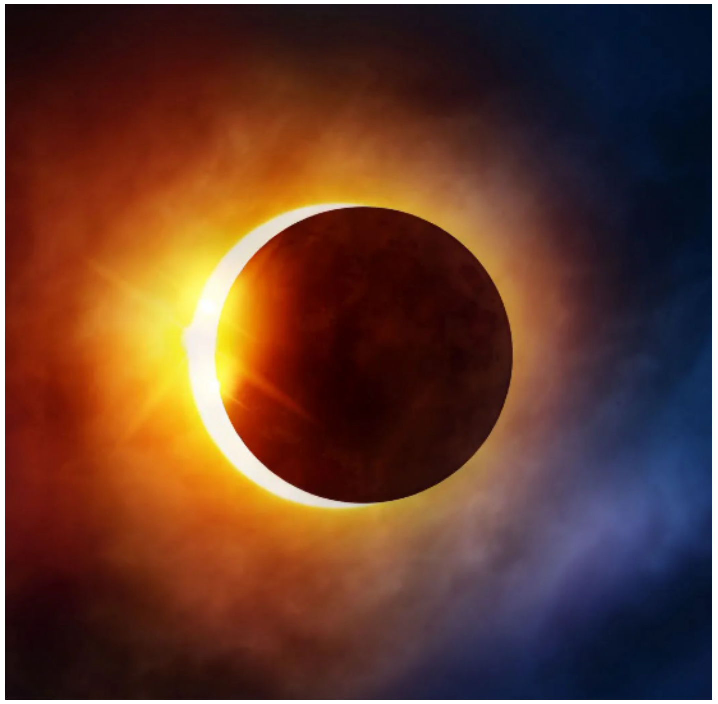
parallax

To measure greater distances, we need:
(1) longer baselines and (2) the ability to measure tiny angles

The Earliest Parallax Measurement by Hipparchus (~150 BC): Baseline limited by the diameter of the Earth

seen in Hellespont (100% obscured)

seen in Alexandria (80% obscured)

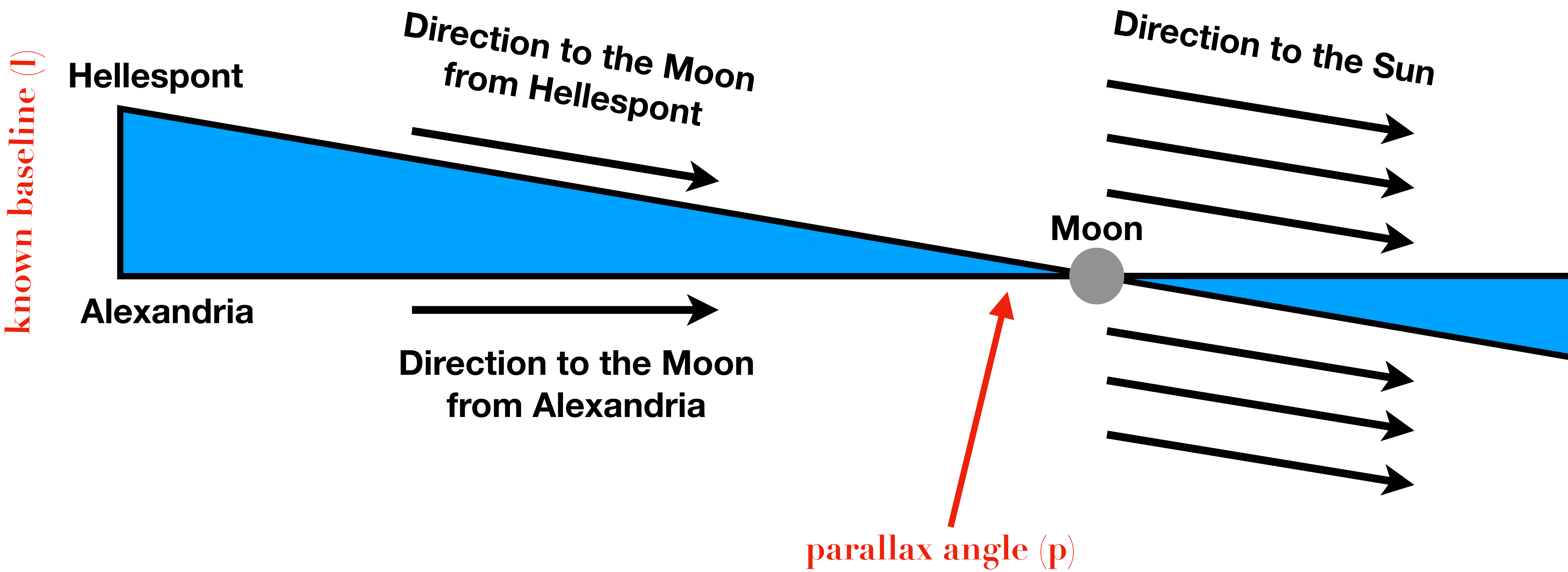


The Solar Eclipse on Mar 14, 190 BC

The Simple Geometry of Parallax

$$D_{\text{moon}} = \text{baseline length/parallax angle in radian} = l/p$$

in the above, we have used the small angle approximation

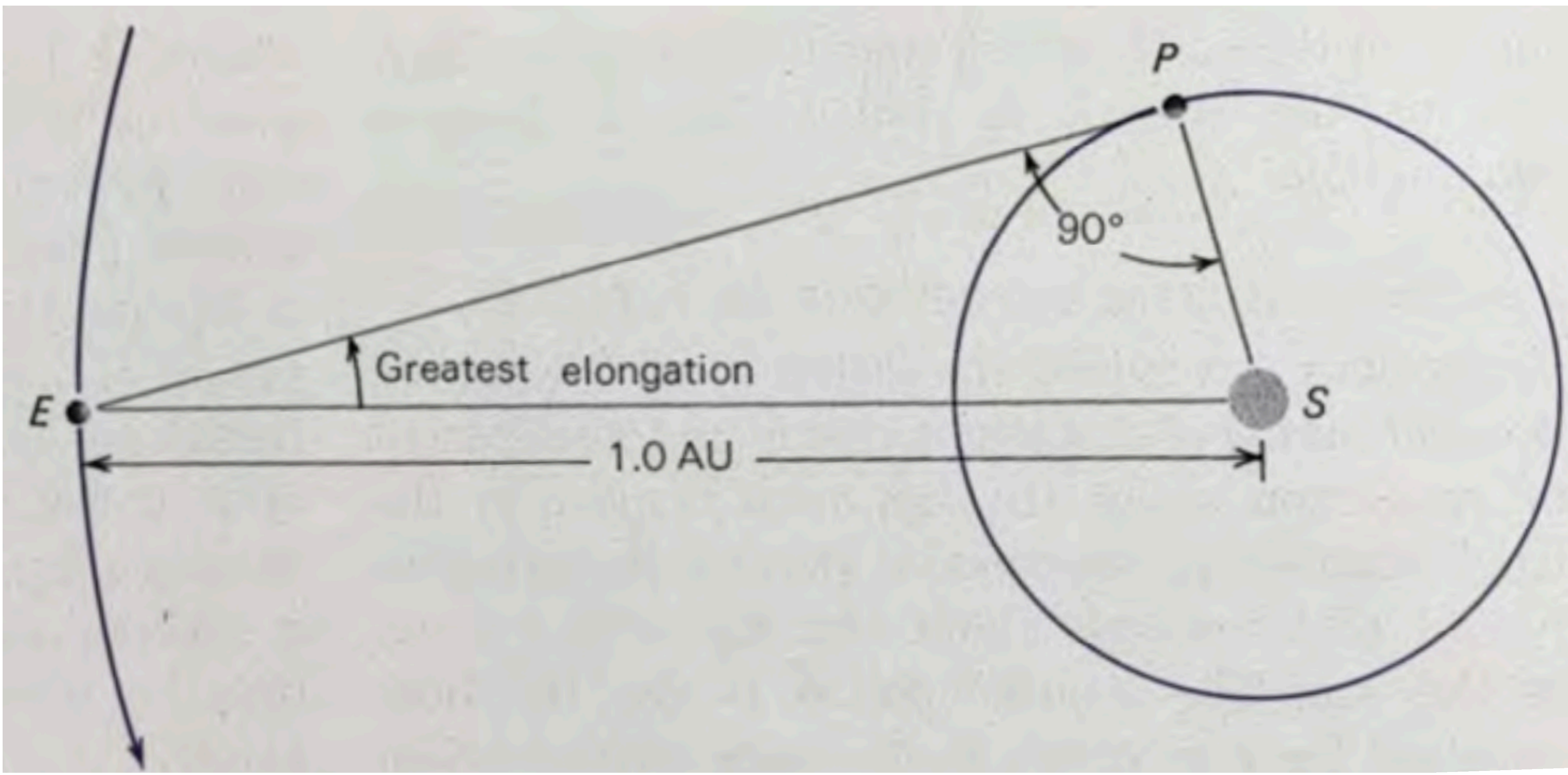


Measure the baseline itself

- How do we measure the Astronomical Unit (AU)? Recall that AU is defined as the average heliocentric distance of the Earth, but how long is it exactly?
- One geometric method involves two components:
  - An inner planet's heliocentric distance in AU
  - A parallax measurement of the distance between the Earth and the inner planet during its transit of the Sun

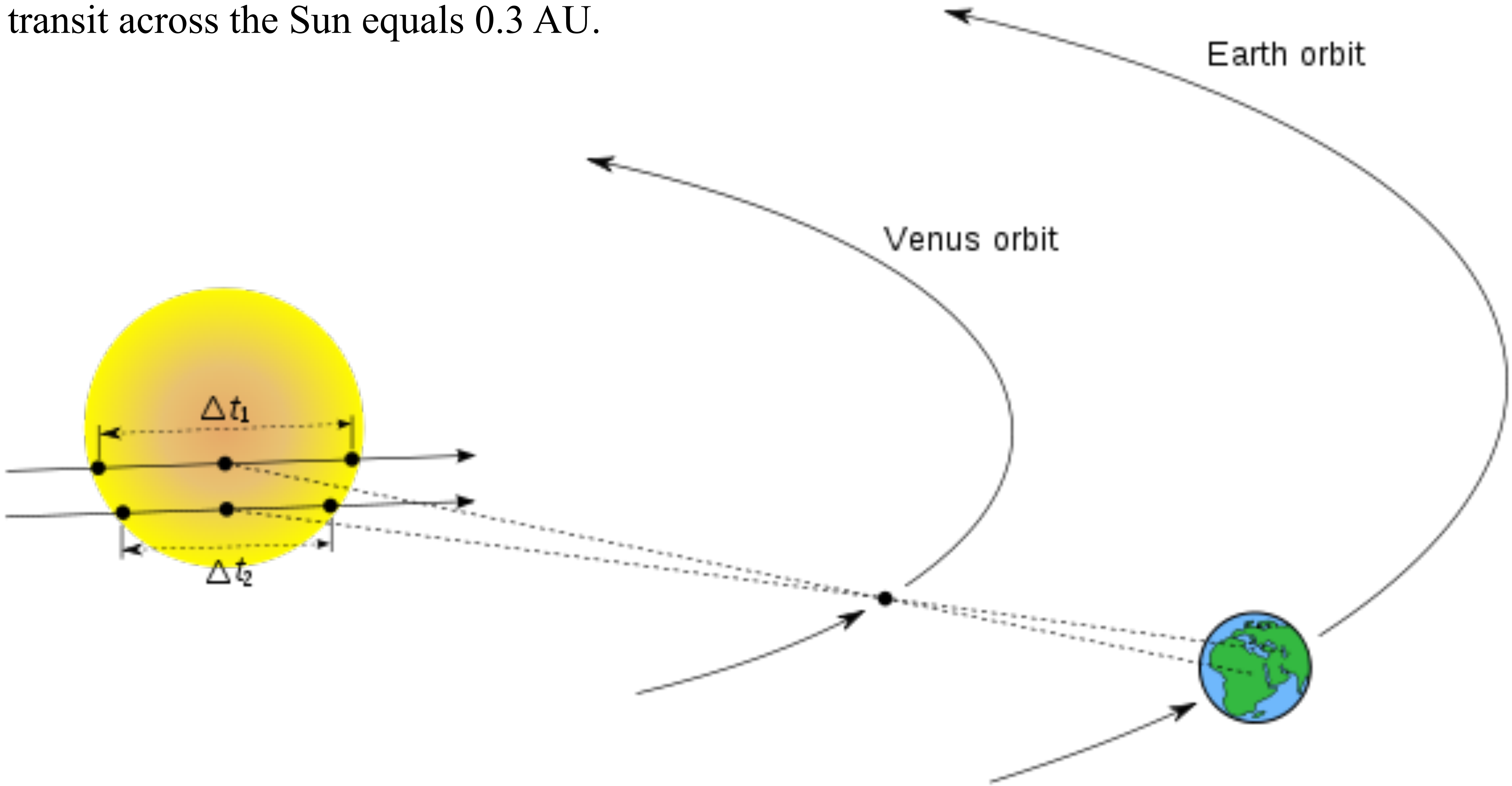
Copernicus' method of determining the heliocentric distance of inner planets

Inner Planets: R_{AU} = sin(Max Elongation)



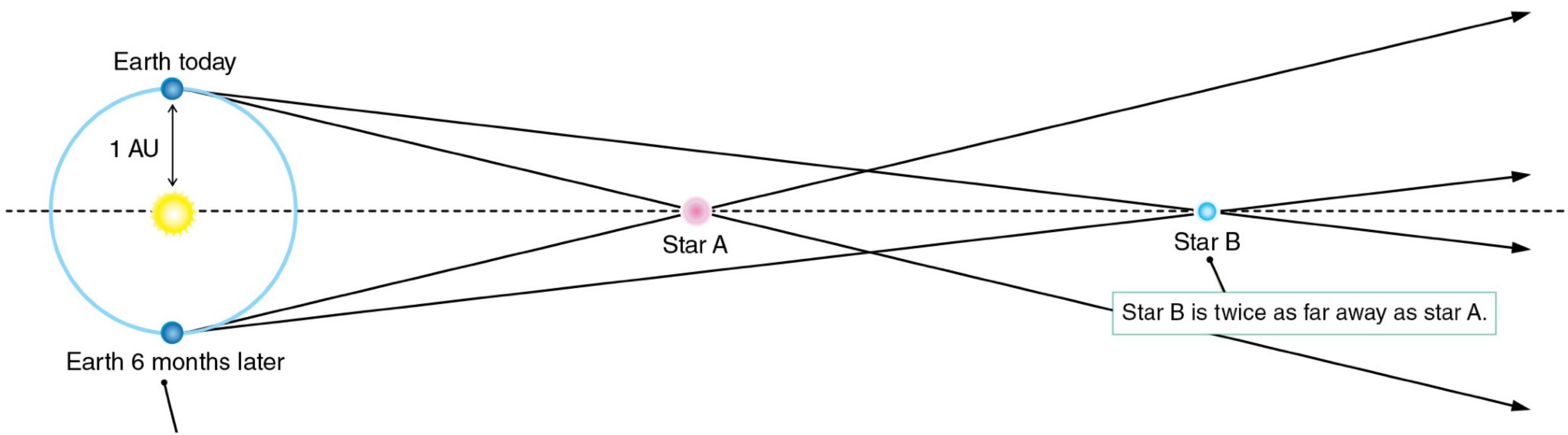
Parallax measurement of the AU

- By observing the transit of Venus at two locations on Earth, one can measure the distance to Venus using the measured **parallax** and the **baseline length**.
- Assuming Venus and Earth are both on circular orbits around the Sun, the greatest elongation angle of Venus tells us its orbit has a radius of 0.7 AU.
- Since the Earth's orbit has a radius of 1 AU, the measured parallax distance of Venus during its transit across the Sun equals 0.3 AU.



Extend the Baseline from Earth Size to Earth's Orbit Size: Stellar Parallax

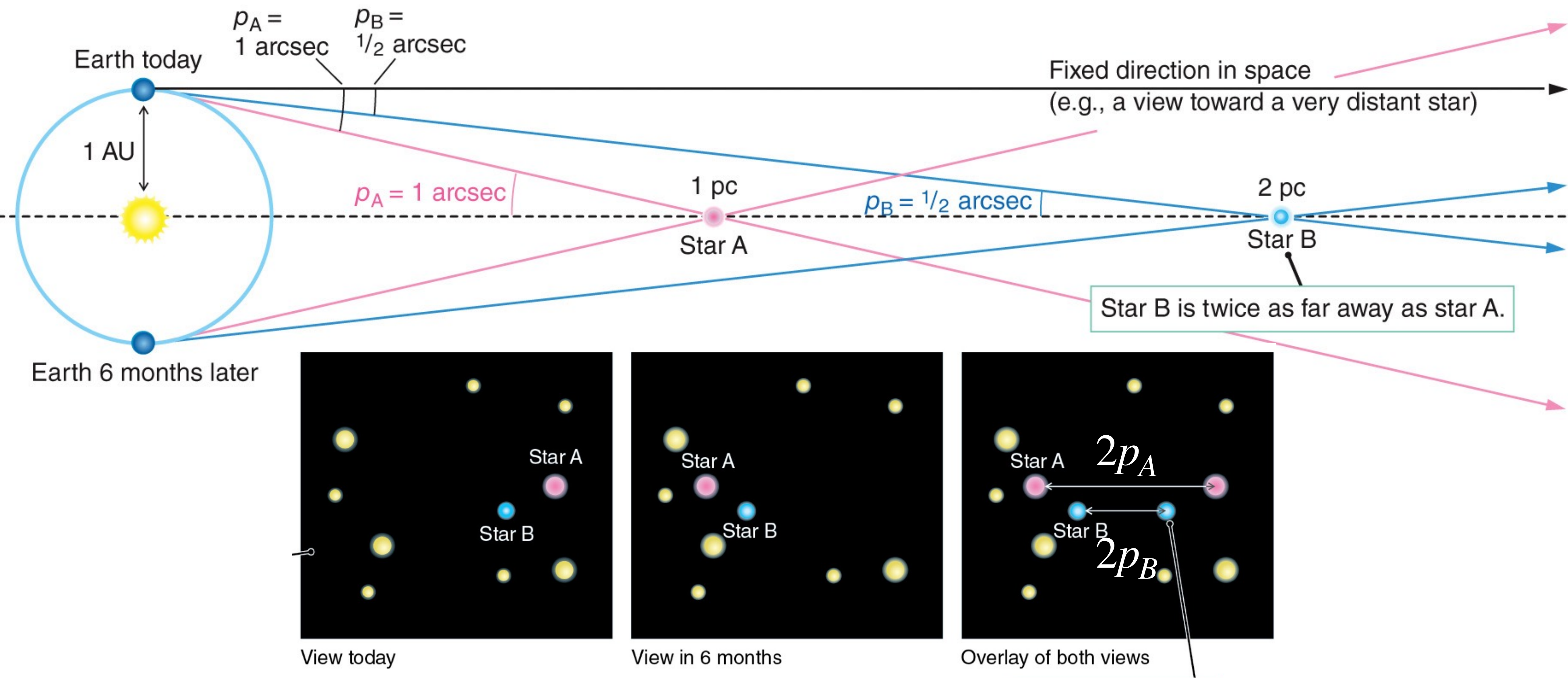
Parallax distance can be measured given the baseline length (l) and the parallax angle (p); by using the orbit of the Earth, *the baseline length is increased by 23500 times from 2x 6400 km to 2x 149.6 million km.*



The Definition of Parallax in Stellar Astronomy

Any directional shift due to a positional shift is a parallax effect, but in **stellar astronomy**, **parallax** is defined as **half of the maximum directional shift** due to Earth's orbital motion.

From this diagram, it's clear that **parallax** is inversely proportional to **distance**: $p \sim 1/d$



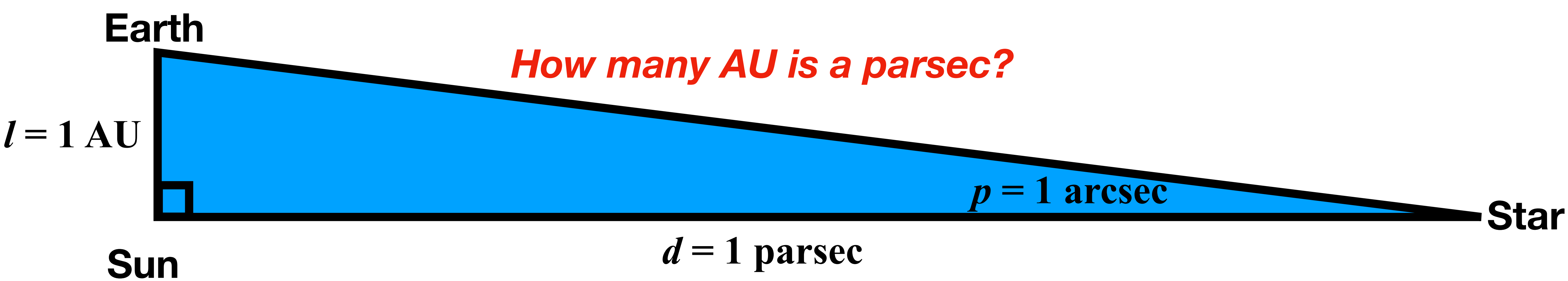
Definition of the unit parsec: the distance at which p = 1 arcsec

Let p be the parallax in arcseconds.

Let d be the distance in parsecs; **the unit parsec is defined as the distance at which p = 1 arcsec**

Given this definition we have:

$$d = 1 \text{ parsec} \left(\frac{1 \text{ arcsec}}{p} \right)$$



From the diagram above, we derived:

1 parsec = 206,205 AU since $l/d = \tan p \sim p$ (in radian)

Practice: convert parallax to distance

The greater the parallax, the smaller the distance.

A star with a parallax of 1 **arcsecond (arcsec)** is at a distance of 1 **parsec (pc)**.

- 1 arcsec = 1/3,600 degree
- 1 pc = 3.26 light-years (o*nly useful when talking to non-astronomers*)

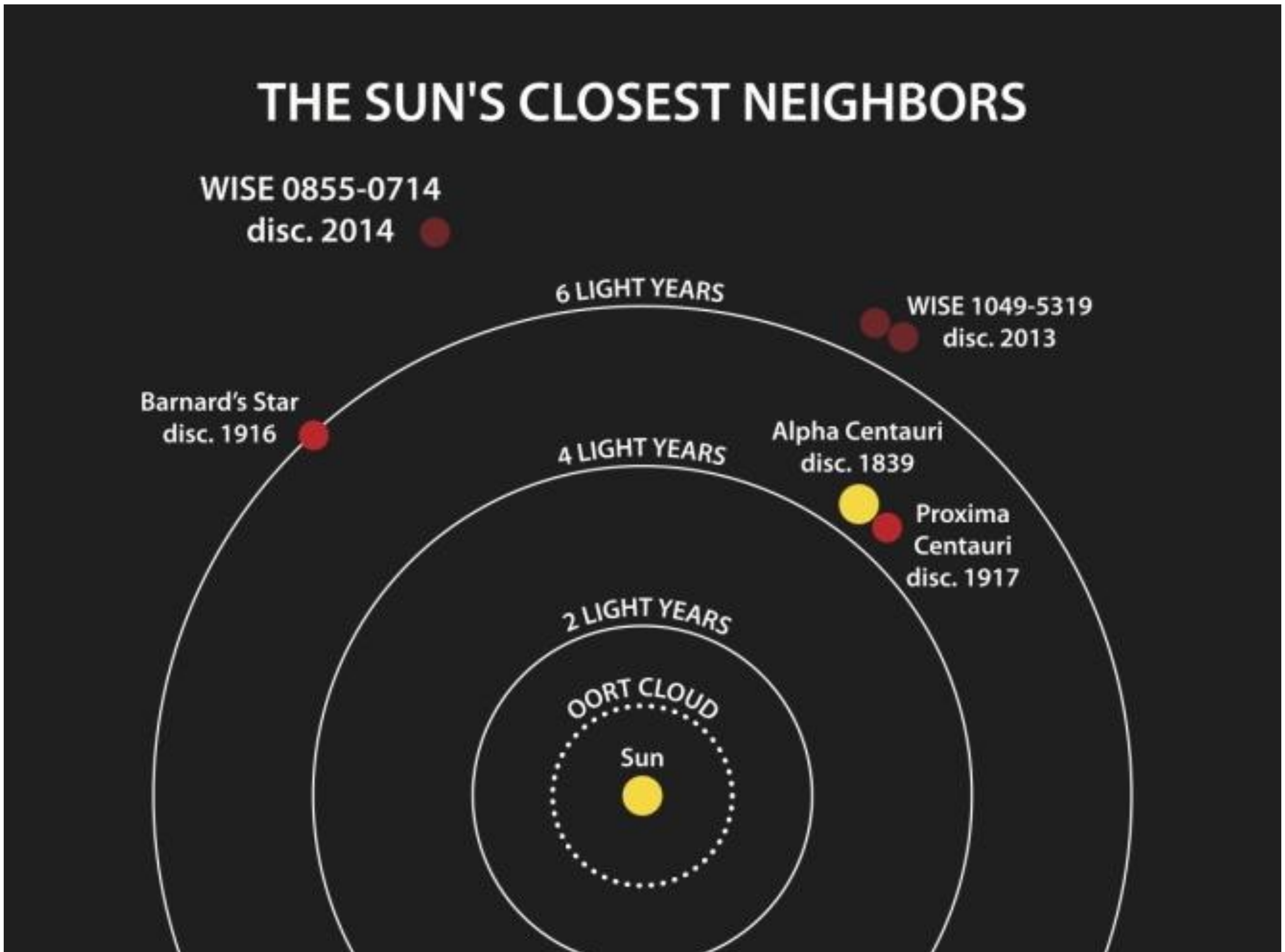
Parallax angles have been measured for >1 billion stars.

The first star with measured parallax was 61 Cygni by Friedrich Bessel in **1838**.
It had a parallax of **0.314 arcsec**, what is its distance in parsec and light-year?

Bessel functions in Mathematics are named after him.

Practice: Convert distance to parallax angle

Let's try a reversed problem. After the Sun, the closest star to Earth is Proxima Centauri, which is 4.24 light-years away. What is the star's parallax in arcsec? (1 pc = 3.26 ly)



Practice: Convert distance to parallax

Let's try a reversed problem. After the Sun, the closest star to Earth is Proxima Centauri, which is 4.24 light-years away. What is the star's parallax in arcsec?

First, we convert light-years to parsecs:

$$d = 4.24 \text{ light--years} \times \frac{1 \text{ parsecs}}{3.26 \text{ light--years}} = 1.30 \text{ parsecs}$$

Then, we plug in to find the distance:

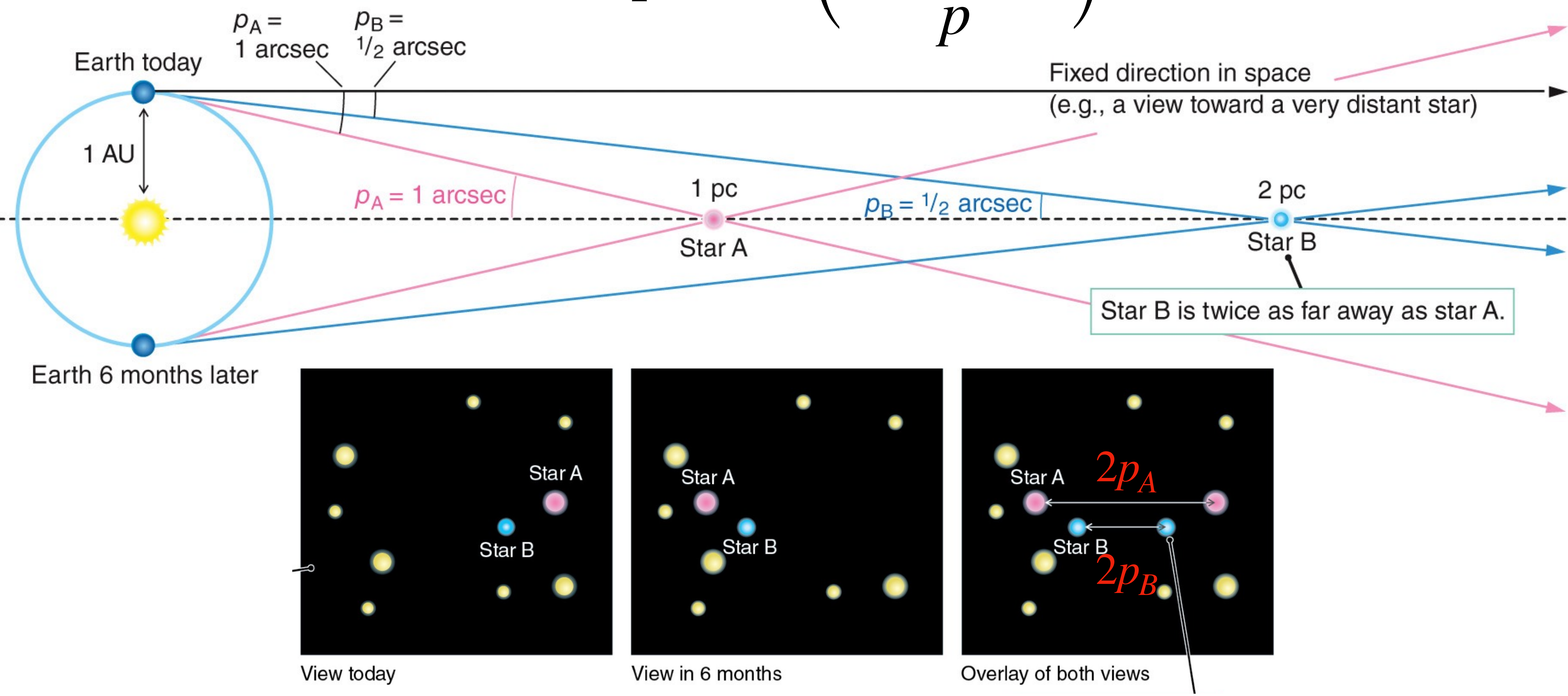
$$p\ (\text{arcsec}) = \frac{1}{1.30 \text{ pc}} = 0.77 \text{ arcsec}$$

The closest star to the Sun has a parallax smaller than 1 arcsec!

Stellar Parallax: One Slide Introduction

In stellar astronomy, **parallax (*p*)** is defined as **half of the maximum directional shift** due to Earth's orbital motion. The unit is typically arcsec or mas (milliarcsec) With this definition and the definition of the **parsec**, we have the following **parallax-distance relation**:

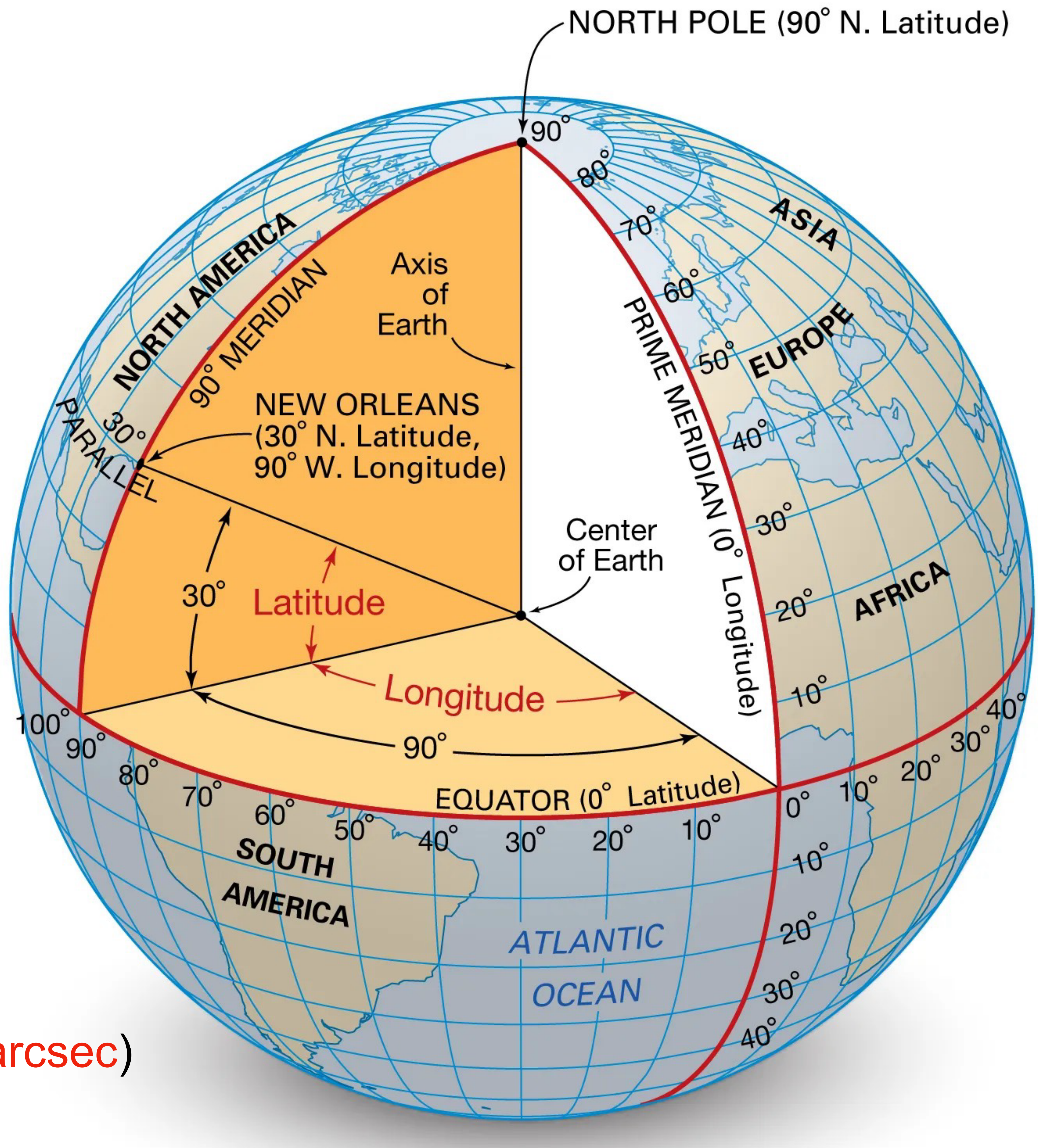
$$d = 1 \text{ parsec} \left(\frac{1 \text{ arcsec}}{p} \right)$$



How to Calculate Parallax from Coordinates?

A star's position is recorded in celestial coordinates (RA, Dec), how to calculate the angular offset between two coordinates?

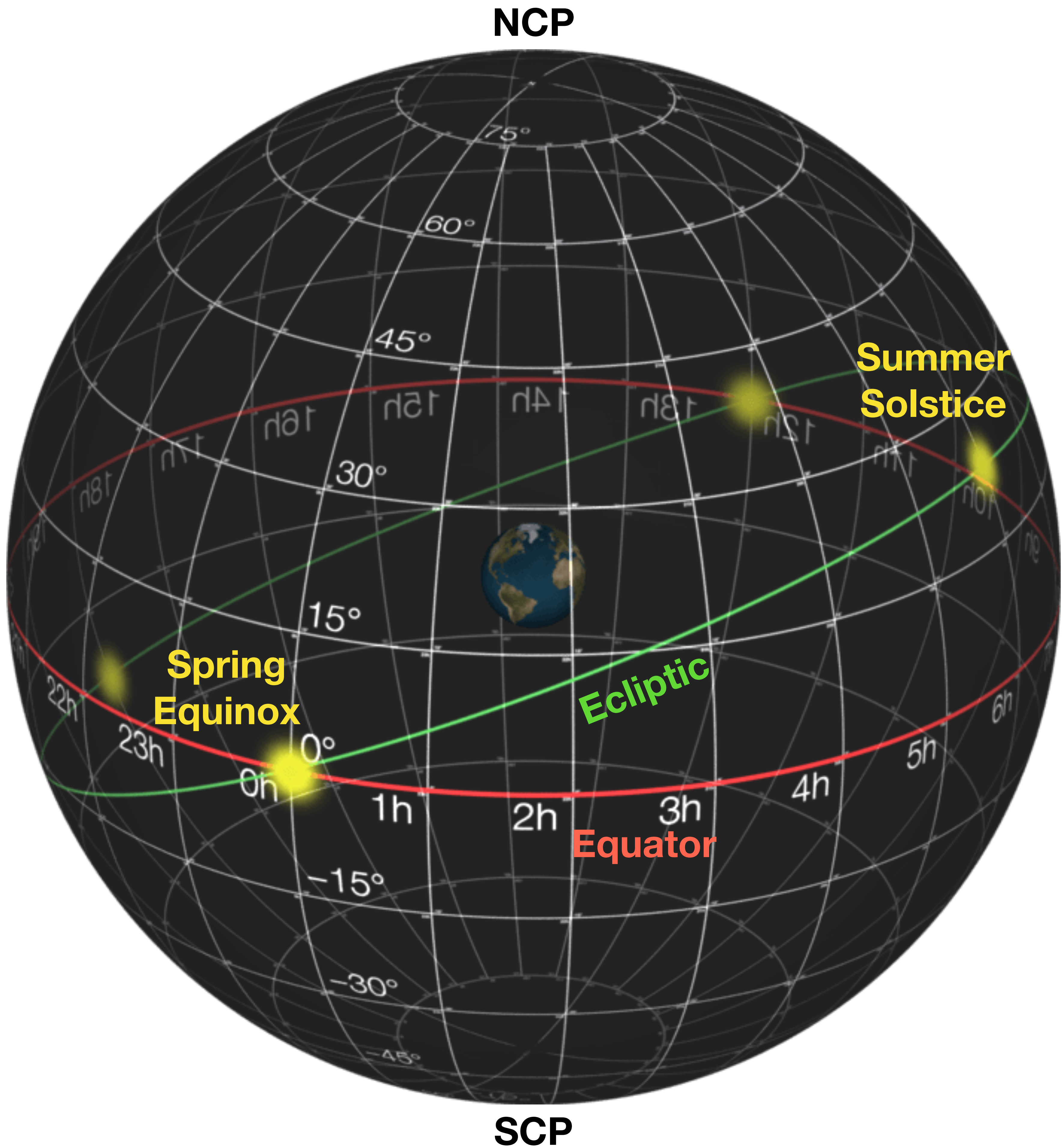
Celestial Coordinates are similar to the **Longitude** and **Latitude** system on Earth's surface



Longitude & Latitude units: (deg, arcmin, arcsec)

Equatorial coordinates

right ascension (RA)
declination (Dec)



RA's units
(hour, minute, second)

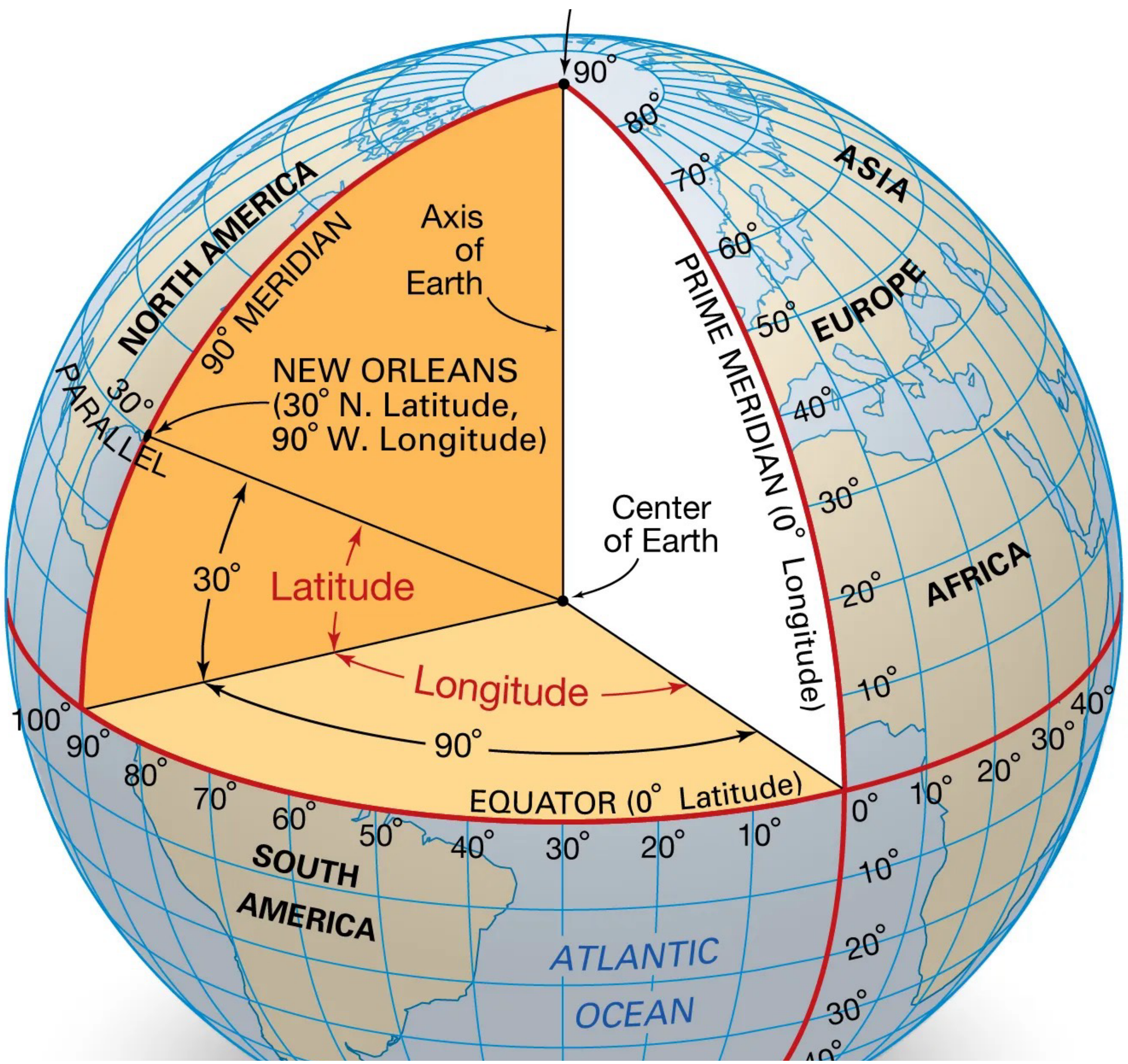
Dec's units:
(deg, arcmin, arcsec)

Practice: Calculate the distance between locations along the same parallel

Iowa City: 41.6578° N, 91.5346° W

Des Moines: 41.5868° N, 93.6250° W

Earth's Radius: 3960 miles

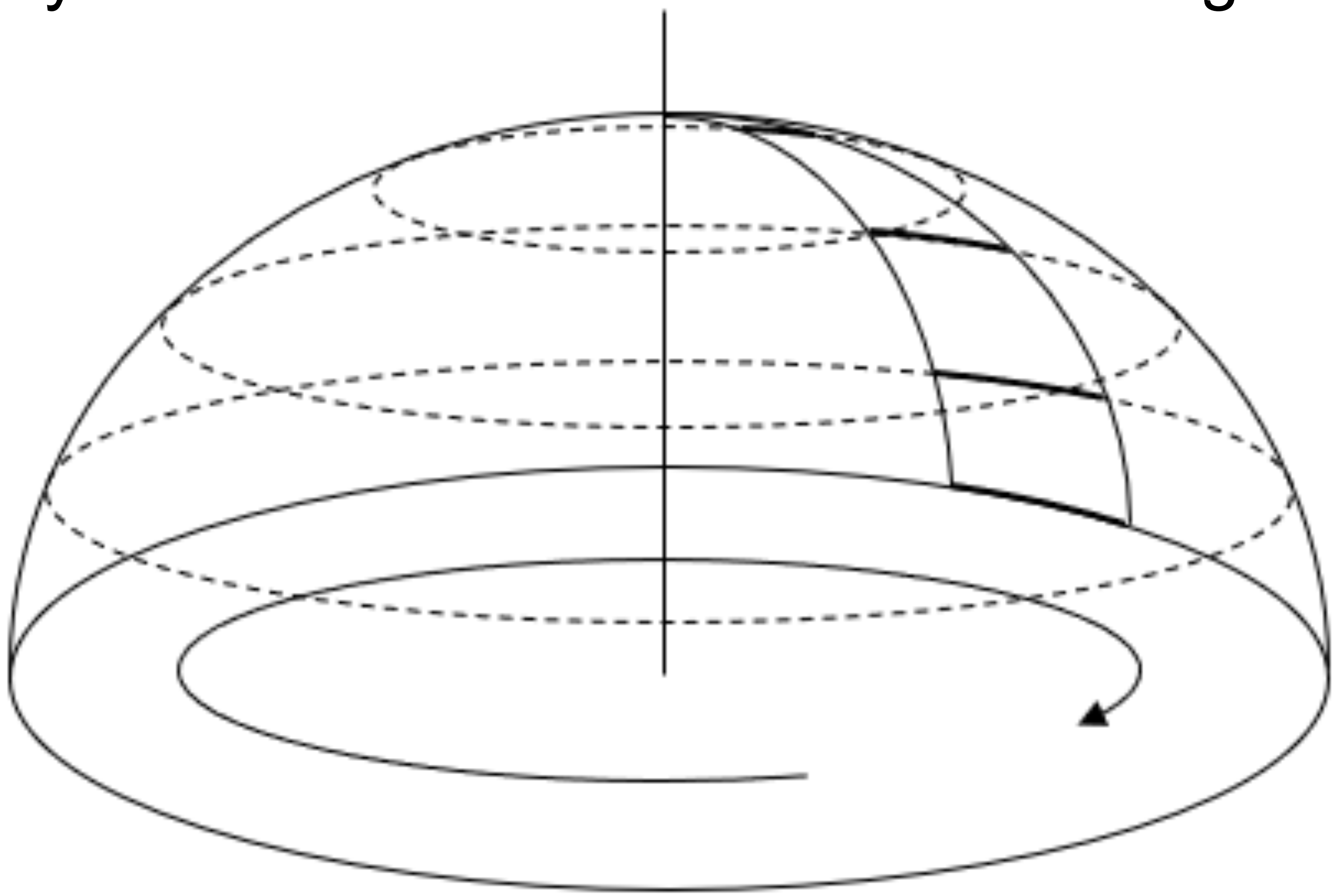


Almost the same latitude, difference in longitude about 2.1 degrees.

- convert 2.1 deg to radian: 0.0366519
- distance = arc length = $R \cdot \theta = 3960\text{miles} \times 0.0366519 = 145\text{miles}$
- Google says the distance is 115 miles (much less than 145 miles), why?

Practice: at fixed Declination, calculate angular offset in R.A.

- Obj 1: RA = 2 hr, Dec = 60 deg, Obj 2: RA = 3 hr, Dec = 60deg; what's their angular distance in **degrees**?
- Note that you'll need to first convert hours to degrees …



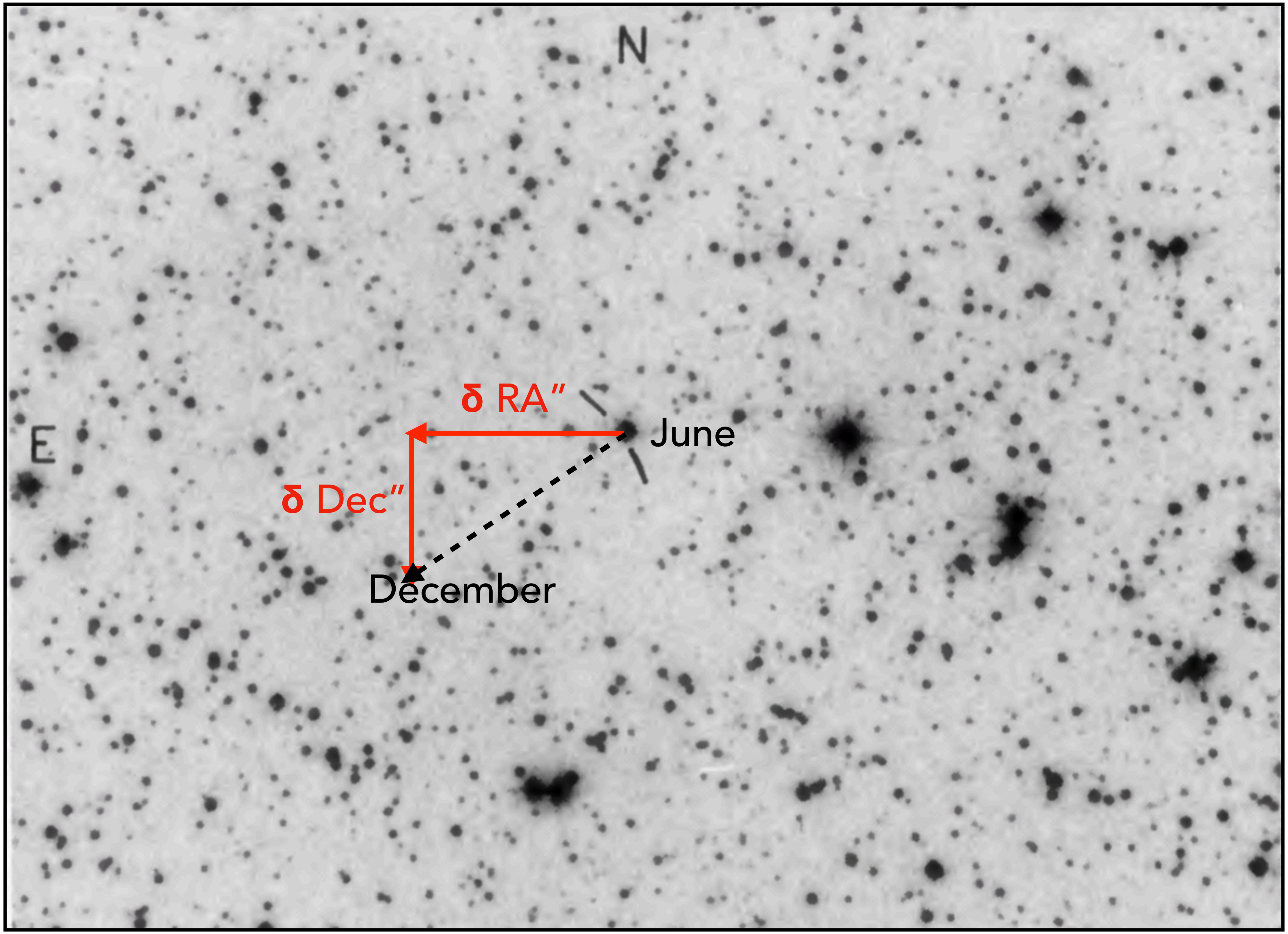
$$\delta \mathrm{RA}^{\circ} = (\mathrm{RA}_1^h - \mathrm{RA}_2^h) \cdot \cos(\mathrm{Dec}_1^{\circ}) \cdot 15^{\circ}/\mathrm{hour}$$

$$\delta \mathrm{Dec}^{\circ} = \mathrm{Dec}_1^{\circ} - \mathrm{Dec}_2^{\circ}$$

Offsets in both RA and Dec, how to calculate the total offset?

When the two coordinates are close together, we can use plane trigonometry to approximate spherical trigonometry:

$$\Delta'' = \sqrt{\delta \mathrm{RA}''^2 + \delta \mathrm{Dec}''^2}$$



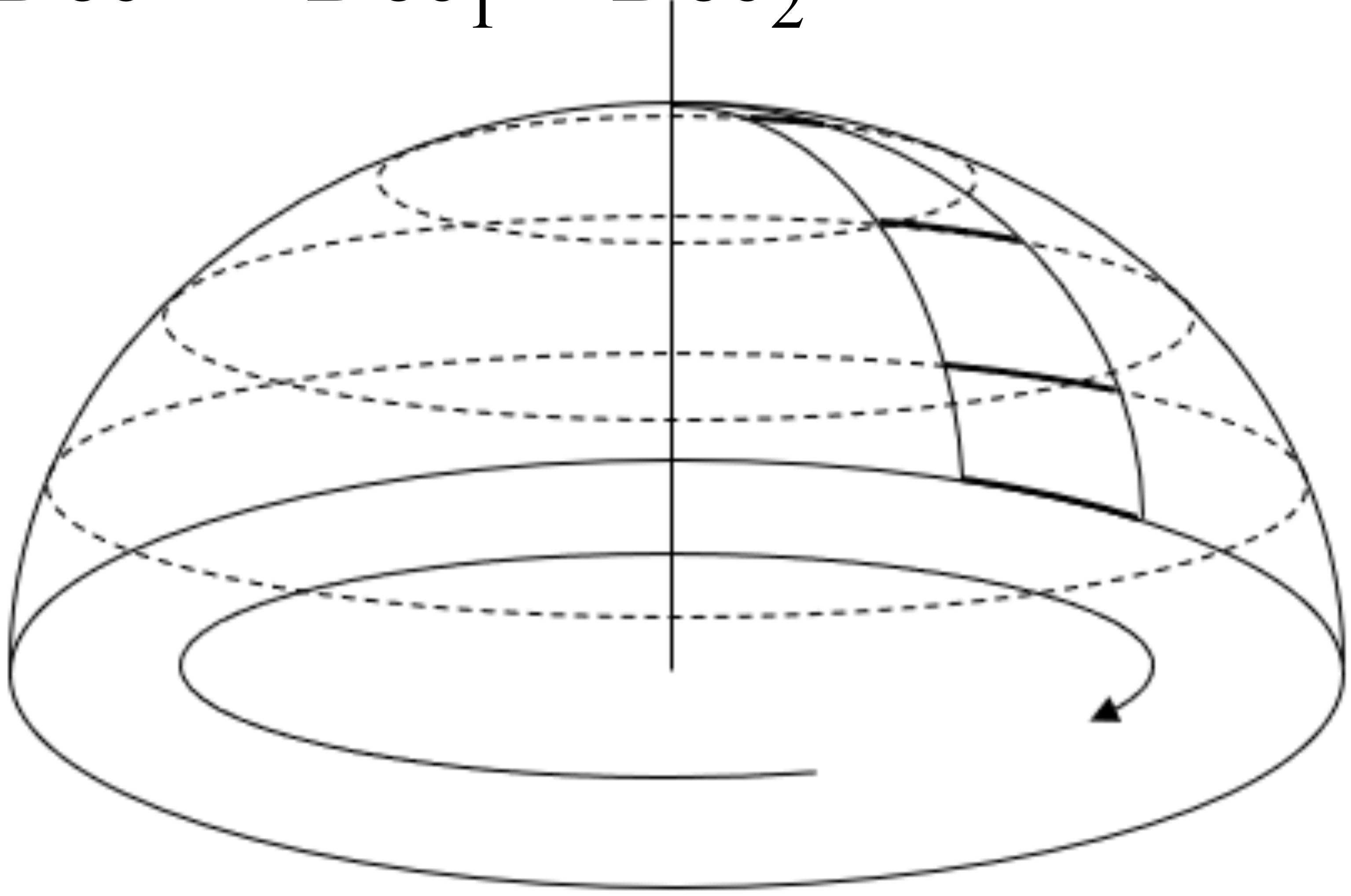
Given two (RA, Dec) coordinates, calculate their angular offset

$$\Delta'' = \sqrt{\delta\text{RA}''^2 + \delta\text{Dec}''^2}$$

Note that (1) RA's units are (hour, minute, second), and Dec's units are (deg, arcmin, arcsec), and (2) the angular distance between two meridians ***decreases*** from the equator to the poles. As a result, we have the following formulae to calculate both the RA offset and the Dec offset in arcsec:

$$\delta\text{RA}'' = (\text{RA}_1^s - \text{RA}_2^s) \cdot \cos(\text{Dec}^\circ) \cdot 15''/\text{s}$$

$$\delta\text{Dec}'' = \text{Dec}_1'' - \text{Dec}_2''$$



Practice: Given two (RA, Dec) coordinates, calculate their angular offset

$$\Delta'' = \sqrt{\delta \mathrm{RA}''^2 + \delta \mathrm{Dec}''^2}$$

$$\delta \mathrm{RA}'' = (\mathrm{RA}_1^s - \mathrm{RA}_2^s) \cdot \cos(\mathrm{Dec}^\circ) \cdot 15''/\mathrm{s}$$

$$\delta \mathrm{Dec}'' = \mathrm{Dec}_1'' - \mathrm{Dec}_2''$$

dRA = 0.03 * cos(23.5 deg) * 15 = 0.413”
dDec = 0.005”
=> p = 0.413”/2 => d = 2.4*2 parsec

A star’s coordinates have been recorded based on images taken on the following dates:

Mar 21 2022: 06h00m15.205s 23d29’15.155”
Sep 21 2022: 06h00m15.235s 23d29’15.160”

- How far has the star moved in RA & in Dec (both in arcsec)?
- How large is the parallax? What’s the distance in parsec?

How to Plan Parallax Observations?

Given a star’s position in equatorial coordinates (RA, Dec), how to decide when to make the two observations to detect the maximum parallax effect?

Are these dates and coordinates arbitrary?
Mar 21 2022: 06h00m15.205s 23d29’15.155”
Sep 21 2022: 06h00m15.235s 23d29’15.160”

Equatorial coordinates

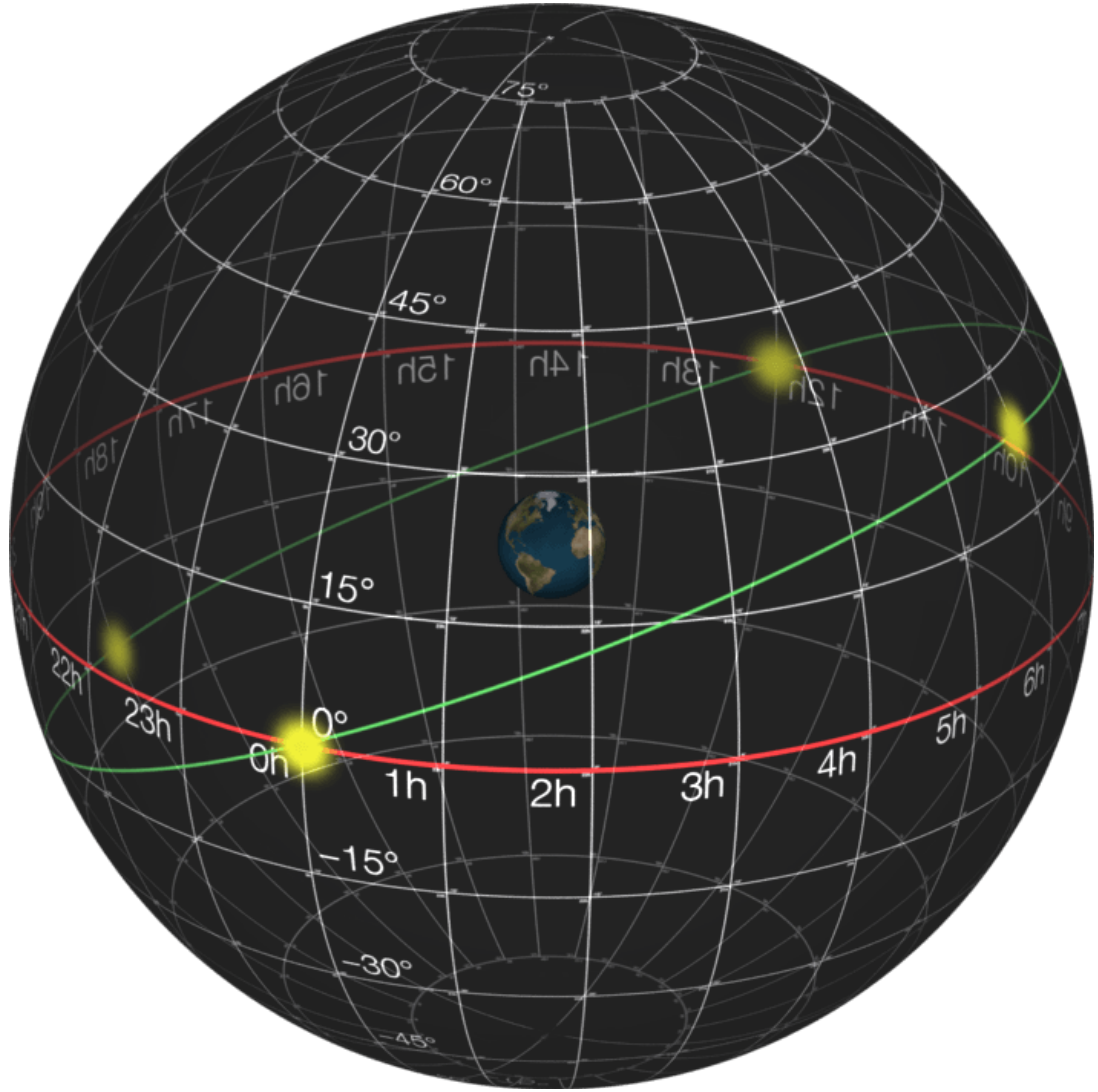
right ascension (RA)
declination (Dec)

Ecliptic coordinates

Longitude
Latitude

Ecliptic Longitude ~ RA
~ means "roughly equal"

|Ecliptic Latitude - Dec| < 23.5 degrees



The Equatorial and Ecliptic Coordinates of the Sun

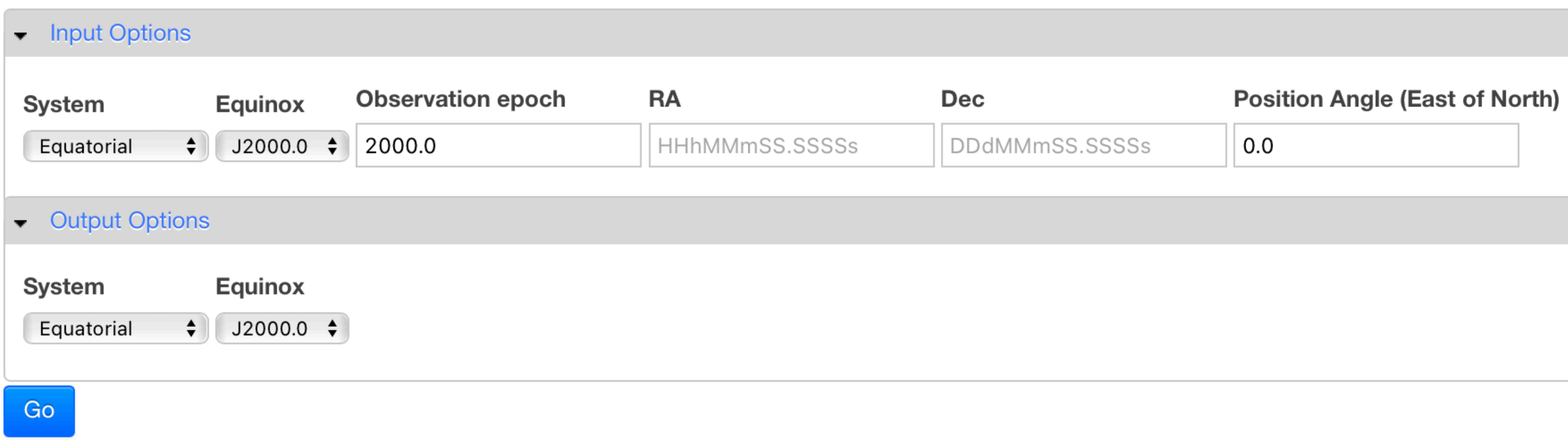
- In the course of a year, the Sun travels on the Ecliptic from Spring Equinox, to Summer Solstice, to Fall Equinox, to Winter Solstice, and back to Spring Equinox

	RA	*Dec*	*Ecliptic Longitude*	*Ecliptic Latitude*	*Notes*
Spring Equinox (Mar 20)	*0 hr*	*0 deg*	*0 hr*	*0 deg*	*Coordinates Origin*
Summer Solstice (Jun 21)	*6 hr*	*+23.5 deg*	*6 hr*	*0 deg*	*longest day in a year*
Fall Equinox (Sep 22)	*12 hr*	*0 deg*	*12 hr*	*0 deg*	*equal day and night*
Winter Solstice (Dec 21)	*18 hr*	*-23.5 deg*	*18 hr*	*0 deg*	*longest night in a year*

Conversion between the Equatorial and Ecliptic Coordinates

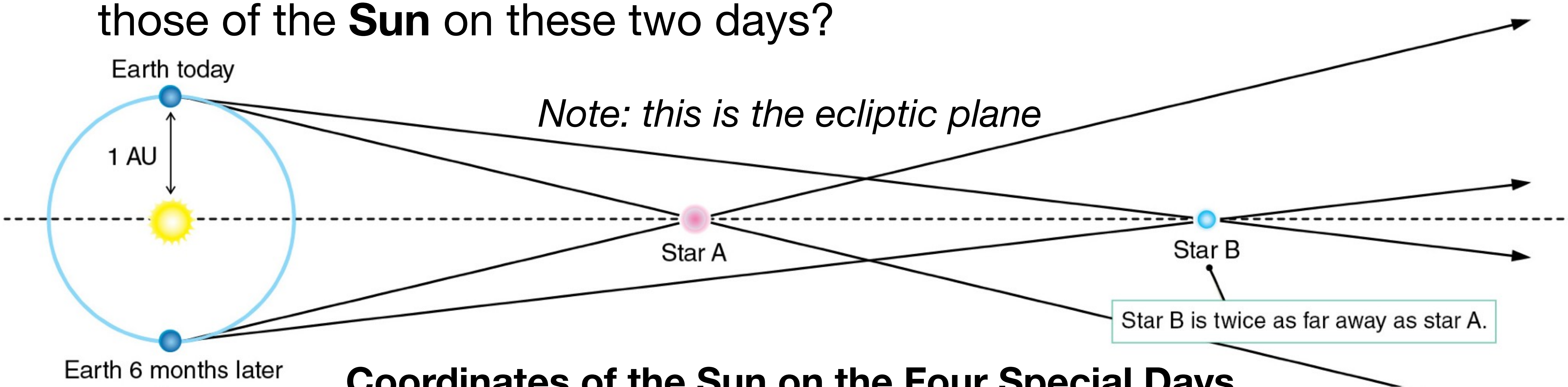
```
from astropy import units as u
from astropy.coordinates import SkyCoord

raval=[112.357708,122.580465,104.726966]
decval=[-12.69596,-5.513852,-10.580455]
coord_eq = SkyCoord(ra=raval*u.degree, dec=decval*u.degree, frame='icrs')
coord_ecl=coord_eq.transform_to('geocentricmeanecliptic')
latval=coord_ecl.lat.degree
longval=coord_ecl.lon.degree
print(latval,longval)
```

Coordinate Converter: https://ned.ipac.caltech.edu/coordinate_calculator



Sources on the Ecliptic Plane: Observational Considerations

- On these two days illustrated in the graph below, at **what local time** do Stars A and B transit the meridian?
- What are the **Ecliptic Longitudes** of Star A and Star B **relative to** those of the **Sun** on these two days?



Coordinates of the Sun on the Four Special Days

	RA	Dec	Ecliptic Longitude	Ecliptic Latitude	Notes
Spring Equinox (Mar 20)	*0 hr*	*0 deg*	*0 hr*	*0 deg*	*Coordinates Origin*
Summer Solstice (Jun 21)	*6 hr*	*+23.5 deg*	*6 hr*	*0 deg*	*longest day in a year*
Fall Equinox (Sep 22)	*12 hr*	*0 deg*	*12 hr*	*0 deg*	*equal day and night*
Winter Solstice (Dec 21)	*18 hr*	*-23.5 deg*	*18 hr*	*0 deg*	*longest night in a year*

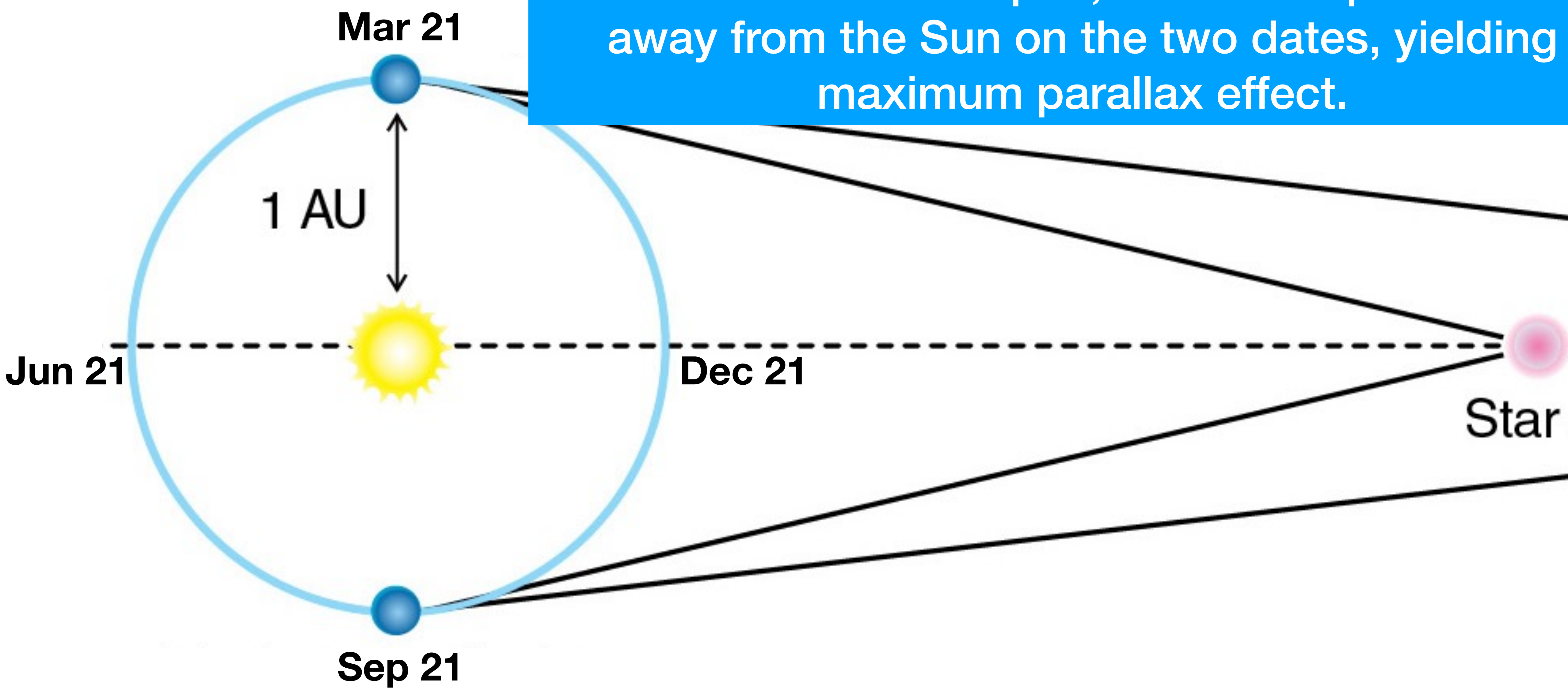
Let’s check the RA, Dec, & Dates in the previous practice example

Are these **dates** and **coordinates** arbitrary? Should its **RA** increase or decrease? Why its **Dec** did NOT change much?

Mar 21 2022: 06h00m15.205s 23d29’15.155”

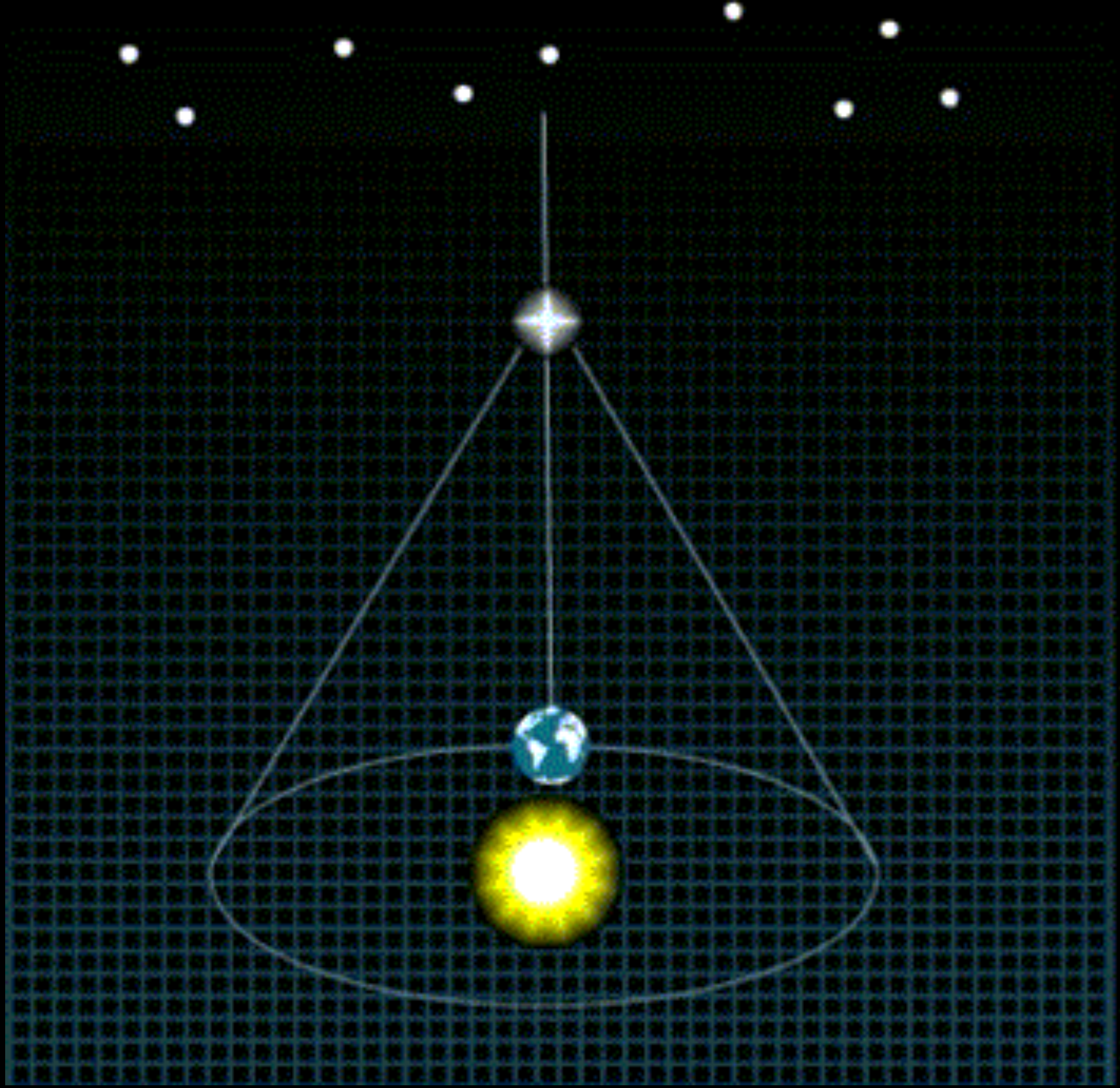
Sep 21 2022: 06h00m15.235s 23d29’15.160”

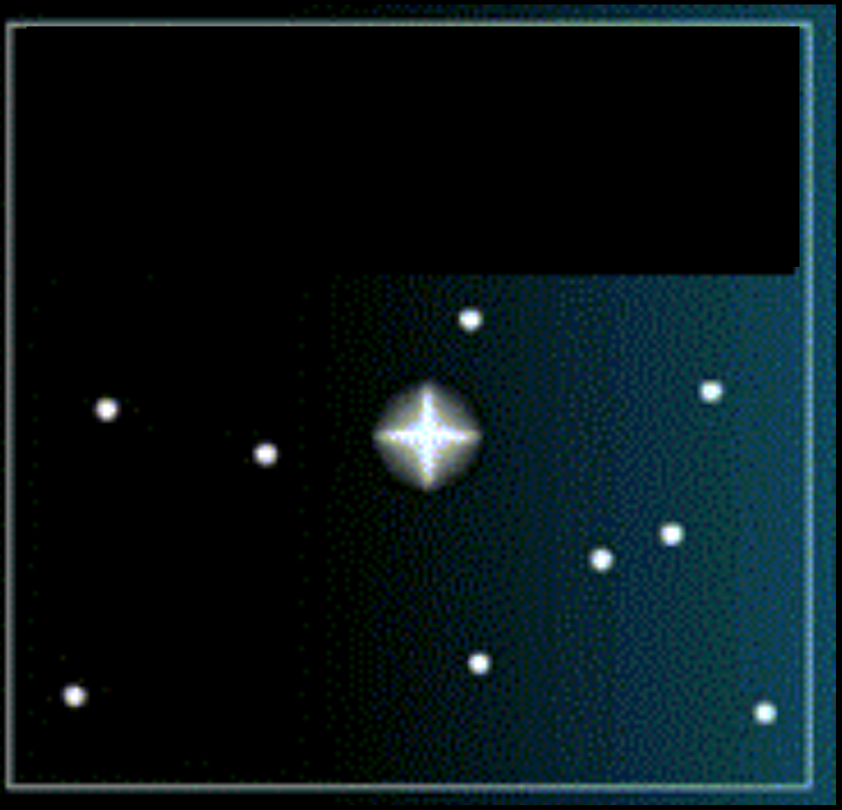
The star is on the ecliptic, and its RA places it 90° away from the Sun on the two dates, yielding maximum parallax effect.



Special case: sources near the ecliptic poles

Do we need to worry about when to observe them to measure parallax?

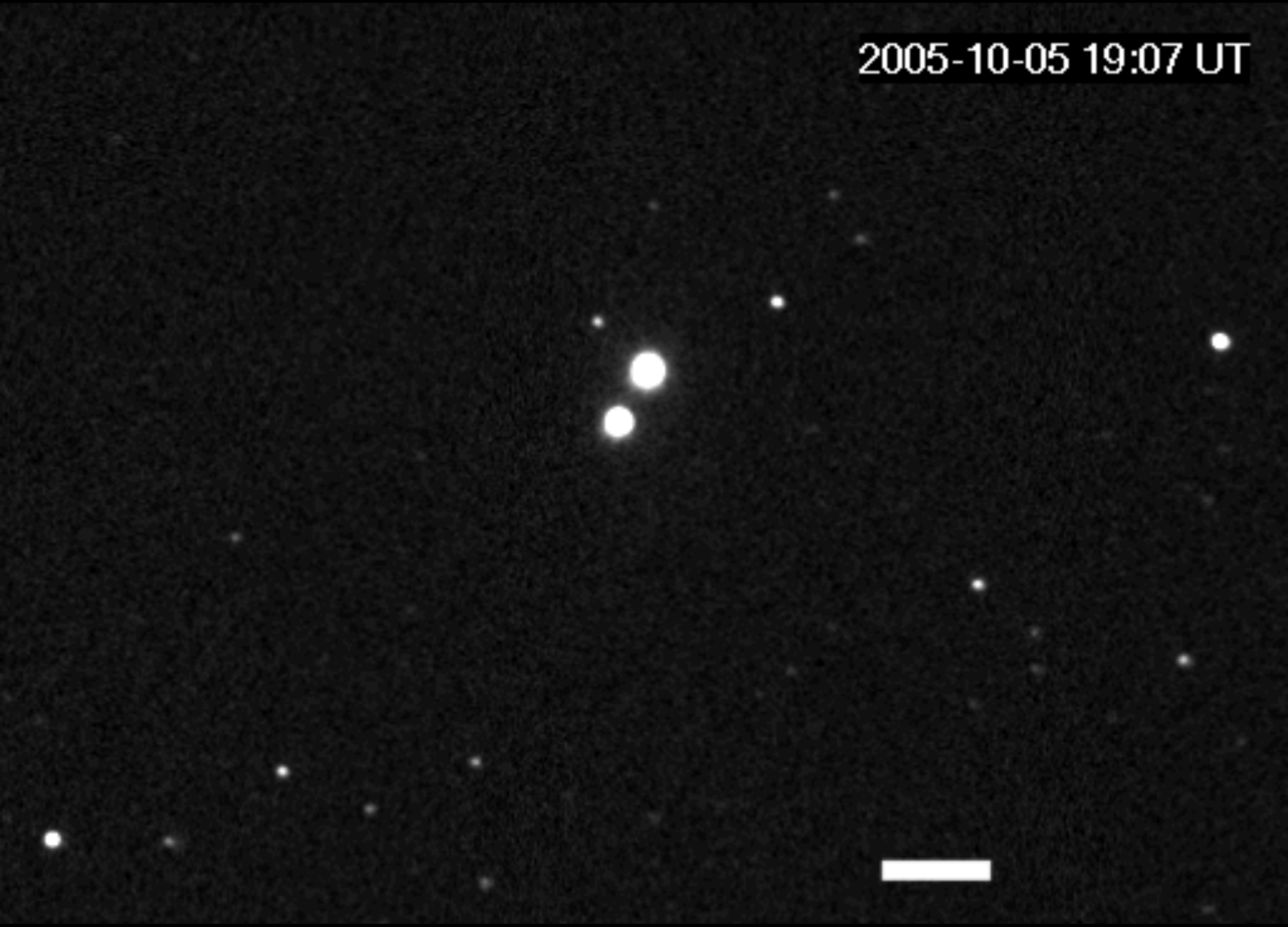




Weten Schaps

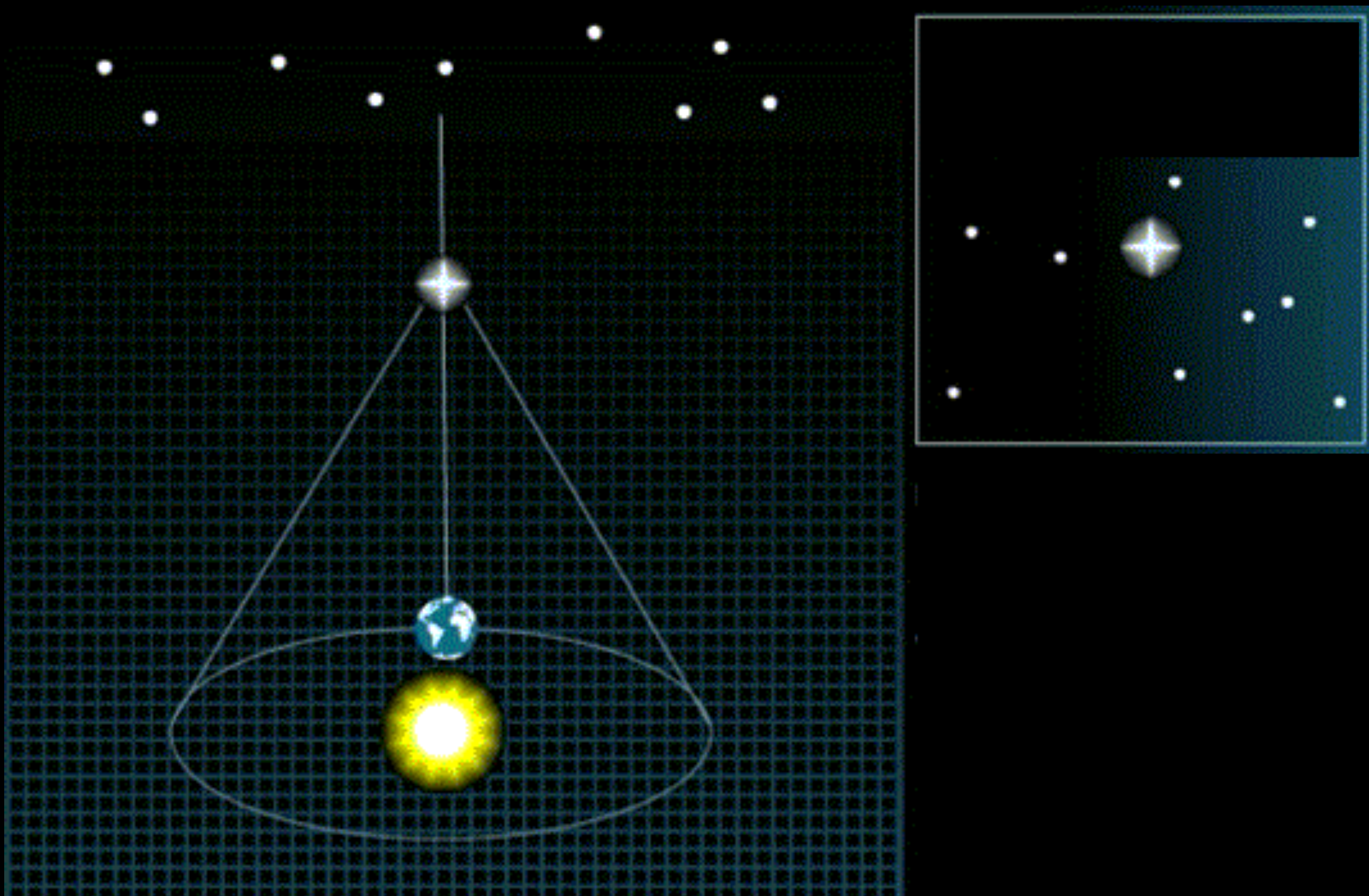
Annual Parallax Traces

What kind of pattern does a star draw on the sky due to Earth's annual motion?
We can record this pattern if we continuously monitor its position over a year


2005-10-05 19:07 UT

Simplest case: sources on the ecliptic poles

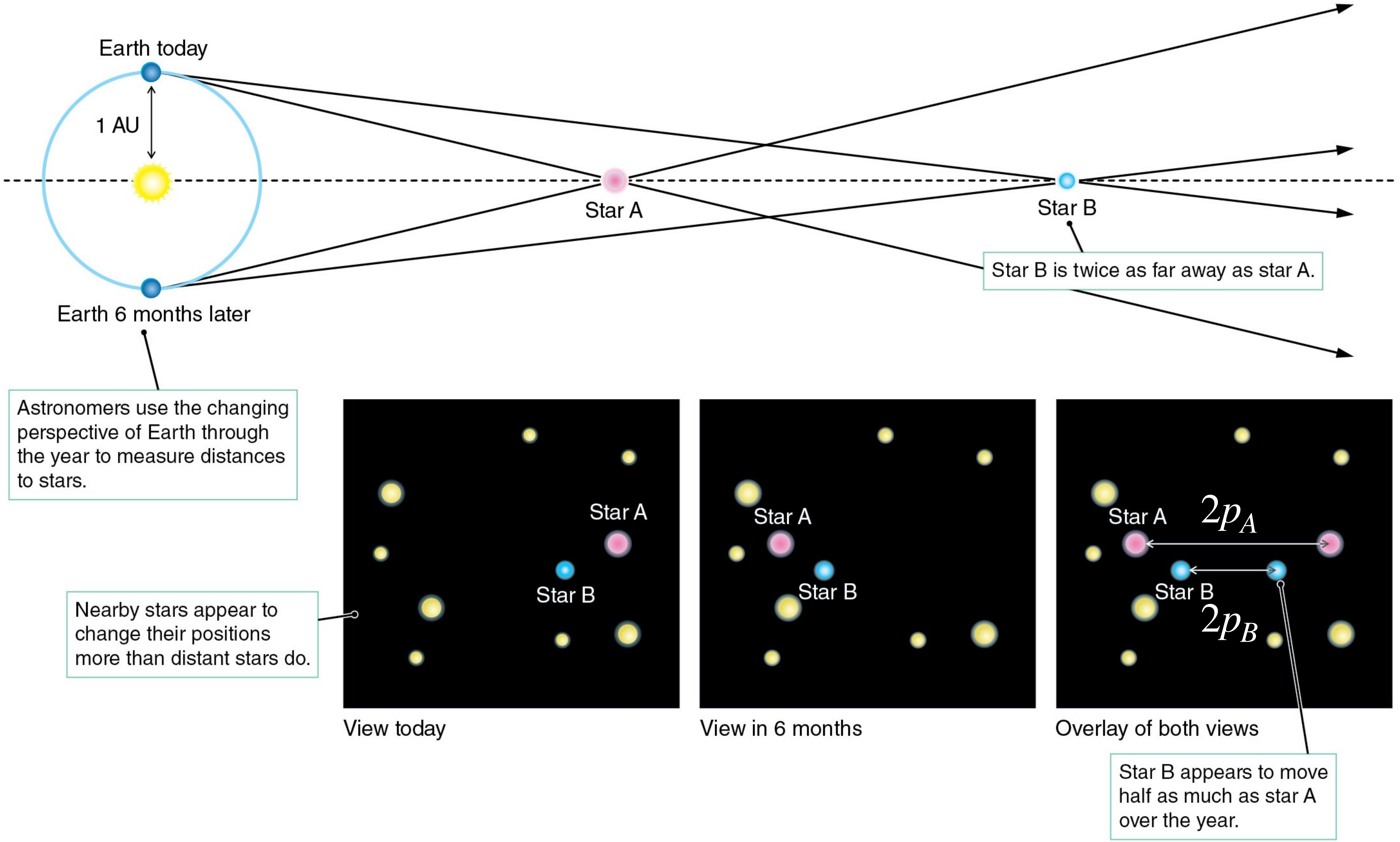
moving along a circle



Weten Schaps

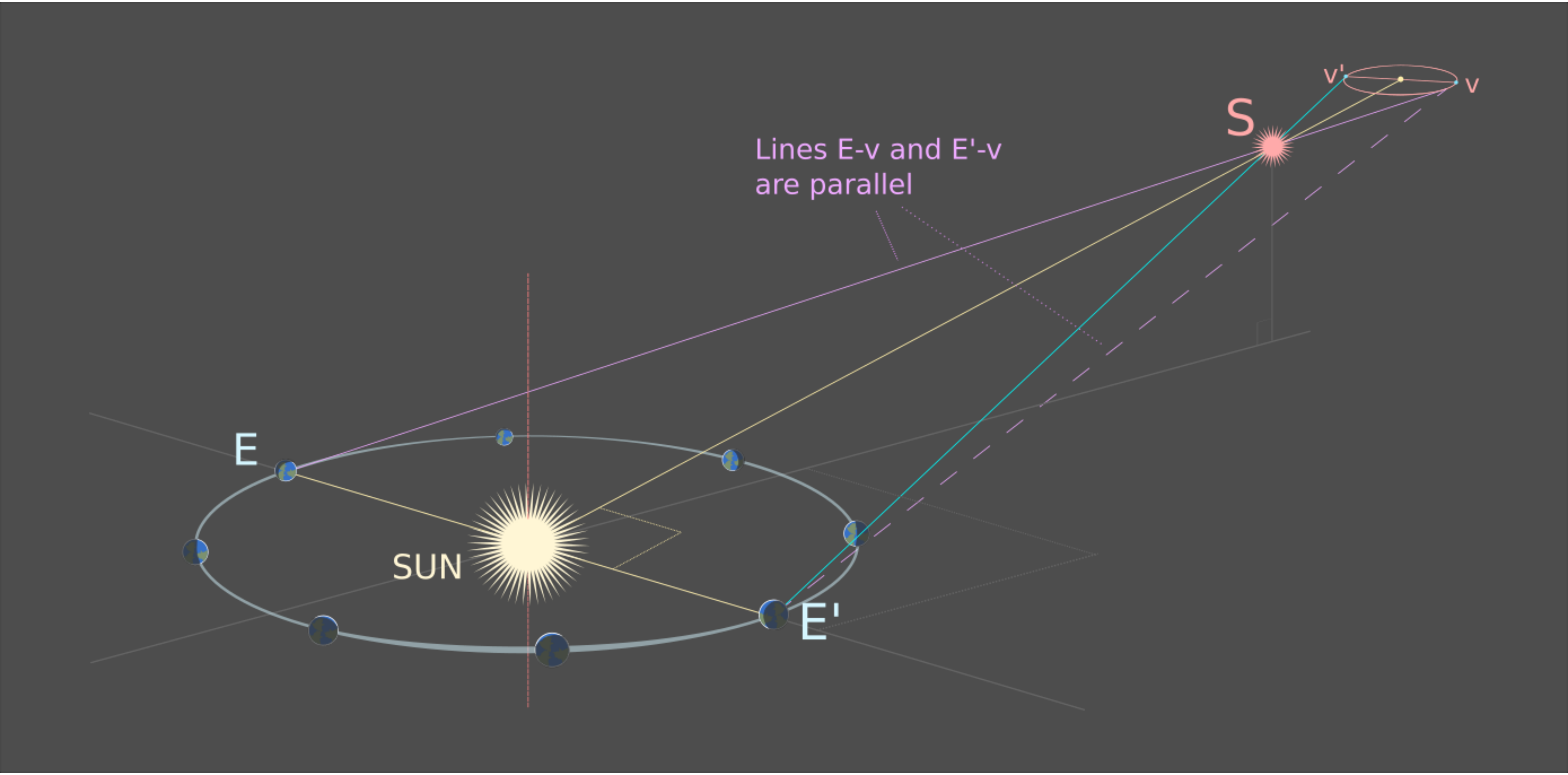
Simpler case: sources on the ecliptic plane

oscillating along a short line



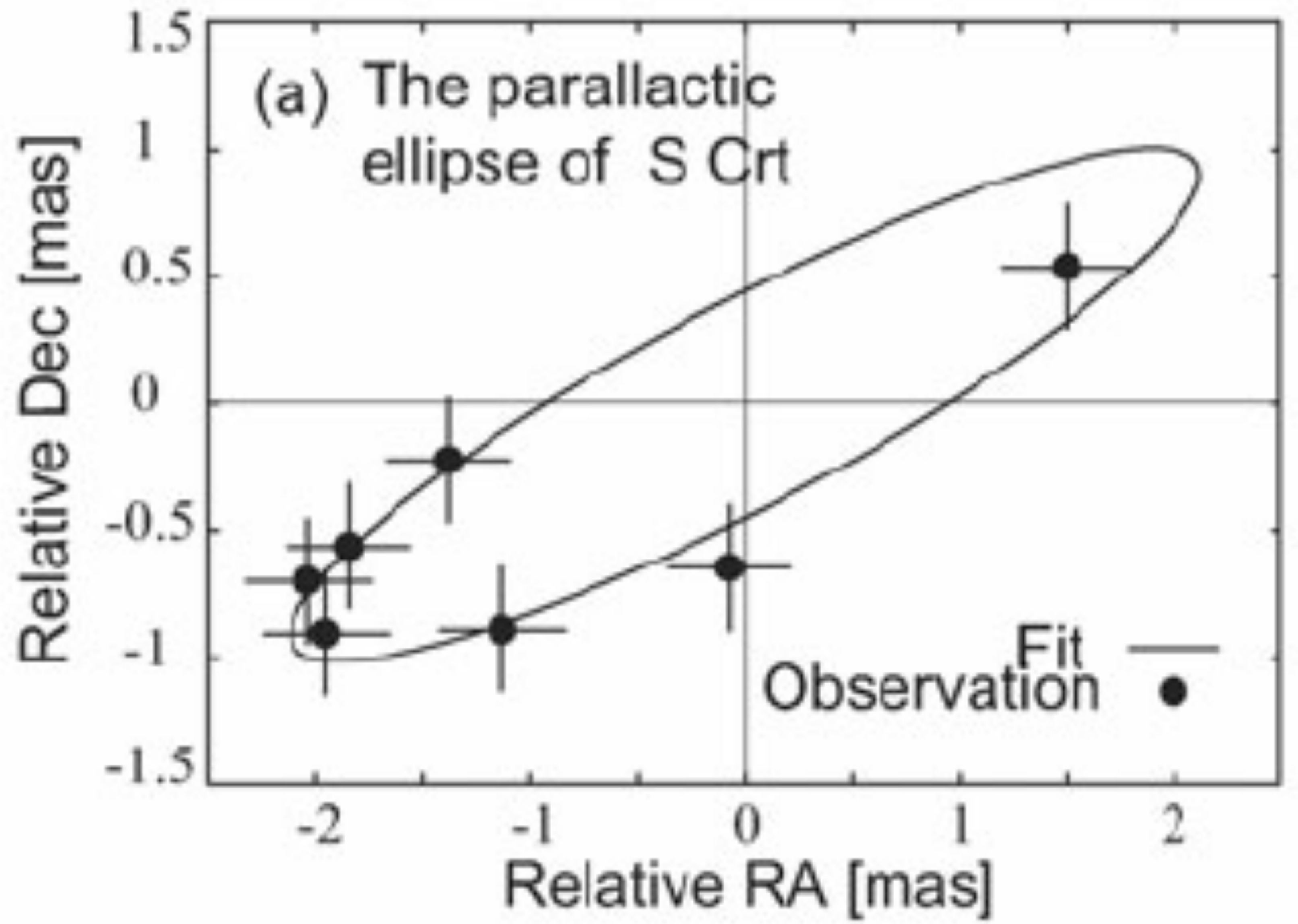
General cases: 0 < ecliptic latitude < 90 deg

moving along an ellipse



Summary: Parallactic Traces & Parallax Measurements

- Sources on the ecliptic oscillate on short lines along the ecliptic; the parallax to measure distance is half of the length of the line.
- Sources on the ecliptic poles draw parallactic circles; the parallax to measure distance is the radius.
- All other sources draw ellipses with major axes parallel to ecliptic; For a parallactic ellipse, what is the parallax to measure distance?



Nakagawa+2008

Summary

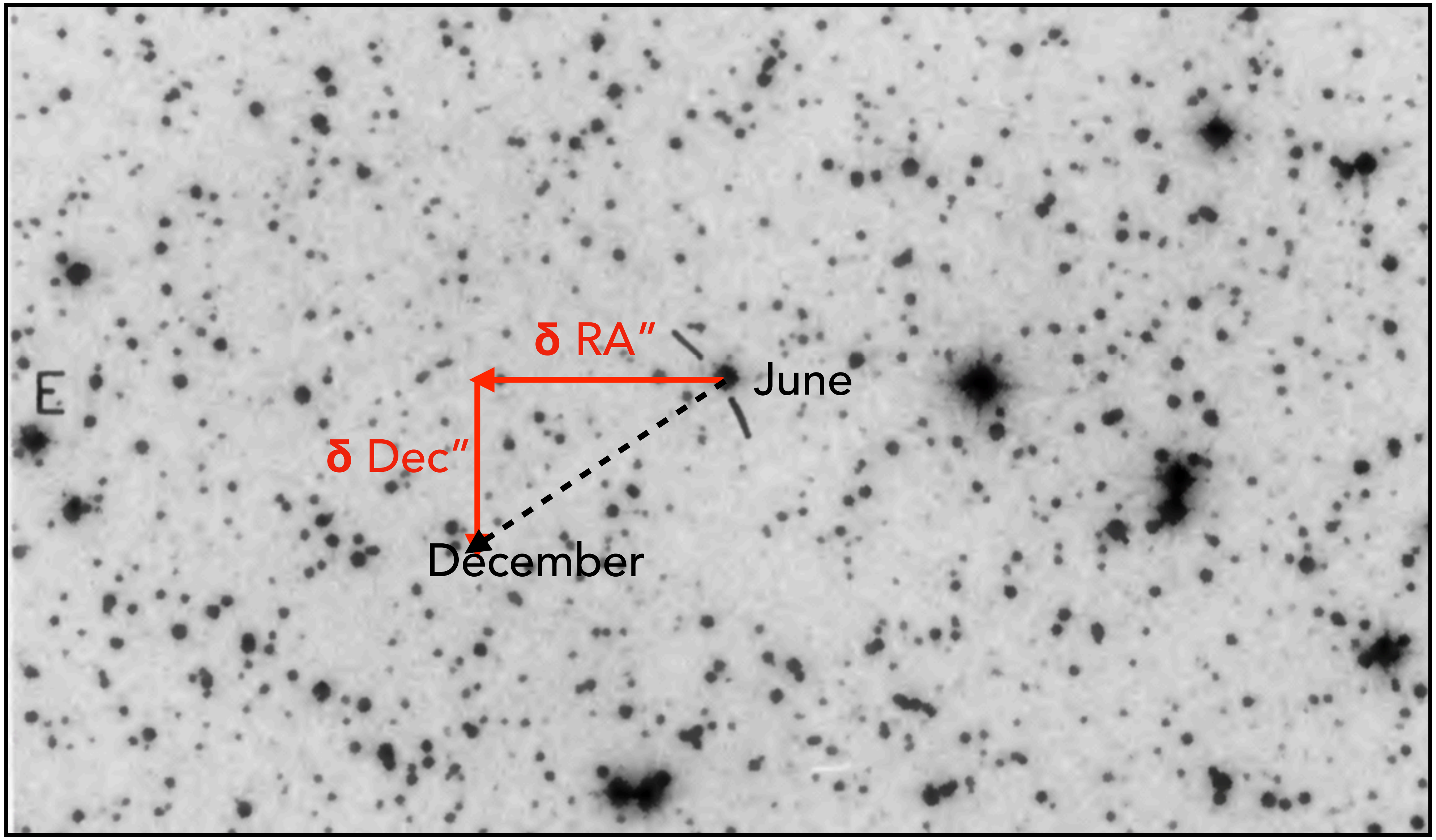
Advanced Topics of Parallax

Calculate angular offset given Equatorial coordinates

$$\Delta'' = \sqrt{\delta \mathrm{RA}''^2 + \delta \mathrm{Dec}''^2}$$

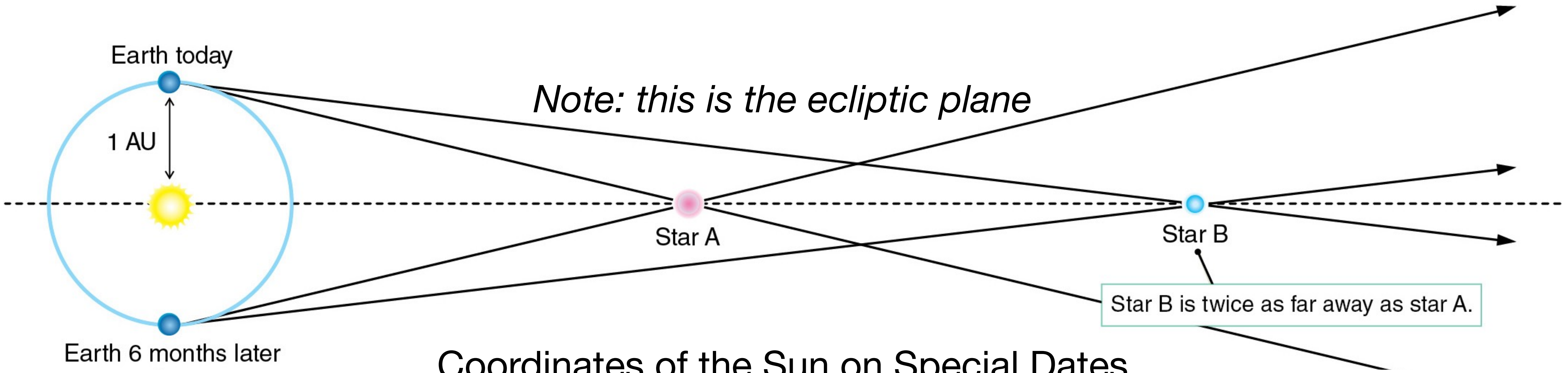
$$\delta \mathrm{RA}'' = (\mathrm{RA}_1^s - \mathrm{RA}_2^s) \cdot \cos(\mathrm{Dec}^\circ) \cdot 15''/\mathrm{s}$$

$$\delta \mathrm{Dec}'' = \mathrm{Dec}_1'' - \mathrm{Dec}_2''$$



Stellar Parallax: Observational Considerations

- To see maximum parallax effect, you must choose two nights when the **Ecliptic Longitudes** of the target is **6 hrs (90 deg)** away from the **Sun**.



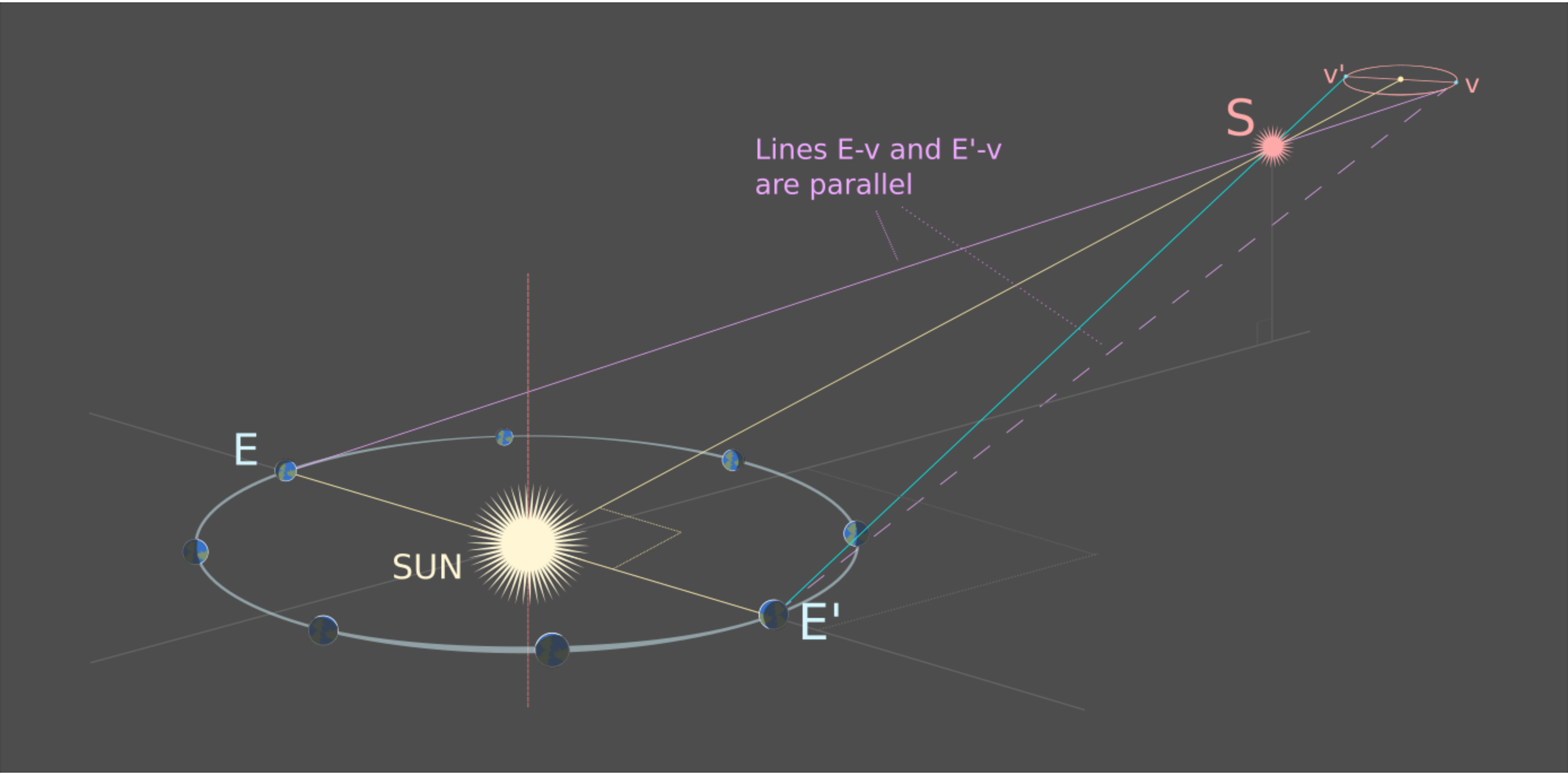
Coordinates of the Sun on Special Dates

	RA	Dec	Ecliptic Longitude	Ecliptic Latitude	Notes
Spring Equinox (Mar 20)	*0 hr*	*0 deg*	*0 hr*	*0 deg*	*Coordinates Origin*
Summer Solstice (Jun 21)	*6 hr*	*+23.5 deg*	*6 hr*	*0 deg*	*longest day in a year*
Fall Equinox (Sep 22)	*12 hr*	*0 deg*	*12 hr*	*0 deg*	*equal day and night*
Winter Solstice (Dec 21)	*18 hr*	*-23.5 deg*	*18 hr*	*0 deg*	*longest night in a year*

Parallax Ellipse: the trace cross a full year's observations

General cases: 0 < ecliptic latitude < 90 deg

moving along an ellipse

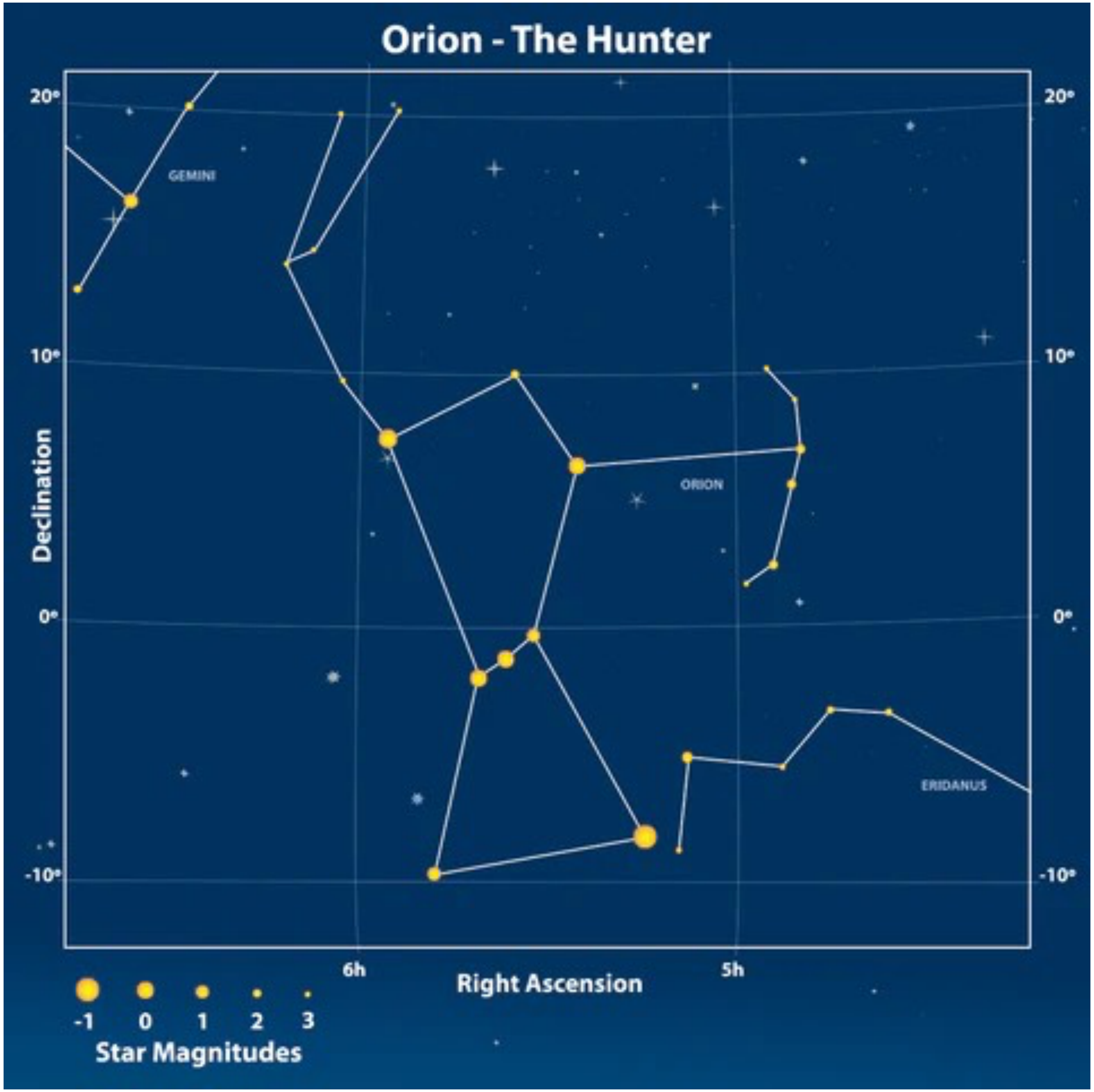


Brightness Measurements: Apparent Magnitude

Visual classification of brightness: The Greek Magnitude System

Ancient Greeks: "*the stars that appear first after sunset are the 1st magnitude stars, the stars that appear second are the 2nd magnitude stars, and so on ……*"

129 BC, first formally introduced by **Hipparchus**, then refined by **Ptolemy** in **150 AD**:
visual classification of stars into 6 classes, **brightest as being of 1st magnitude, faintest of 6th magnitude**



MAGNITUDE & ENERGY FLUX

A BRIEF HISTORY

- 129 BC, first Hipparchus, then refined by Ptolemy in 150 AD: visual classification of stars into 6 classes, brightest as being of 1st magnitude, faintest of 6th magnitude
- 1856, Norman Pogson: 5 magnitude difference = 100x in **energy flux**, while preserving historically classified 6th mag stars, some brightest stars have negative magnitudes (e.g., **Sirius**, V-band mag = -1.5). Summarized in an equation, we have **Pogson's ratio**: $m_{\lambda,1} - m_{\lambda,2} = -2.5 \log(f_{\lambda,1}/f_{\lambda,2})$
- 1850s - 1990s: photographic glass plates
- 1940s, photoelectric cells, tubes, photomultipliers
- 1969, Boyle & Smith: CCD detectors (2009 Nobel Prize for Physics). First used in astronomy in 1976 at U. of Arizona

Observed Brightness of Stars show a HUGE range

- The Sun is the brightest star, which dominates the sky during the day, rendering it impossible to see any other stars
- The faintest star your eye can see is $\mathbf{10^{13}}$ fainter than the Sun
- The faintest star that can be detected by the Hubble space telescope is $\mathbf{10^{20}}$ fainter than the Sun.

- How do we deal with such a large range? We put everything on a logarithmic scale similar to that used by the Greeks, thus preserving the history started from Hipparchus in 129 BC.
- As a result, brighter stars still have lower magnitudes (*a minor annoyance astronomy students have to live with*).

- Mathematically we have the Pogson's ratio**:**

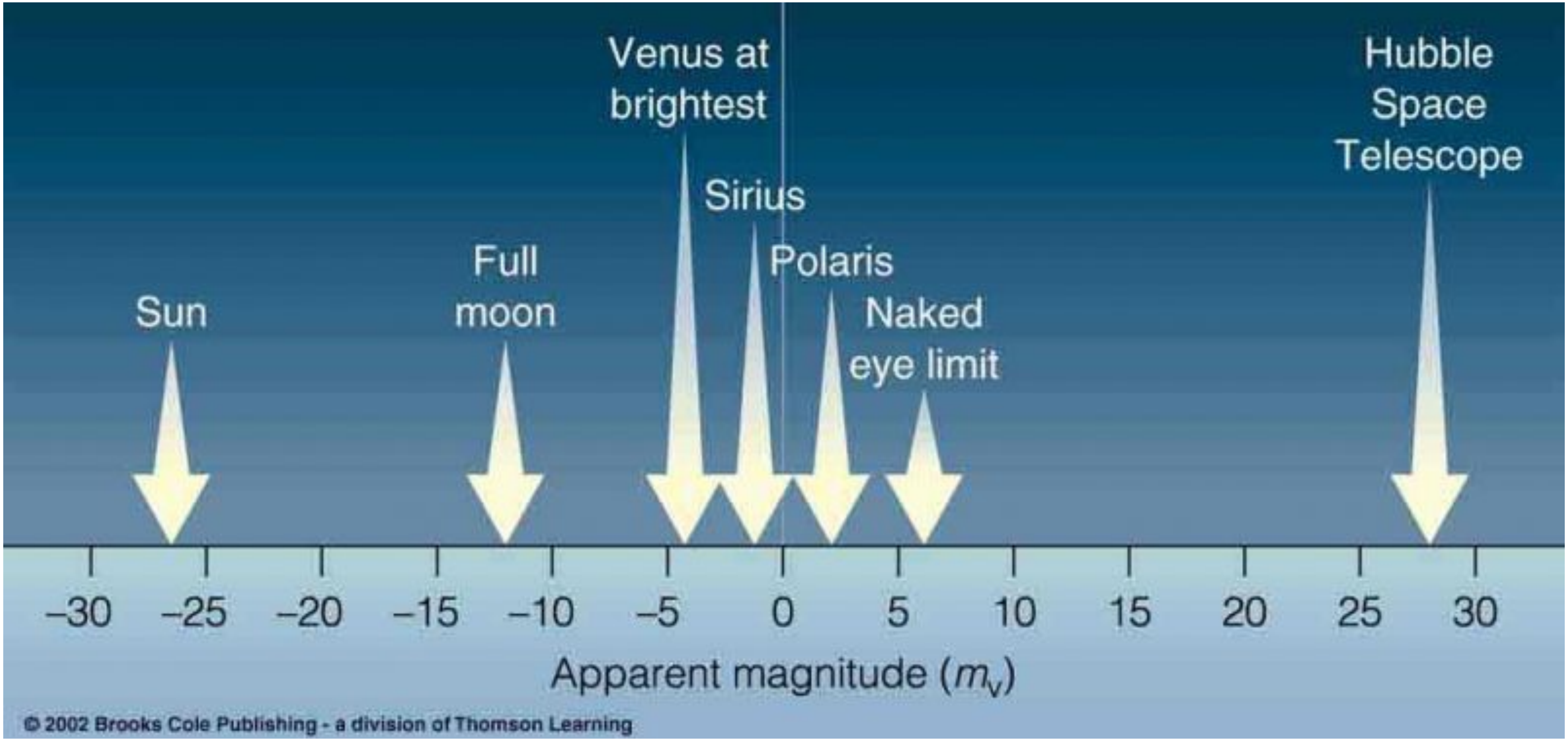
$$m_{\lambda,1} - m_{\lambda,2} = -2.5\log(f_{\lambda,1}/f_{\lambda,2})$$

to tell the **magnitude difference** between two objects, but how do we put the magnitudes on a universal scale so that a given magnitude means the same flux to all astronomers?

The universal magnitude system based on reference objects

$$m_\lambda - m_{\lambda,0} = -2.5\log(f_\lambda / f_{\lambda,0})$$

- where **_0** indicate the chosen **reference source**'s magnitude and flux at wavelength lambda. In optical wavelengths, the reference star is **Vega**.



The universal magnitude system based on Vega

$$m_\lambda - m_{\lambda,0} = -2.5\log(f_\lambda / f_{\lambda,0})$$

- Normally in the optical wavelengths, the reference star is **Vega**.
- For simplicity, **Vega's** magnitude is set to be **zero** at all wavelengths
- As a result, we have the Vega magnitude defined in the following equation:

$$\text{Vega magnitude}: m_\lambda = -2.5\log(f_\lambda / f_{\lambda,\text{Vega}})$$

Practice: From flux ratio to apparent magnitude relative to Vega

- What's the magnitude of a star that is 50x fainter than Vega at 500nm?
- What's the magnitude of a star that is 30x fainter than Vega?

m(50x fainter) = 4.25
m(30x fainter) = 3.69

Practice: From apparent magnitude to flux ratio

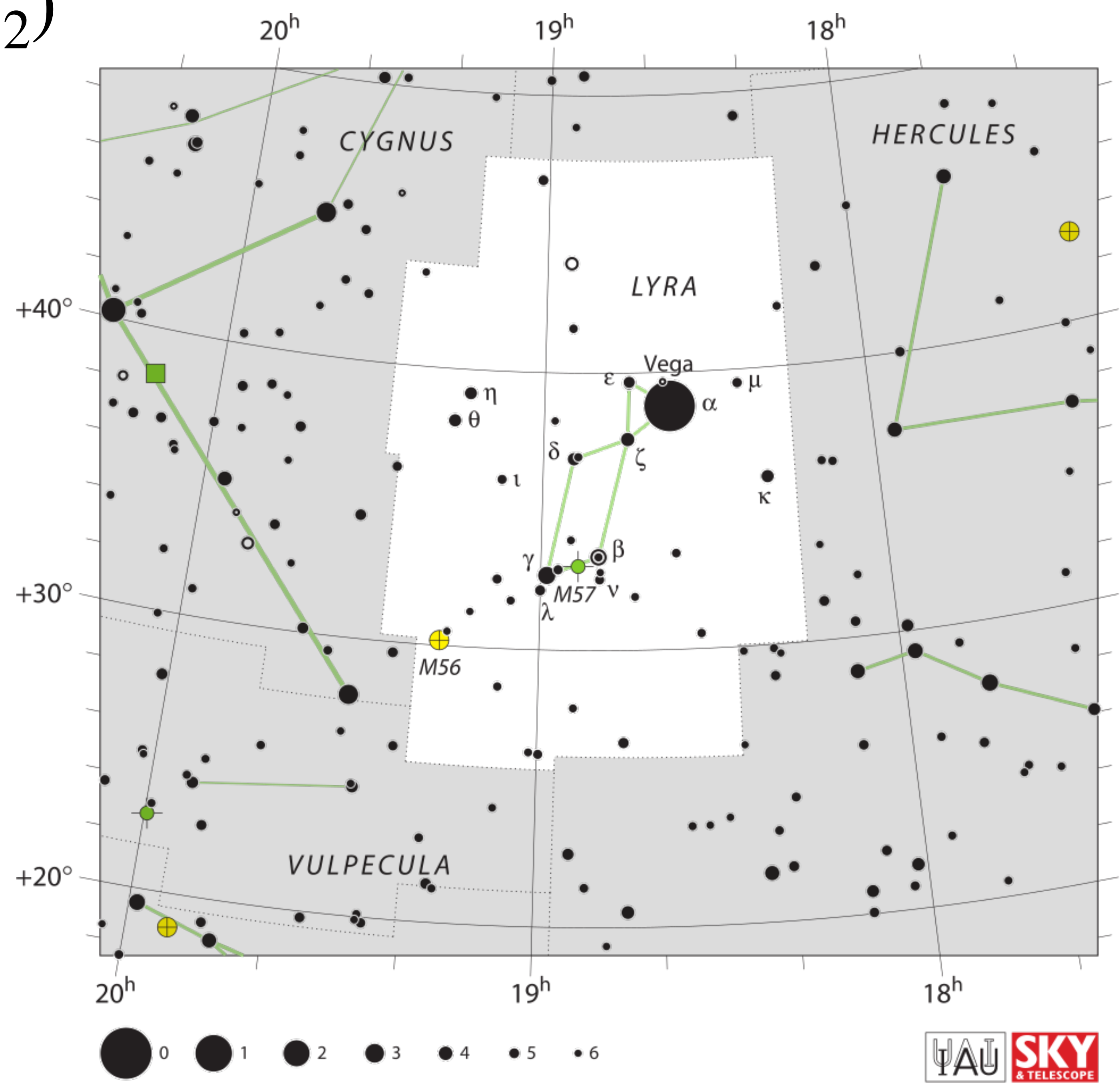
$$\text{Pogson's ratio}: m_{\lambda,1} - m_{\lambda,2} = -2.5 \log\left(\frac{f_{\lambda,1}}{f_{\lambda,2}}\right)$$

$$\Rightarrow \frac{f_{\lambda,1}}{f_{\lambda,2}} = 10^{-0.4(m_{\lambda,1} - m_{\lambda,2})}$$

- **δ Lyrae** has an apparent magnitude of **4.2** in V-band (551 nm), how many times fainter is it compared to Vega (α Lyrae)?
- **17 Lyrae** has an apparent magnitude of **5.2** in V-band, how many times fainter is it compared to **δ Lyare**?

10^(0.4*4.2) = 47.9

10^(0.4*(5.2-4.2)) = 2.512



Summary: Apparent Magnitude and Flux Ratio

- 100x in flux ratio corresponds to a magnitude difference of 5
- 1 magnitude difference corresponds to 2.514x (=10^0.4) difference in flux

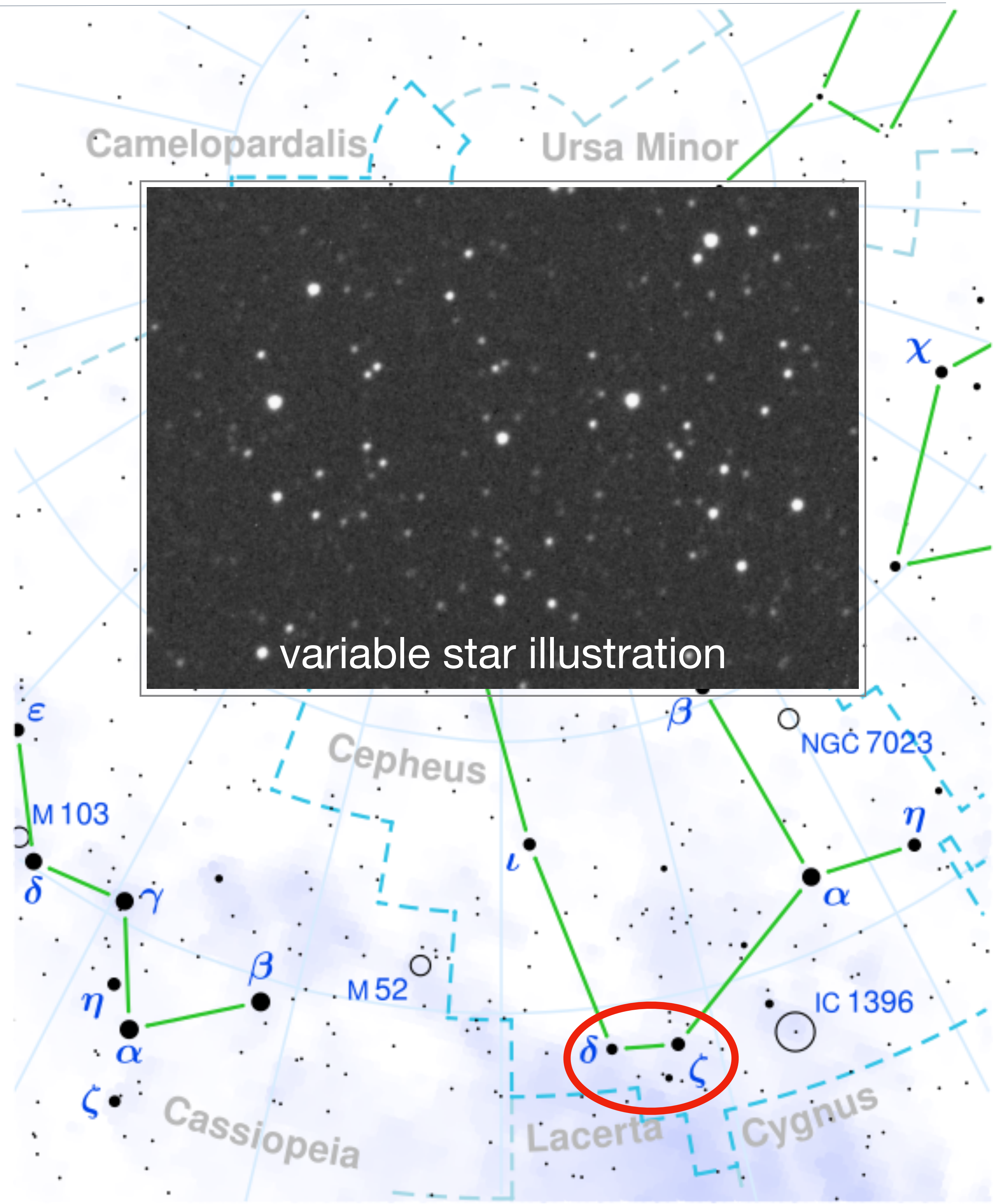
- To determine the magnitude of one source, you must know the magnitude and flux of another source (reference or standard) and compare the fluxes of the two sources

$$m_{\lambda,1} - m_{\lambda,2} = -2.5 \log\left(\frac{f_{\lambda,1}}{f_{\lambda,2}}\right)$$

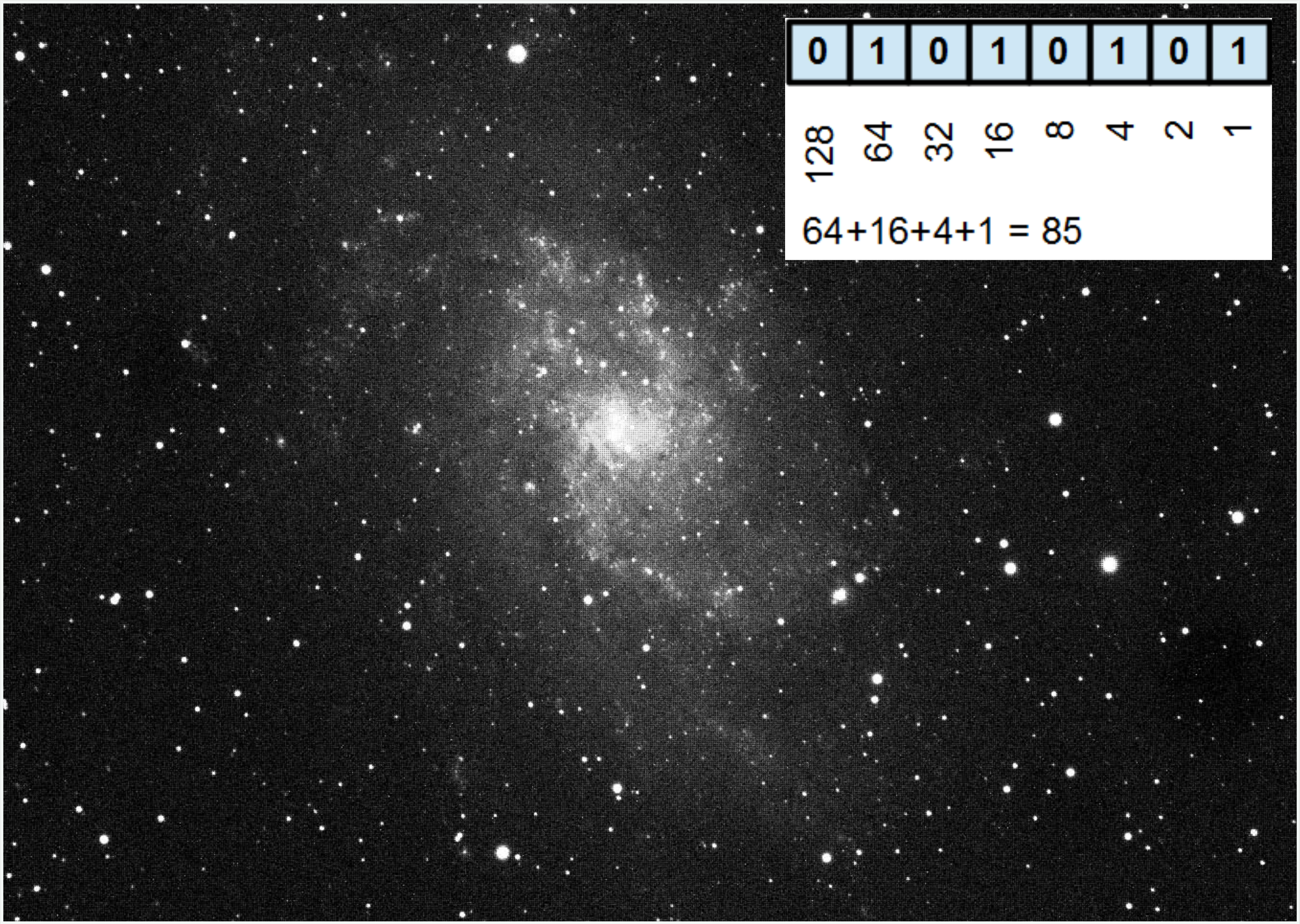
$$\Rightarrow \frac{f_{\lambda,1}}{f_{\lambda,2}} = 10^{-0.4(m_{\lambda,1} - m_{\lambda,2})}$$

Differential Photometry: e- Count Rates to Magnitudes

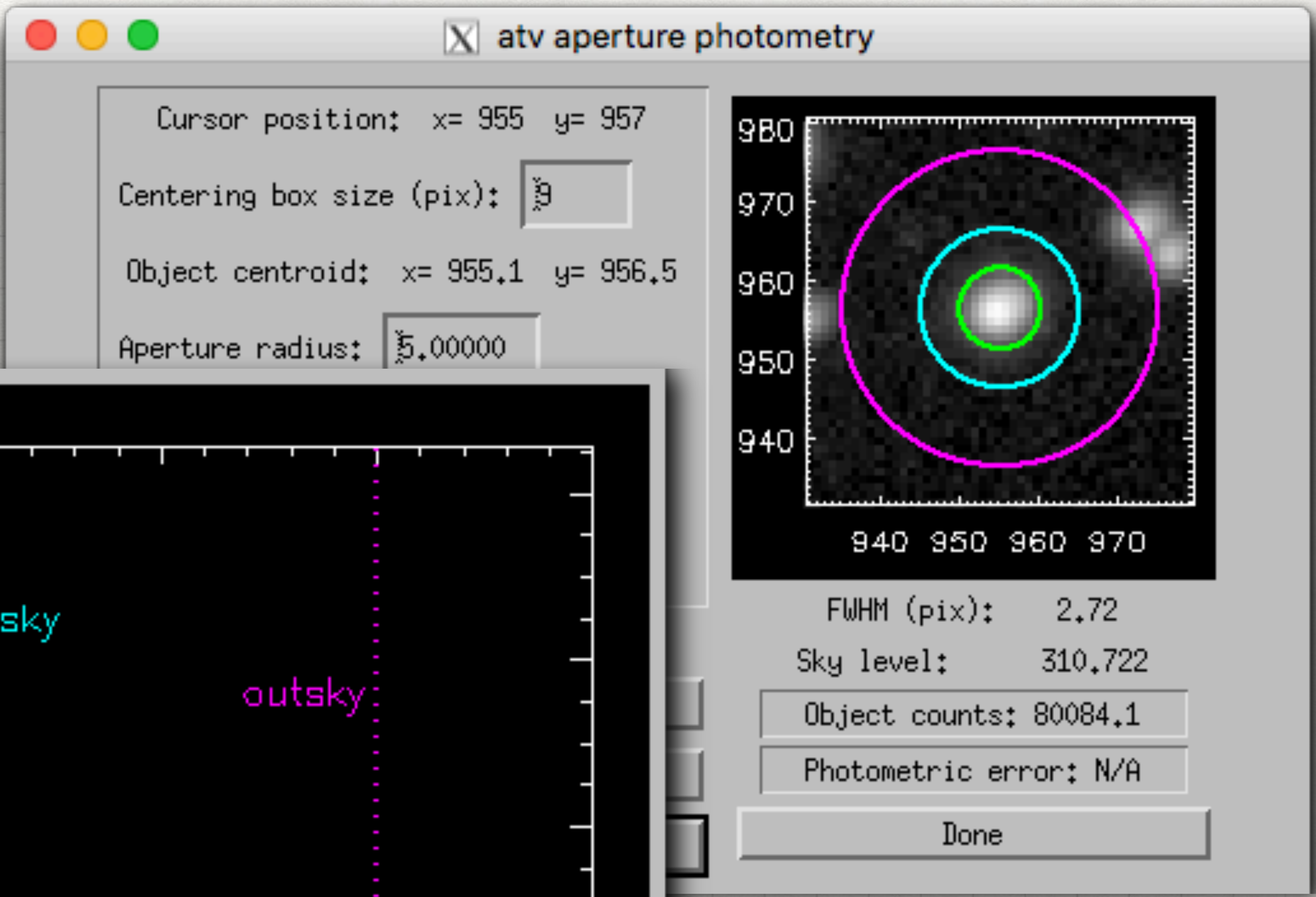
- We can point the same telescope at **two different sources *simultaneously*** and measure the **ratio** of their **count rates**.
- This approach is easier because all instrumental effects in the two measurements cancel out.
- If we know the magnitude from one of the sources, we can infer the magnitude of the other source using this relative measurement.

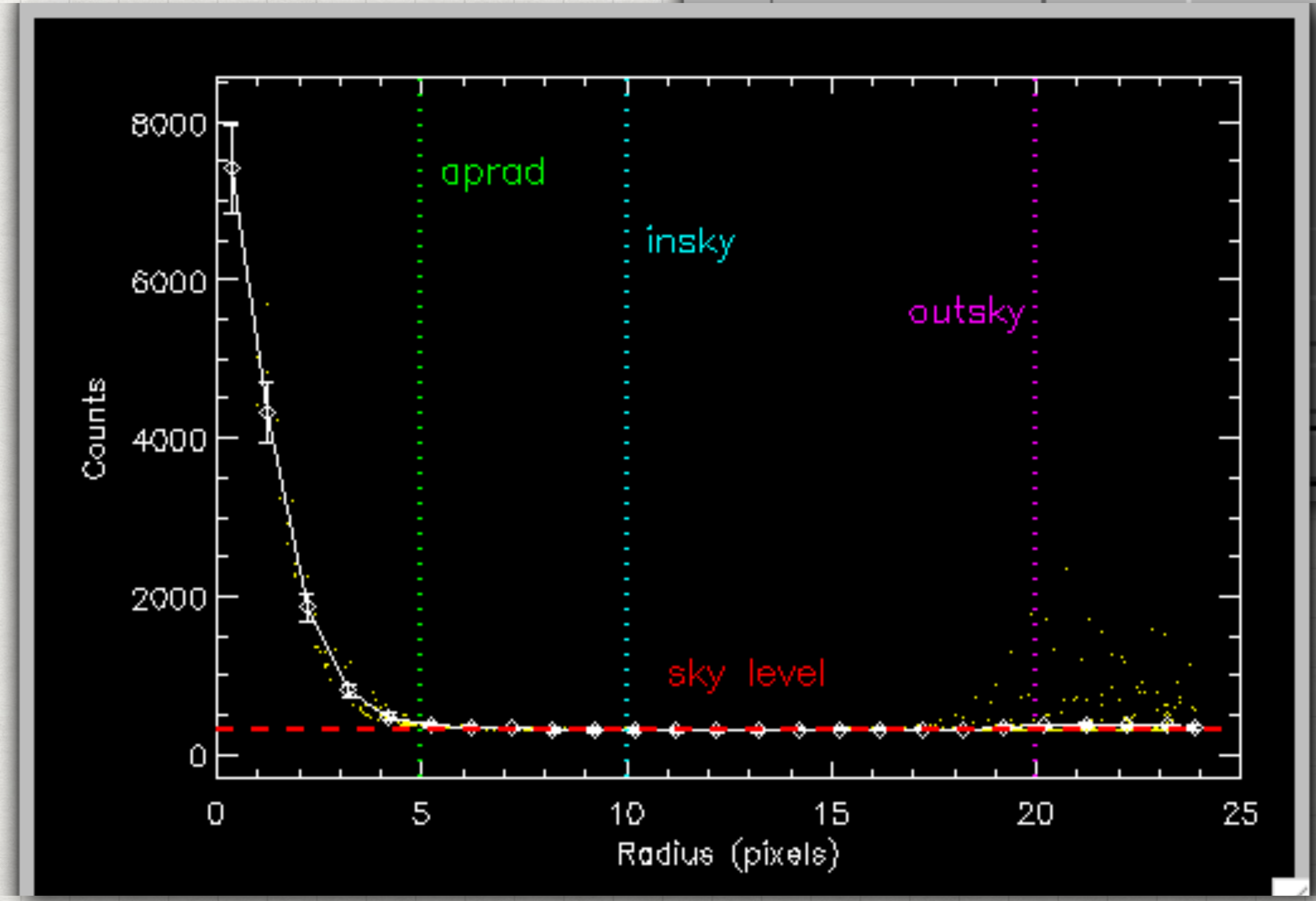


A typical CCD image - data illustrated with DS9: the number of e- collected in each pixel (from 0 to ~65k; 16 bit) is represented by only 256 shades of gray (8 bit)



TO COUNT ELECTRONS FROM A SOURCE, WE USE APERTURES





Definition of Magnitudes is based on Differential Photometry

$$m_\lambda = m_{\lambda\mathrm{ref}} - 2.5\log(f_\lambda / f_{\lambda,\mathrm{ref}})$$

the reference source here does not need to be Vega, it just needs to be a relatively stable source with a known magnitude

Count rates to magnitude difference

$$m_a - m_b = -2.5\log\left(\frac{F_a}{F_b}\right) = -2.5\log\left(\frac{Q_a/t_a}{Q_b/t_b}\right)$$

where object a is your science target and object b is the reference source with known magnitudes.

Practice: from count rates to magnitude

$$m_a - m_b = -2.5 \log\left(\frac{F_a}{F_b}\right) = -2.5 \log\left(\frac{Q_a/t_a}{Q_b/t_b}\right)$$

where object a is your science target and object b is the standard star with known magnitudes.

Your standard star has a magnitude of 10.5 mag in V-band, you took a CCD image of the standard star with a V-band filter and you got a total of 1500 counts in 10 seconds.

Next, you slew the telescope to take a V-band image of your science target, say a random galaxy far away, and with 30 min exposure, you could barely see it. The total count from the galaxy is 50.

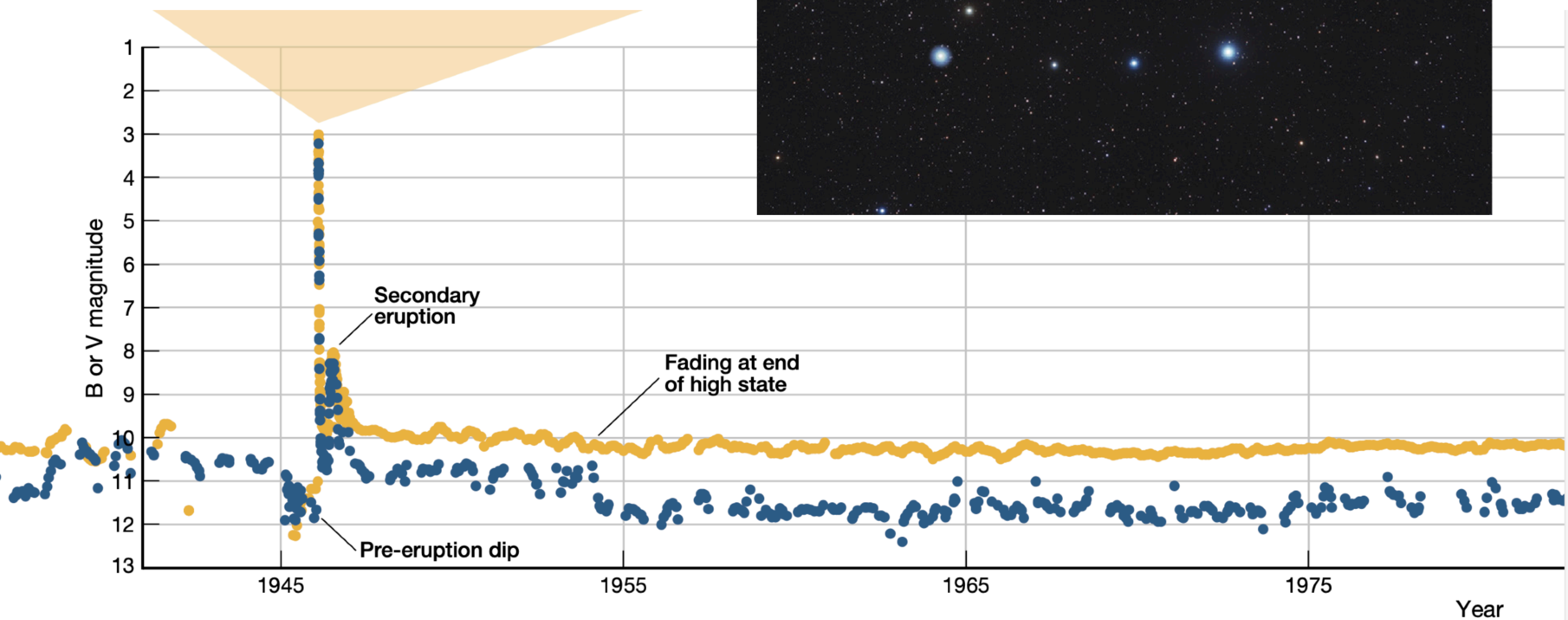
What's the V-band magnitude of the galaxy?

$V_{galaxy} = 10.5 - 2.5 \log((50/1800)/(1500/10)) = 19.83$

Get Ready for a Nova's Bright Return in 2024

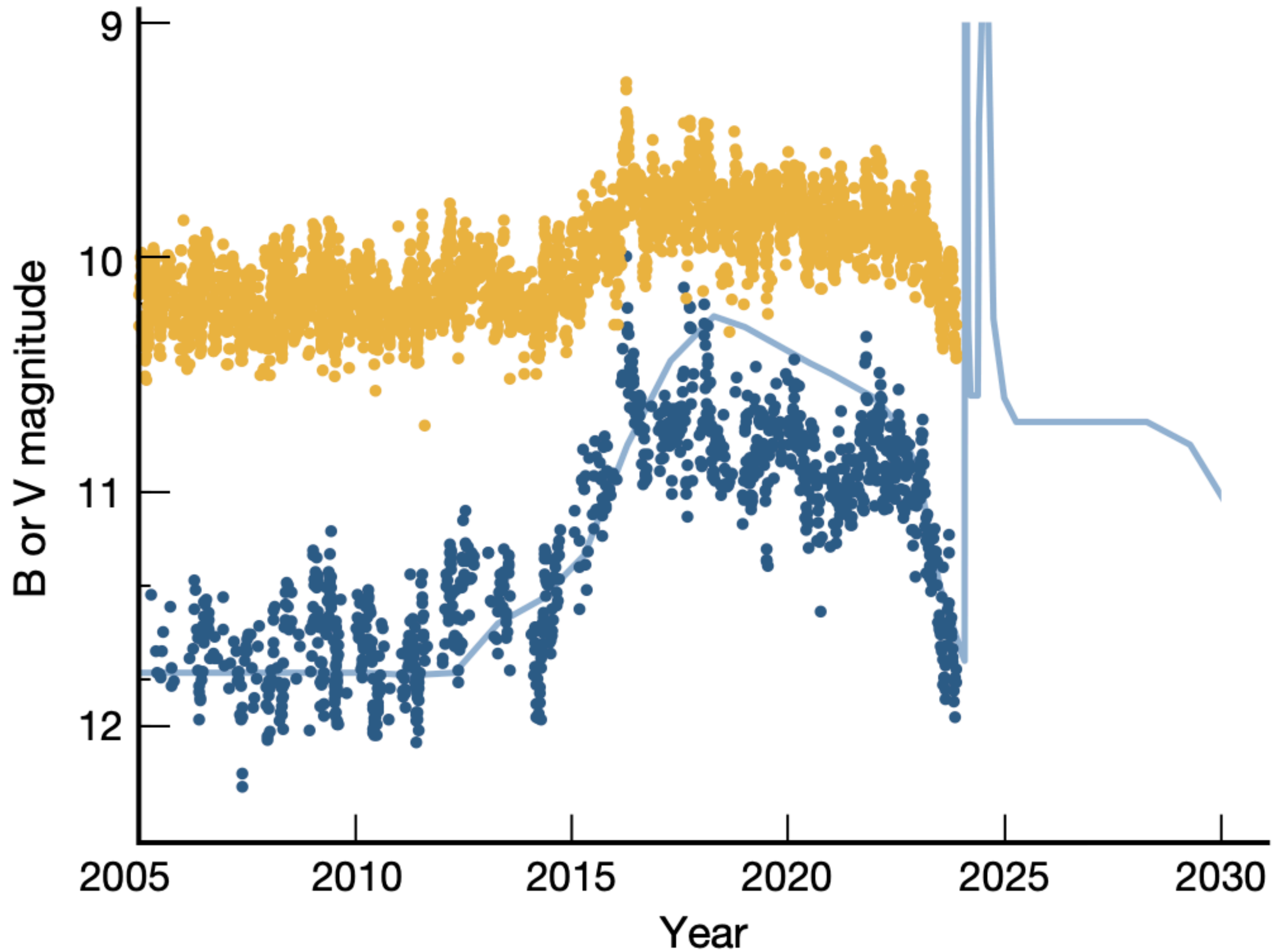
Recurrent Nova
T Coronae Borealis

The Northern Crown



38 MARCH 2024 • SKY & TELESCOPE

Get Ready for a Nova's Bright Return in the Northern Crown



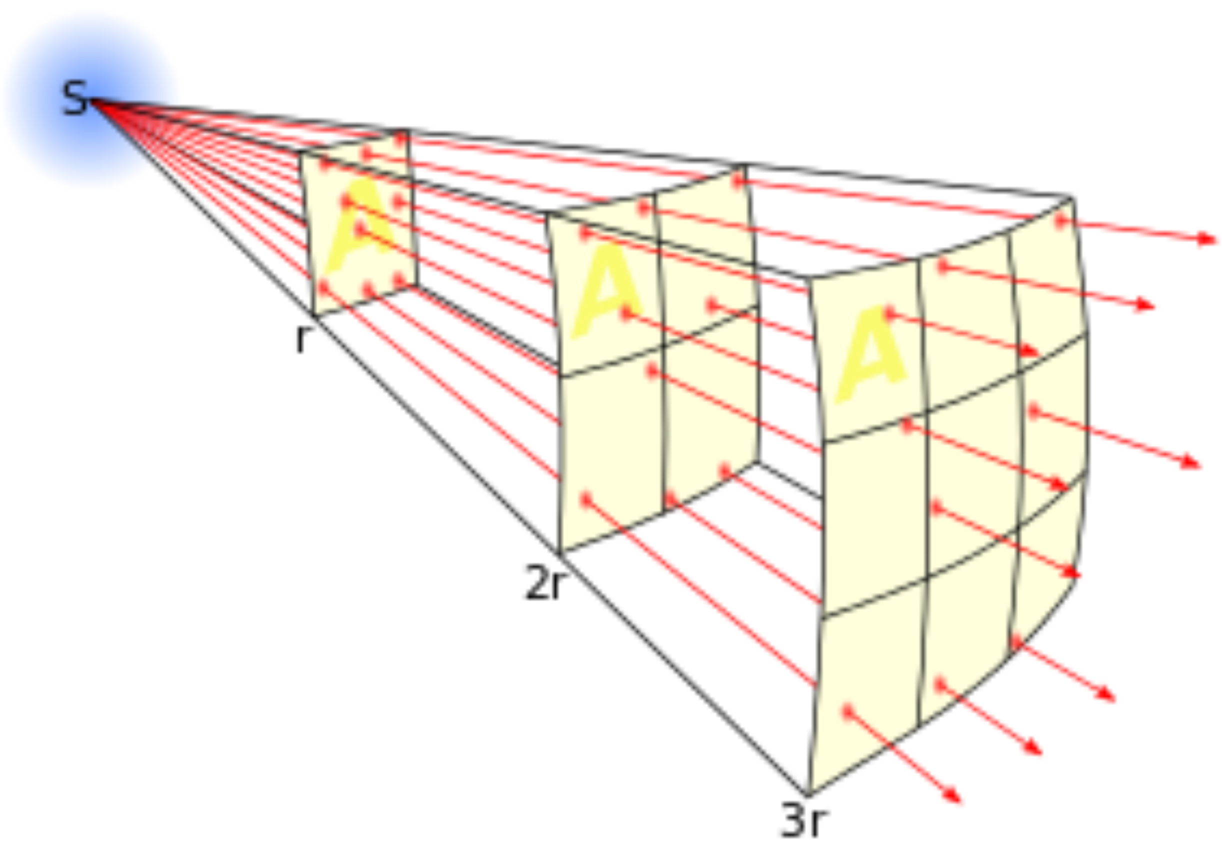
▲ **LEAD-UP TO 2024** This closeup shows T CrB's brightness in visual (V, in orange) and blue light (B, in blue) from 2005 to 2023. The rise from the low state to the high state (starting around 2015) and the decline in the pre-eruption dip (starting around March 2023) are both more prominent in blue light. The 1927-to-1952 blue light curve is shown as the faint blue line. Assuming the nova follows the same pattern it did in the 1930s and 1940s, it should erupt soon.

Luminosity Measurements: Absolute Magnitude

(requires Distance & Brightness)

The Inverse Square Law of Flux

- **Luminosity** is the total amount of energy per unit time (i.e., power) emitted by the source (unit: Watt = Joule/s)
- **Flux** is the amount of arriving energy per unit time per unit area (unit: Watt/m^2) at a distance *d* from source
- **Flux** decreases as the **distance** from the source increases, obeying an **inverse square law**:

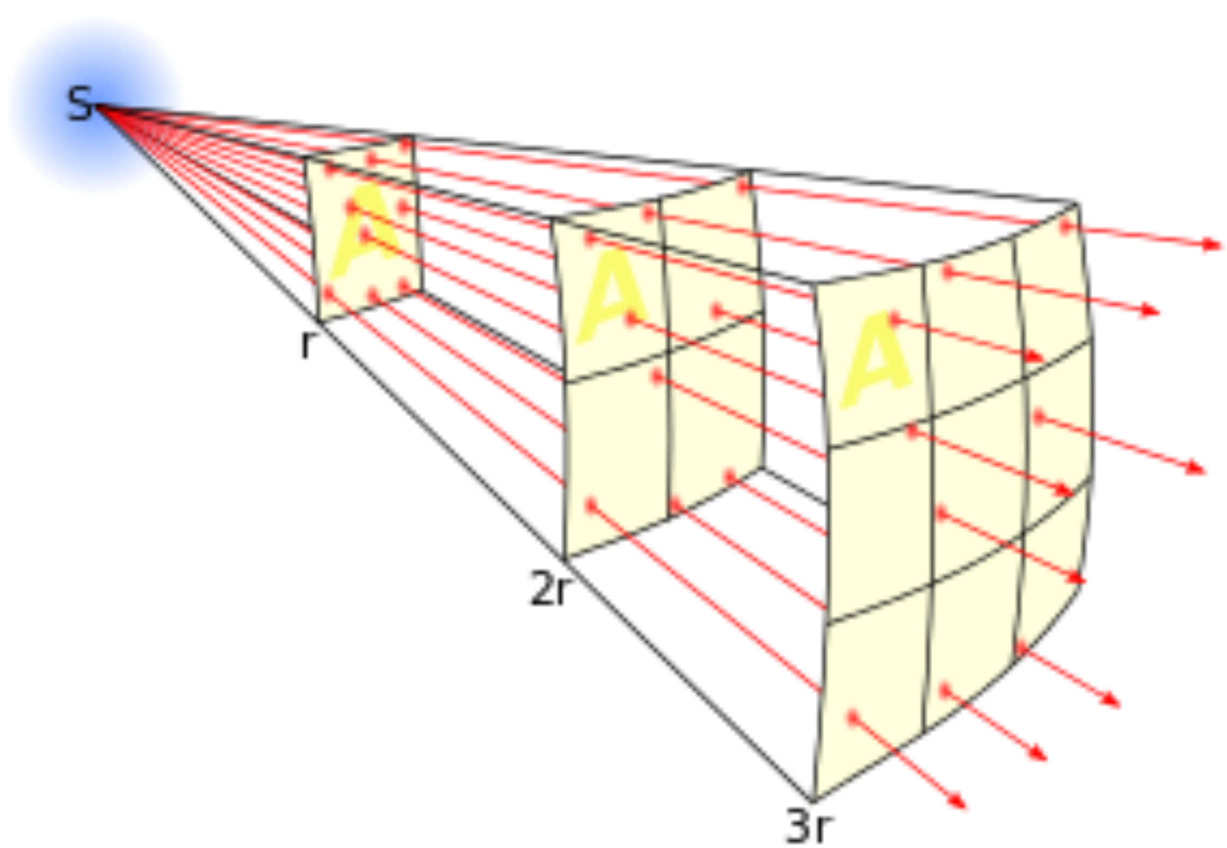


$$F = \frac{L}{4\pi d^2}$$

The Invariability of Luminosity

- **Luminosity** is the total amount of energy per unit time (i.e., power) emitted by the source (unit: Watt = Joule/s)
- **Flux** is the amount of arriving energy per unit time per unit area (unit: Watt/m^2) at a distance *d* from source
- **Flux** decreases as the **distance** from the source increases, obeying an **inverse square law,** which **preserves the luminosity**

$$L = F(d_1)4\pi d_1^2 = F(d_2)4\pi d_2^2$$

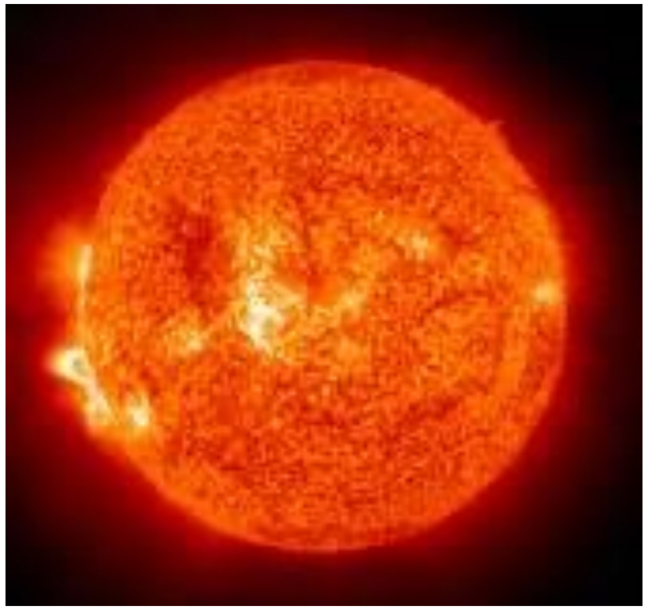


Definition: Absolute Magnitude (M) vs. Apparent Magnitude (m)

- apparent magnitude (m) is the magnitude of the source at its actual distance (d)
- absolute magnitude (M) is defined as the apparent magnitude of the source if it were at a distance of **10 parsec**

Practice: Calculate the absolute magnitude of the Sun

- The Sun has an apparent magnitude of -26.74 (d = 1 AU = 1/206265 pc)
- What's its absolute magnitude? Think about how faint it would appear at 10 pc



$$\text{Pogson's ratio}: m_{\lambda,1} - m_{\lambda,2} = -2.5 \log\left(\frac{f_{\lambda,1}}{f_{\lambda,2}}\right)$$

- 26.74 + 2.5 * log(2062650^2) = 4.83

Derivation: Absolute Magnitude (M) vs. Apparent Magnitude (m)

- apparent magnitude (m) is the magnitude of the source at its actual distance (d)
- absolute magnitude (M) is defined as the apparent magnitude of the source if it were at a distance of 10 parsec
- because both are measurements of the same source, we can express the same luminosity (L) using its actual flux (f) and its presumed flux (F) at 10 parsec:

$$L_\lambda = 4\pi d^2 f_\lambda = 4\pi (10\ \text{parsec})^2 F_\lambda \quad \Rightarrow \frac{F_\lambda}{f_\lambda} = \frac{d^2}{(10\ \text{parsec})^2}$$

$$m_\lambda - m_{\lambda,0} = -2.5 \log(f_\lambda / f_{\lambda,0})$$

$$M_\lambda - m_{\lambda,0} = -2.5 \log(F_\lambda / f_{\lambda,0})$$

$$m_\lambda - M_\lambda = 2.5 \log\left(\frac{d}{10\ \text{parsec}}\right)^2 = 5\ [\log \frac{d}{1\ \text{parsec}} - 1]$$

This, **m-M**, is called the **distance modulus**, because it only depends on distance

Practice: What's the absolute magnitude of the Sun?

- distance = 1 AU, V-band magnitude = -26.74
- What's its absolute magnitude in V-band?

M = -26.74-5*(log(1/206265)-1)
= 4.83

$$m_\lambda - M_\lambda = 5\ [\log d(\text{parsec}) - 1]$$

$$\Rightarrow M_\lambda = m_\lambda - 5\ [\log d(\text{parsec}) - 1]$$

Practice: Calculate absolute magnitude from p and m

- Suppose you measured a star's apparent magnitude in V-band (550 nm) to be $m_V = 10.5$
- You also measured its parallax to be $p = 5$ *mas* (milli-arcsec).
- What's its distance in parsec?

$$d = 1 \text{ parsec} \left(\frac{1 \text{ arcsec}}{p} \right)$$

- What's its absolute magnitude in V-band (M_V)?

$$m_\lambda - M_\lambda = 5 \; [\log d(\text{parsec}) - 1]$$

$$\Rightarrow M_\lambda = m_\lambda - 5 \; [\log d(\text{parsec}) - 1]$$

d = 200 parsec
M = 10.5 - 5 * (log(200) - 1) = 4.0

Distance from Distance Modulus:

The Standard Candle Methods

Distance Modulus: the difference between m and M

- The definition of absolute magnitude and the inverse distance square law for an **isotropic emitter** give us this equation:

$$m - M = 5\ [\log d(\text{parsec}) - 1]$$

- The term on the left side, **m-M**, is called the **distance modulus**, because it only depends on distance
- **m-M** offers us a group of methods to measure distances called **the standard candle**

$$d_{\text{pc}} = 10^{1+0.2(m-M)}$$

The Standard Candle Methods

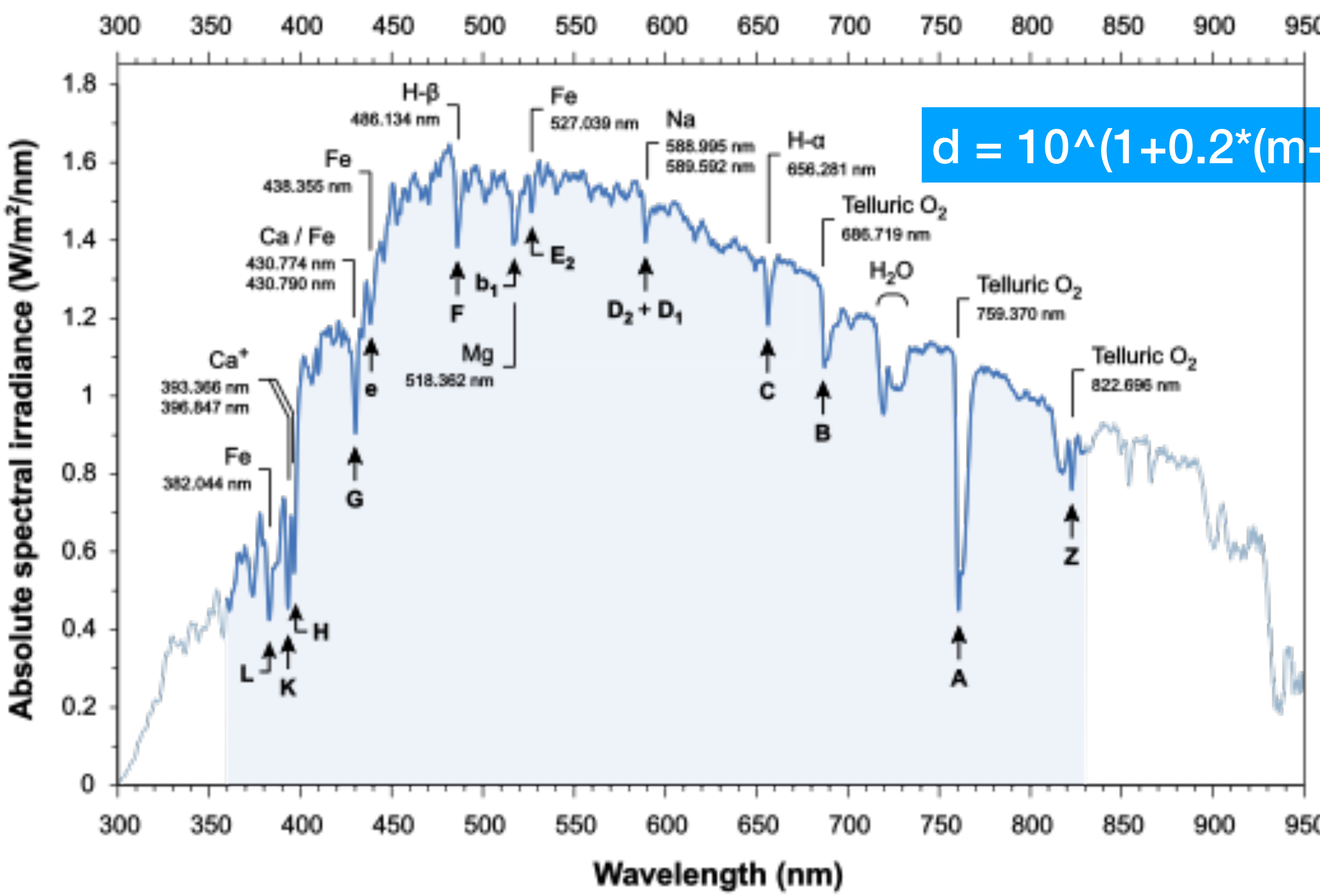
- If we had measured or inferred the absolute magnitude of a class of astrophysical objects, we can get the distance modulus (m-M) from its apparent magnitude.
- The distance modulus then gives us the distance:

$$m - M = 5\ (\log d_{\rm pc} - 1) \Rightarrow d_{\rm pc} = 10^{1+0.2(m-M)}$$

Standard Candle Method 1 — Spectroscopic “Parallax”

Suppose we find a solar-type star in the constellation Ursa Major, its spectrum looks just like that of the Sun, so we assume that this star has the same luminosity as the Sun. Given the Sun has **M_V = 4.83** and this star has **m_V = 10.5**, can you estimate its distance?

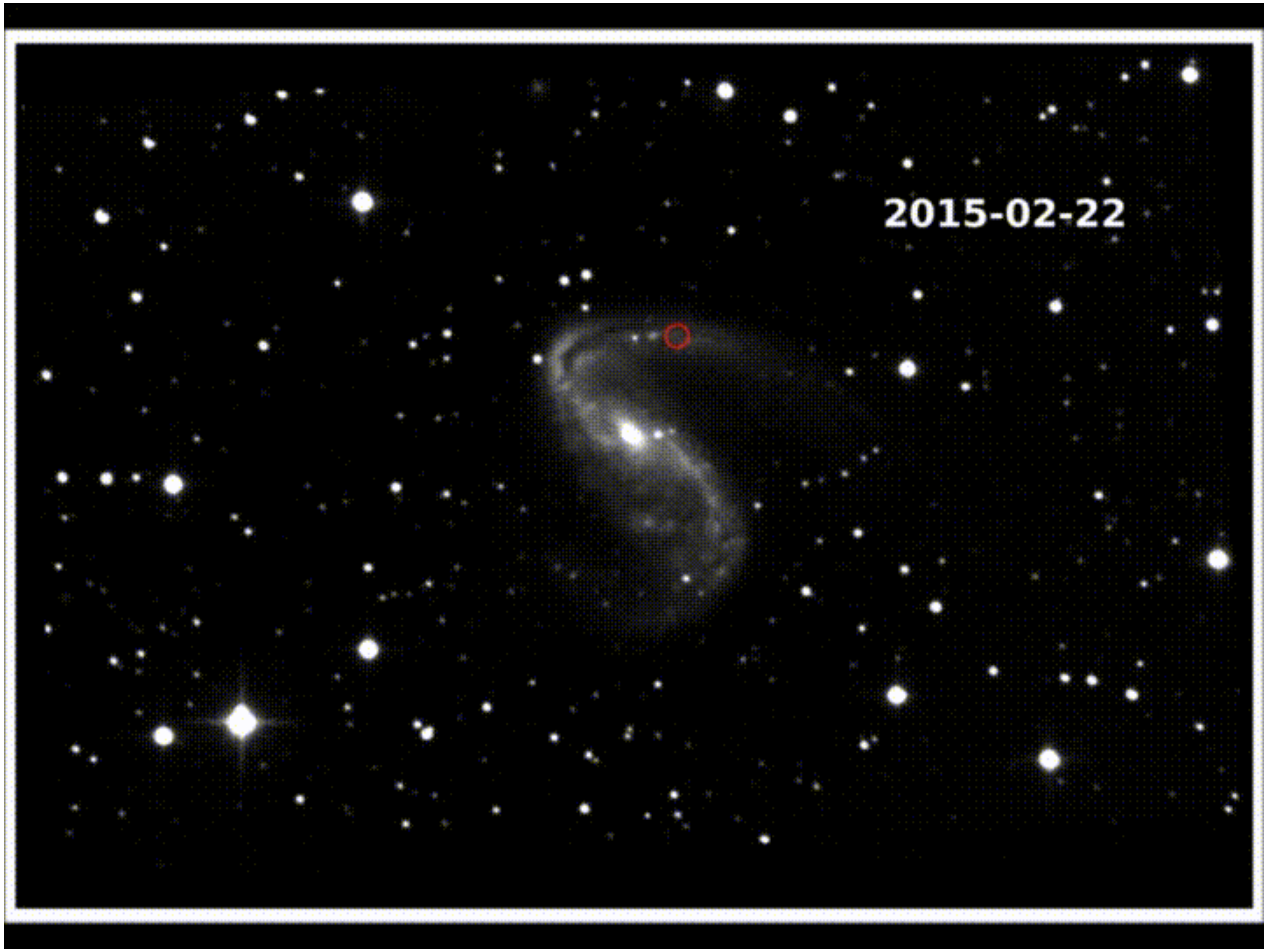
$$d(\text{parsec}) = 10^{1+0.2(m-M)}$$



d = 10^(1+0.2*(m-M)) = 136 parsec

The Standard Candle Method 2 — Type Ia SNe

- Type Ia supernovae (SNe) have been used as standard candles to measure cosmological distances to other galaxies.
- They work as standard candles because presumably the white dwarfs have to reach 1.44 solar mass (the Chandrasekhar mass) to trigger the thermonuclear explosion, reaching a peak absolute magnitude of $M_V = -19$.



Practice: The Standard Candle Method of Distance Measurement

- Type Ia supernovae (SNe) have been used as standard candles to measure cosmological distances to other galaxies.
- They work as standard candles because presumably the white dwarfs have to reach 1.44 solar mass (the Chandrasekhar mass) to trigger the thermonuclear explosion
- At its peak, the absolute magnitude in V-band (550 nm) is ***M_V = -19***, and you measured a peak apparent magnitude of ***m_V = 10***, what's the distance in parsec?

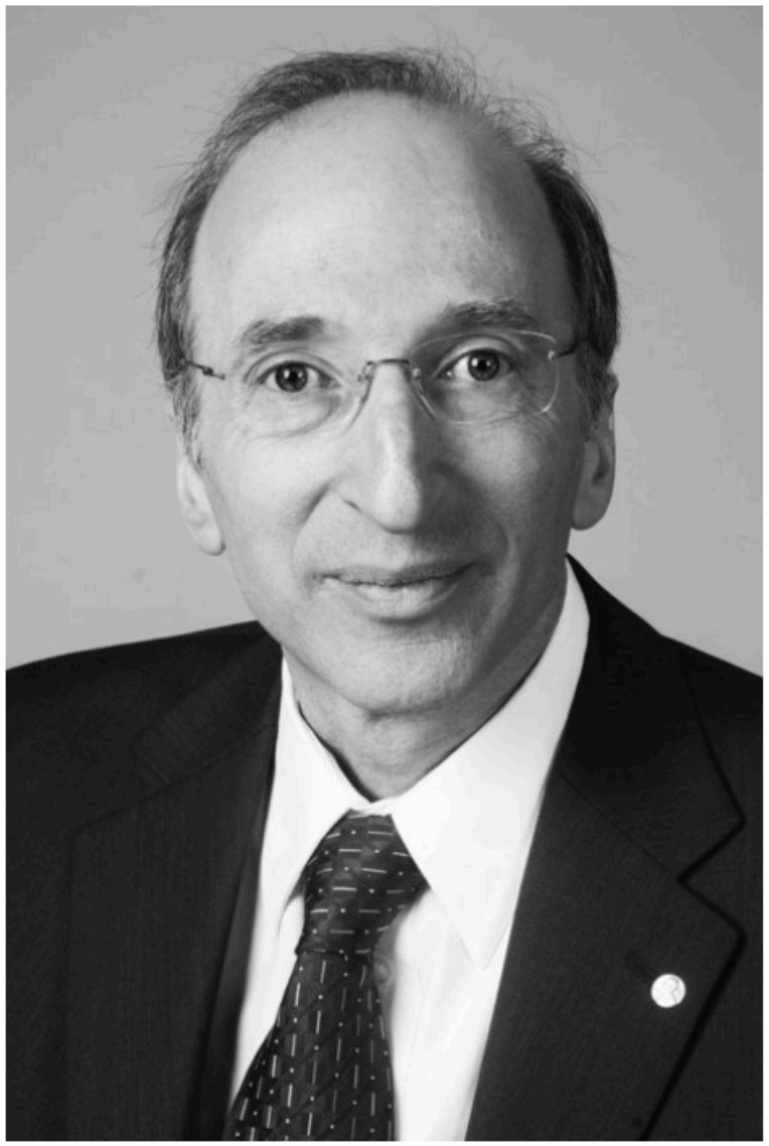
$$m - M = 5\ [\log d(\text{parsec}) - 1]$$

$$d(\text{parsec}) = 10^{1+0.2(m-M)}$$

10 parsec * 10^(0.2*(10-(-19))) = 6.3 Mpc

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

The Nobel Prize in Physics 2011

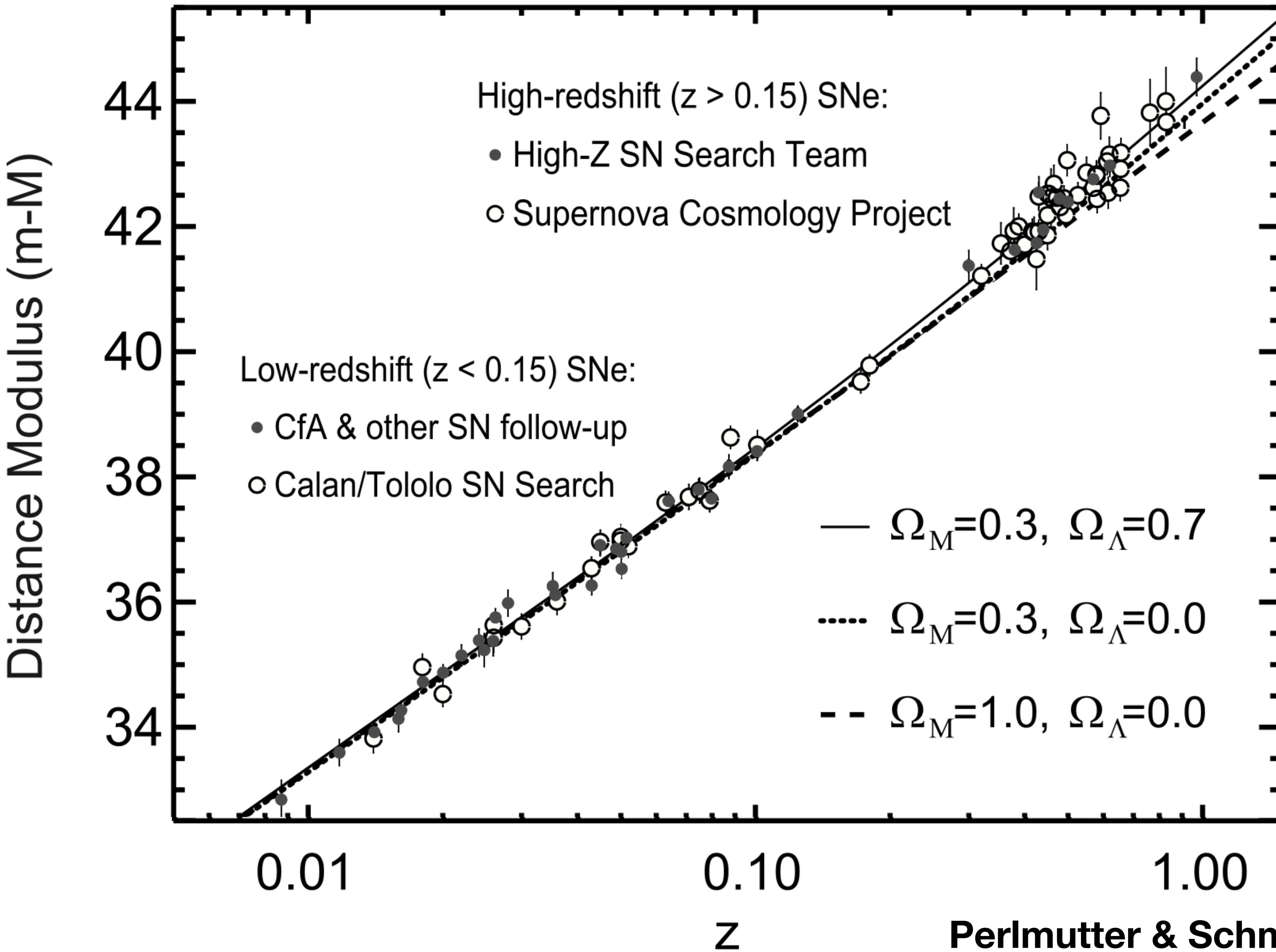


Saul Perlmutter

Brian P. Schmidt

Adam G. Riess

Distance Modulus vs. Cosmological Redshifts (Hubble Diagram)



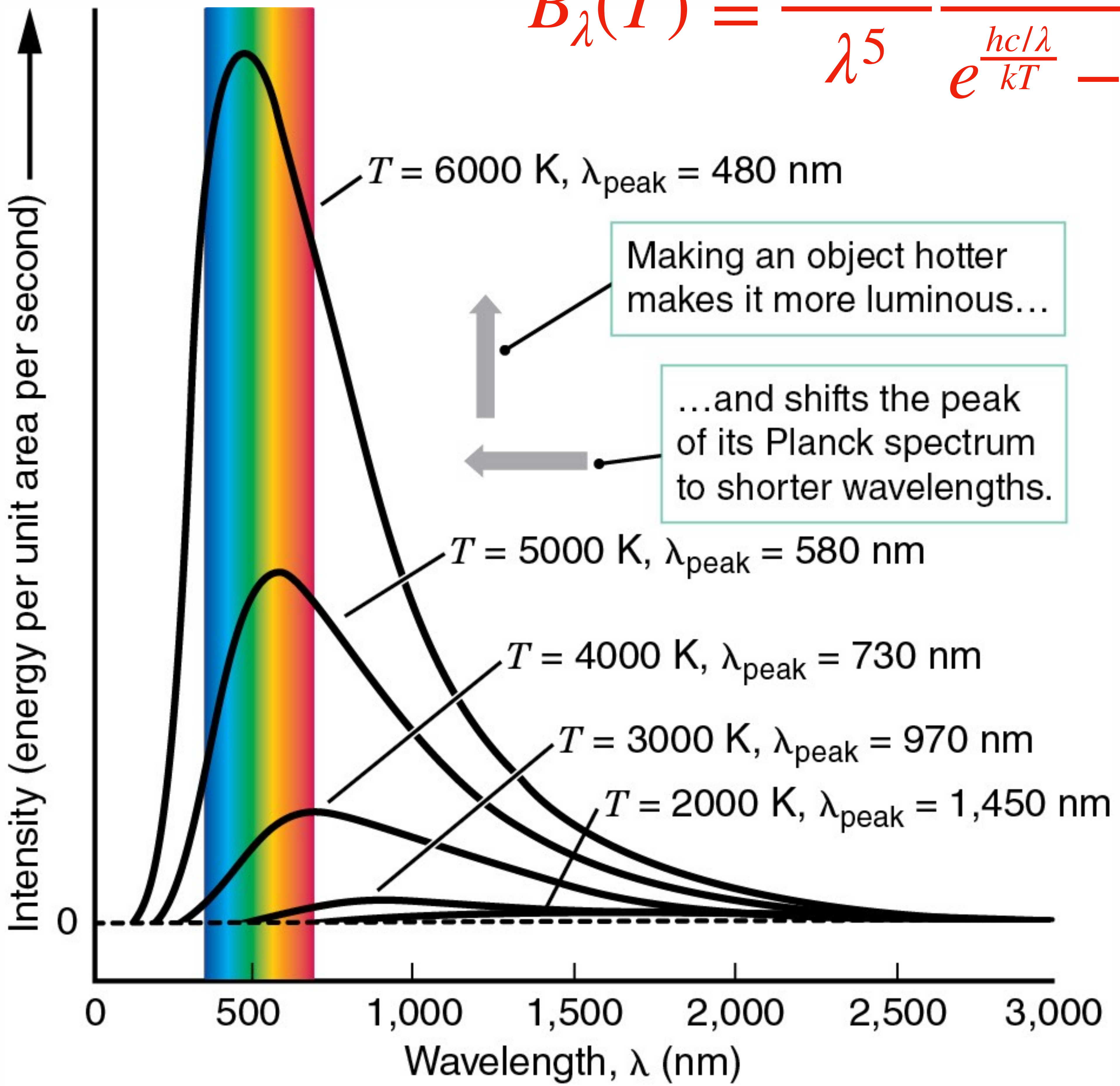
Perlmutter & Schmidt 2003

Surface Temperatures of Stars

spectroscopic methods: Wien's law and spectral classification

Planck Curves at Various T

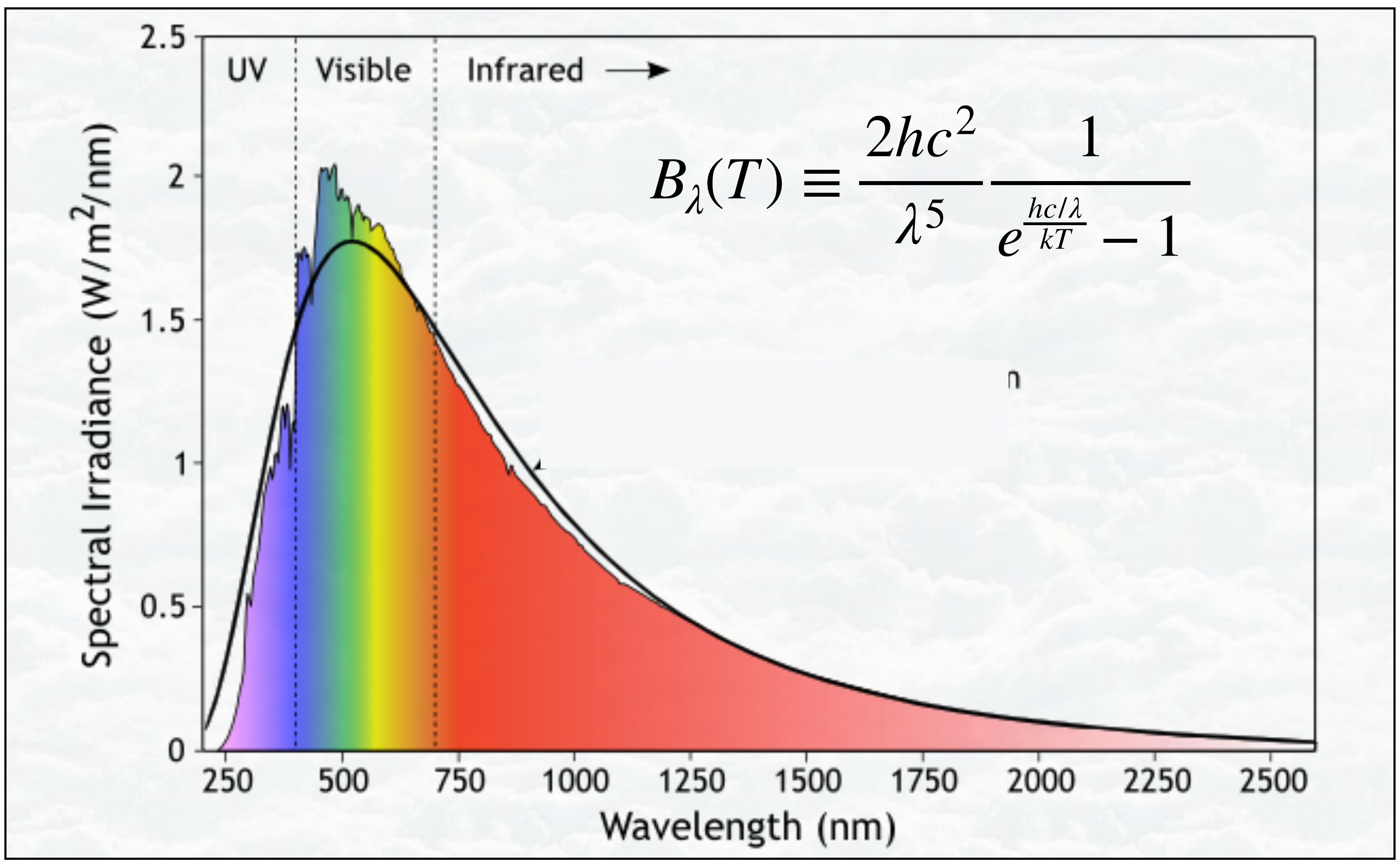
$$B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc/\lambda}{kT}}-1}$$



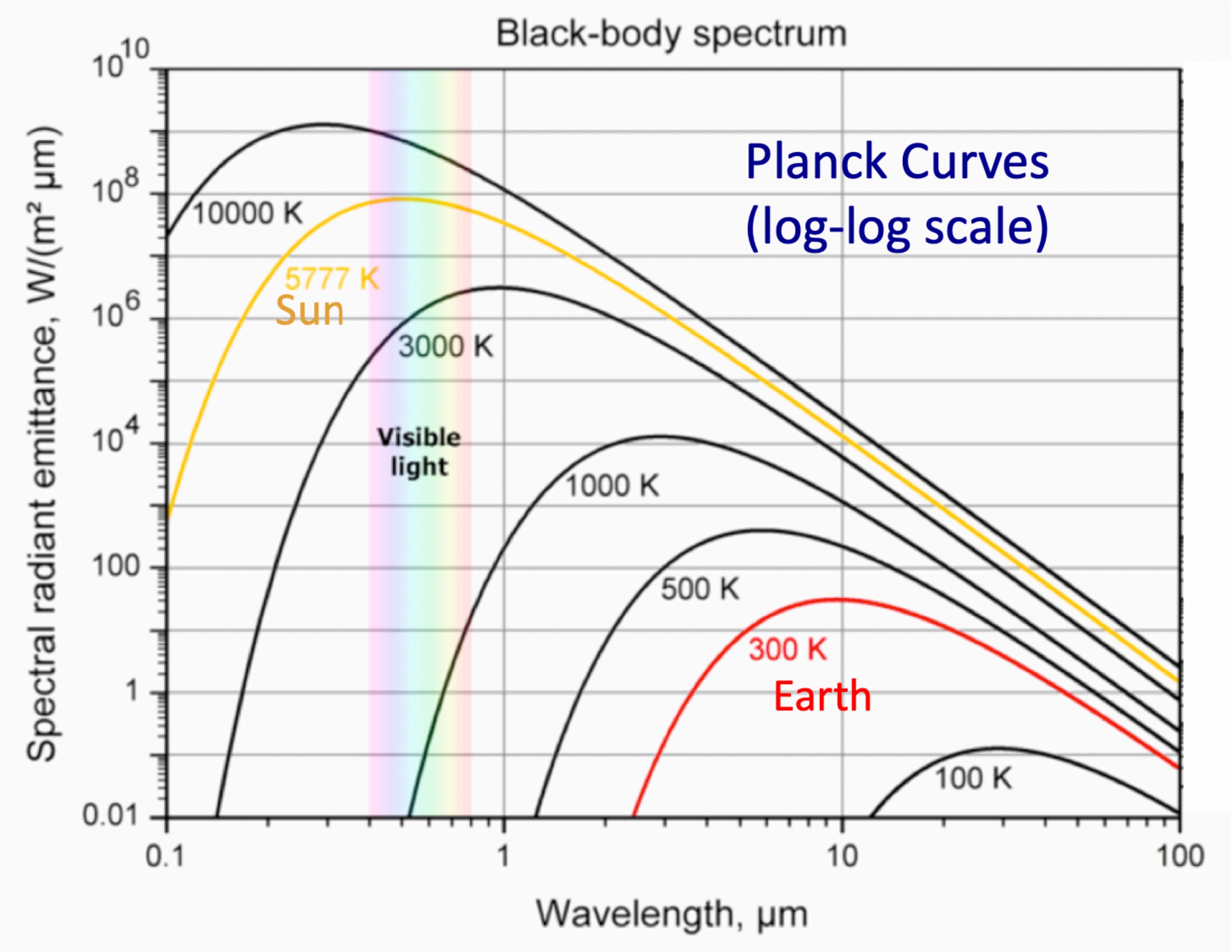
Temperature from Wien's Displacement Law

$$\lambda_{\text{peak}} = \frac{2.9 \text{ mm K}}{T} \Rightarrow T = \frac{2.9 \text{ mm K}}{\lambda_{\text{peak}}}$$

- Given a temperature, calculate the wavelength at which the BB emission's flux density peaks; Or given a peak wavelength, calculate the temperature.

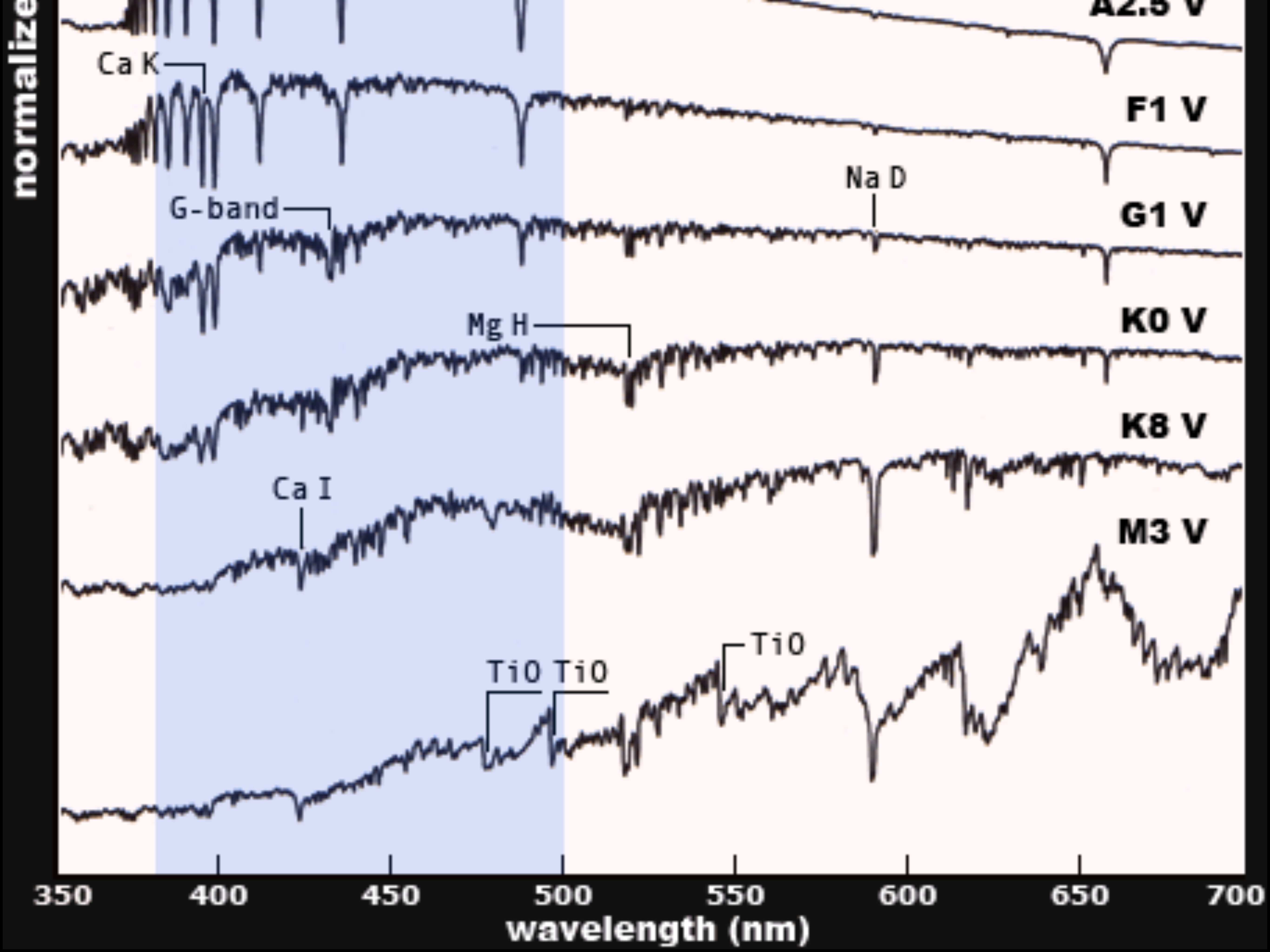


What to do when the peak shifts outside of the visible light window?
e.g. when T > 9000 K or T < 3000 K

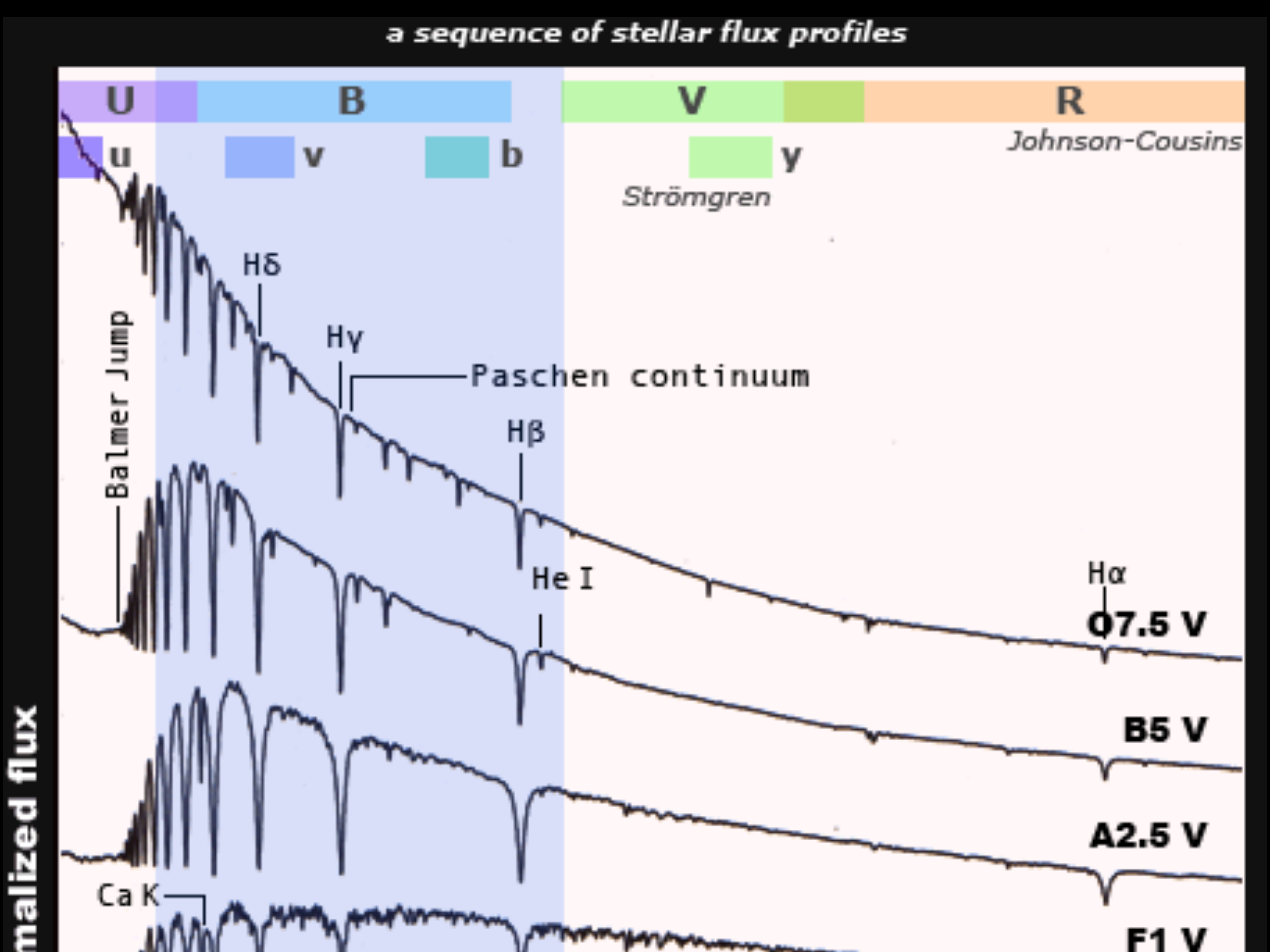


Optical spectral classification of stars

- **The strength of absorption lines from different elements depend mainly on the temperature (because of ionization equilibrium).**
- The current classification scheme was **re-ordered** and **simplified** by **Annie Jump Cannon** (1863–1941) at Harvard College Observatory.
- The full sequence is **O B A F G K M**, which are further subdivided by adding numbers to the letter. The Sun is a **G2** spectral-type star.

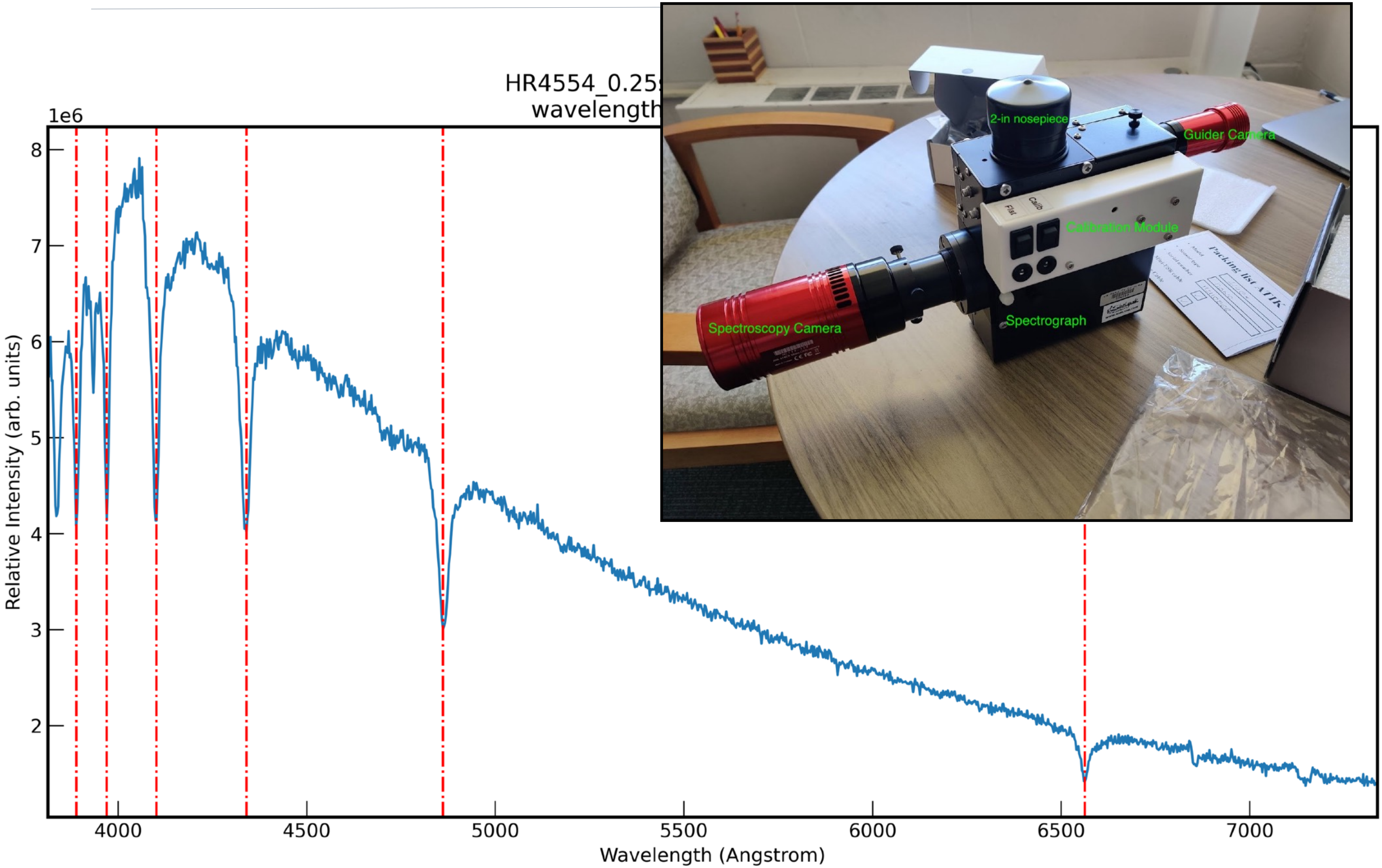


normalize
A2.5 V
Ca K
F1 V
Na D
G-band
G1 V
Mg H
K0 V
K8 V
Ca I
M3 V
TiO
TiO TiO
350
400
450
500
550
600
650
700
wavelength (nm)

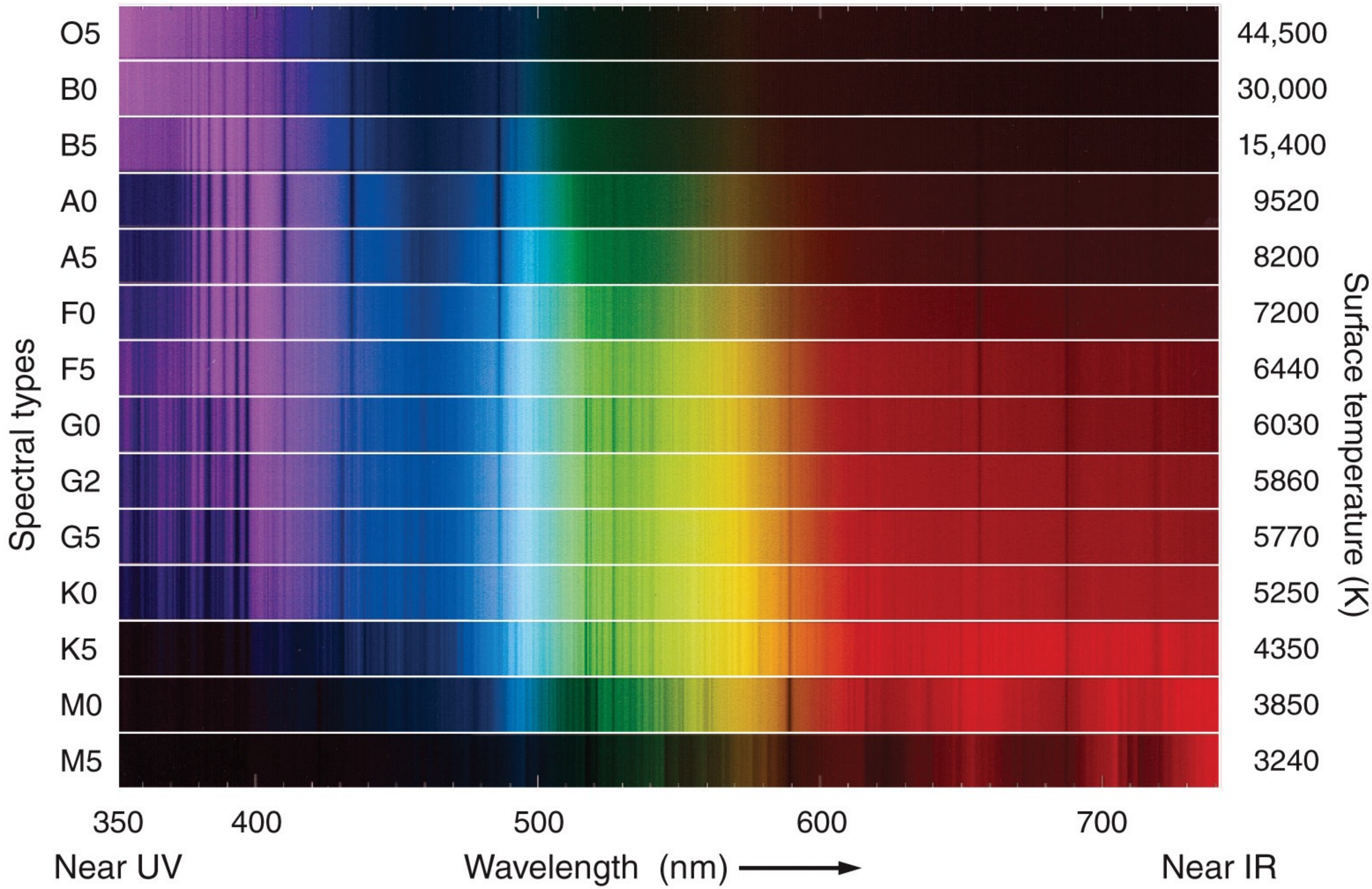


a sequence of stellar flux profiles
U
B
V
R
u
v
b
y
Johnson-Cousins
Strömgren
Hδ
Hγ
Paschen continuum
Hβ
Balmer Jump
He I
Hα
O7.5 V
B5 V
A2.5 V
Ca K
F1 V
malized flux

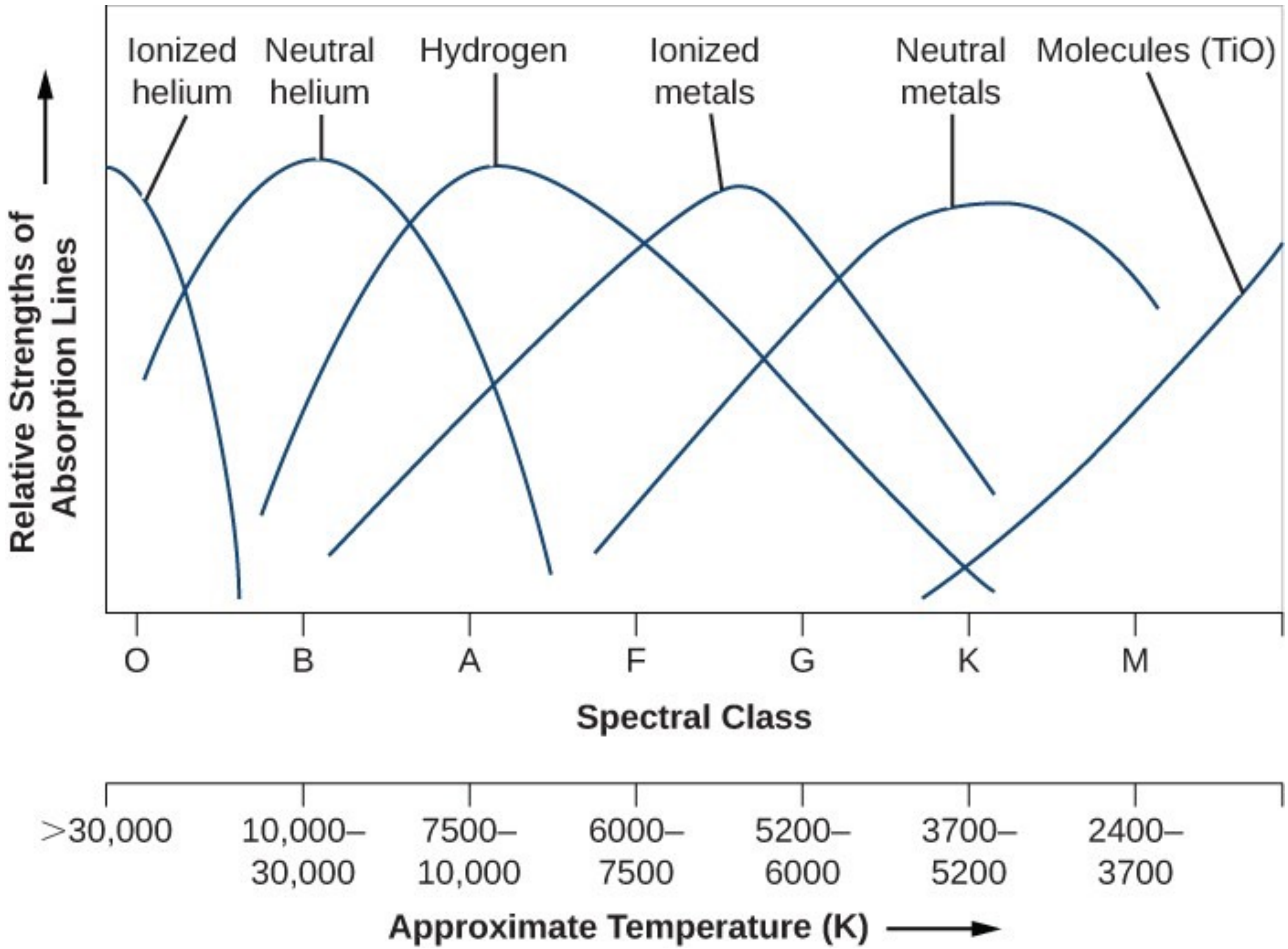
An A-type star's spectrum taken by the Van Allen Observatory



Temperature from Spectral Classes



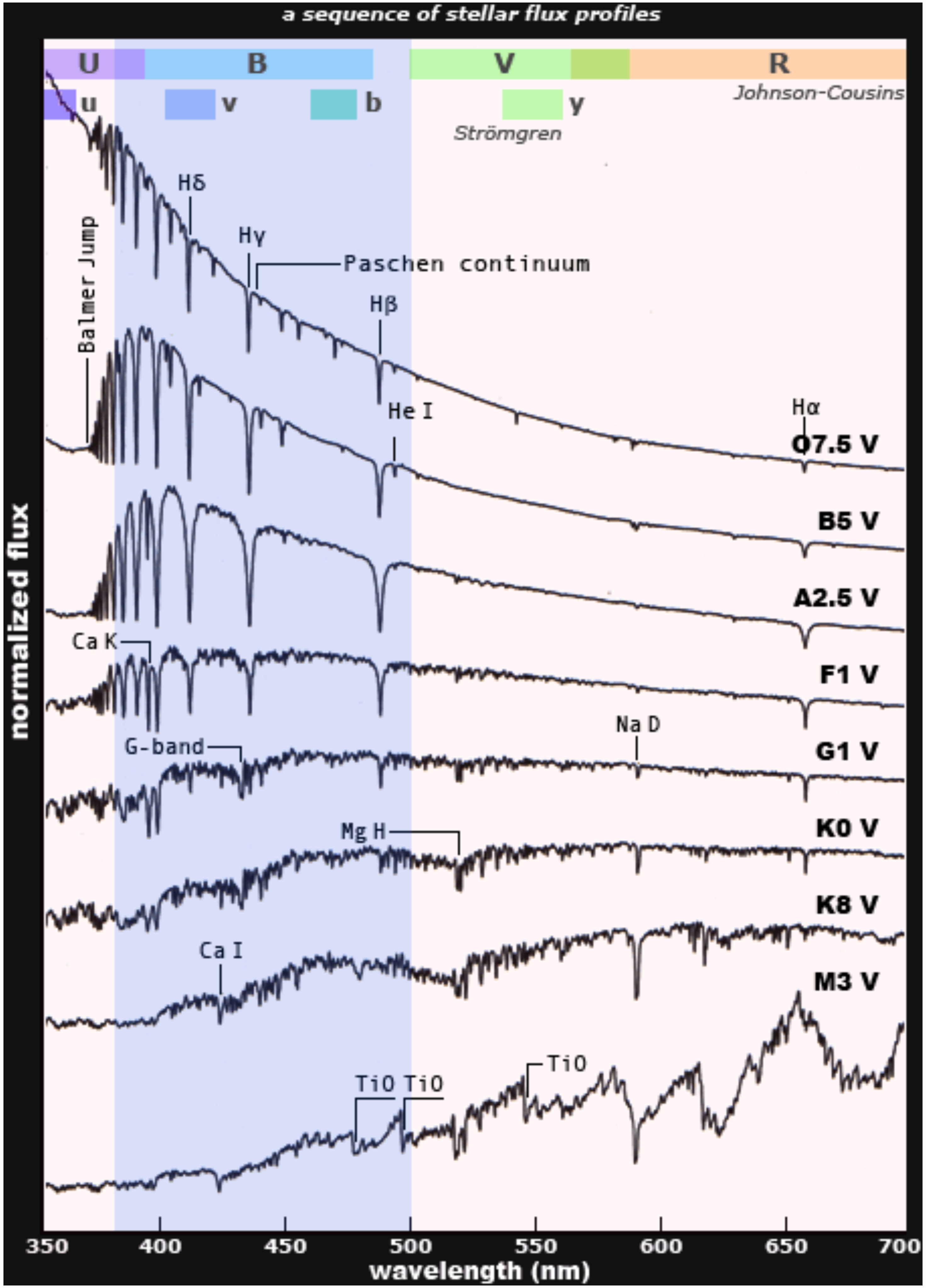
Relative Strengths of Absorption Lines vs. Atmospheric Temperature



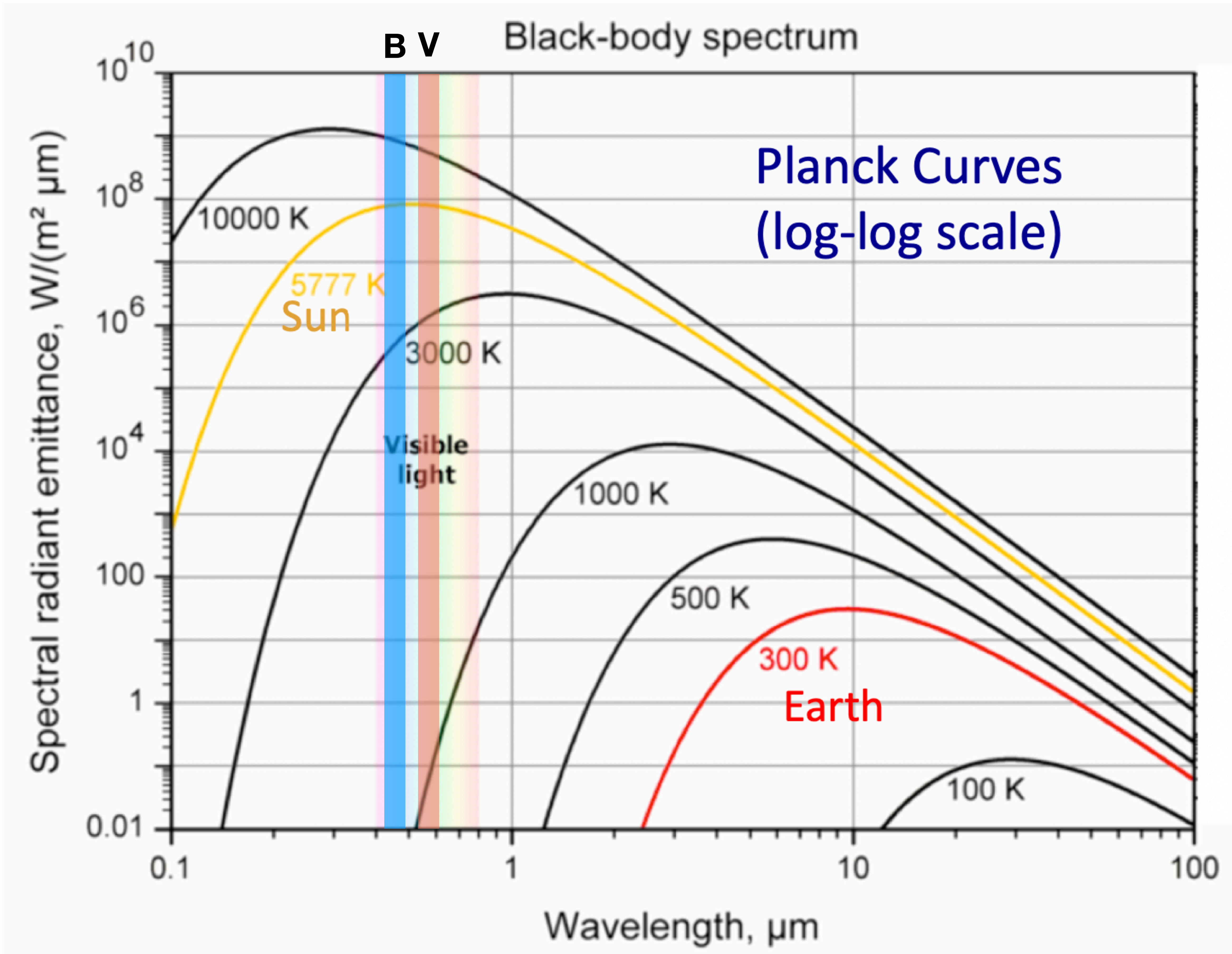
Temperature

photometric method: color index

Spectroscopy takes longer time to acquire, because each star would require its own spectroscopic observations with a traditional longslit spectrograph



Two-band photometry offers a much simpler way to estimate temperature



Temperature from Color Index

- **Color index** is defined as the magnitude difference of the same object at two different wavelengths.

- According to **Pogson**, the magnitude difference corresponds to a flux ratio at two different wavelengths:

$$m_B - m_V = -2.5[\log(f_B/f_V) - \log(f_{B,0}/f_{V,0})]$$

or simply

$$B - V = -2.5[\log(f_B/f_V) - \log(f_{B,0}/f_{V,0})]$$

- Typically, we subtract a bluer magnitude (e.g., B) to a redder magnitude (e.g., V), so that **the higher the value of the color index, the redder the object appears** (i.e., the object appears much fainter in B-band than in V-band)

Practice: From flux ratio to color index

$$B - V = -2.5[\log(f_B/f_V) - \log(f_{B,0}/f_{V,0})]$$

- **Vega** is the usual reference star that sets the zero point of the apparent magnitude system. Its surface temperature is at 9600 K, much hotter than that of the Sun (5800 K).
- Consider a star that is 100x fainter than Vega at 440nm (B-band) and also 100x fainter than Vega at 550nm (V-band),
  - What are the magnitudes of the star in B and V?
  - What is the color index?
  - What is its surface temperature?

Practice: From flux ratio to color index

$$B - V = -2.5[\log(f_B/f_V) - \log(f_{B,0}/f_{V,0})]$$

- Vega is the usual reference star that sets the zero point of the apparent magnitude system. Its surface temperature is at 9600 K, much hotter than that of the Sun (5800 K).
- Consider another star that is 100x fainter than Vega at 440nm but 200x fainter than Vega at 550nm (V-band), what are the B and V magnitudes? What is the color index? Is this star hotter or cooler than Vega?

B = 5, V = 5.75; B-V = -0.75

A table that gives the color indices at a range of temperatures

Sample calibration colors[1]

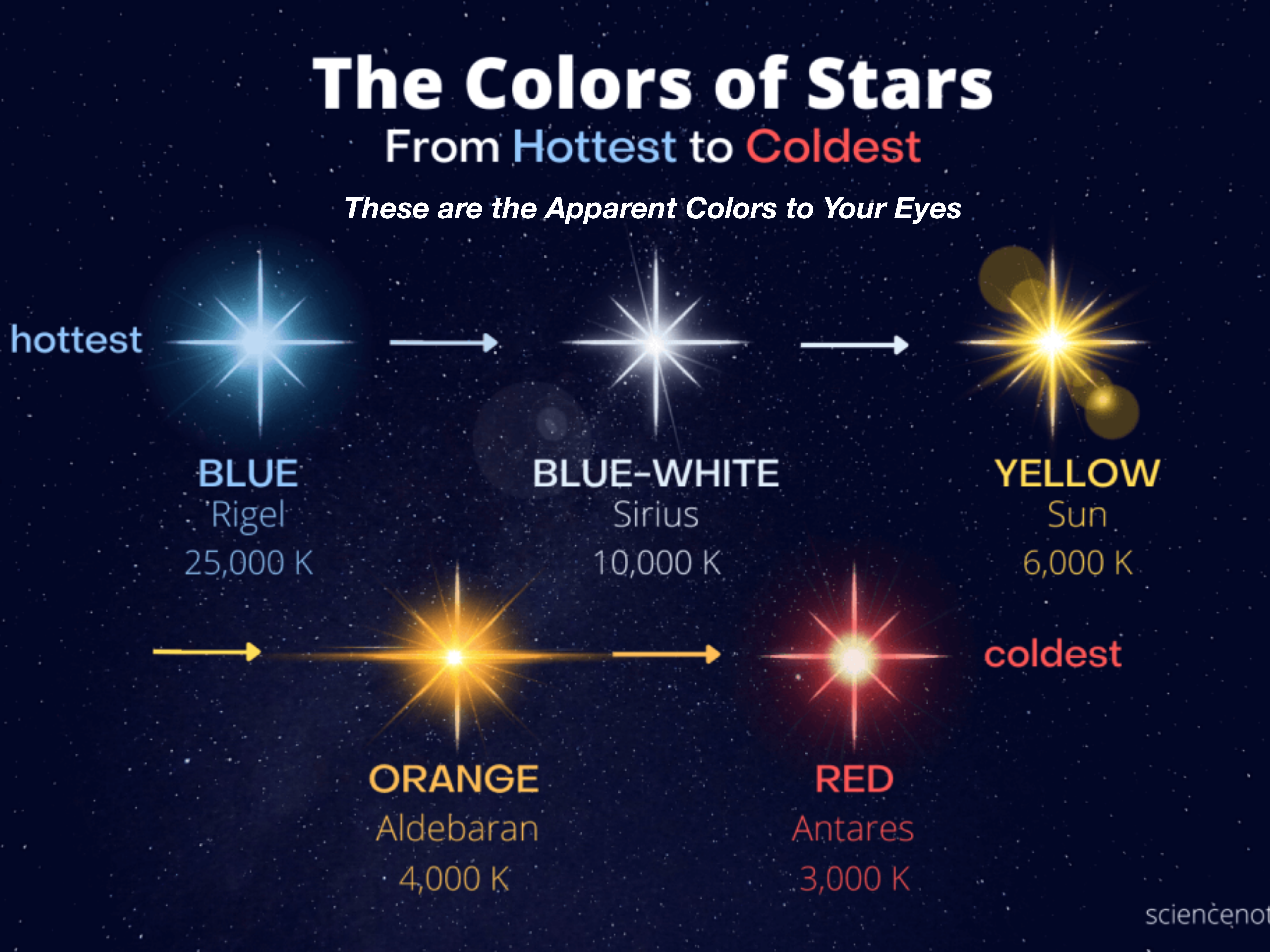
Class	B–V	U–B	V–R	R–I	T_{eff} (K)
O5V	−0.33	−1.19	−0.15	−0.32	42,000
B0V	−0.30	−1.08	−0.13	−0.29	30,000
A0V	−0.02	−0.02	0.02	−0.02	9,790
F0V	0.30	0.03	0.30	0.17	7,300
G0V	0.58	0.06	0.50	0.31	5,940
K0V	0.81	0.45	0.64	0.42	5,150
M0V	1.40	1.22	1.28	0.91	3,840

Apparent Colors of Stars

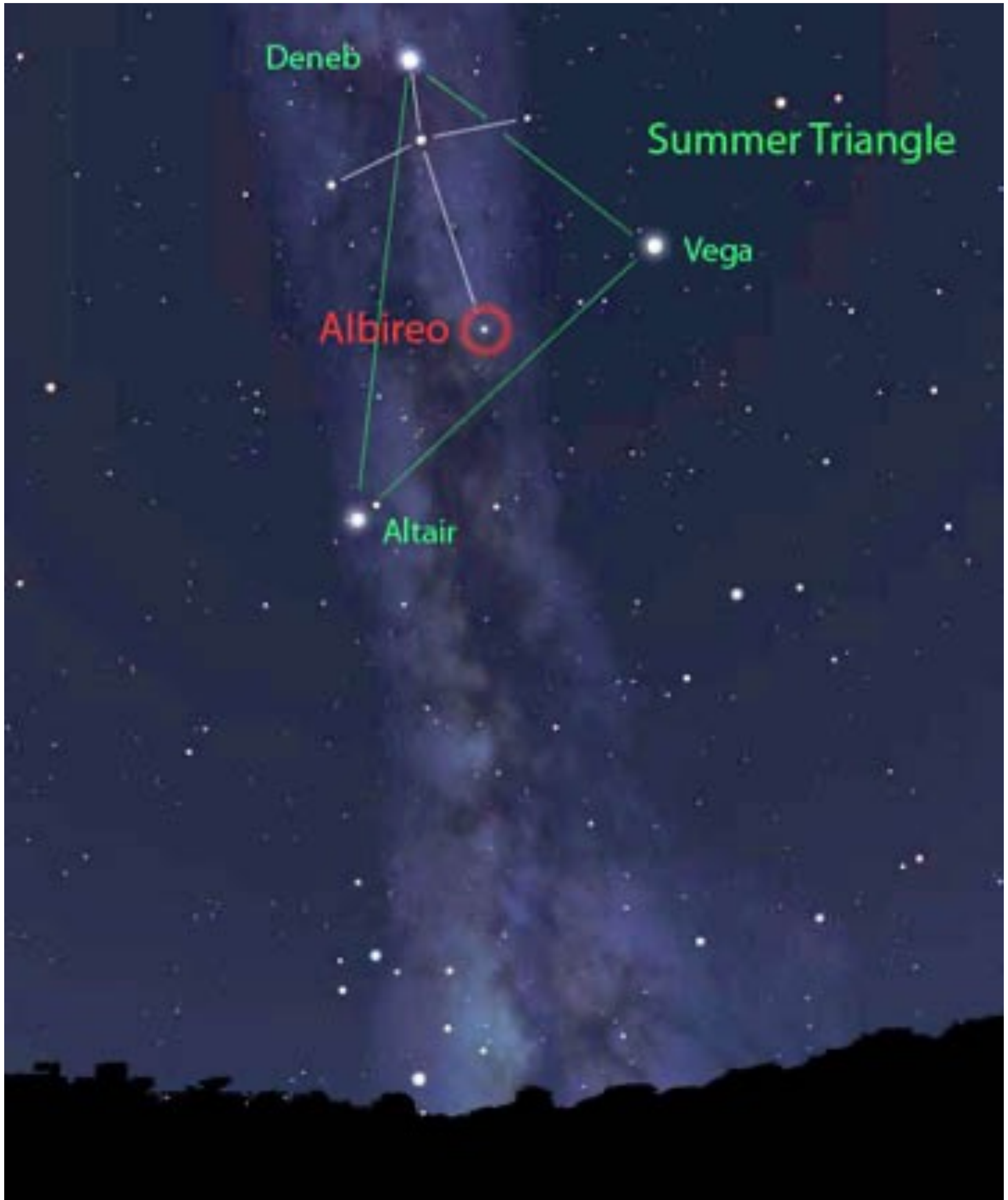
Temperature vs. Color Index vs. Apparent Color

$$B - V = -2.5[\log(f_B/f_V) - \log(f_{B,0}/f_{V,0})]$$

Spec Type	Surface Temperature	B–V	Apparent Color
O	≥ 33,000 K	−0.33	blue
B	10,000–30,000 K	−0.30	blue white
A	7,500–10,000 K	−0.02	white to blue white
F	6,000–7,500 K	0.30	white
G	5,200–6,000 K	0.58	yellowish white
K	3,700–5,200 K	0.81	yellow orange
M	≤ 3,700 K	1.40	orange red


The Colors of Stars
From Hottest to Coldest
These are the Apparent Colors to Your Eyes
hottest
BLUE
Rigel
25,000 K
BLUE-WHITE
Sirius
10,000 K
YELLOW
Sun
6,000 K
ORANGE
Aldebaran
4,000 K
RED
Antares
3,000 K
coldest
sciencenot

Albireo A & B (beta Cygni)



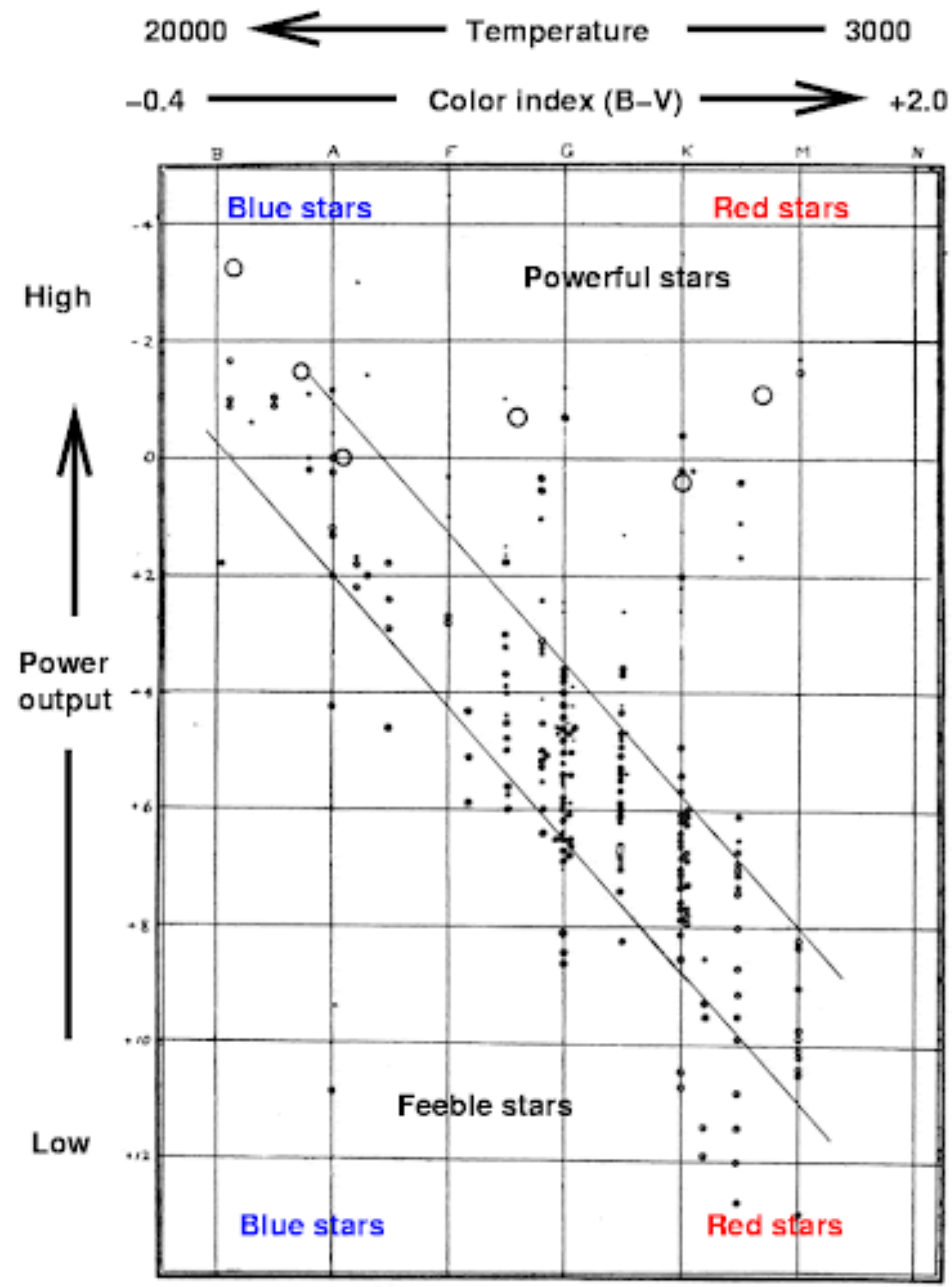
The Hertzsprung-Russell Diagram:

Luminosity-Temperature Diagram

The Hertzsprung-Russell Diagram

- In **1905, Hertzsprung** first published the measurements in a Table instead of a Figure. **Almost nobody noticed this remarkable finding**.
- In **1914, Russell** published his independent measurements in the format of a Figure on the journal *Nature,* **making big waves in Astronomy.**

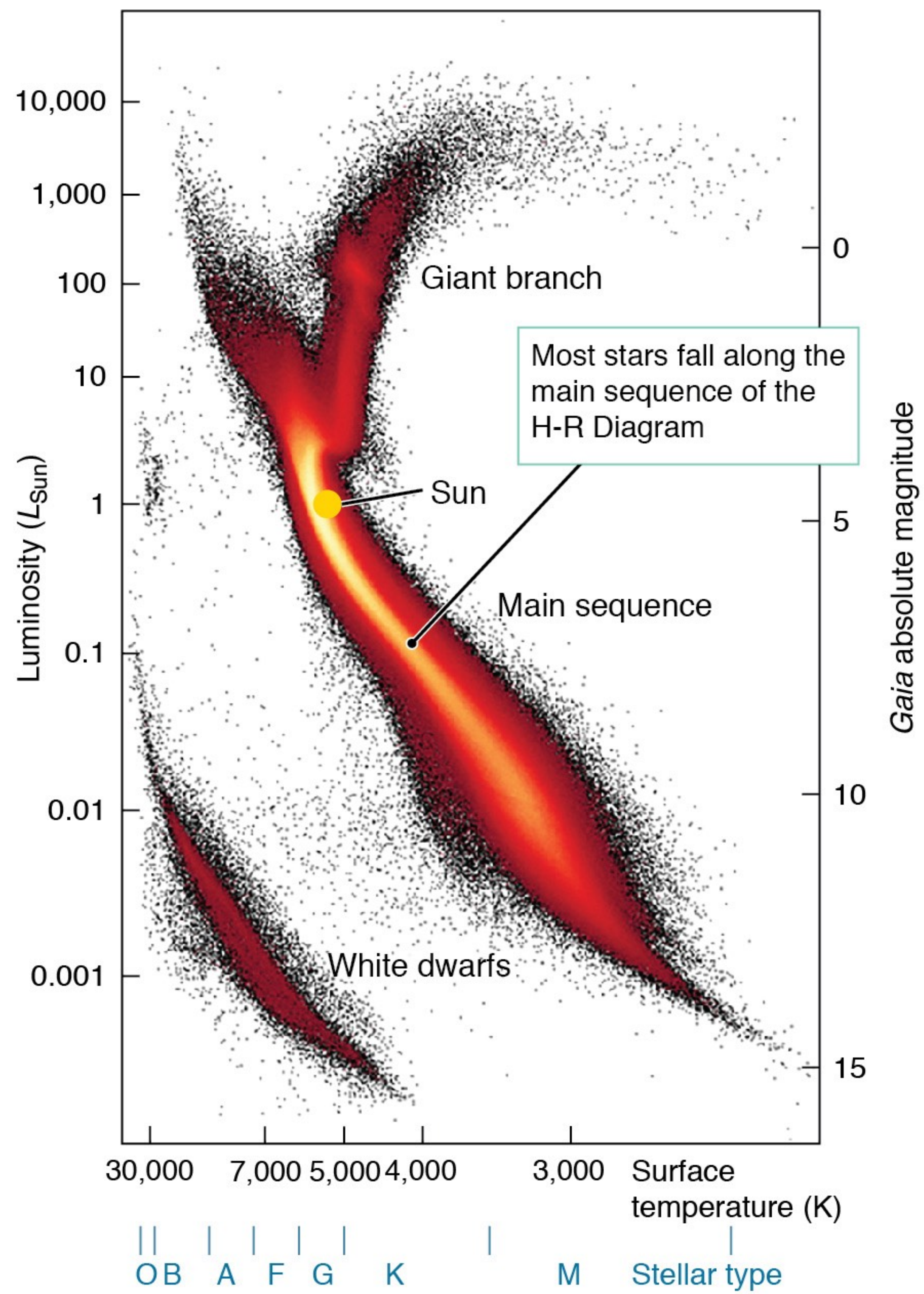
- *Lesson for Astronomy students:* **how you present your data matters**



The Hertzsprung-Russell Diagram

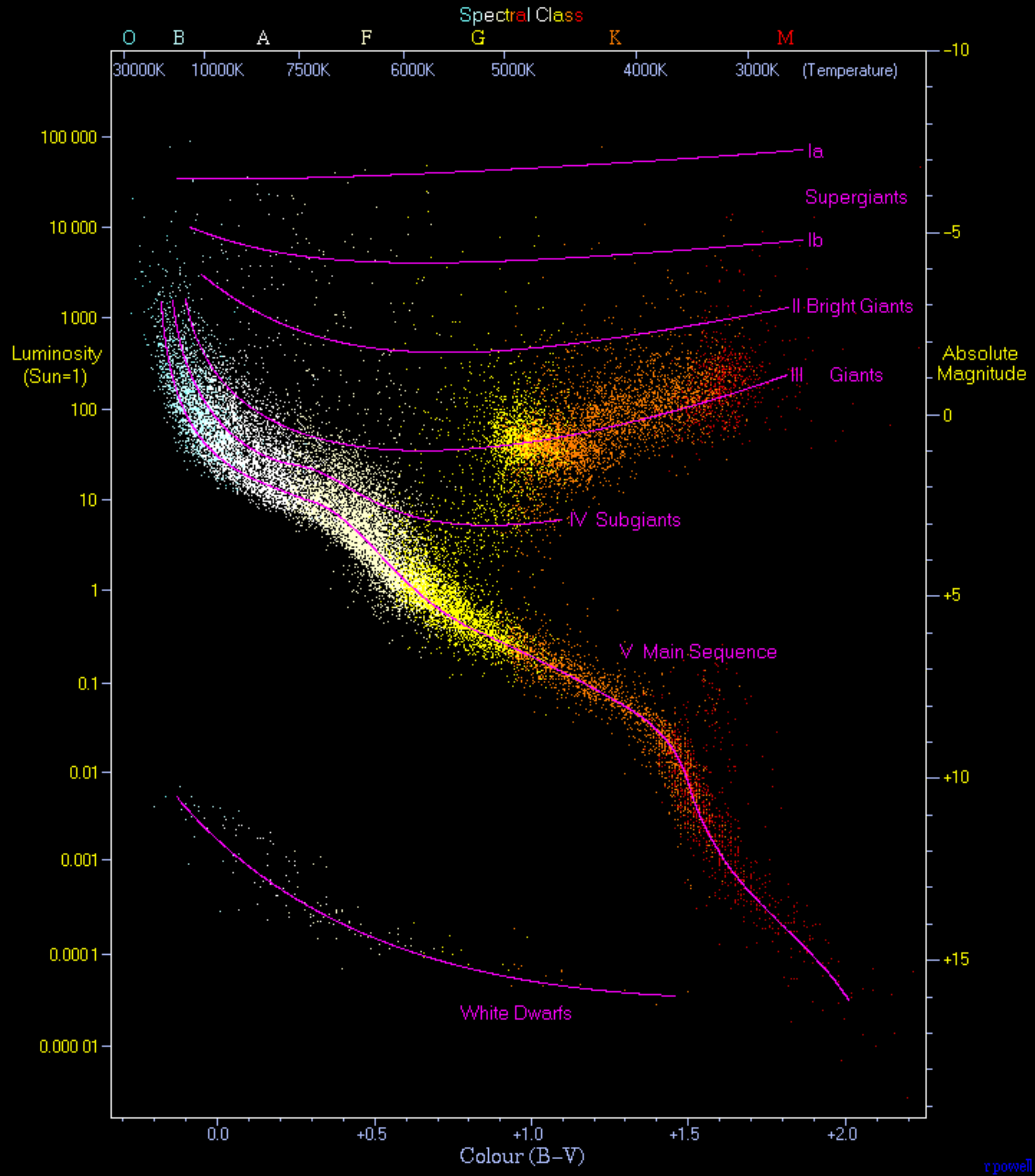
- In **1905, Hertzsprung** first published the measurements in a Table instead of a Figure. **Almost nobody noticed this remarkable finding**.
- In **1914, Russell** published his independent measurements in the format of a Figure on the journal *Nature,* **making big waves in Astronomy.**

- *Lesson for Astronomy students:* **how you present your data matters**



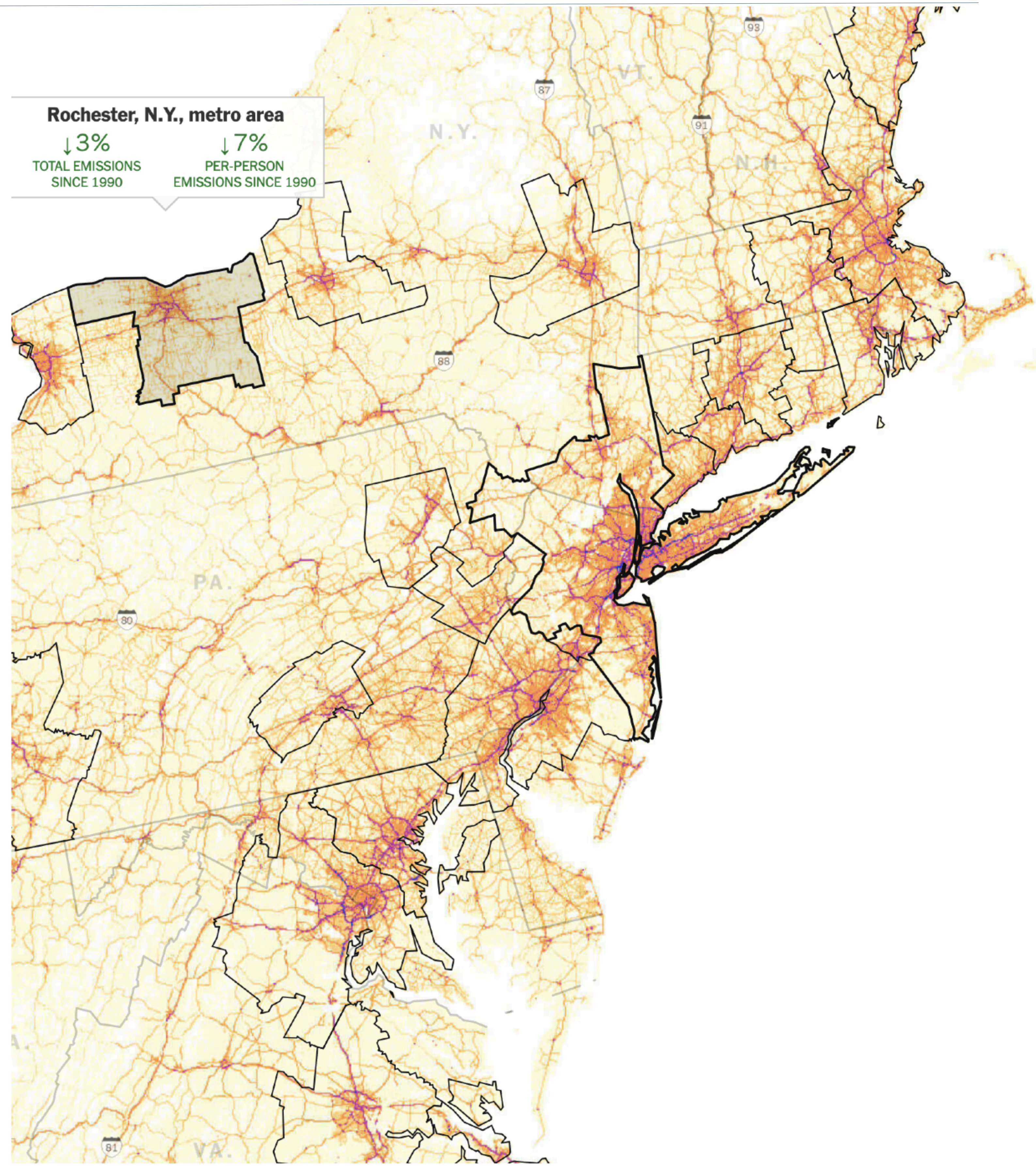
HR Diagram

- The luminosity-temperature diagram is the most important graph in stellar astronomy and is the key to unraveling stellar evolution.
- Two questions to discuss today:
  - What does the concentration of stars in certain areas imply?
  - What does the location of any star on the HRD tell us about its physical properties?



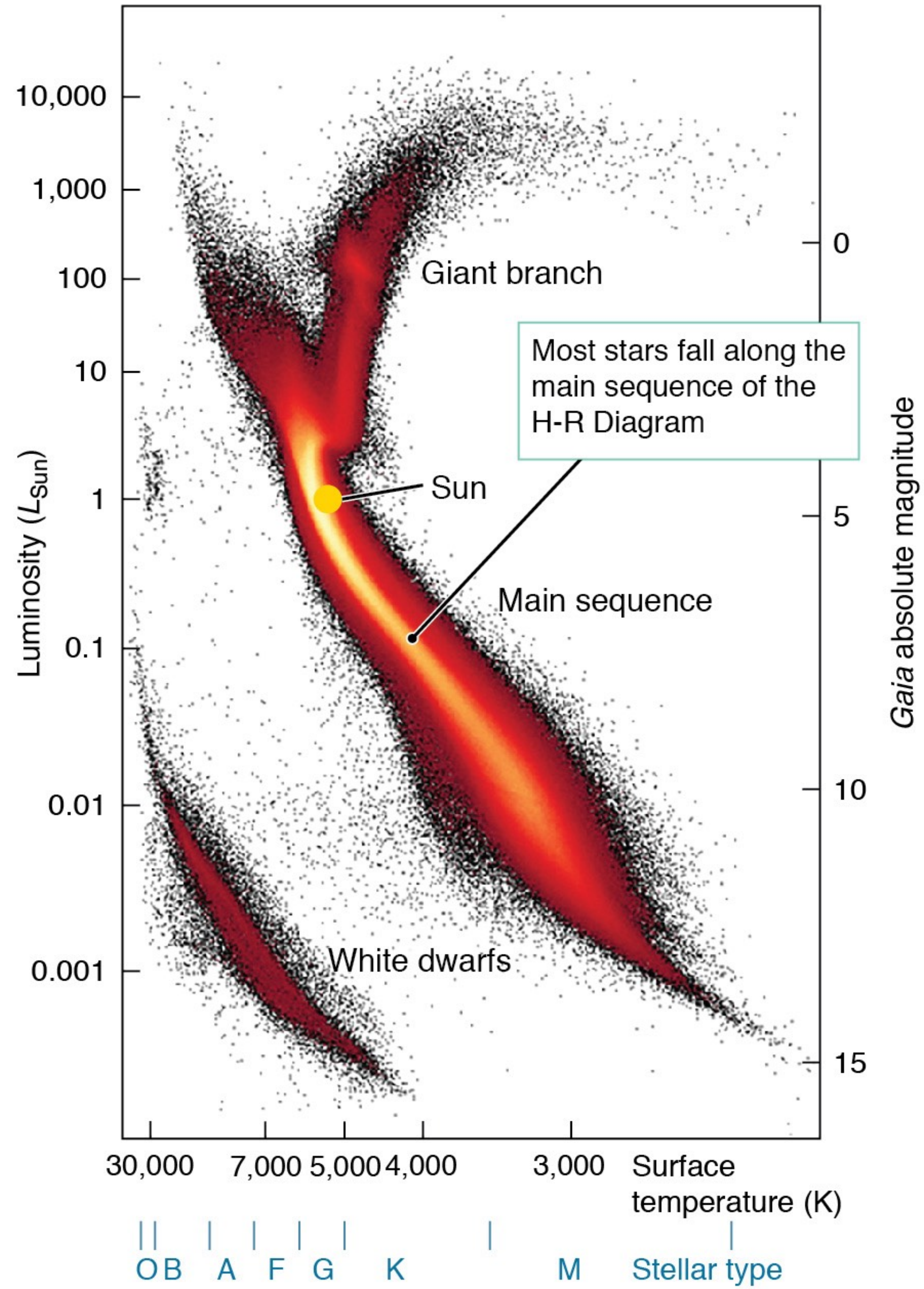
I - Why stars concentrate in certain areas on the HRD

- On the right is the most detailed auto emission map of the US east coast
- It's made by the New York Times in 2019.
- High emission areas are high concentration areas of internal combustion engines
- Conclusions:
  - People spend much more time in cities than in-between.
  - People spend a lot of time commuting along the DC-NY line



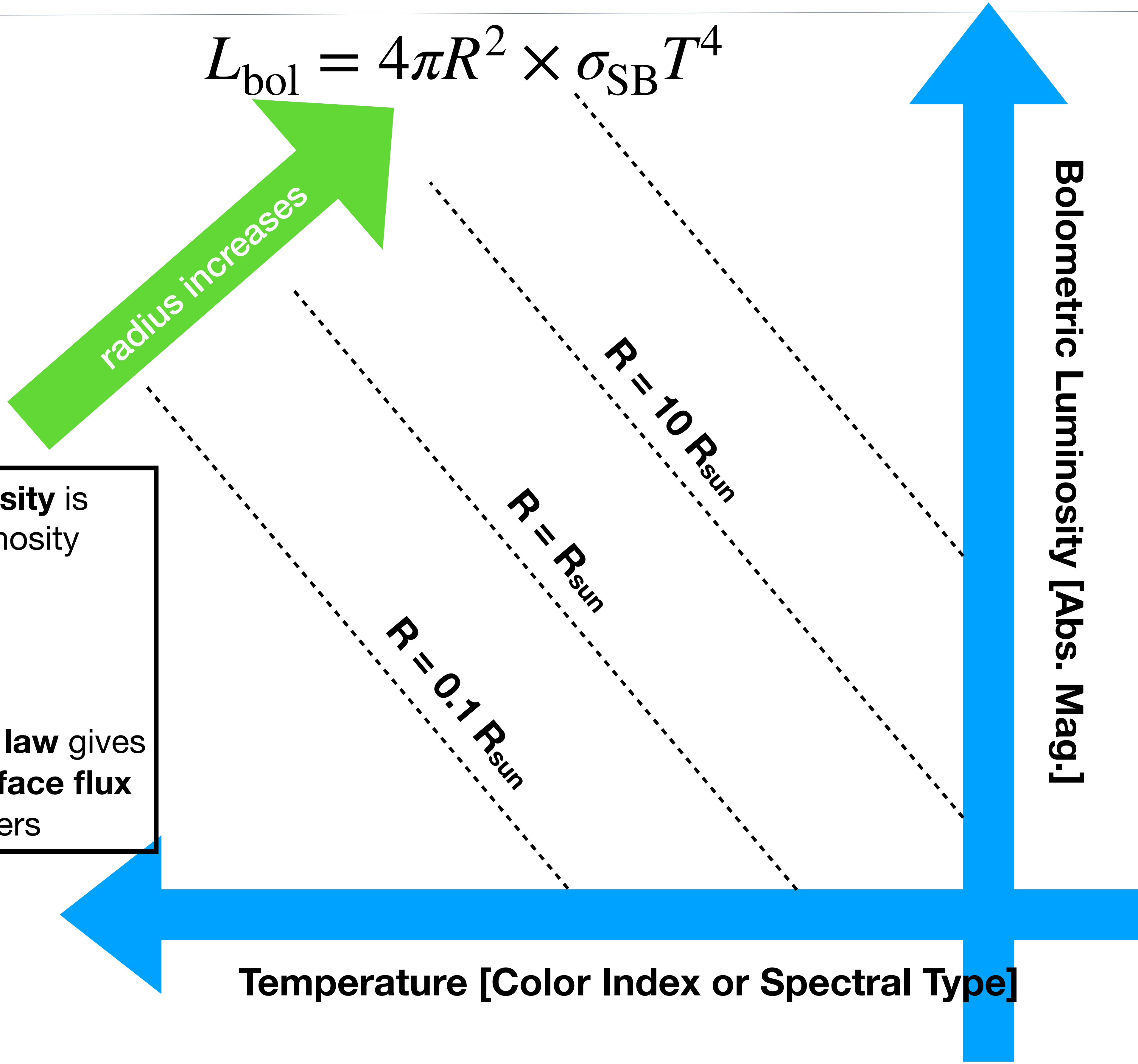
I - Why stars concentrate in certain areas on the HRD

- On the right is the HR diagram made using the Gaia data.
- Bright color indicates high concentration areas of stars:
  - Main sequence
  - Giant branch
  - White dwarfs
- Conclusions:
  - Stars spend much more time in the main sequence, the giant branch, and the white dwarf branch



II - Reading Physical Properties of Stars from the HRD

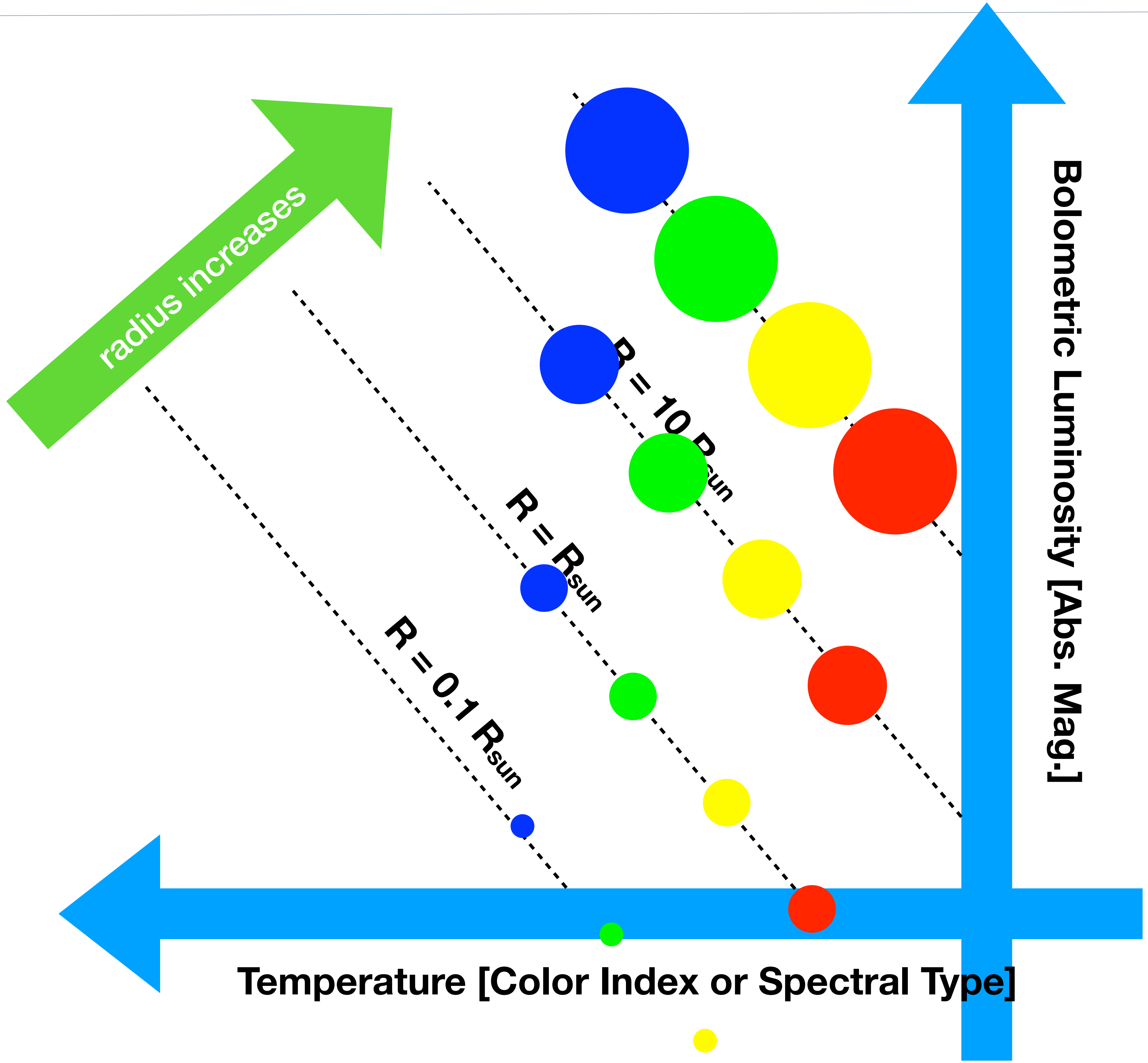
$$L_{\rm bol} = 4\pi R^2 \times \sigma_{\rm SB} T^4$$



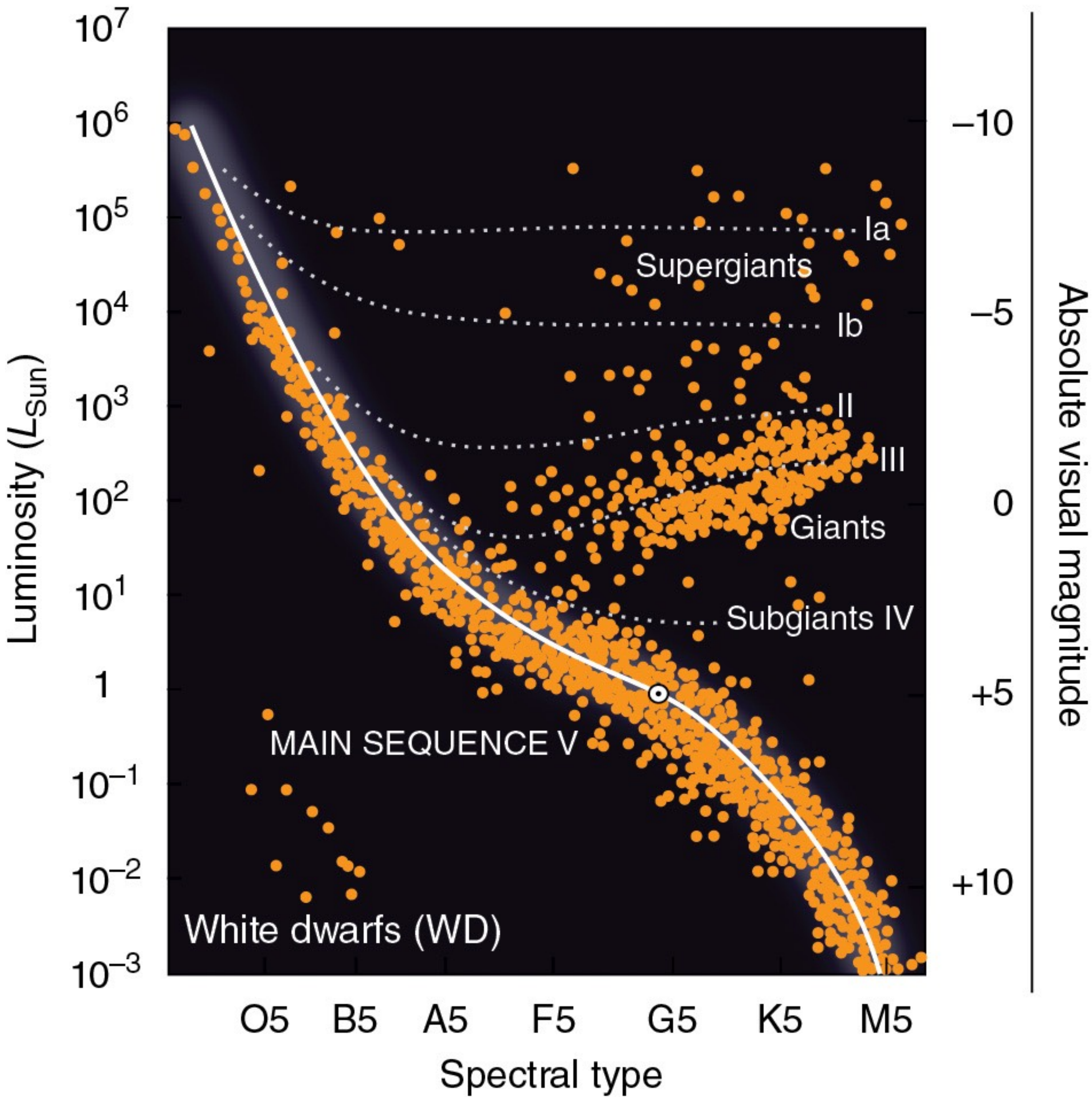
Bolometric luminosity is defined as the luminosity density of a source ***integrated over all wavelengths***.

Stefan-Boltzmann law gives the **bolometric surface flux** of **blackbody** emitters

Size Estimates using Stefan-Boltzmann Law



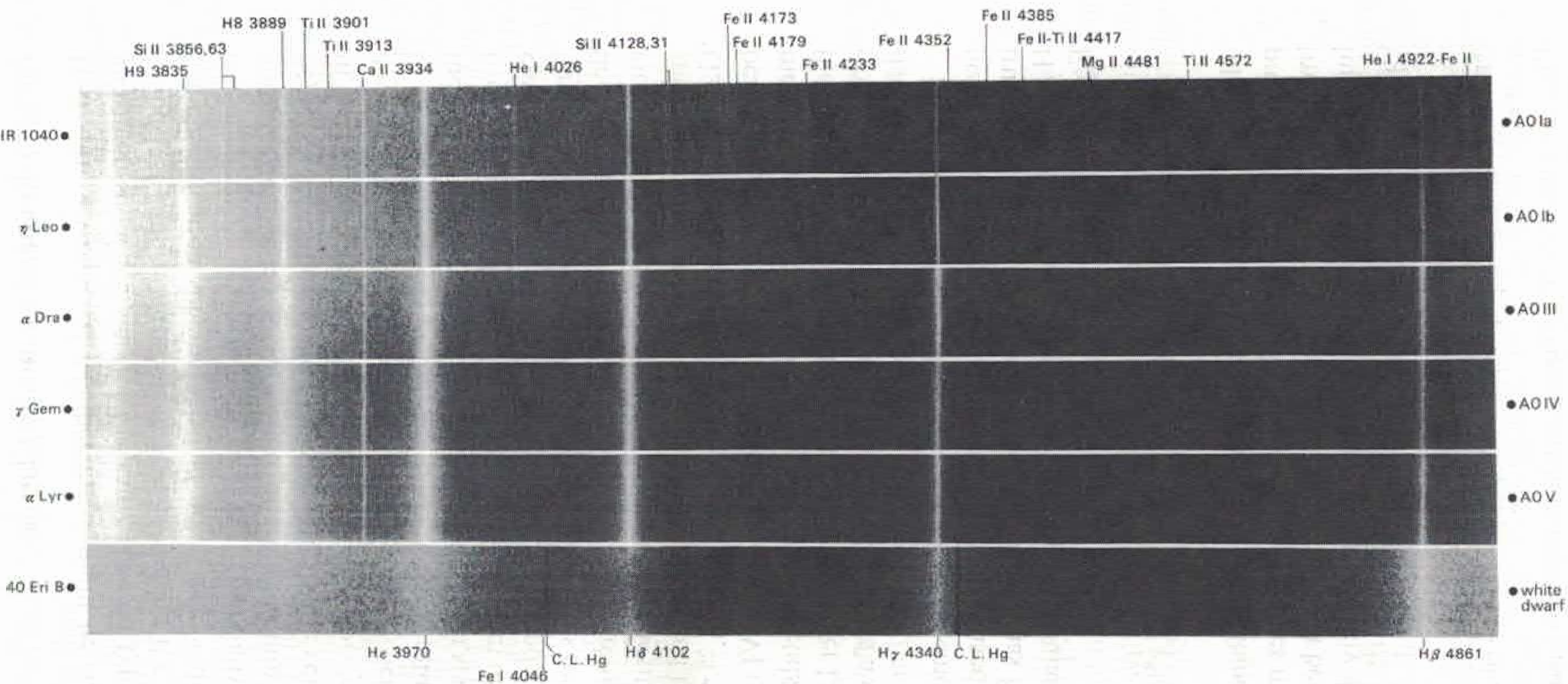
Luminosity Classes I-V: from Giant Stars and Dwarf Stars



- *Broad* **luminosity classes** are defined roughly along the luminosity axis.
- This makes spectral classification of stars in a **two-dimensional** parameter space: T & L
- The Sun is a **G2V** star:
  G2 - spectral type
  V - luminosity class
- Betelgeuse is a **M1Ia**:
  M1 - spectral type
  Ia - luminosity class

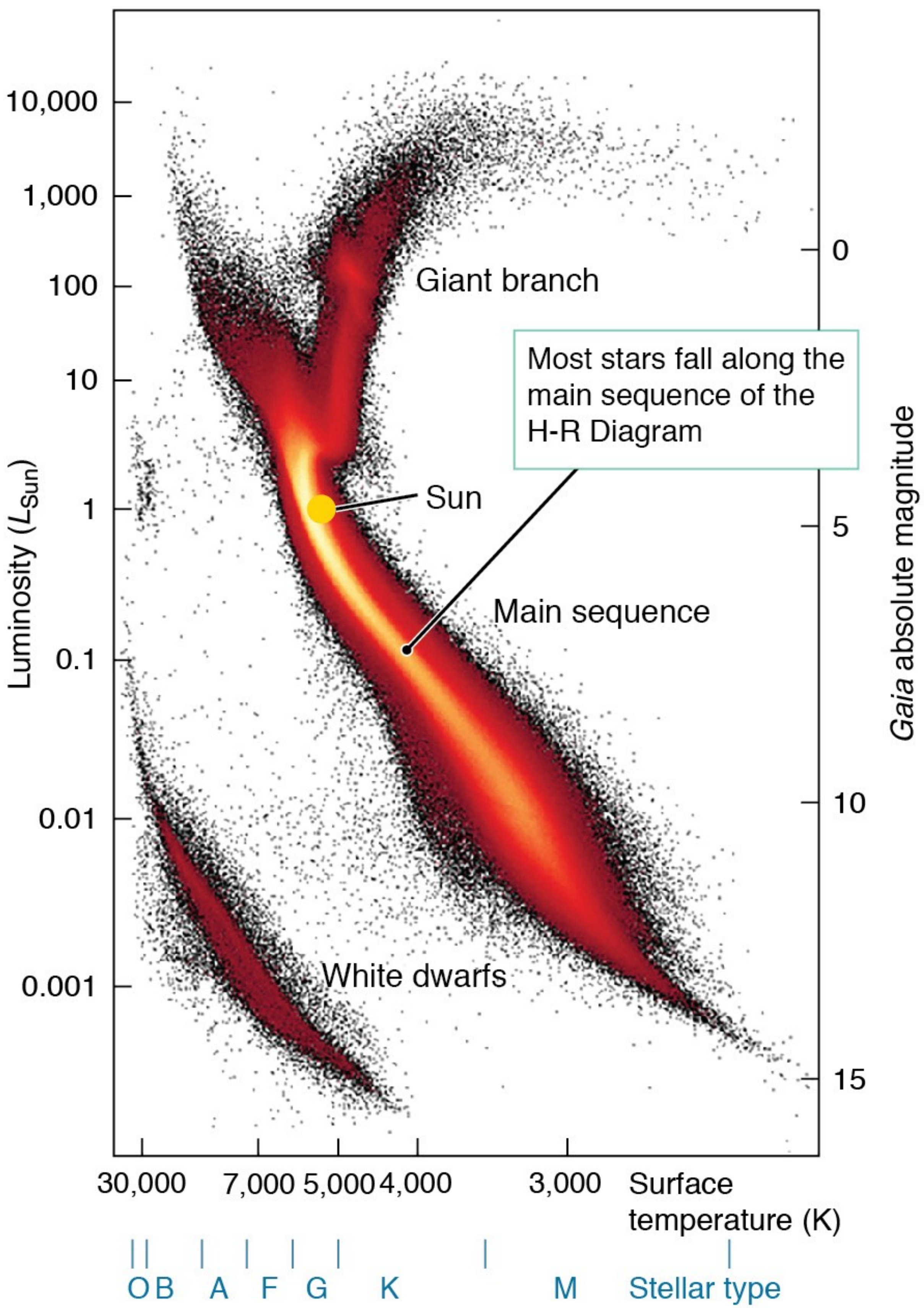
Luminosity Classes - Spectral signatures

- Stars in higher luminosity classes are denser, so they have larger surface gravity. As a result, the absorption lines appear broader.
- Below are examples of A0-type stars from class I to V, plus a white dwarf.



Carroll & Ostlie, Fig 8.15

HR Diagram: Main Sequence, Giant Branch, White Dwarfs, & Luminosity Classes



- Most stars, incl. the Sun, are found on the **main sequence,** which runs from luminous/hot stars in upper left corner to low-luminosity/cool stars in lower right corner
- It covers a temperature range of **~10**, and a radius range of **~100**, but a luminosity range of **~10^9**

- The **Giant branch** is connected to the main sequence but branches off to the lower temperature side. That is where the **red giant stars** live

- A separate branch parallel to the main sequence to the lower left, these stars have low luminosities but hot temperatures; this is where the **White Dwarfs** live

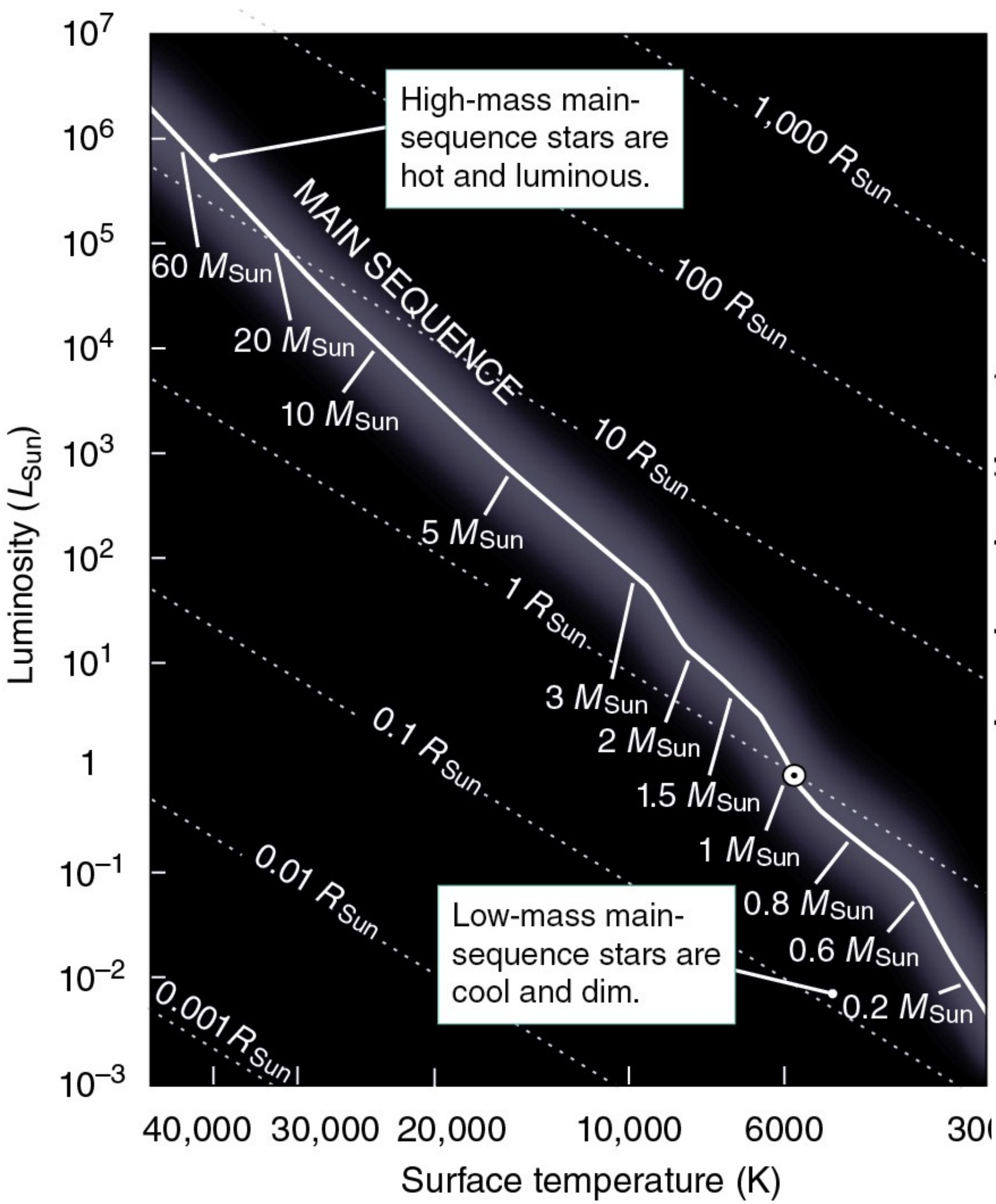
- **Spectral classification is 2-dimensional**: (1) temperature (OBAFGKM), and (2) luminosity (Ia, Ib, II, III, IV, V)

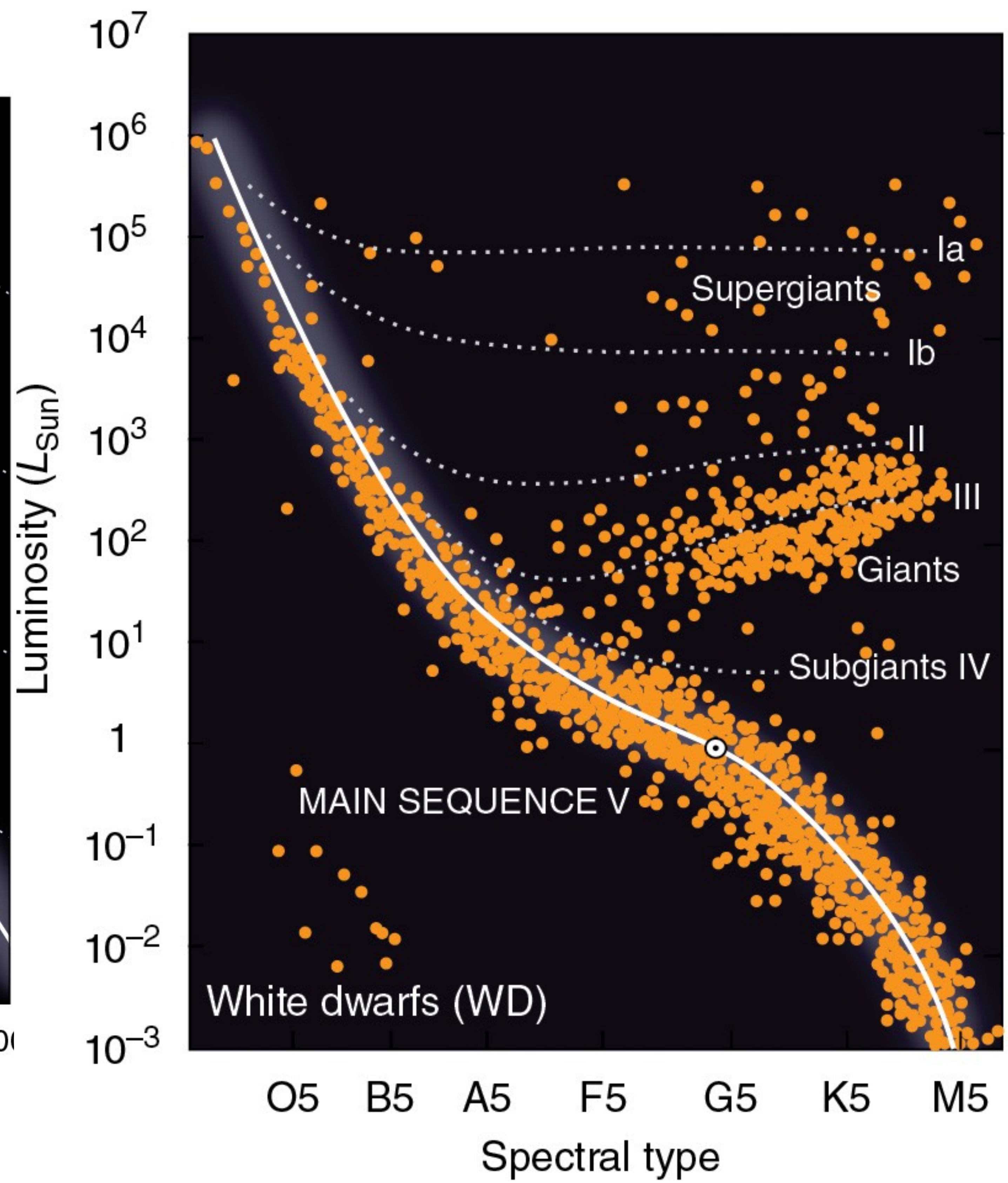
More advanced topics

radius estimates & filtered photometry

Size Estimates using Stefan-Boltzmann Law (Req. bolometric luminosity)

$$L_{\rm bol} = 4\pi R^2 \times \sigma_{\rm SB} T^4$$



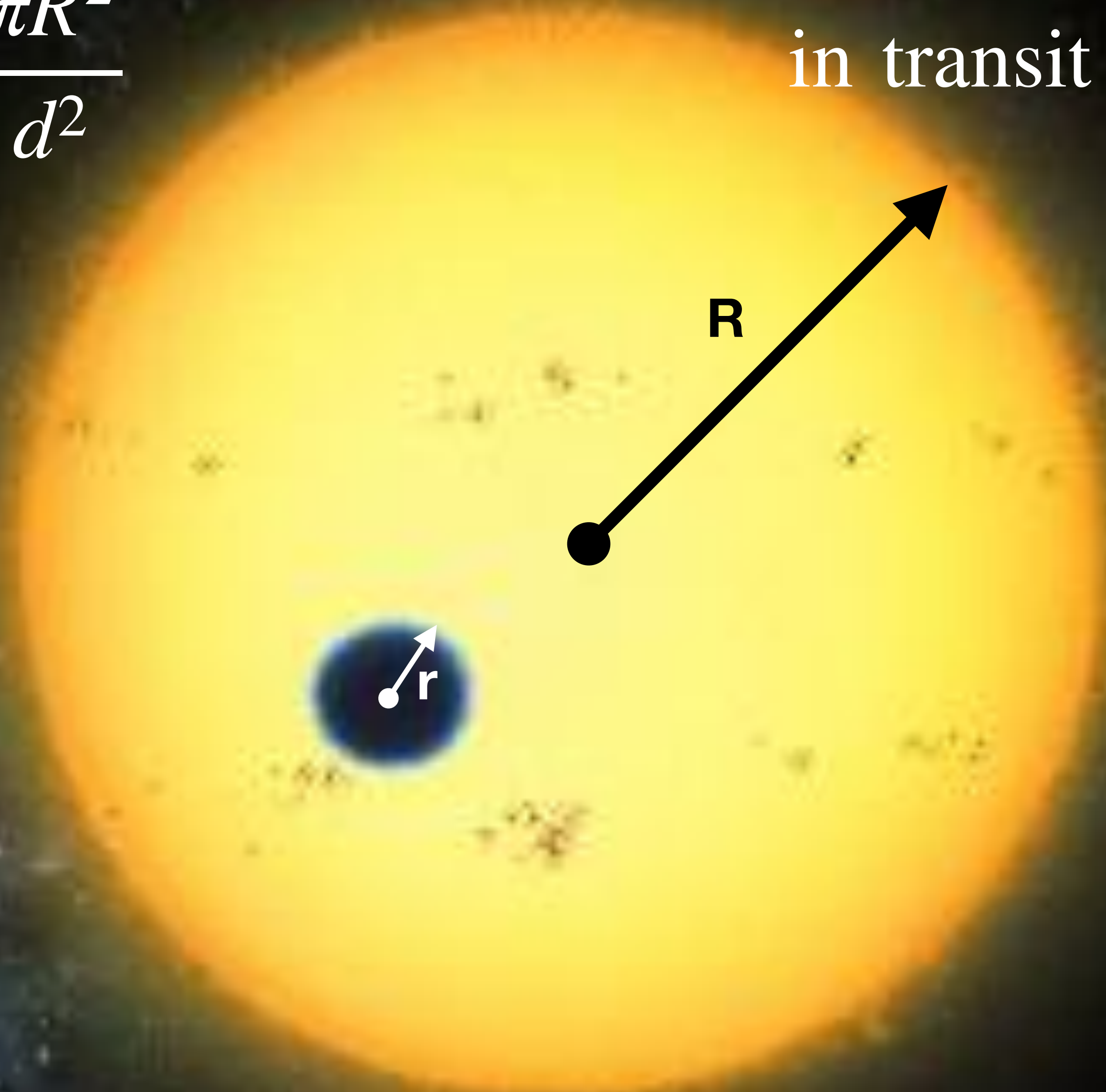


Alternative Method: Radius of the Star from Transit Depth

*Once we know the planet radius from transit ingress/egress time, we can use the **transit depth** to estimate the radius of the star.*

$$\text{no transit}: F_0 = \frac{F_S}{\pi}\frac{\pi R^2}{d^2}$$

$$\text{in transit}: F_t = \frac{F_S}{\pi}\frac{\pi(R^2 - r^2)}{d^2}$$



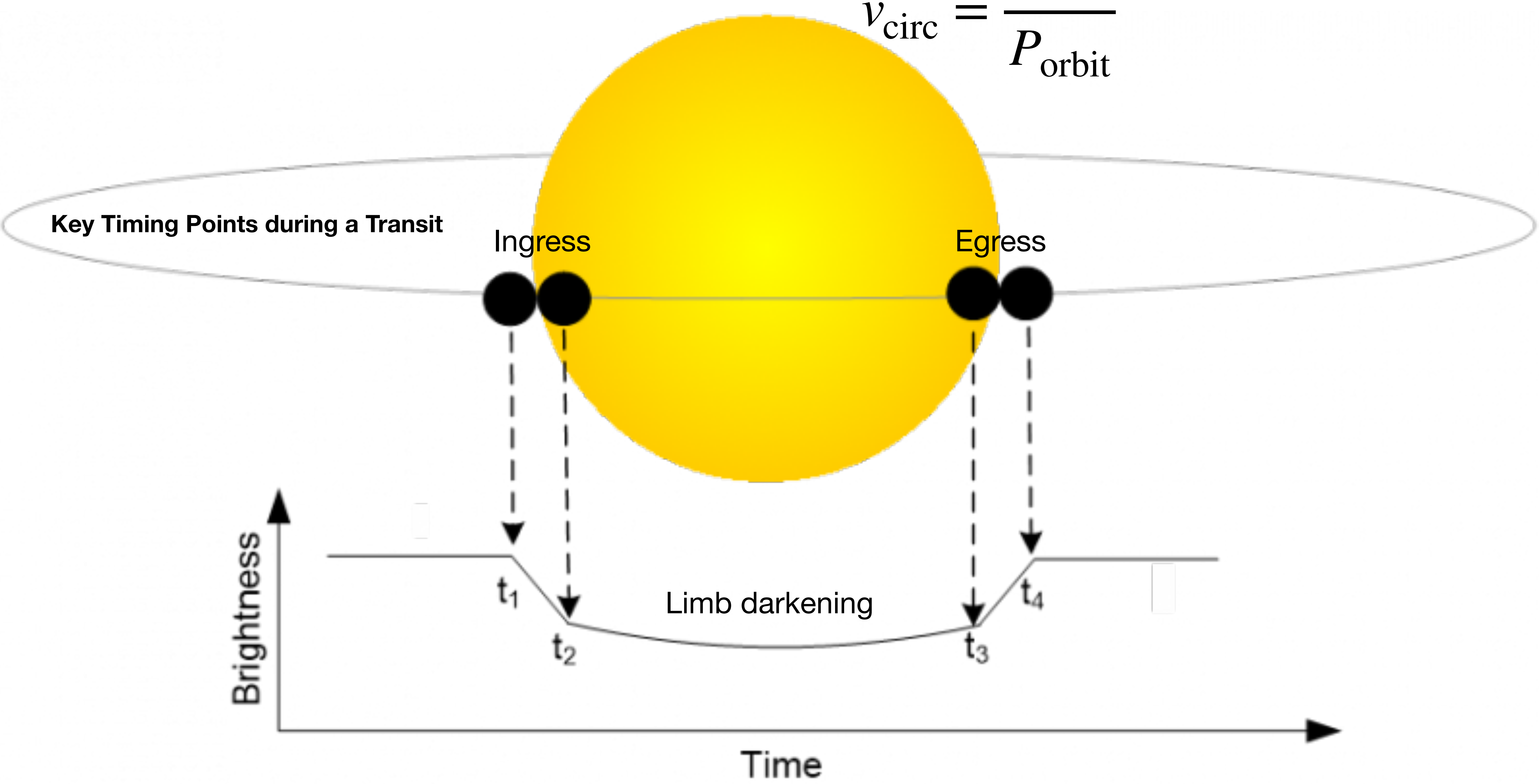
$$\text{fractional reduction in flux}: \frac{F_0 - F_t}{F_0} = \frac{r^2}{R^2}$$

Planet Size from Ingress & Egress time (requires Mass of the Star)

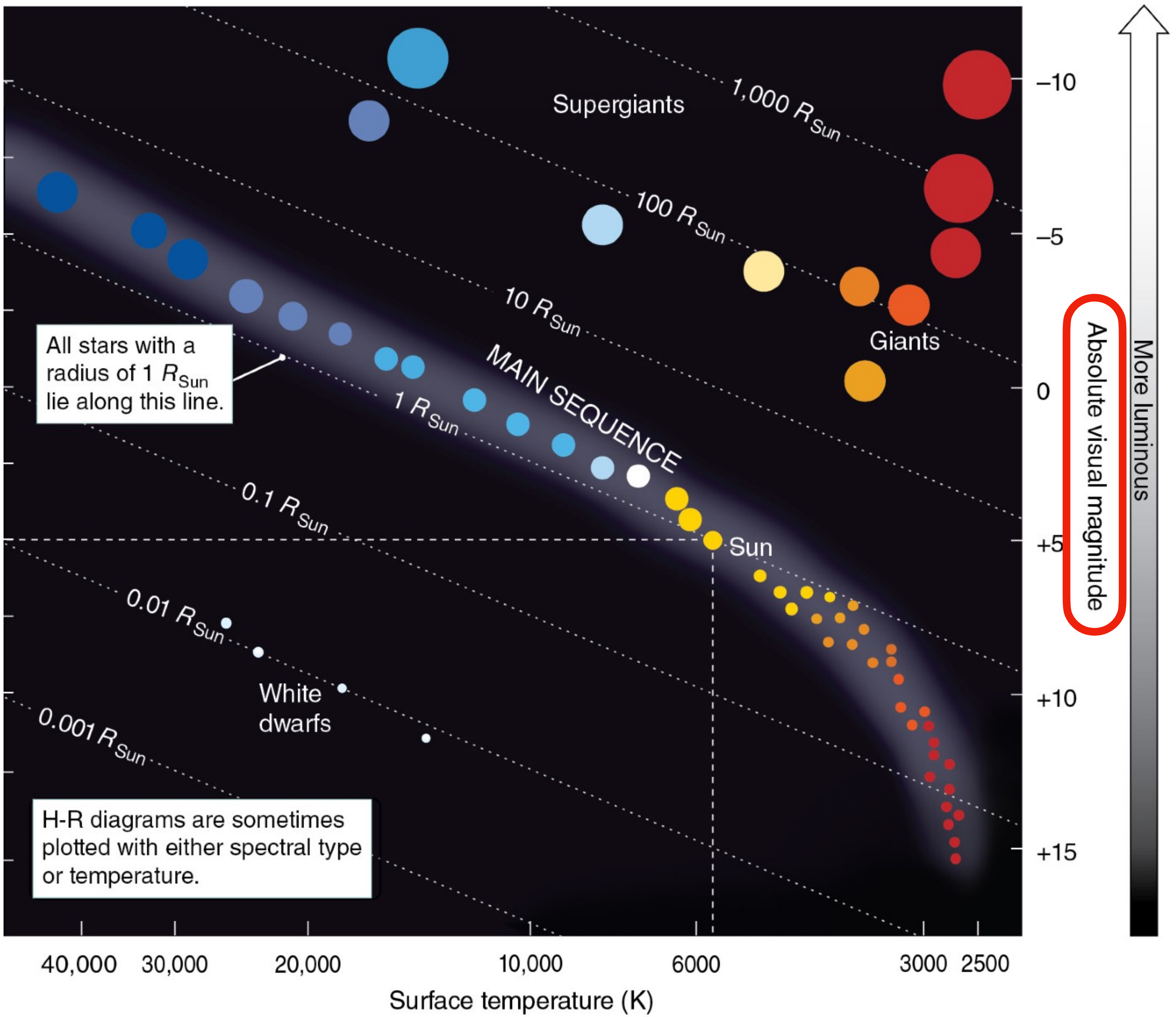
$$r = v_{\mathrm{circ}}(t_2 - t_1)/2$$

$$a_{AU} = (M_{\mathrm{solar-mass}} P_{\mathrm{year}}^2)^{1/3}$$

$$v_{\mathrm{circ}} = \frac{2\pi a}{P_{\mathrm{orbit}}}$$



Side Note: Why this textbook diagram is incorrect?



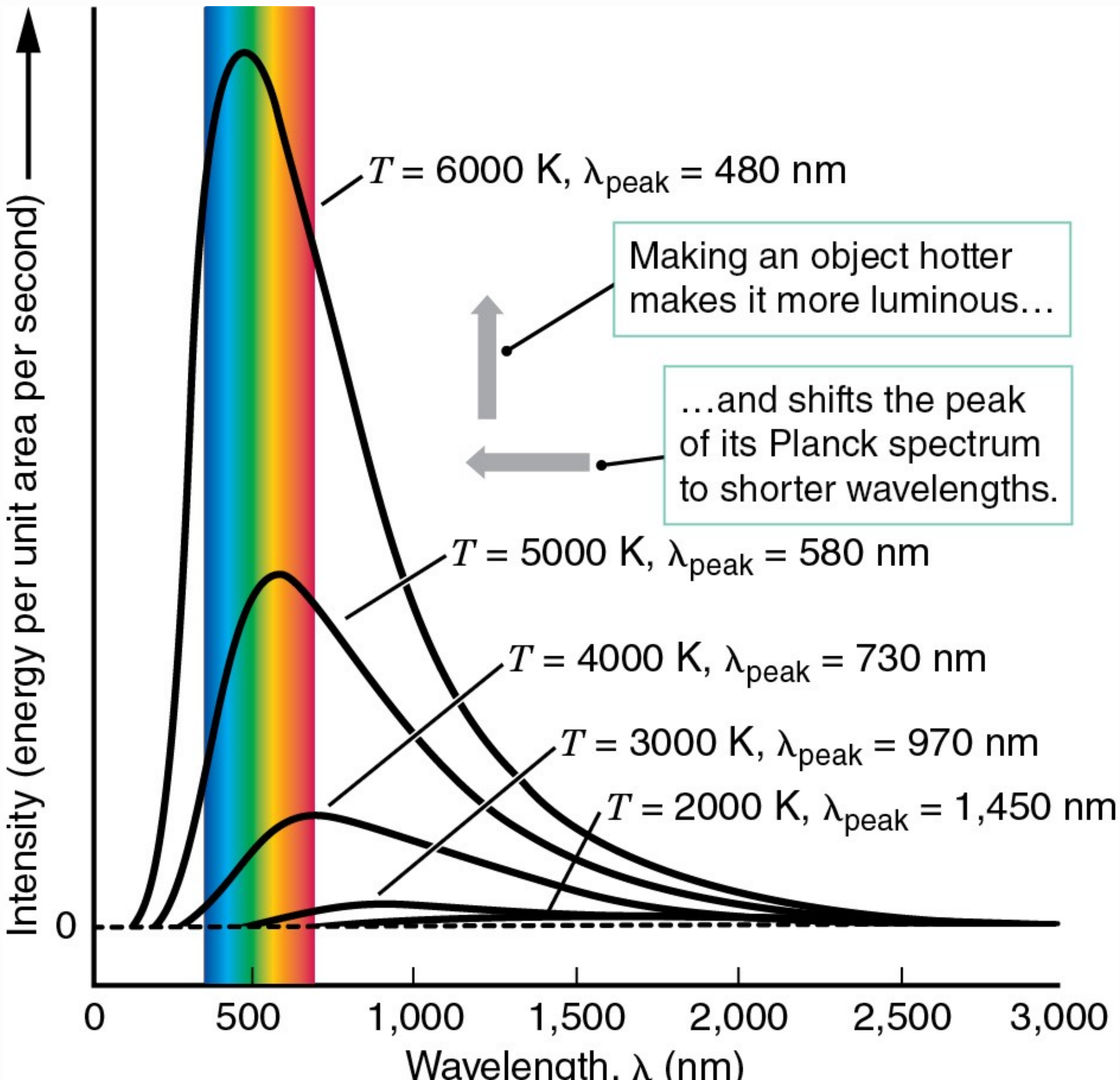
Luminosity Density vs. Bolometric Luminosity

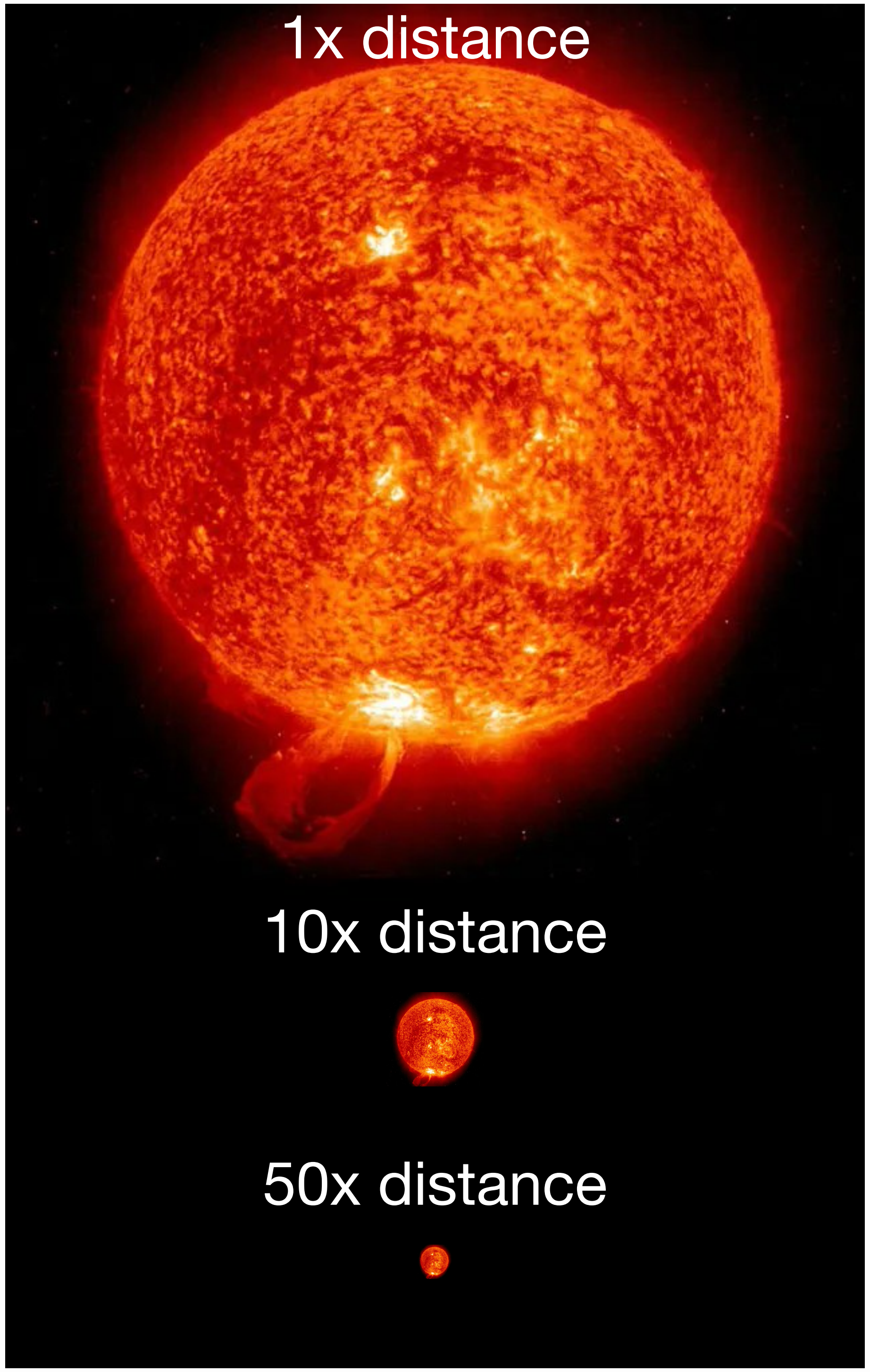
- **Luminosity Density** is the luminosity measured at a given wavelength:
  $L_\lambda = dL/d\lambda$
- For spherical blackbody radiators of uniform surface temperature, we have:
  $L_\lambda = 4\pi R^2 \times \pi B_\lambda(T)$,
  where
  $$B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc/\lambda}{kT}} - 1}$$ is the **Planck function**

- **Bolometric Luminosity** is the luminosity density integrated over all wavelengths:
  $$L_{\text{bol}} = \int_0^\infty L_\lambda d\lambda$$
- For spherical blackbody radiators, we have:
  $L_\lambda = 4\pi R^2 \times \pi B_\lambda(T)$
  as a result,
  $$L_{\text{bol}} = 4\pi R^2 \int \pi B_\lambda(T) d\lambda$$
  $$= 4\pi R^2 \sigma_{\text{SB}} T^4$$

So … What $B_\lambda(T)$ actually is to an observer?

- **Flux density** can be derived from **luminosity density** using the **inverse distance square law**: $F_\lambda = L_\lambda / 4\pi d^2 = B_\lambda(T)\pi R^2/d^2$
- Since $\pi R^2/d^2$ is the **angular area** of the source, hence **Planck function** $B_\lambda(T)$ gives the **surface brightness** of the source *(at λ), which is **distance invariant.***





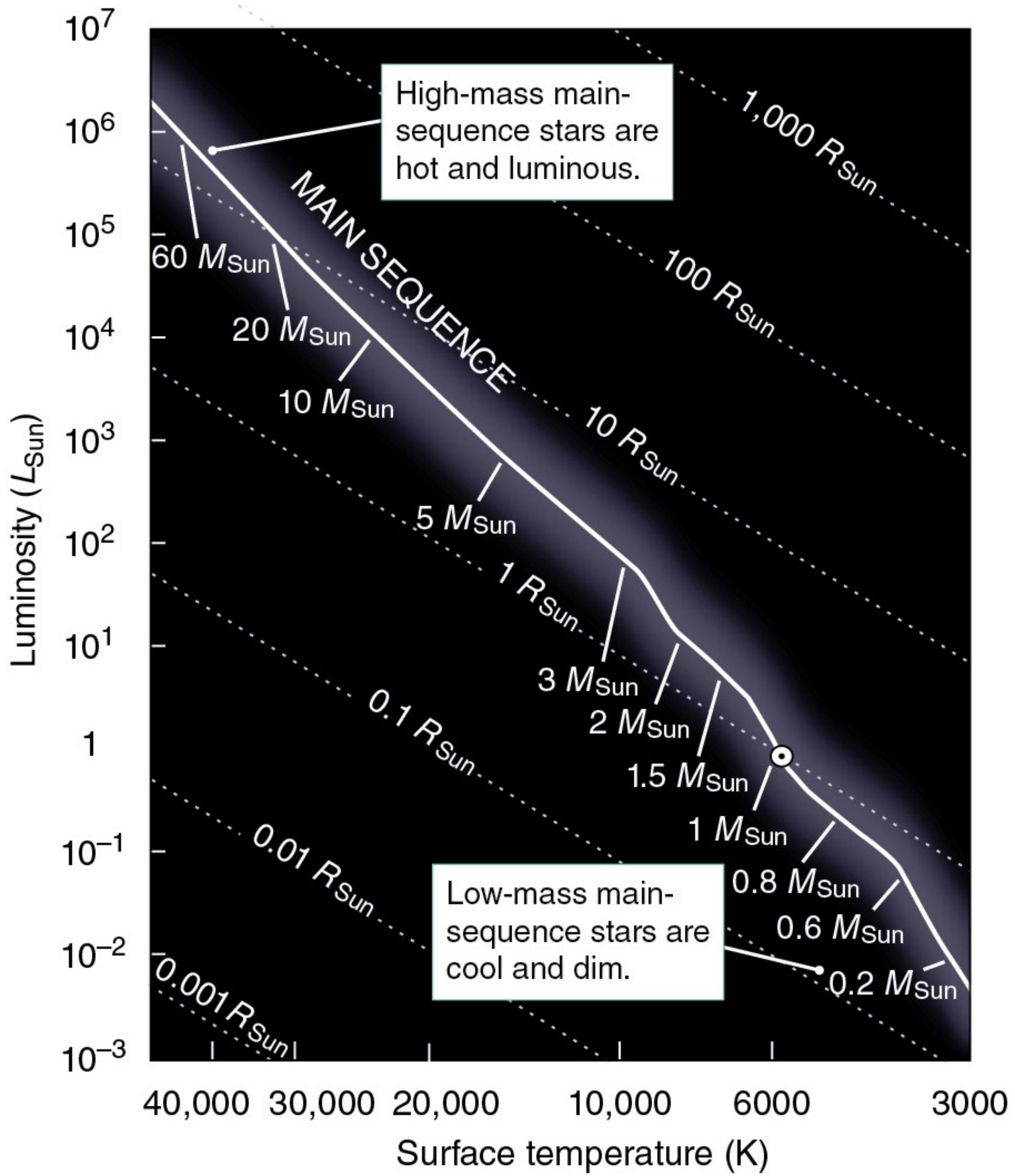
How to measure stellar mass?

Binary stars and Kepler's Laws

How did we know that the main sequence stars cover a range of masses?

TABLE 13.2 Properties of Main-Sequence Stars

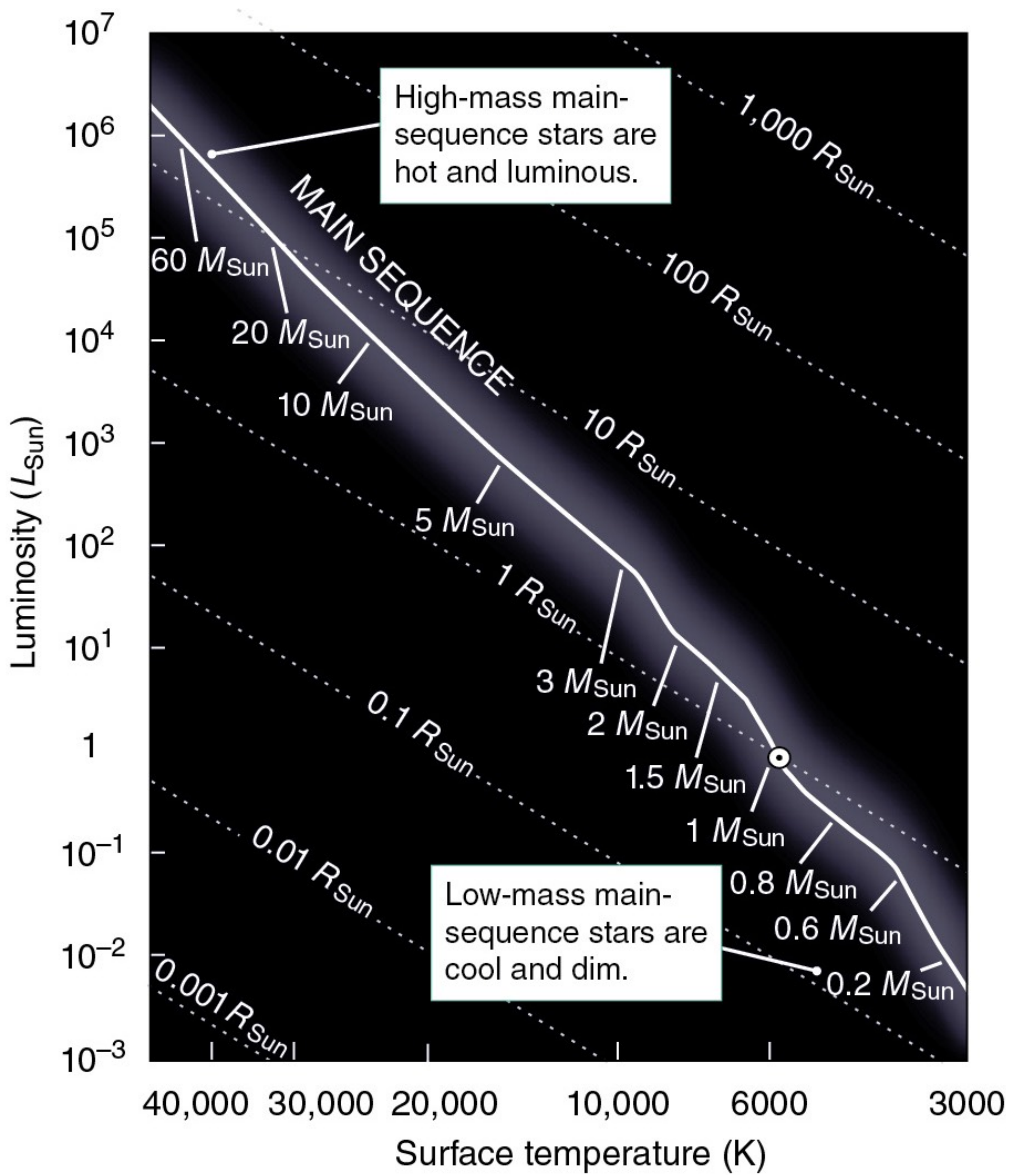
Spectral Type	Temperature (K)	Mass (M_{Sun})	Radius (R_{Sun})	Luminosity (L_{Sun})
O5	42,000	60	13	500,000
B0	30,000	17.5	6.7	32,500
B5	15,200	5.9	3.2	480
A0	9800	2.9	2.0	39
A5	8200	2.0	1.8	12.3
F0	7300	1.6	1.4	5.2
F5	6650	1.4	1.2	2.6
G0	5940	1.05	1.06	1.25
G2 (Sun)	5780	1.00	1.00	1.0
G5	5560	0.92	0.93	0.8
K0	5150	0.79	0.93	0.55
K5	4410	0.67	0.80	0.32
M0	3840	0.51	0.63	0.08
M5	3170	0.21	0.29	0.008



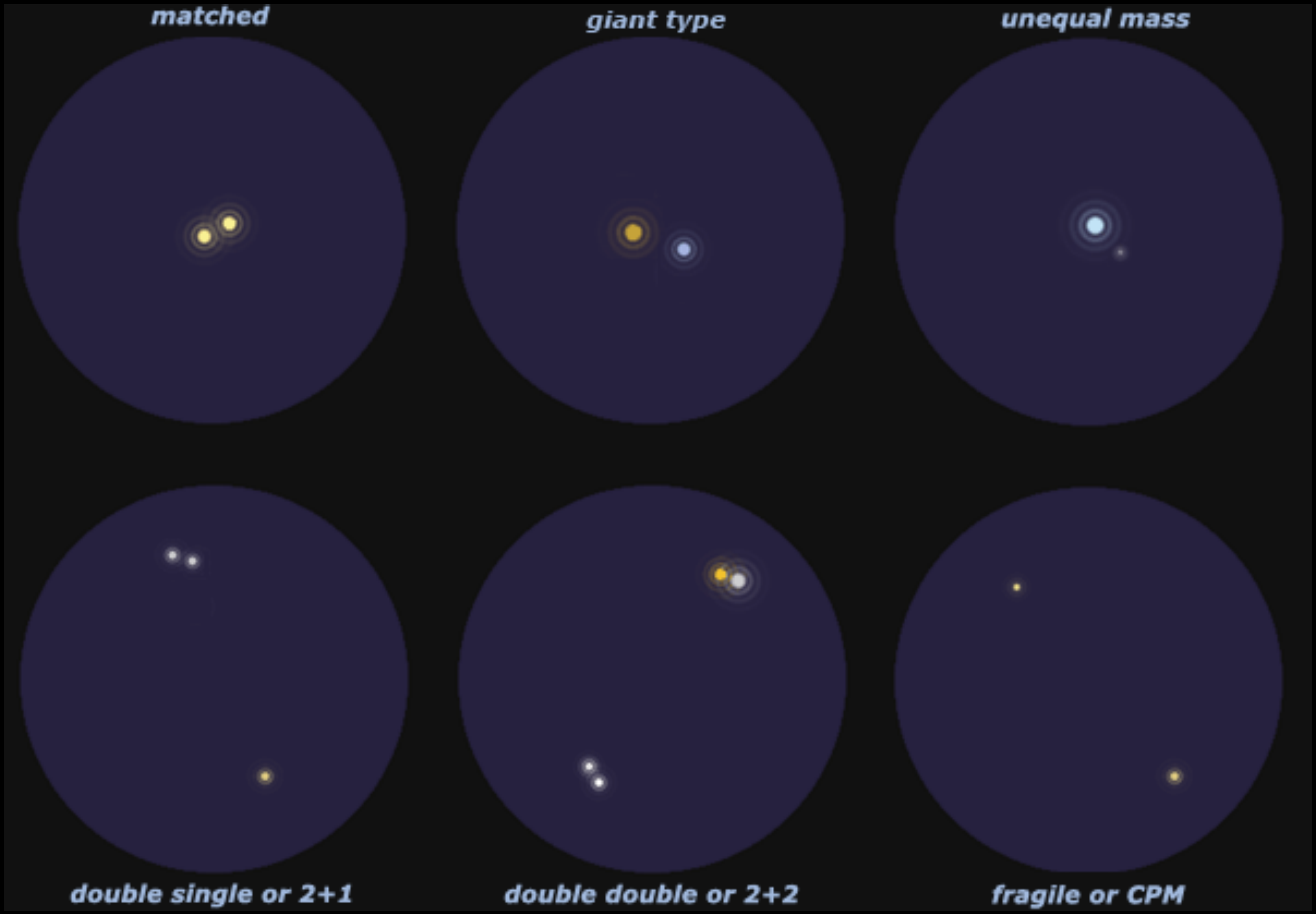
which led us to conclude that the MS stars have drastically different lifetimes

Table 16.1 Main-Sequence Lifetimes

Spectral Type	Mass (M_{Sun})	Luminosity (L_{Sun})	Main-Sequence Lifetime (years)
O5	60	500,000	3.6×10^5
B0	17.5	32,500	7.8×10^6
B5	5.9	480	1.2×10^8
A0	2.9	39	7×10^8
A5	2.0	12.3	1.6×10^9
F0	1.6	5.2	3.1×10^9
F5	1.4	2.6	4.3×10^9
G0	1.05	1.25	8.9×10^9
G2 (Sun)	1.0	1.0	1.0×10^{10}
G5	0.92	0.8	1.2×10^{10}
K0	0.79	0.55	1.8×10^{10}
K5	0.67	0.32	2.7×10^{10}
M0	0.51	0.08	5.4×10^{10}
M5	0.14	0.008	4.9×10^{11}
M8	~0.08	0.0003	1.1×10^{12}

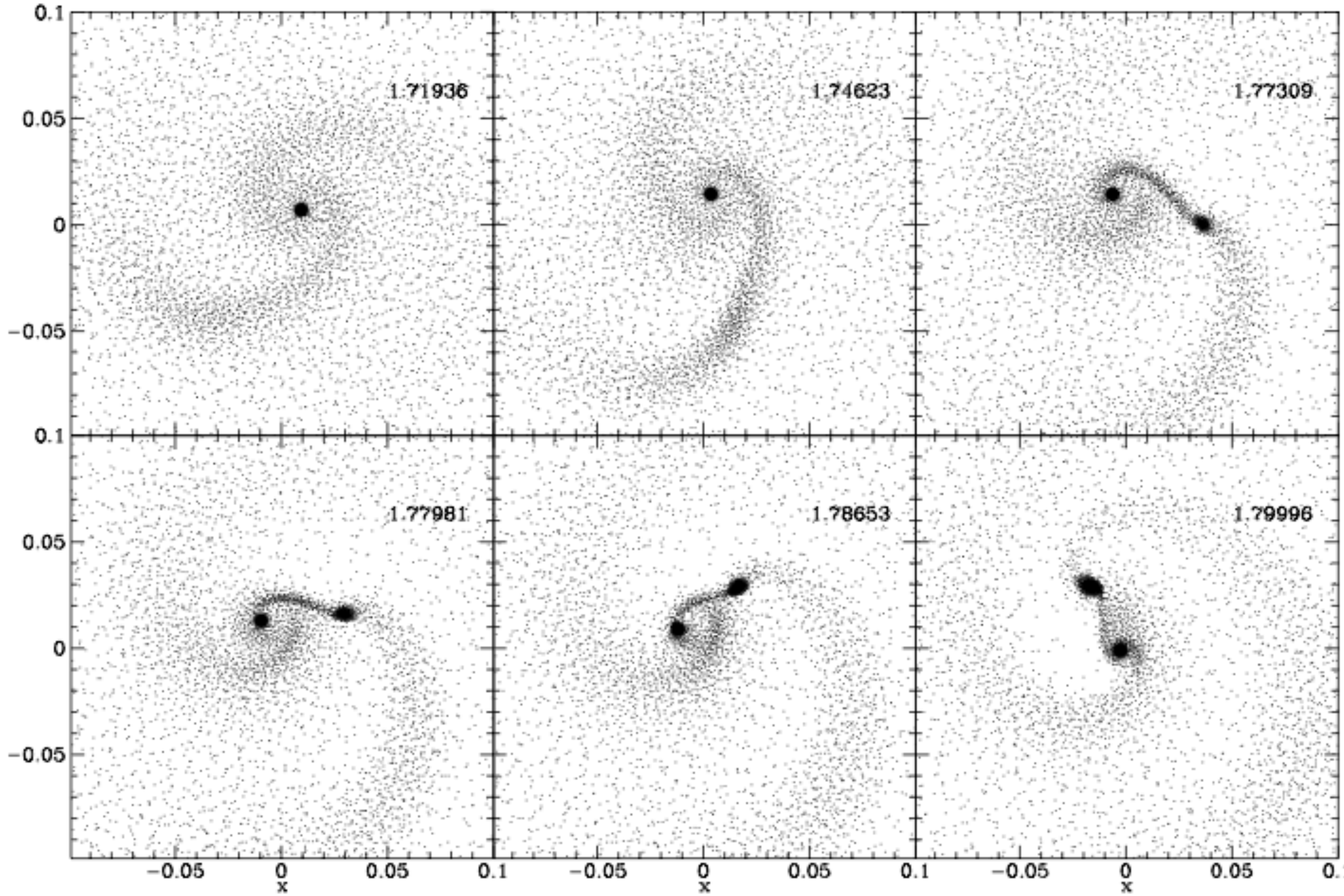


The various configurations of visual binaries and multiples



B. MacEvoy 2012

Binary Star Formation: Accretion Disk Fragmentation



Fragmentation of the protostar accretion disk is believed to be a frequent if not the most common path to binary formation at distances of around 40 AU … a massive spiral arm forces the protostar off the center of mass to produce a binary structure; the spiral arms draw more mass into the accretion disk while reducing the binary orbital momentum via gravitational (and possibly magnetic) torque (Source: Bonnell & Bate, 1994)

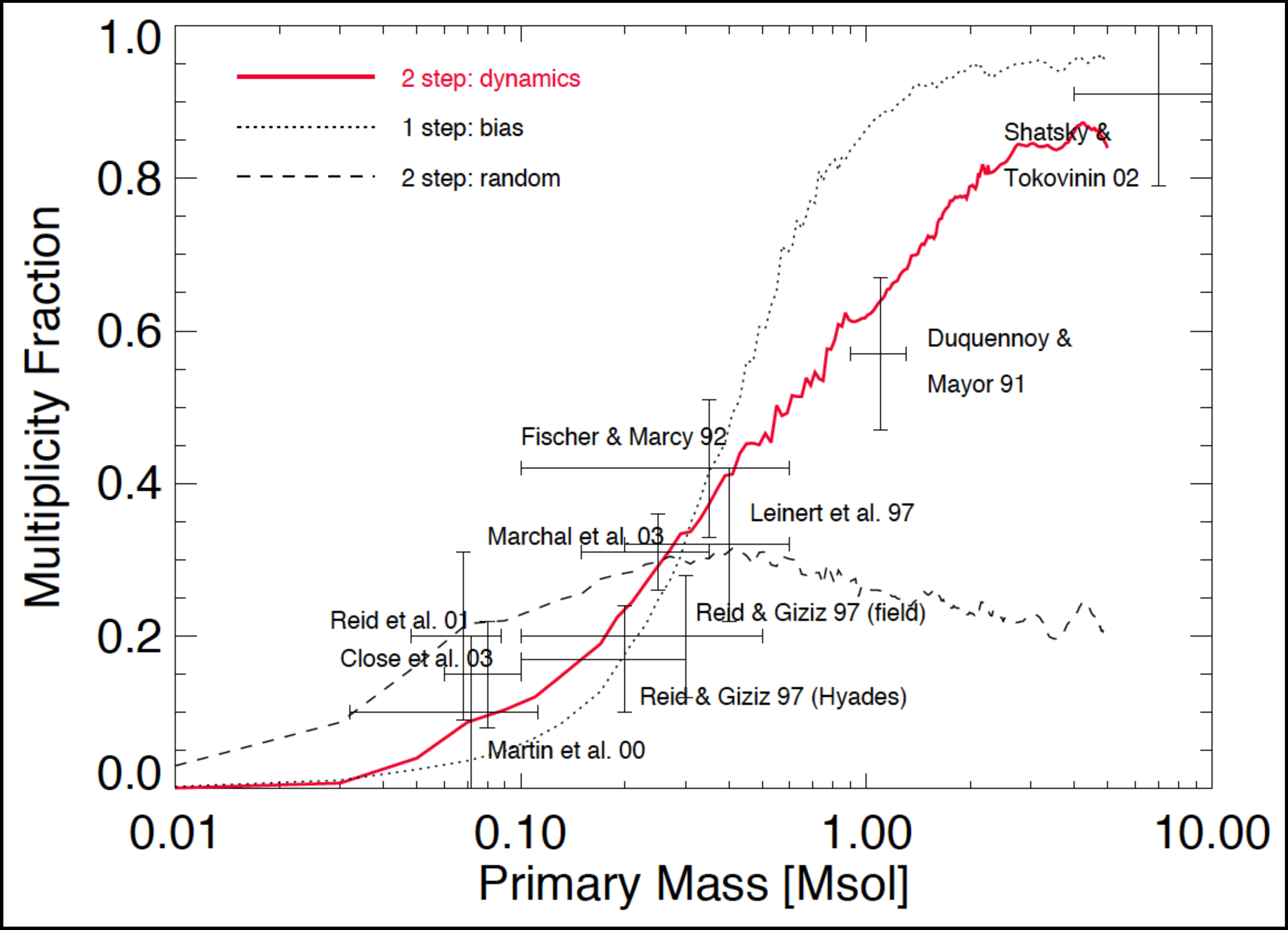
What fraction of stars are binaries?

For solar-type stars, ~60% of star systems are single star systems, yet only ~40% of all stars are single stars (i.e., ~60% of all stars are components of binary or multiple star systems)

	Kuiper (1942)	*Heintz (1969)*	*Abt & Levy (1976)**	*Duquennoy & Mayor (1991)*	*Nordström et al. (2004)*	*Raghavan et al. (2010)*
Systems (N)	274	*n.a.*	123	164	16682	454
Single Star Systems	70%	30%	45%	**57%**	66%	**56%**
Binary	25%	47%	46%	38%	34%	33%
3	4%	16%	8%	4%	.	8%
4+	1%	7%	1%	1%	.	3%
Double Star Systems	30%	70%	55%	43%	34%	**44%**
Median R		50 *AU*		35 *AU*		40 *AU*
Stars in Doubles	52%	85%	73%	**62%**	51%	**65%**

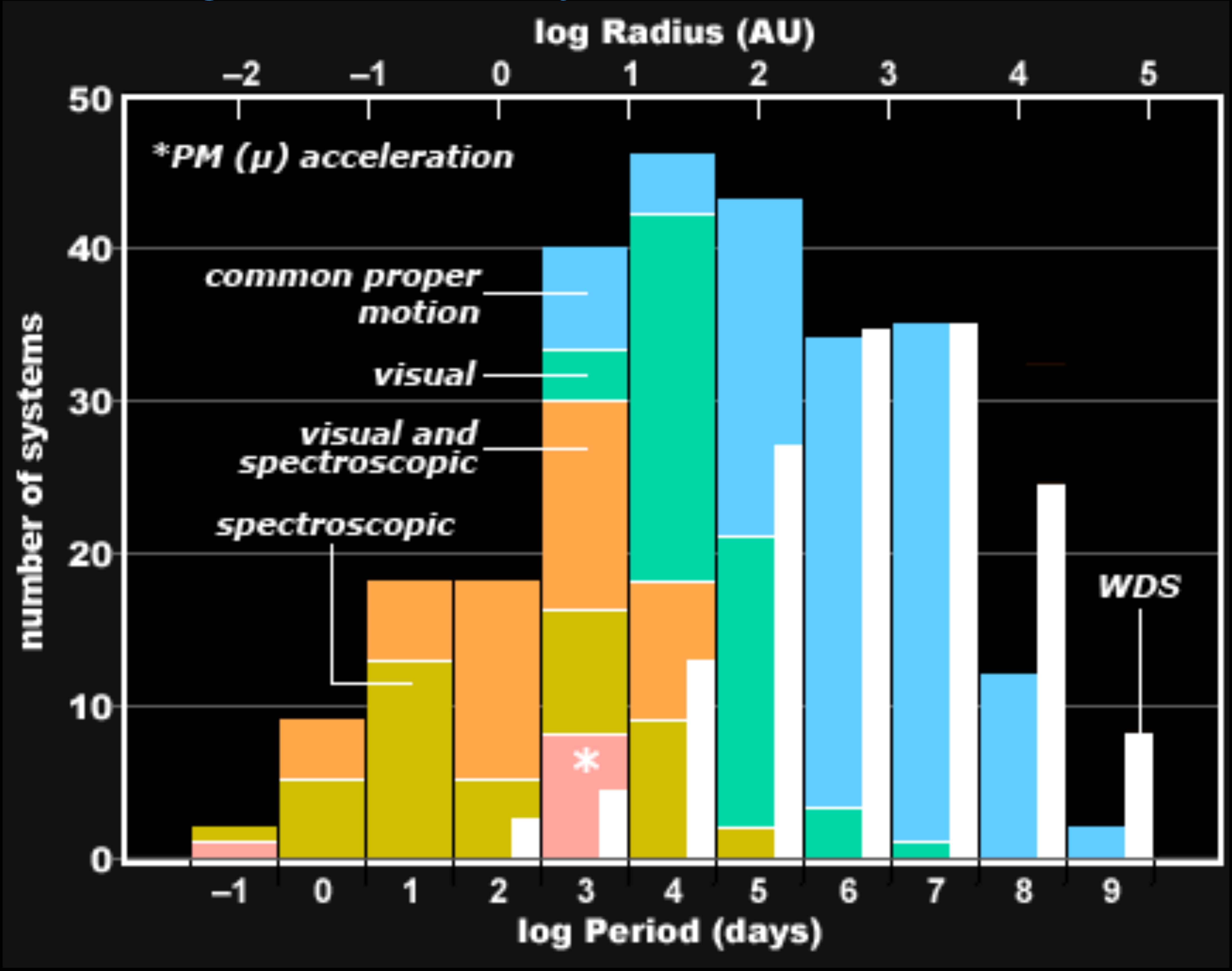
B. MacEvoy 2012

The multiplicity fraction increases with the mass of the primary



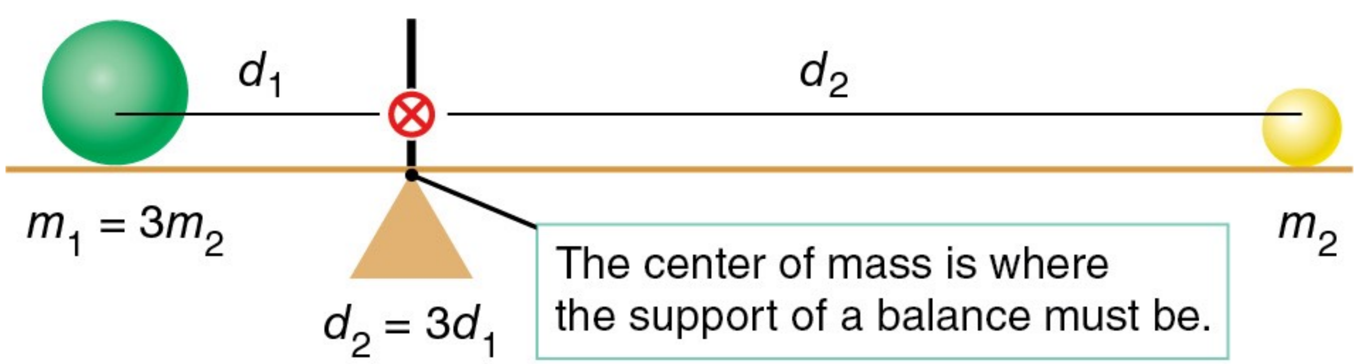
B. MacEvoy 2012

The logarithmic of Binary Periods follow a "Bell" curve

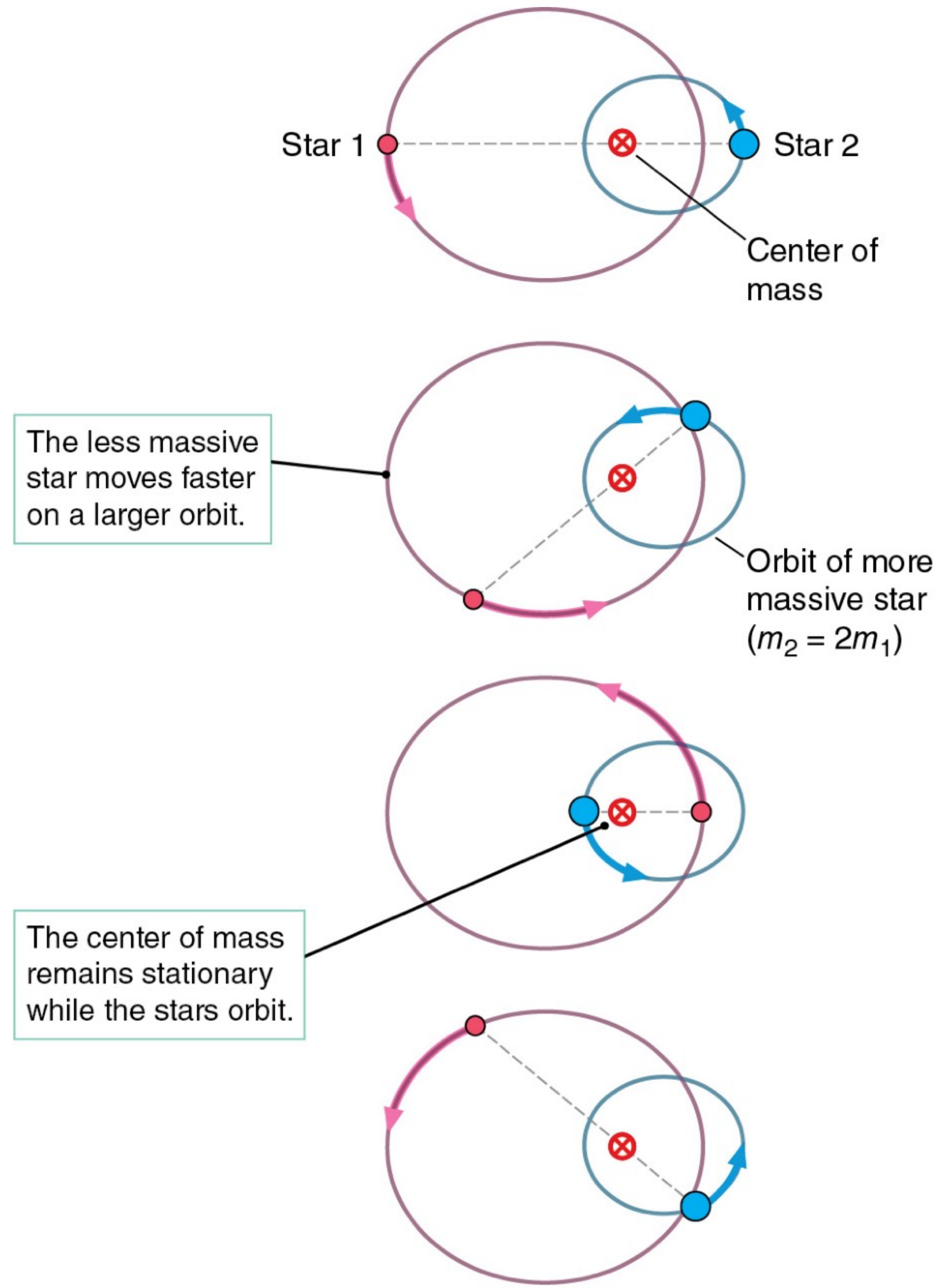


Binary Star - Center of Mass

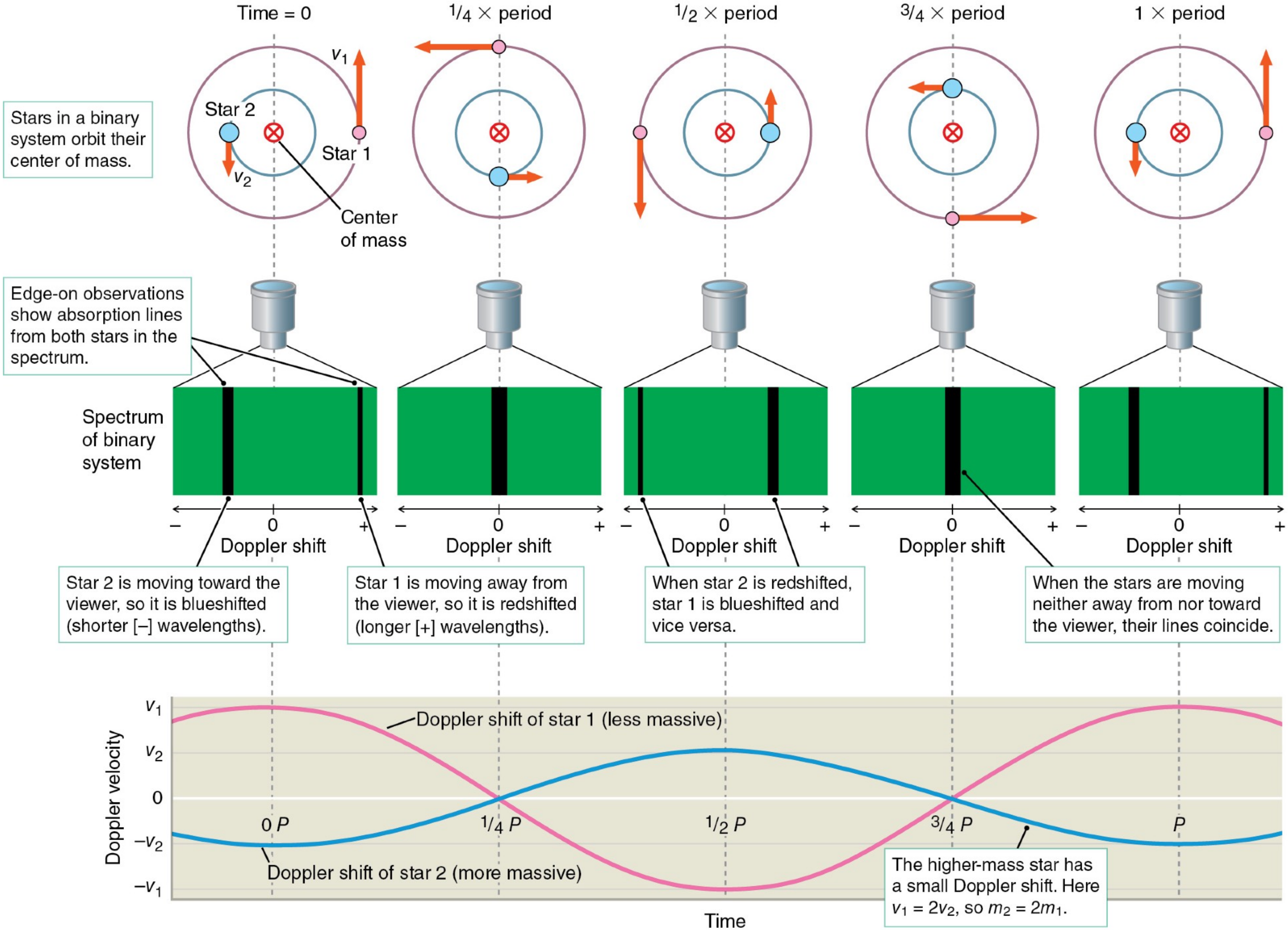
- To measure mass, we must look for the effects of gravity.
- Many stars are **binary stars** orbiting a common **center of mass**.
- A less massive star moves faster on a larger orbit.



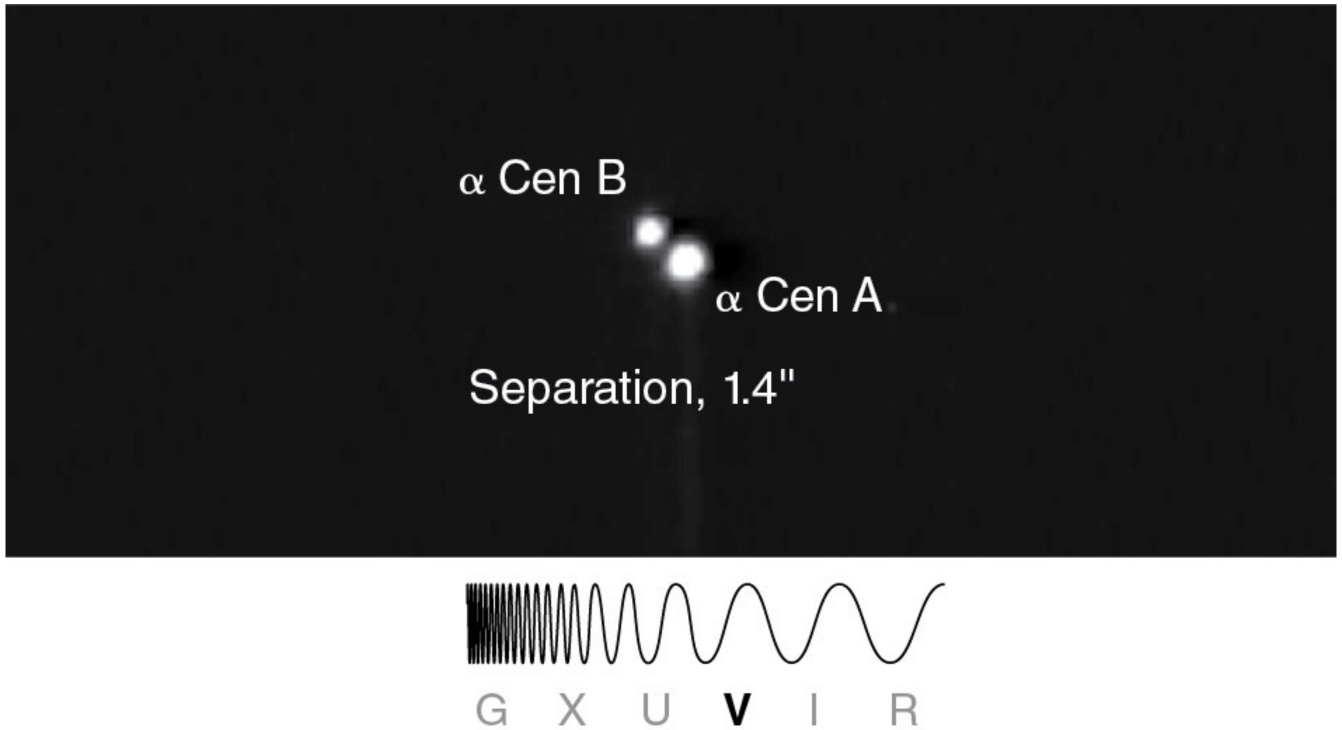
Center of mass "seesaw" equation:
$m_1 d_1 = m_2 d_2$



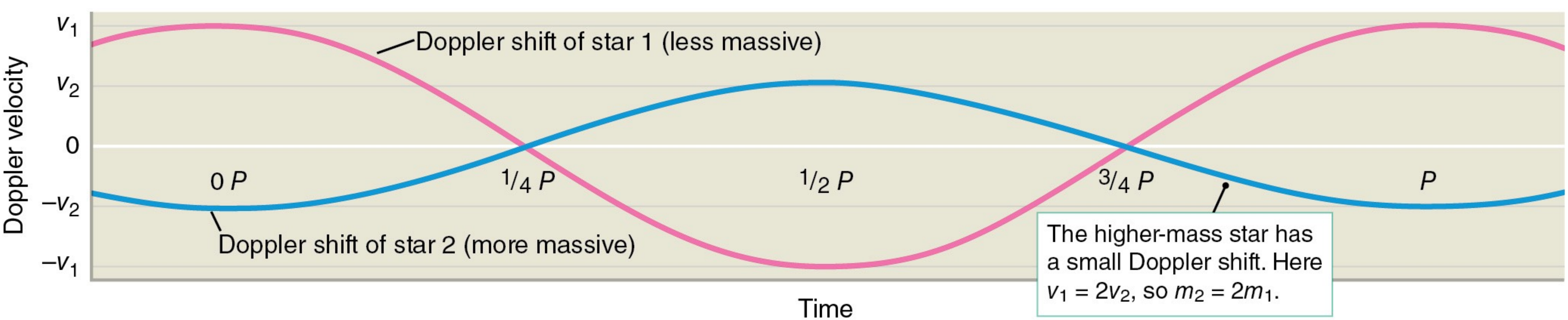
Binary Star - Doppler Shift Measurements vs. Time



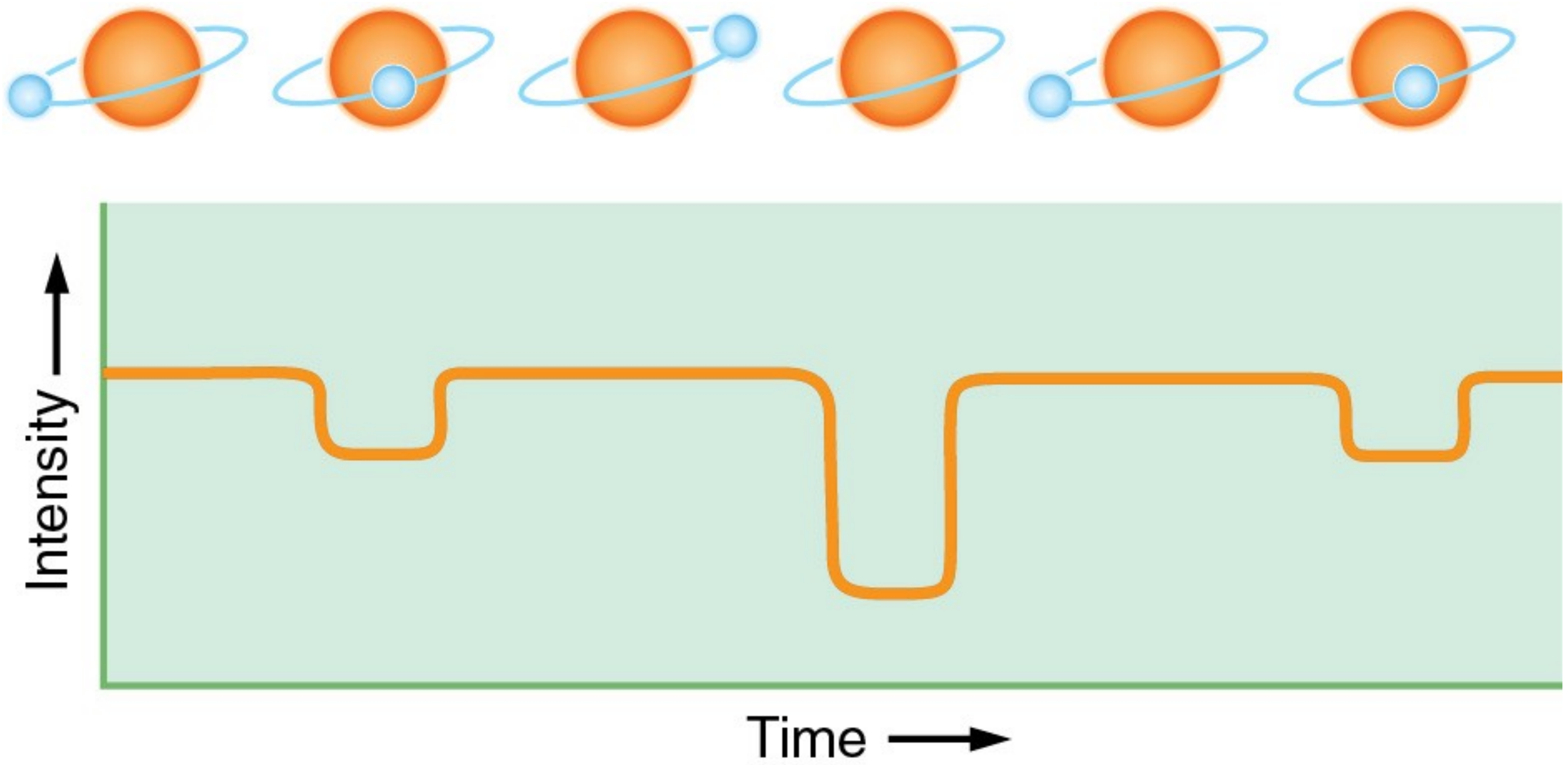
Binary Stars: Doppler shift curves from spectroscopy



- A **visual binary** system is one in which both stars are distinguished visually.
- In a **spectroscopic binary** system, stars are too far away to distinguish; pairs of Doppler-shifted lines trade places.



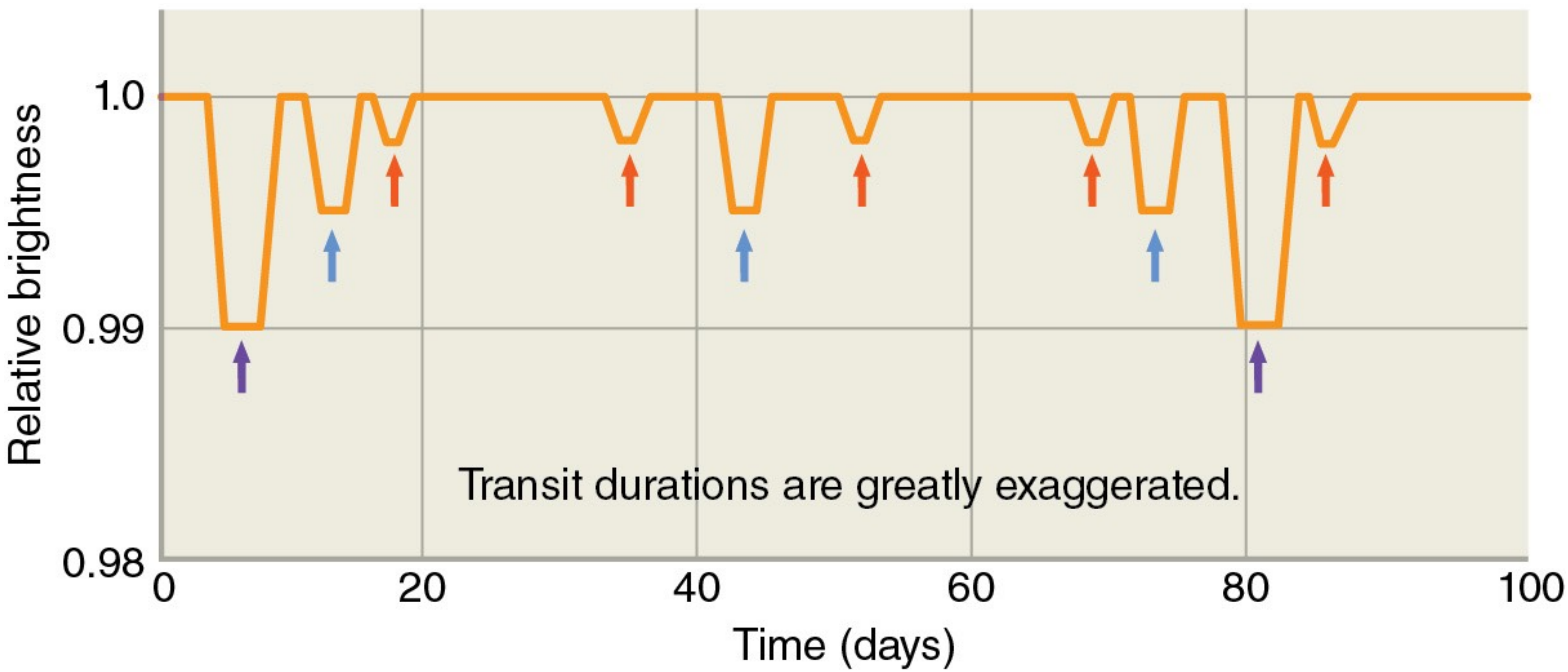
Eclipsing Binary Stars - Light curve from photometry



- In an **eclipsing binary** system, the total light coming from the star system decreases when *either* star passes in front of the other.
- But there are two eclipses (A in front of B vs. B in front of A), why one is deeper than the other? What can the eclipse light curve tell us?
- Can we also measure the radii of the stars in these systems?

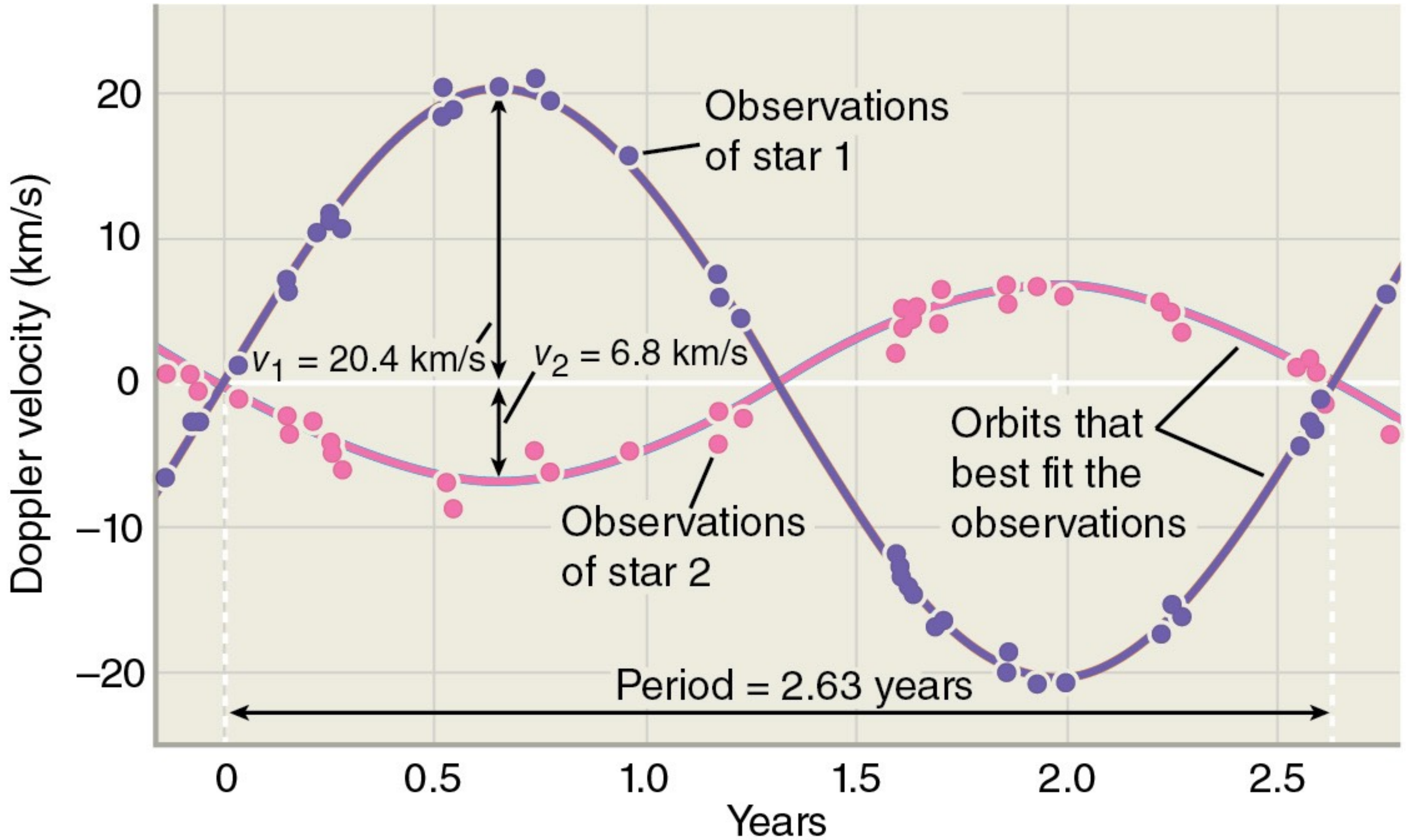
Planet Size from the Depth of the Transit and Radius of the Star

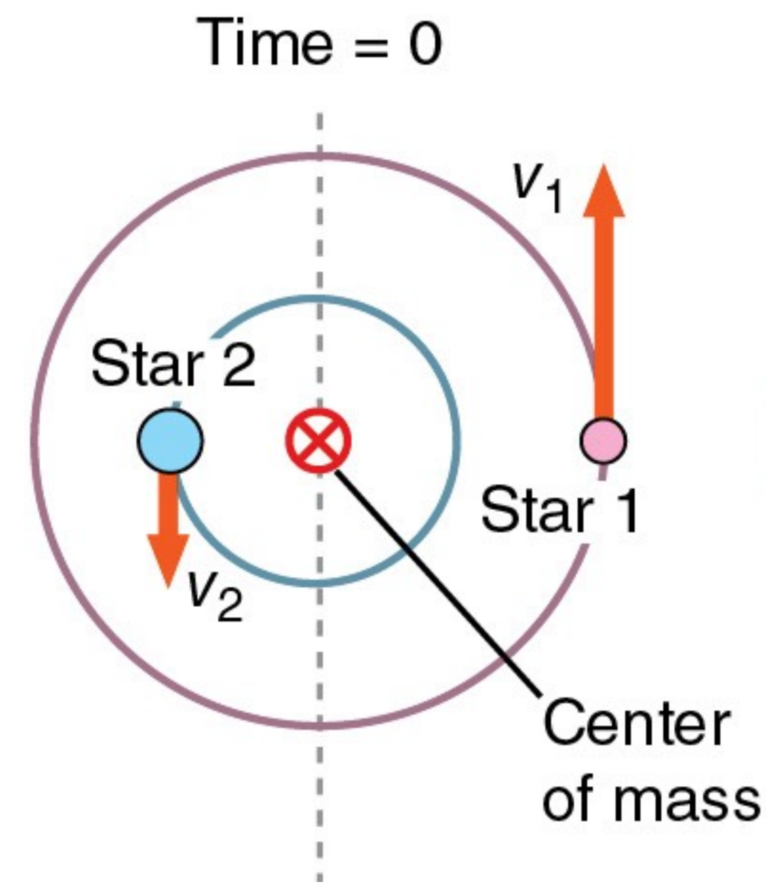
- Notice the difference between eclipsing binary stars and transiting planets: ***both stars emit light, while planets only reflects light from the side that faces the star.***



$$\text{fractional reduction in flux}: \frac{F_0 - F_t}{F_0} = \frac{r^2}{R^2}$$

Measuring the Masses of Stars in Eclipsing+Spectroscopic Binaries





- Being an eclipsing binary implies that their orbits are **viewed edge-on**
- The **Doppler shift** results shown above give key parameters:
  - The period of the binary (***P***)
  - The orbital velocities of star 1 and star 2 ($\boldsymbol{V_1}$ and $\boldsymbol{V_2}$)
- What are the circumferences and radii of the two orbits?

$$C_1 = V_1 \times P = 2\pi a_1$$

$$C_2 = V_2 \times P = 2\pi a_2$$

Kepler's 3rd Law for One-Body Problem (Planets are massless)

3rd Law: period-distance relation

$$\frac{a^3}{P^2} = \frac{GM}{4\pi^2} \qquad \frac{a^3_{AU}}{P^2_{\text{year}}} = M_{\text{solar-mass}}$$

But there are two masses (m1 and m2), and two semimajor axes (r1 & r2), how should we use the Kepler's 3rd law to estimate mass?

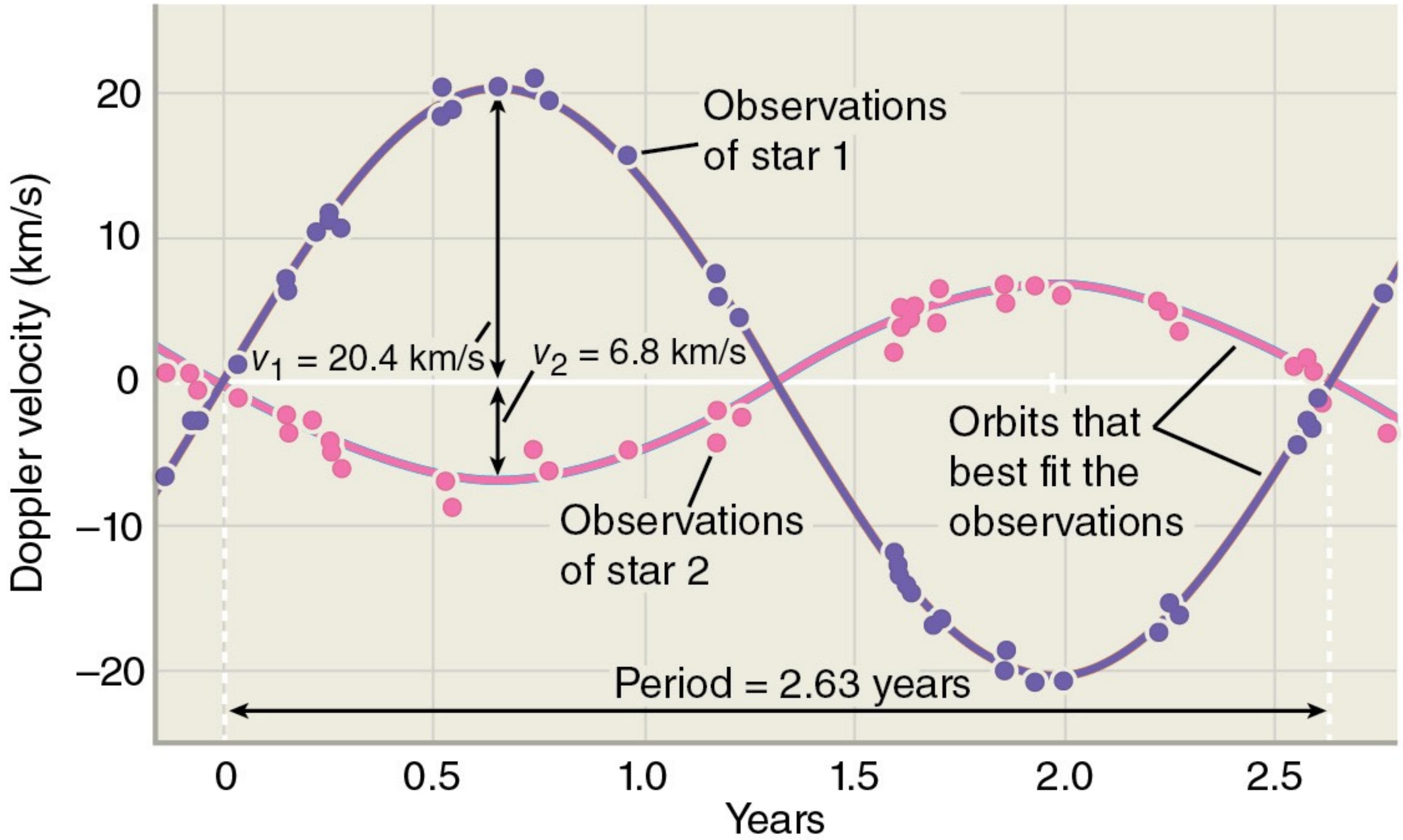
One-body problem:

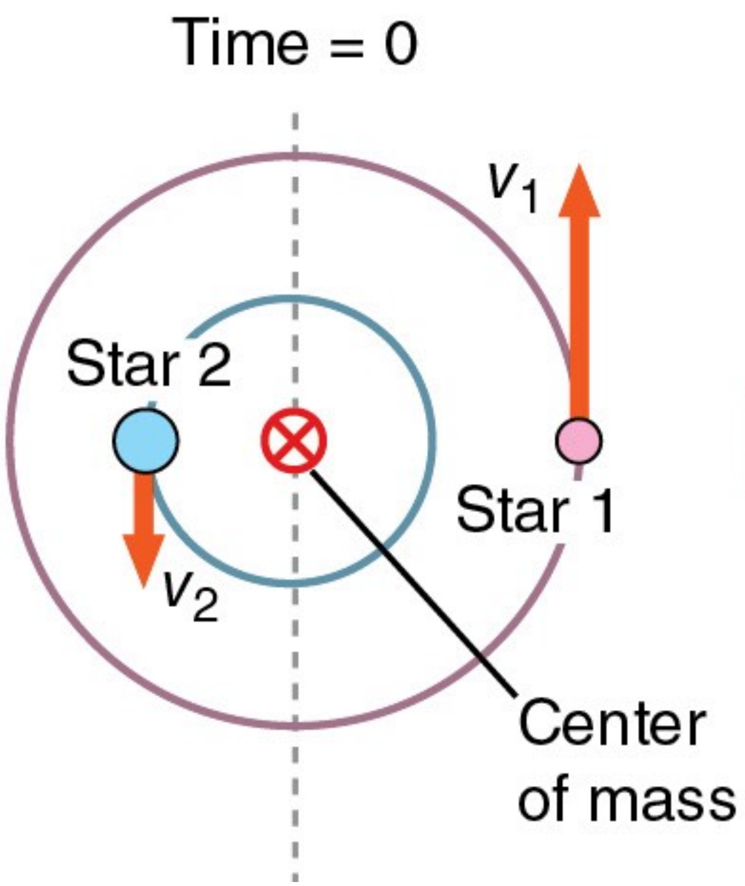
$$\frac{M}{1\ M_{\text{sun}}} = \left(\frac{a}{1\ \text{AU}}\right)^3 \left(\frac{P}{1\ \text{year}}\right)^{-2}$$

Two-body problem:

$$\frac{M_1 + M_2}{1\ M_{\text{sun}}} = \left(\frac{a_1 + a_2}{1\ \text{AU}}\right)^3 \left(\frac{P}{1\ \text{year}}\right)^{-2}$$

Measuring the Masses of Stars in Eclipsing+Spectroscopic Binaries





- Next, we can calculate the total mass using Kepler's third law:

$$\frac{M_1 + M_2}{1\ M_{sun}} = \left(\frac{a_1 + a_2}{1\ \mathrm{AU}}\right)^3 \left(\frac{P}{1\ \mathrm{year}}\right)^{-2}$$

- Finally, we obtain the individual masses based on the velocity ratio:

$$\frac{M_1}{M_2} = \frac{a_2}{a_1} = \frac{V_2}{V_1}$$

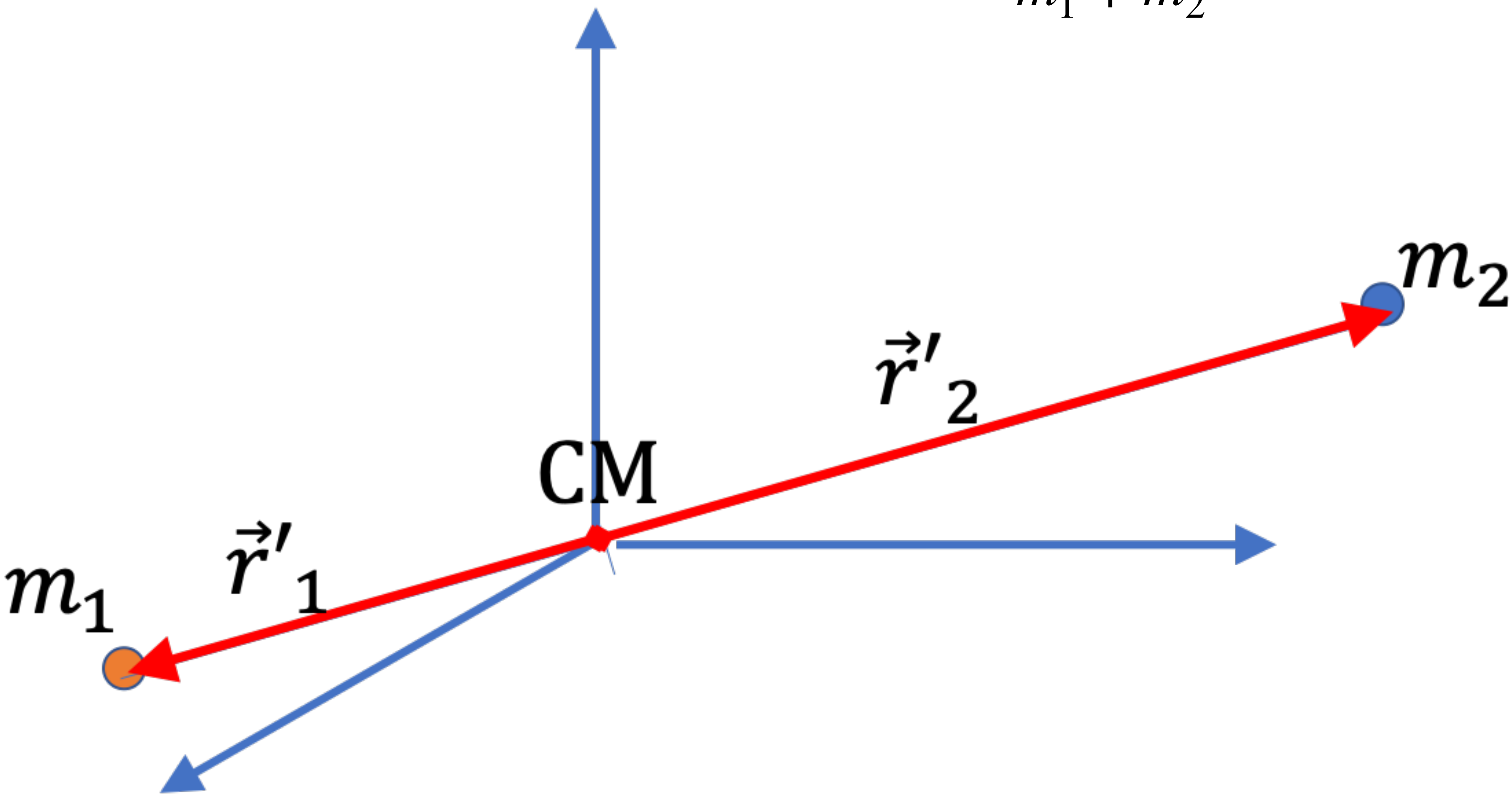
Kepler’s 3rd Law for Two-body Problems

Derivation

Two-body Problem Derivation - The Center-of-Mass Reference Frame

$$m_1\overrightarrow{r_1} + m_2\overrightarrow{r_2} = 0$$
$$\overrightarrow{r_2} - \overrightarrow{r_1} = \vec{r}$$

$$\vec{r}_1 = -\frac{m_2}{m_1 + m_2}\vec{r}$$
$$\vec{r}_2 = \frac{m_1}{m_1 + m_2}\vec{r}$$



Two-Body Problem can be reduced to One-Body Problem

define reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\vec{r}_1 = -\frac{m_2}{m_1 + m_2}\vec{r} = -\frac{\mu}{m_1}\vec{r}$$

$$\vec{r}_2 = \frac{m_1}{m_1 + m_2}\vec{r} = \frac{\mu}{m_2}\vec{r}$$

$$\vec{v}_1 = -\frac{\mu}{m_1}\vec{v}$$

$$\vec{v}_2 = \frac{\mu}{m_2}\vec{v}$$

Then write down the total kinetic and gravitational potential energy

$$E = \frac{1}{2}m_1|\vec{v}_1|^2 + \frac{1}{2}m_2|\vec{v}_2|^2 - G\frac{m_1 m_2}{|\vec{r}_2 - \vec{r}_1|}$$

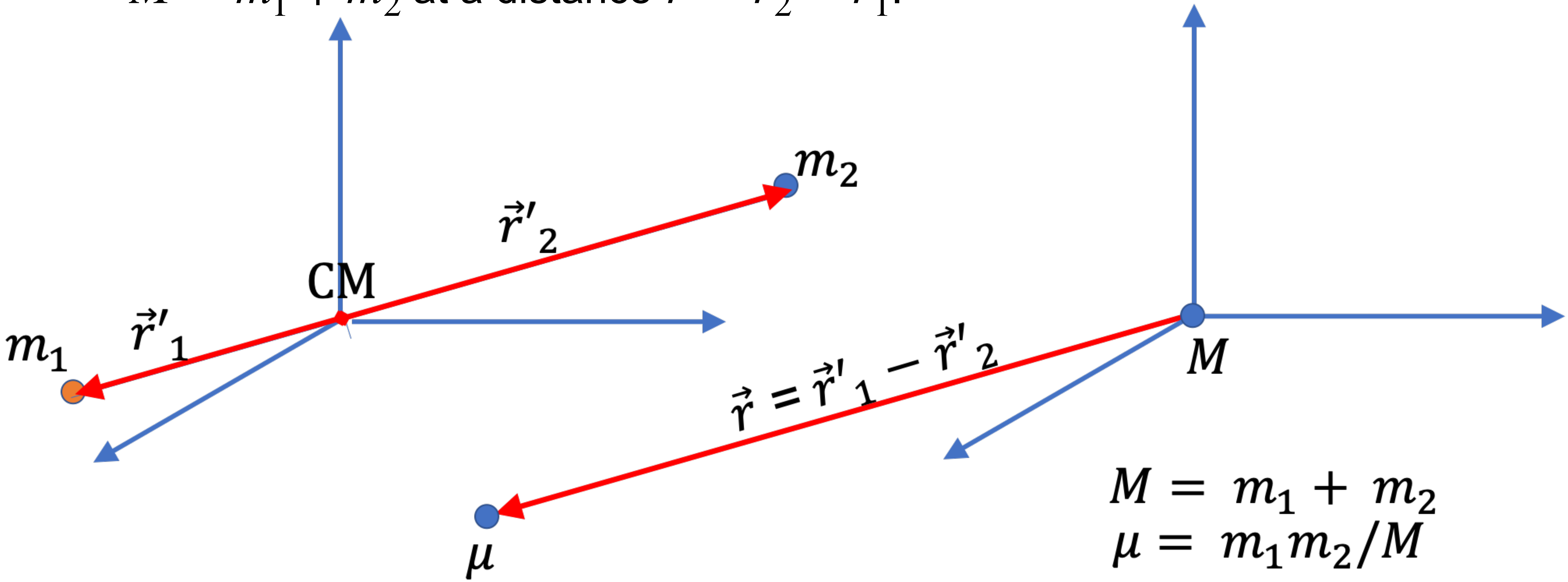
$$= \frac{1}{2}m_1\left(\frac{\mu}{m_1}\right)^2 v^2 + \frac{1}{2}m_2\left(\frac{\mu}{m_2}\right)^2 v^2 - G\frac{(m_1 + m_2)\cdot m_1 m_2/(m_1 + m_2)}{r}$$

$$= \frac{1}{2}\mu\left(\frac{\mu}{m_1} + \frac{\mu}{m_2}\right)v^2 - G\frac{M\mu}{r} \Rightarrow E = \frac{1}{2}\mu v^2 - G\frac{M\mu}{r}$$

- The two-body problem is equivalent to a one-body problem with the reduced mass $\mu = m_1 m_2/(m_1 + m_2)$ moving about a **fixed** total mass $M = m_1 + m_2$ at a distance $\vec{r} = \vec{r}_2 - \vec{r}_1$.

Kepler's 3rd Law for Binary Stars (Two-body Problem)

- The two-body problem is equivalent to a one-body problem with the reduced mass $\mu = m_1 m_2/(m_1 + m_2)$ moving about a fixed total mass $M = m_1 + m_2$ at a distance $\vec{r} = \vec{r}_2 - \vec{r}_1$.



$$M = m_1 + m_2$$
$$\mu = m_1 m_2 / M$$

One-body problem:

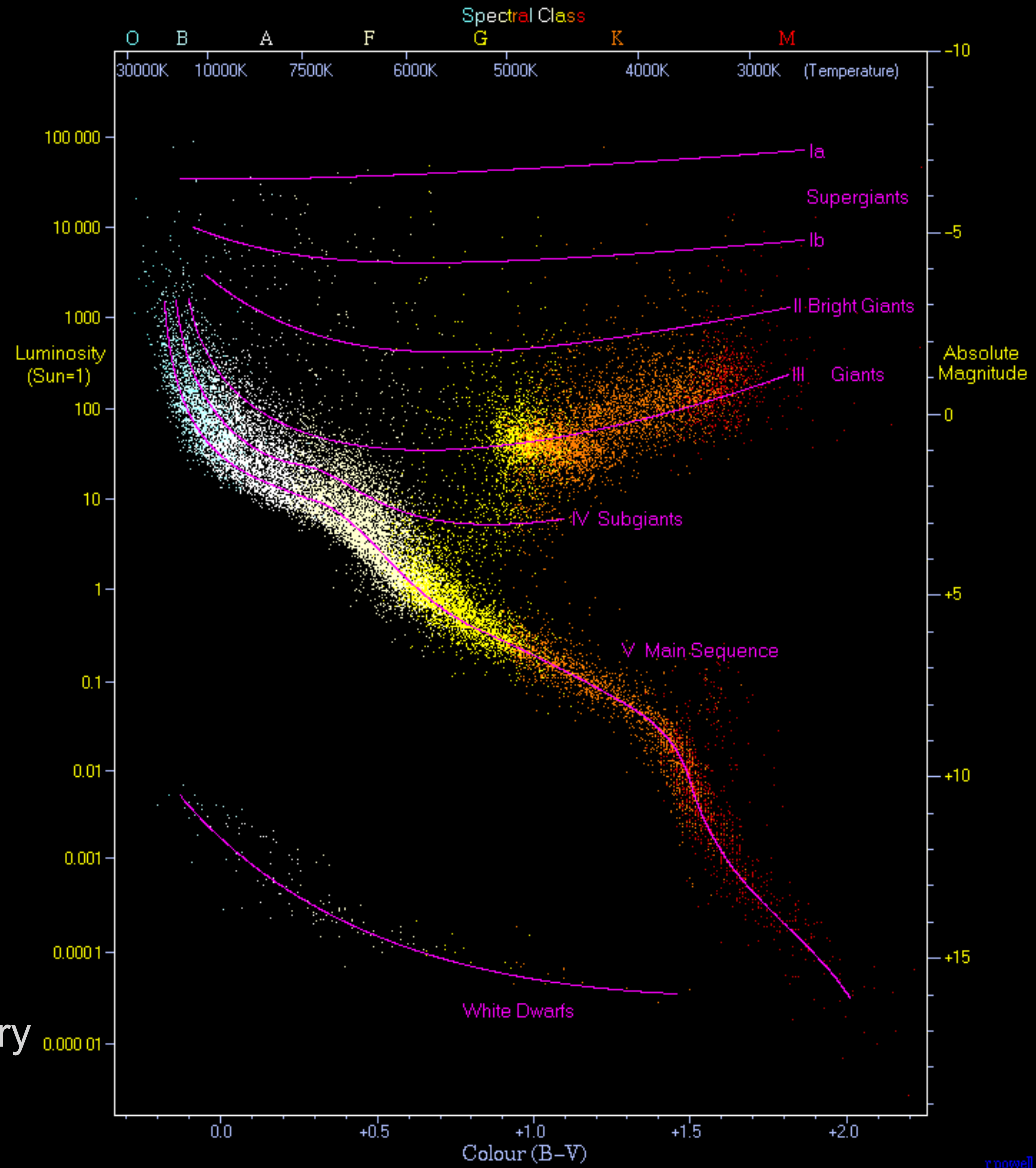
$$\frac{m}{1\ M_{\rm sun}} = \left(\frac{a}{1\ \rm AU}\right)^3 \left(\frac{P}{1\ \rm year}\right)^{-2}$$

Two-body problem:

$$\frac{m_1 + m_2}{1\ M_{\rm sun}} = \left(\frac{a_1 + a_2}{1\ \rm AU}\right)^3 \left(\frac{P}{1\ \rm year}\right)^{-2}$$

Chap 1: Key Concepts

- stellar parallax
- Unit parsec defined by AU
- Pogson's ratio:
  apparent magnitude and flux ratio
- CCD photometry: count rate to magnitude
- absolute magnitude
- distance modulus (m-M)
- standard candle methods
  - spectroscopic parallax
  - type Ia supernovae
- color index and temperature
- luminosity-temperature-size relation
- HR diagram: the main sequence
- Eclipsing and spectroscopic binaries: Kepler's 3rd law for binary systems (two-body problem)



Chap 1: Key Equations

$$d = 1 \text{ parsec} \left(\frac{1 \text{ arcsec}}{p}\right)$$

$$m_{\lambda,2} - m_{\lambda,1} = -2.5 \log(f_{\lambda,2}/f_{\lambda,1})$$

$$m_\lambda - M_\lambda = 2.5 \log\left(\frac{d}{10 \text{ parsec}}\right)^2 = 5 \; [\log d(\text{parsec}) - 1]$$

$$d(\text{parsec}) = 10^{1+0.2(m-M)}$$

$$L_{\text{bol}} = 4\pi R^2 \sigma_{\text{SB}} T^4 \qquad L_\lambda = 4\pi R^2 \pi B_\lambda(T)$$

$$\frac{R}{R_\odot} = \sqrt{\frac{L_{\text{bol}}}{L_\odot}} \left(\frac{T}{T_\odot}\right)^{-2} \qquad \frac{M_1 + M_2}{1 \; M_{\text{sun}}} = \left(\frac{a_1 + a_2}{1 \text{ AU}}\right)^3 \left(\frac{P}{1 \text{ year}}\right)^{-2}$$